The Cause of the Great Recession: What Caused the Downward Shift of the GDP Trend in the United States?

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Abstract

The trend of the gross domestic product (GDP) of the United States clearly shifted downward after the Great Recession of 2008. This shift indicates that the cause of the Great Recession was a change in a fundamental factor that had the potential to significantly affect the steady state. In this paper, I examine three possible causes for the shift: a change in technology, a change in preferences, and a sudden malfunctioning of the price mechanism. I conclude that an upward shift of the expected rate of time preference is the most likely cause of the Great Recession. In addition, I estimated the yearly expected rate of time preference of the United States and found that the expected rate of time preference shifted upwards by 1–2 percentage points when the Great Recession began. I also estimated the expected rate of time preference for Japan and found that the rate increased prior to the extended period of economic stagnation during the 1990s.

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Keywords: The Great Recession; Time preference; Economic fluctuations

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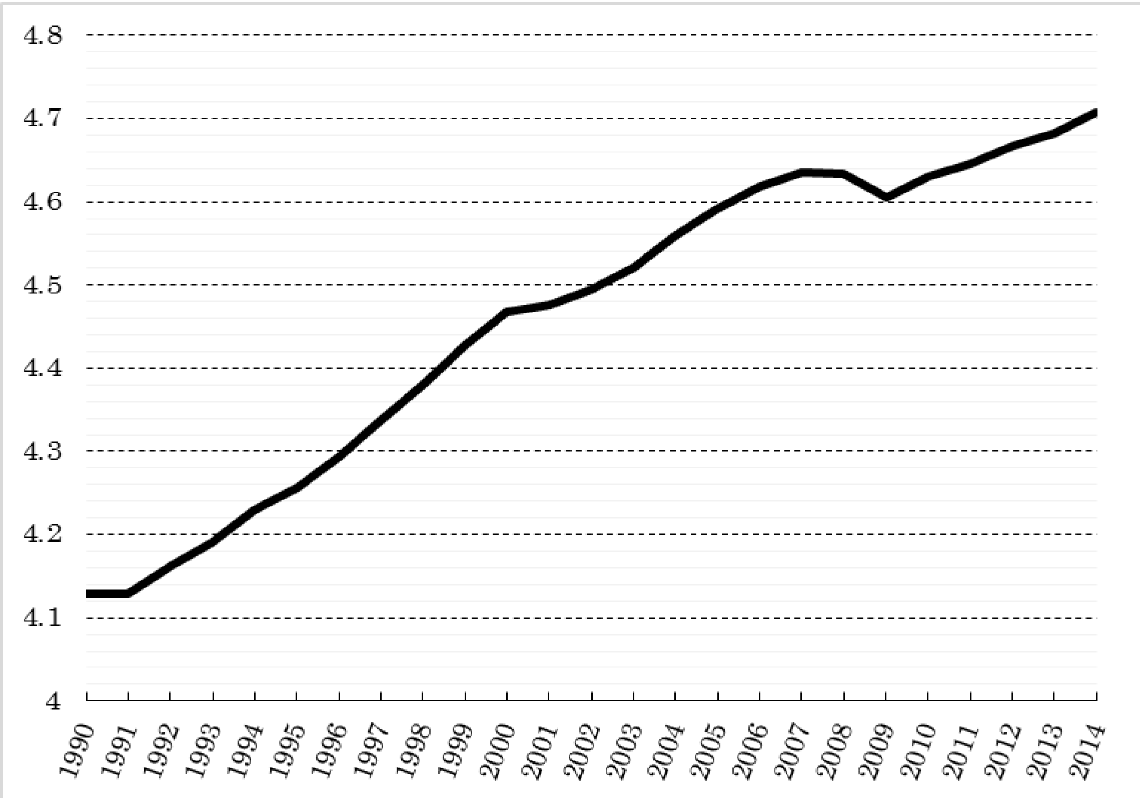
1 INTRODUCTION

The Great Recession that began in about 2008 had lasting impacts on the U.S. economy. The most significant impact is the apparent downward shift of the trend in gross domestic product (GDP). The trend here means the exponential growth path that best fits the data. The trend in GDP after 2009 has not yet returned to the pre-recession trend (see Figure 1). Any explanation of the cause of the Great Recession should therefore be consistent with this observed phenomenon (see Martin et al., 2015). Many explanations of the Great Recession have been presented, although not a few of them are narrative (e.g., Guerrieri and Lorenzoni, 2011; Hall, 2011; Eggertsson and Krugman, 2012; Mian and Sufi, 2012; Christiano et al., 2015; Martin et al., 2015). Many of these studies particularly emphasize financial factors, but most of these explanations seem to be rather superficial because the apparent downward shift in the GDP trend indicates that some fundamental factor changed around 2008; that is, the steady state shifted substantially. Any explanation, therefore, should include a change in a fundamental factor that has the potential to shift the steady state to a large extent. In theory, these fundamental factors are limited to technology, preferences, and the price mechanism, so the Great Recession must have been a phenomenon generated by a change in technology or preferences, or a suddenly malfunctioning price mechanism.

Figure 1: Logarithm of real GDP in the United States

(2009 dollars)



Source: *National Economic Accounts*, the U.S. Department of Commerce, Bureau of Economic Analysis

Studies on changes in technology and malfunctions in the price mechanism (i.e., technology shocks and frictions) have a long history, and a huge amount of research has been conducted on these topics. It is doubtful, however, whether a technology shock or some sort of price friction was the cause of the Great Recession because sufficiently persuasive micro-foundations of either a large technological regression or persistent (e.g., over several years) malfunctioning of the price mechanism have yet to be presented.

A change in preferences has remained relatively unexplored (or rather neglected) as a source of large economic fluctuations because economic researchers have generally held the conviction or preconception that preferences must be temporally unchangeable. However, temporal invariability of preferences has not been proven, and there have been theoretical and empirical studies that have indicated that preferences, particularly the rate of time preference (RTP), are temporally variable (e.g., Böhm-Bawerk, 1889; Fisher, 1930; Uzawa, 1968; Epstein and Hynes, 1983; Lucas and Stokey, 1984; Epstein, 1987; Parkin, 1988; Obstfeld, 1990; Lawrance, 1991; Drugeon, 1996; Becker and Mulligan, 1997; Frederick et al., 2002).

Using Harashima (2014a, 2014b) as a basis, I examined the problems associated with the explanation that the Great Recession was caused by a change in preferences and show that these problems can be solved. An important point is that it is the expected RTP of the representative household (RTP RH), not intrinsic RTP RH, that matters in economic activities, and expectations by nature can change as relevant new information is obtained. A second important point is that households behave intrinsically non-cooperatively. This nature generates a Nash equilibrium that consists of strategies that generate Pareto inefficient payoffs, and this path can generate a high unemployment rate, which was observed during the Great Recession. Given these points, I conclude that an upward shift of RTP is the most likely cause of the Great Recession.

Finally, I estimated a time-series of the RTP RH of the United States to validate the above conclusion. The estimates indicated that the RTP RH of the United States did indeed shift upwards by 1–2 percentage points when the Great Recession began. This empirical result supports my theoretical conclusion. In addition, I estimated the RTP RH of Japan, and the estimates indicated that the RTP of Japan shifted upwards by 2–3 percentage points just before Japan fell into the economic stagnation of the 1990s.

2 THE SHIFTING GDP TREND

2.1 *The apparent GDP trend shift after the Great Recession*

Figure 1 clearly indicates that the GDP trend in the United States shifted downwards around 2008. What kind of shock can explain this large downward shift? Many superficial reasons have been presented, but from a theoretical point of view, such a large shift must be caused by a change in a fundamental variable or element, because GDP would soon return to the pre-recession trend if the shift was caused by one or more non-fundamental factors. This lasting change indicates that the steady state must have shifted. As stated in Section 1, the only fundamental variables and elements that can significantly affect steady states are technology, preferences, and the price mechanism.

2.2 *Suspected causes*

Hence, the cause of the large downward shift of the GDP trend (i.e., a large shift of the steady state) should be explained by one of the following reasons: (1) a large regression in technology, (2) significant friction to price adjustments, or (3) a large change in a preference. Explanation (1) is a supply-side explanation; that is, if technology notably regresses, production will be greatly reduced. Explanation (2) is a demand-side explanation; that is, if the price mechanism does not work well, demand cannot necessarily match supply and thus production will be reduced. Explanation (3) is also a demand-side explanation. If preferences of households change, the steady state will shift and the level of production will change.

Some researchers may argue that there are other possible explanations, such as phenomena known as indeterminacy, multiple-equilibria, or sunspots. There are many types of multiple-equilibria models that depend on various types of increasing returns, externalities, or complementarities, but they are vulnerable to a number of criticisms (e.g., insufficient explanation of the switching mechanism; see, e.g., Morris and Shin, 2001). These explanations may be interesting from a strictly mathematical point of view, but they are somewhat divorced from economic reality. Therefore, I do not consider these possibilities in this paper.

2.3 Validity and criticism of the suspected causes

2.3.1 Technology

If technology greatly regresses, the GDP trend will clearly shift downwards because the steady state shifts "downward." Here "technology" indicates the total factor productivity (TFP) in the aggregated production function of a country. If innovations are steadily generated and technology progresses constantly even after a great regression in technology (i.e., a period of reduced TFP), the inclination angle of the GDP trend will not change even though the intercept changes.

The most serious problem with this explanation is whether or not technology can actually regress (i.e., TFP can decrease) suddenly and greatly. A micro-foundation of technological regression is needed. Most endogenous growth models present a micro-foundation of technological progress in which existing innovations, knowledge, and human capital usually do not vanish easily or suddenly; that is, they are basically accumulated. To the best of my knowledge, no micro-foundation of technological regression in modern industrial economies has been presented. Machines and equipment for production will become obsolete as time passes, but new innovations are generated in every period and obsolete machines and equipment will be replaced with new advanced machines and equipment. Hence, the overall level of technology in an economy will not generally regress to a large extent; that is, TFP will not notably decrease from a scientific or technological point of view.

A decline in TFP may occur, however, if other elements associated with TFP malfunction. For example, a decline in the efficiency of institutions or systems (e.g., banks, legal systems, or transportation networks) may cause a decline in TFP. The efficiency of these institutions or systems is regarded to be an important element in determining the level of TFP (e.g., Levine, 1997; Levine et al., 2000; Easterly and Levine, 2003; Wachtel, 2003; Do and Levchenko, 2007). If some institutions or systems suddenly begin to malfunction to a large extent, TFP will eventually decrease. This change, however, will not occur immediately because the institutional elements in TFP do not affect current production capacity. A suddenly malfunctioning institution or system, therefore, will negatively affect TFP, but the negative effects will take time to be realized. For example, if the efficiency of banks degenerates, ongoing new business projects will be delayed and improper investments will be approved more often. As a result of the decreased efficiency in the banking system, TFP will gradually decline, and the negative effects will only be clearly observed in the long run. Many financial institutions in the United States were in crisis around 2008, but the level of physical production capacity (capital and labor) in most U.S. industries basically remained the same as before 2008.

In sum, it is very difficult to envision any micro-foundation for a sudden and large regression in technology, which makes it highly unlikely that technology was the cause of the downward shift in the GDP trend.

2.3.2 The price mechanism

If the price mechanism does not work well, many unusual phenomena will inevitably occur. For example, if there is friction in the process of price adjustments, the economy will not soon return to the steady state when the economy accidentally deviates from it; thus, the deviation will persist. Therefore, a shift in the GDP trend can be explained by assuming that some type of friction exists in the price mechanism. After a large negative shock, a large economic downturn will persist if the price mechanism is not working properly. In addition, persistent large amounts of unused resources will also be observed.

It is not easy, however, to present a persuasive rationale for a malfunctioning price mechanism because the price mechanism is one of the most fundamental principles behind economic activities of rational agents. Rationality, in fact, guarantees a well-functioning price adjustment mechanism. Unless some type of irrationality is assumed, it is difficult to show a malfunctioning price mechanism. Many Keynesian economists, however, have tried to overcome this difficulty. Although some of these models can trace fluctuations in GDP, the micro-foundations they present do not seem to be sufficiently persuasive (e.g., Mankiw, 2001). Humans are generally considered to be clever and rational such that they cannot be persistently cheated; for example, they are assumed to exploit the opportunities provided by a friction, and the friction will thereby soon disappear. It is therefore difficult to envision friction as the cause of the GDP trend shift after the Great Recession.

More importantly, the persistence of the effect of friction is a serious problem. As shown in Figure 1, the GDP trend did not return to the pre-recession trend even 7 years after the shift. This fact indicates that the magnitude of the effect of friction has not diminished for 7 years. Such a long period cannot be rationalized by any micro-foundation of friction in price adjustments. The Calvo staggered contracts model, which provides one of the most prominent micro-foundations of price friction, usually assumes that the effect of friction gradually diminishes as time passes and that most of the effect disappears a few years after the shock. In this case, 7 years seems to be a long-term phenomenon, or at least not a short-term one, and most researchers agree that models of friction cannot be applied to long-term phenomena. Therefore, it is unlikely that friction in price adjustments can explain the GDP trend shift after the Great Recession.

In addition, there is another important issue with this explanation. What shock triggered the sudden malfunction of the price mechanism? Frictions alone cannot generate a phenomenon such as the Great Recession. Initially, some type of huge negative shock must have occurred. The disruption that occurred in financial markets (e.g., the subprime mortgage crisis) around 2008 may have been such a shock. However, disruption in financial markets is not a deep parameter of a Ramsey-Cass-Koopmans economy in the optimization behavior of households. This type of shock may have temporarily affected households' behavior, but its effects would have soon disappeared because households are basically indifferent to this type of shock when generating their expected future utilities, and the economy would soon return to the former steady state. Therefore, the shock that triggered the Great Recession remains uncertain with this explanation.

2.3.3 A change in preferences

If a fundamental preference (e.g., RTP), changes sufficiently, a large economic fluctuation will be generated because this change significantly affects the steady state. A changed steady state requires that many economic variables adjust to the new steady state. Consequently, a boom or a recession is generated. Therefore, a change in a fundamental preference can intrinsically be an important source of economic fluctuations. In addition, some preference shocks will generate persistent large amounts of unused resources (e.g., persistently high rates of unemployment), the generation mechanism of which is shown in Harashima (2004, 2013) and also in Appendix B.

An important criticism of this explanation is that preferences are assumed to remain constant. This conviction or preconception has been so widely shared by economists that preference shocks have rarely been studied as a source of economic fluctuations, even though preferences have never been proven to actually remain constant. Another problem is that, after RTP changes, consumption has to change in a direction that is not intuitively acceptable if Pareto optimality is to be held. For example, suppose that RTP shifts upwards. To keep Pareto optimality, consumption must increase greatly at the time of the shift and then it needs to gradually decrease to the level appropriate for the new steady state and this level will be lower than it was before the shift. This upward jump of consumption does not seem to be intuitively acceptable because steady state consumption must eventually decrease. Compared with the first two explanations, however, the problems with this third explanation may be more surmountable. Although technological and monetary shocks have long been studied, preference shocks have rarely been studied as an important source of economic fluctuations. If the possibility of temporally variable RTP is examined in detail without preconceptions, it may be possible to easily solve the problems associated with this explanation.

Note that a change in risk preference (the degree of risk aversion) does not affect steady states in Ramsey-type dynamic models. Therefore, this preference is not considered in this paper.

3 THE MOST LIKELY CAUSE OF THE GREAT RECESSION

3.1 Solutions to the problems in explanation (3)

3.1.1 Temporally changing expected preferences

3.1.1.1 Necessity of expected preferences

As noted previously, temporal variability of preferences has been indicated in many studies, but the question of the magnitude of its effects remains. Although an individual's preferences may change a great deal, the average preferences of households may not change. However, Harashima (2014a, 2014b) showed that it is the expected preferences of the representative household, not the intrinsic preferences, that are important for households to behave optimally. The average intrinsic preferences of households may remain almost unchanged, but the expected preferences of the representative households may occasionally change by a large amount when conditions change because households change expectations if important new information is obtained. Expectations by nature, therefore, can change over time.

It is also important to note that the representative household should not be assumed to be the same as the average household if households are heterogeneous in preferences in a dynamic model. As Becker (1980) showed, if RTP is heterogeneous across households in a dynamic model, all capital will eventually be owned by the most patient household; thus, the average household is almost represented by the most patient household that monopolizes returns of capitals. Therefore, the representative household defined as the average household becomes meaningless in a dynamic model with households having heterogeneous RTPs. An alternatively defined representative household is needed when a dynamic model is used. In a dynamic macro-economic model, this newly defined representative household is indispensable, and all households must know the preferences of the representative household to achieve their optimality. Because they cannot know its intrinsic preferences, they must "expect" them.

3.1.1.2 An alternative definition of the representative household's preferences

Harashima (2014a, 2014b) presented an alternative definition of the representative household that can be used in a dynamic model with households having heterogeneous preferences. The representative household is defined such that the behavior of the representative household is the collective behavior of all households under "sustainable heterogeneity." Sustainable heterogeneity indicates the state at which all optimality conditions of all heterogeneous households are satisfied.

The concept of sustainable heterogeneity is explained in Appendix A and Harashima (2010) in detail. Suppose that there are $H(\in N)$ groups of households in an economy.

Households in each group are identical, and all groups are identical except for RTP, the degree of risk aversion, or the productivity of each group's households. A household includes a laborer who is one of the factors that determine the productivity of the group. The population growth rate is zero in all groups. The groups are fully open to each other, and goods, services, and capital are freely transacted among them, but labor is immobilized in each group.

Sustainable heterogeneity is achieved, that is, all optimality conditions of all heterogeneous households are satisfied, if and only if

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right)^{-1}\left\{\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha}-\frac{\sum_{q=1}^{H}\theta_q\omega_q}{\sum_{q=1}^{H}\omega_q}\right\} \tag{1}$$

for any group i (= 1, 2, … , H) is satisfied, and at this state,

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}}=\lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}}=\lim_{t\to\infty}\frac{\dot{A}_t}{A_t}=\lim_{t\to\infty}\frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}}=\lim_{t\to\infty}\frac{\frac{d\int_0^t\tau_{i,j,s}ds}{dt}}{\int_0^t\tau_{i,j,s}ds} \tag{2}$$

for any i and j ($i\neq j$) where $c_{i,t}$, $k_{i,t}$, and $y_{i,t}$ are per capita consumption, capital, and output of group i in period t, respectively; θ_i, ε_i, and ω_i are RTP, the degree of risk aversion, and productivity of group i, respectively; A_t is technology in period t; and α, m, v, and ϖ are constants. In addition, $\tau_{i,j,t}$ is the current account balance of group i with group j. The production function is a Harrod-neutral production function such that

$$y_t=A_t^{\alpha}k_t^{1-\alpha}. \tag{3}$$

When sustainable heterogeneity is achieved, all heterogeneous households are connected (in the sense that all households behave by considering other households' optimality) and appear to be behaving collectively as a combined supra-household that unites all households. The supra-household is unique and its behavior is time-consistent. Its actions always and consistently represent those of all households. Even if households are heterogeneous, they can be represented by a representative household as defined above. Unlike the representative household defined as the average household, the collective representative household reaches a steady state where all households satisfy all of their optimality conditions in dynamic models.

All households need to set their initial consumption to be consistent with sustainable heterogeneity for their optimality. Before setting their initial levels of consumption, households must calculate and expect the economic path under sustainable heterogeneity. To calculate and expect this path, each household first must know the RTP RH. However, although a household naturally knows its own RTP, it does not intrinsically know RTP RH. To know RTP RH, a household has to know the values of all the other households' RTPs. Hence, the expected RTP RH must somehow be generated utilizing all other relevant available information. The necessity of an expected RTP RH is critically important because RTP plays a crucial role as the discount factor in dynamic models.

Note that, if we assume that RTP is identical for all households, an expected RTP RH is no longer needed because any household's own RTP is equal to RTP RH. This solution is still problematic, however, because the assumption is not merely expedient for the sake of simplicity; rather, it is a critical requirement to eliminate the need for an expected RTP RH.

Therefore, any rationale for assuming identical RTPs needs to be validated; that is, it should be demonstrated that identical RTPs do exist and are universally observed. However, RTP is unquestionably not identical among households. Therefore, households must use an expected RTP RH.

Although households must generate an expected RTP RH to reach optimality, Harashima (2014a, 2014b) showed that the expected RTP RH cannot be generated based on a structural model of RTP RH, but rather it is created based on a belief. In addition, the belief can be influenced by heuristic considerations. The point is that it is very difficult to know the correct parameter values in any structural model of RTP RH. Beliefs based on heuristic considerations can be easily and largely revised over time as new information is acquired. This nature indicates that the expected RTP RH will change more frequently and to a greater extent than the intrinsic RTP RH.

A household's expected RTP RH will of course change if the intrinsic RTP RH changes. Even if the intrinsic RTP RH does not change, however, a household's expected RTP RH will change if its belief is changed. That is, the expected RTP RH can change independently of intrinsic changes in RTP RH. Therefore, even if intrinsic changes in RTP RH occur infrequently, changes in the expected RTP RH can occur frequently. Even a small piece of additional information about a relevant belief can significantly change the path of the economy. Hence, a large RTP shock can occur occasionally, which solves one of the two problems posed by explanation (3).

3.1.2 Non-jump path of consumption

3.1.2.1 The micro-foundation of a non-jump path

The second important problem with the third explanation was the intuitively unacceptable jump path of consumption after a preference shock if households maintain a Pareto optimal path. However, Harashima (2004, 2013a) showed a mechanism whereby households rationally do not engage in this type of jump consumption after an RTP shock. Harashima (2004, 2013a) demonstrated that there is a Nash equilibrium that consists of strategies that generate Pareto inefficient payoffs (i.e., a Nash equilibrium of a Pareto inefficient path) because households are intrinsically risk averse and not cooperative. In a strategic environment, this nature generates the possibility that, if consumption needs to be substantially and discontinuously increased to keep Pareto optimality, a non-cooperative household's strategy to deviate from the Pareto optimal path gives a higher expected utility than the strategy of choosing the Pareto optimal path. If households are cooperative, they will always proceed on Pareto efficient paths because they will coordinate with each other to perfectly utilize all resources. Conversely, if they do not coordinate with each other, they may strategically not utilize all resources; that is, they may select a Nash equilibrium of a Pareto inefficient path. In fact, households are intrinsically not cooperative—they act independently of one another.

Suppose that an upward shift of RTP occurs. All households will be knocked off the Pareto efficient path on which they were proceeding prior to the shift. At that moment, each household must decide how to proceed. Because they are no longer on a Pareto efficient path, households strategically choose a path on the basis of their expected utility calculated considering other households' choices; that is, each household behaves non-cooperatively in its own interest considering other households' strategies. Harashima (2004, 2013a) showed that, if a household is sufficiently risk averse, its expected utility is higher when its consumption does not jump than it is when it does. The mechanism of this outcome is also explained in detail in Appendix B. Nevertheless, this outcome depends on the expectation of other households' behavior. All households generate the same expectation, but they behave non-cooperatively. This situation can be described by a non-cooperative mixed strategy game, and there is a Nash equilibrium of a Pareto inefficient path as a pure-strategy Nash equilibrium in this game. That is, the intuitively difficult response of consumption initially moving in the wrong direction will not usually be generated.

Hence, the two problems noted for the third explanation (a preference shock) of the GDP trend shift can be resolved. Because it is much more difficult to solve the problems posed by the first two explanations (technology and the price system), a preference shock is the most likely cause of the Great Recession.

3.1.2.2 Persistent and large amounts of unused resources

An important feature of a Nash equilibrium of a Pareto inefficient path is that the path is not Pareto efficient and therefore large amounts of unused resources are persistently generated, for example, large unemployment rates and a large amount of idle capital. The mechanism of this type of phenomena was first examined by Keynes for the period of the Great Depression. A similar (but less severe) phenomenon was observed during the Great Recession. A Nash equilibrium of a Pareto inefficient path naturally explains persistent and large amounts of unused resources in an economy.

3.2 Time preference or leisure preference shock?

Although both main problems with the preference shock explanation can be resolved, the question remains: which preference changed—RTP, leisure preference (LP), or risk preference? As stated previously, risk preference is basically indifferent to economic fluctuations so it is not a factor. Changes in the expected RTP and LP, however, can both generate a Nash equilibrium of a Pareto inefficient path. Thus, both preferences can be a source of economic fluctuations.

A major difference between the expected RTP and LP is the likely range of change. A reasonable range of RTP will not be small, e.g., from about 2% to 8% annually (e.g., Frederick et al., 2002). Hence, there is room for the expected RTP to shift upward to a large extent (e.g., double from 3% to 6%). With LP, however, there is a consensus that a 10% increase in the wages leads to a 1% decrease in hours of work on average (e.g., Borjas, 2012). Hours of work are very inelastic and do not change to a large extent; that is, the average number of hours worked will not double or be halved in a given year. Therefore, there is little room for LP to substantially increase. In addition, even a small percentage point change in the expected RTP (e.g., from 3% to 5%) generates very large impacts because RTP is the discount factor in calculations of expected utility. A small change in the expected RTP can greatly change the expected utility and thereby also change the steady state. Considering this substantial difference between the expected RTP and LP, it is likely that a severe recession caused by a large shift in the steady state can only be generated by an upward shift of the expected RTP. A change in the expected LP may cause small-scale economic fluctuations, but it most likely will not be the ultimate source of a severe recession.

There is another difference between expected RTP and LP—the response to new information. It is likely that the Great Recession and the financial crisis that occurred around 2008 were related. Financial crises may raise uncertainty about future economic conditions. In general, an increase in uncertainty will raise the expected RTP (see Harashima, 2004) and decrease the preference for leisure. An increase in RTP will generate a recession, but a decrease in the preference for leisure (i.e., an increase in the preference for work) will not. Therefore, if the Great Recession and the financial crisis that occurred around 2008 were related, an LP shock was not the cause of the Great Recession.

A change in the expected RTP around 2008 triggered by information that had surfaced about many large financial institutions certainly could have caused many U.S. households to determine that their expected RTP RHs were wrong and needed to be corrected based on the newly obtained information. The upward RTP shock explanation is also consistent with the co-occurrence of severe recessions in other countries during this period. These coincidental recessions no doubt were at least partly generated through diminishing trade with the world's largest economy (i.e., the United States) where the Great Recession initially broke out, but also partly through upward changes in the expected RTP RH by households in these other countries upon obtaining information about the financial crisis and recession in the United States.

4 ESTIMATES OF EXPECTED RTP RH

4.1 *Estimated expected U.S. RTP RH*

In this section, I examine whether the theoretical conclusions drawn in Section 3 are supported empirically. A problem in doing so is that time-series data of expected RTP RH cannot be directly obtained. Therefore, I estimated them indirectly based on the Euler equation such that

$$\theta_t = \frac{\partial y_t}{\partial k_t} - \frac{\dot{c}_t}{c_t} \tag{4}$$

using various macro-economic data. If we can obtain time-series data for $\frac{\dot{c}_t}{c_t}$ and $\frac{\partial y_t}{\partial k_t}$, we can estimate those of the expected RTP RH based on equation (4). The data of $\frac{\dot{c}_t}{c_t}$ can easily be obtained from the System of National Accounts.

The more difficult task is obtaining time-series data for $\frac{\partial y_t}{\partial k_t}$. One possibility is to use the real interest rate as a substitute, but this method has important drawbacks. The rate of real interest also cannot be directly observed and has to be estimated by subtracting inflation rates from nominal interest rates, but there are various kinds of nominal interest rates and inflation rates, and estimates of real interest rates vary significantly depending on which rates are used. In addition, nominal interest rates are one of the most important instruments in monetary policies, and are therefore usually significantly biased by interventions of central banks in financial markets. If agents regard monetary policies as a temporary manipulation, they will not respond naively to these policies; thus, the estimated real interest rates may provide biased information. As a result, it is likely that estimated rates of real interest do not necessarily correctly reflect $\frac{\partial y_t}{\partial k_t}$.

For that reason, I directly estimated $\frac{\partial y_t}{\partial k_t}$ using capital stock data and an assumed rate of average technological progress. The production function was assumed to be the same as equation (3) (i.e., a Harrod-neutral production function) such that $y_t = A_t^{\alpha} k_t^{1-\alpha}$; thus,

$$\frac{\partial y_t}{\partial k_t} = A_t^{\alpha}(1-\alpha)k_t^{-\alpha} . \tag{5}$$

I estimated the time-series data of $\frac{\partial y_t}{\partial k_t}$ based on equation (5), with the k_t data and the assumed values of A_t and α. Using these estimated values of $\frac{\partial y_t}{\partial k_t}$ and the published c_t data, I then estimated the time-series data of expected RTP RH (θ_t) based on equation (4).

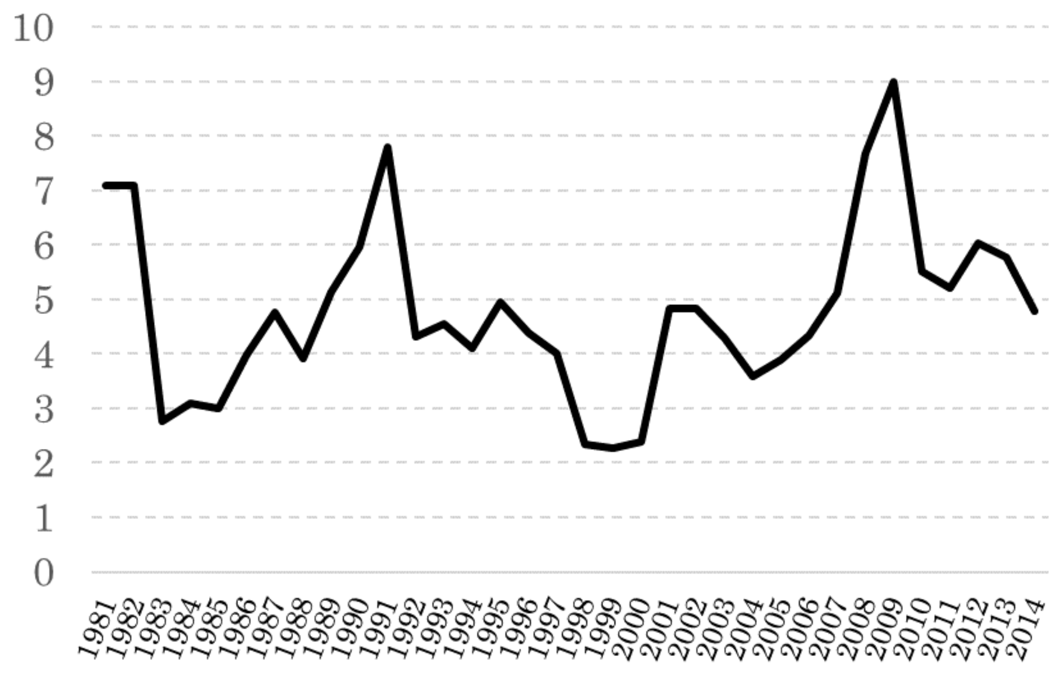
Data for c_t were derived from *National Economic Accounts* distributed by the U.S.

Department of Commerce, Bureau of Economic Analysis, and k_t data were derived from the chain-type quantity index for private nonresidential fixed assets in *National Economic Accounts*. α as the labor share was set at 0.7 that is a typical value of labor share, and A_t^{α} was assumed to grow constantly at 1.25% annually, meaning that technology was assumed to basically progress constantly. This rate of growth (1.25%) was adopted based on an average per capita GDP growth rate of 1.8% annually because, if sustainable heterogeneity is satisfied, equation (2) holds; that is, the growth rate of A_t is equal to the growth rate of y_t on a balanced growth path. Therefore, by equation (3), the growth rate of A_t^{α} is $\left(1.018^{0.7}-1\right)\times 100 = 1.25\,\%$ annually. Because my primary focus is fluctuations of the expected RTP RH and not the absolute level of A_t, A_t was set to make the level of the expected RTP RH equal 0.03 in 1985. The expected RTP RH of the United States in 1985 may not have been 0.03, but the actual level itself is not the important point—the range of the temporal changes in the values are. Setting the level in this manner further illustrates how difficult, if not impossible, it is to know the actual RTP RH.

The estimation results are shown in Figure 2. The expected RTP RH was relatively high in the periods of the early 1980s, the early 1990s, and around 2008—all periods of recession. The expected RTP RH before 2008 was clearly lower than that after 2008 and lower in general than at other times, except for the recession periods of the early 1980s and the early 1990s. In the latter part of the 1990s and the first half of 2000s, the era of the so-called "New Economy," the expected RTP RH continued to be relatively low, but it rose suddenly when the Great Recession began. The difference of the average expected RTP RH between the period of post-2008 and the period of 1992–2007 (i.e., from the end of the early 1990s recession to the beginning of the Great Recession) is 1–2 percentage points. This result is consistent with explanation (3); that is, an upward 1–2 percentage point RTP shock was the cause of the Great Recession.

Figure 2: The estimated RTP RH of the United States

(%)

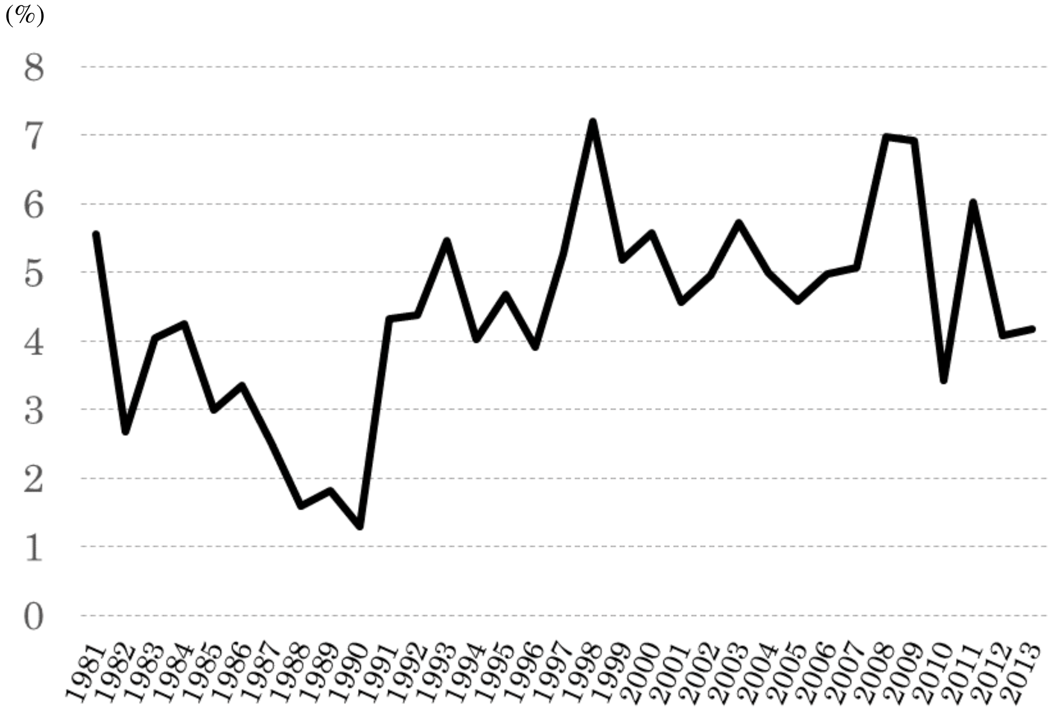


4.2 *Japan's estimated expected RTP RH*

Japan experienced a long-lasting period of economic stagnation in the 1990s. To examine whether this stagnation was also generated by an upward RTP shock, I estimated the time-series of the expected RTP RH of Japan by using the same method as I did for the United States. k_t data were derived from the non-financial produced tangible fixed assets in the National Accounts of Japan, and c_t data were also derived from the National Accounts of Japan. α was assumed to be 0.7 and A_t^{α} is assumed to grow constantly at 1.25% annually for the same reasons as described in the U.S. case. Because my primary focus is fluctuations of the expected RTP RH and not the absolute level of A_t, the initial level of A_t was also set to make the level of the expected RTP RH be 0.03 in 1985.

The estimation results are shown in Figure 3. The average expected RTP RH before 1991 was lower than that after 1991 by about 2–3 percentage points. In the second half of 1980s, the era of the so-called "bubble economy" in Japan, the expected RTP RH was particularly low, but it rose sharply in 1991 when the "bubble" burst. The estimated expected RTP RH of Japan is consistent with the explanation that an upward RTP shock generated the stagnation of the Japanese economy in the 1990s. The estimated upward shift was larger than that of the U.S. case during the Great Recession by about 1 percentage point, which implies that the negative impact of the upward RTP shock in the 1990s in Japan was far greater than that of the Great Recession in 2008 in the United States

Figure 3: The estimated RTP RH of Japan



5 CONCLUDING REMARKS

The GDP trend of the United States shifted notably downward after the Great Recession but has not yet returned to the pre-recession trend. This shift indicates that the cause of the Great Recession was a change in a fundamental factor that has the potential to shift the steady state. In this paper, I examined three possible causes for the shift: a change in technology, a change in preferences, and a sudden malfunctioning of the price mechanism. A change in preferences has generally been unexplored as a source of large economic fluctuations because of the prevailing preconception that preferences must be temporally stable. However, this temporal stability has not been proven, and there have been theoretical and empirical studies that indicate that preferences, particularly RTP, in fact are temporally variable.

I showed that, unlike the technology shock and price mechanism explanations, there are no theoretical problems with an upward RTP shock as the cause of the Great Recession. This is true because it is the expected RTP RH, not the intrinsic RTP RH, that is of importance in economic activities. In addition, households behave intrinsically non-cooperatively, which generates a Nash equilibrium that consists of strategies that generate Pareto inefficient payoffs. I therefore concluded that an upward expected RTP RH shock is the most likely cause of the Great Recession. To validate this conclusion, I estimated the yearly expected RTP RH of the United States and found that the expected RTP RH shifted upwards by 1–2 percentage points when the Great Recession began. This empirical result supports the explanation that the Great Recession was caused by an upward RTP shock.

APPENDIX A

A1 The representative household

A1.1 The representative household in dynamic models

A1.1.1 The assumption of the representative household

The concept of the representative household is a necessity in macroeconomic studies. It is used as a matter of course, but its theoretical foundation is fragile. The representative household has been used given the assumption that all households are identical or that there exists one specific individual household, the actions of which are always average among households (I call such a household "the average household" in this paper). The assumption that all households are identical seems to be too strict; therefore, it is usually assumed explicitly or implicitly that the representative household is the average household. However, the average household can exist only under very strict conditions. Antonelli (1886) showed that the existence of an average household requires that all households have homothetic and homogeneous utility functions. This type of utility function is not usually assumed in macroeconomic studies because it is very restrictive and unrealistic. If more general utility functions are assumed, however, the assumption of the representative household as the average household is inconsistent with the assumptions underlying the utility functions.

Nevertheless, the assumption of the representative household has been widely used, probably because it has been believed that the representative household can be interpreted as an approximation of the average household. Particularly in static models, the representative household can be seen to approximate the average household. However, in dynamic models, it is hard to accept the representative household as an approximation of the average household because, if RTPs of households are heterogeneous, there is no steady state where all of the optimality conditions of the heterogeneous households are satisfied (Becker, 1980). Therefore, macroeconomic studies using dynamic models are fallacious if the representative household is assumed to approximate the average household.

A1.1.2 The representative household in static models

Static models are usually used to analyze comparative statics. If the average household is represented by one specific unique household for any static state, there will be no problem in assuming the representative household as an approximation of the average household. Even though the average household is not always represented by one specific unique household in some states, if the average household is always represented by a household in a set of households that are very similar in preferences and other features, then the representative household assumption can be used to approximate the average household.

Suppose, for simplicity, that households are heterogeneous such that they are identical except for a particular preference. Because of the heterogeneous preference, household consumption varies. However, levels of consumption will not be distributed randomly because the distribution of consumption will correspond to the distribution of the preference. The consumption of a household that has a very different preference from the average will be very different from the average household consumption. Conversely, it is likely that the consumption of a household that has the average preference will nearly have the average consumption. In addition, the order of the degree of consumption will be almost unchanged for any static state because the order of the degree of the preference does not change for the given state.

If the order of consumption is unchanged for any given static state, it is likely that the household with consumption that is closest to the average consumption will also always be a household belonging to a group of households that have very similar preferences. Hence, it is possible to argue that, approximately, one specific unique household's consumption is always average for any static state. Of course, it is possible to show evidence that is counter to this argument, particularly in some special situations, but it is likely that this conjecture is usually true in normal situations, and the assumption that the representative household approximates the average household is acceptable in static models.

A1.1.3 The representative household in dynamic models

In dynamic models, however, the story is more complicated. In particular, heterogeneous RTPs pose a serious problem. This problem is easily understood in a dynamic model with exogenous technology (i.e., a Ramsey growth model). Suppose that households are heterogeneous in RTP, degree of risk aversion (ε), and productivity of the labor they provide. Suppose also for simplicity that there are many "economies" in a country, and an economy consists of a household and a firm. The household provides labor to the firm in the particular economy, and the firm's level of technology (A) varies depending on the productivity of labor that the household in its economy provides. Economies trade with each other: that is, the entire economy of a country consists of many individual small economies that trade with each other.

A household maximizes its expected utility, $E\int_0^\infty u(c_t)\exp(-\theta t)dt$, subject to $\dot{k}_t = f(k_t) - c_t$, where $u(\bullet)$ is the utility function; $f(\bullet)$ is the production function; θ is RTP; E is the expectation operator; $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, and $c_t = \frac{C_t}{L_t}$; Y_t (≥ 0) is output, K_t (≥ 0) is capital input, L_t (≥ 0) is labor input, and C_t (≥ 0) is consumption in period t. The optimal consumption path of this Ramsey-type growth model is

$$\frac{\dot{c}_t}{c_t} = \varepsilon^{-1}\left(\frac{\partial y_t}{\partial k_t} - \theta\right) ,$$

and at steady state,

$$\frac{\partial y_t}{\partial k_t} = \theta \quad . \tag{A1}$$

Therefore, at steady state, the heterogeneity in the degree of risk aversion (ε) is irrelevant, and the heterogeneity in productivity does not result in permanent trade imbalances among economies because $\frac{\partial y_t}{\partial k_t}$ in all economies is kept equal by market arbitrage. Hence, heterogeneity in the degree of risk aversion and productivity does not matter at steady state. Therefore, the same logic as that used for static models can be applied. Approximately, one specific unique household's consumption is always average for any time in dynamic models, even if the degree of risk aversion and the productivity are heterogeneous. Thus, the assumption of the representative household is also acceptable in dynamic models even if the degree of risk aversion and the productivity are heterogeneous.

However, equation (A1) clearly indicates that heterogeneity in RTP is problematic. As Becker (1980) shows, if RTP is heterogeneous, the household that has the lowest RTP will eventually possess all capital. With heterogeneous RTPs, there is no steady state where all households achieve all of their optimality conditions. In addition, the household with consumption that is average at present has a very different RTP from the household with consumption that is average in the distant future. The consumption of a household that has the average RTP will initially be almost average, but in the future the household with the lowest RTP will be the one with consumption that is almost average. That is, the consumption path of the household that presently has average consumption is notably different from that of the household with average consumption in the future. Therefore, any individual household cannot be almost average in any period and thus cannot even approximate the average household. As a result, even if the representative household is assumed in a dynamic model, its discounted expected utility $E\int_0^\infty u(c_t)\exp(-\theta t)dt$ is meaningless, and analyses based on it are fallacious.

If we assume that RTP is identical for all households, the above problem is solved. However, this solution is still problematic because that assumption is not merely expedient for the sake of simplicity; rather, it is a critical requirement to allow for an assumed representative household. Therefore, the rationale for identical RTPs should be validated; that is, it should be demonstrated that identical RTPs are actually and universally observed. RTP is, however, unquestionably not identical among households. Hence, it is difficult to accept the representative household assumption in dynamic models based on the assumption of identical RTP.

The conclusion that the representative household assumption in dynamic models is meaningless and leads to fallacious results is very important, because a huge number of studies have used the representative household assumption in dynamic models. To solve this severe problem, an alternative interpretation or definition of the representative household is needed.

Note that in an endogenous growth model the situation is even more complicated. Because a heterogeneous degree of risk aversion also matters, the assumption of the representative household is more difficult to accept, so an alternative interpretation or definition is even more important when endogenous growth models are used.

A1.2 Sustainable heterogeneity

A1.2.1 The model

Suppose that two heterogeneous economies—economy 1 and economy 2—are identical except for their RTPs. Households within each economy are assumed to be identical for simplicity. The population growth rate is zero. The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy.

Each economy can be interpreted as representing either a country (the international interpretation) or a group of identical households in a country (the national interpretation). Because the economies are fully open, they are integrated through trade and form a combined economy. The combined economy is the world economy in the international interpretation and the national economy in the national interpretation. In the following discussion, a model based on the international interpretation is called an international model and that based on the national interpretation is called a national model. Usually, the concept of the balance of payments is used only for the international transactions. However, because both national and international interpretations are possible, this concept and terminology are also used for the national models in this paper.

RTP of household in economy 1 is θ_1 and that in economy 2 is θ_2, and $\theta_1 < \theta_2$. The production function in economy 1 is $y_{1,t} = A^\alpha f(k_{1,t})$ and that in economy 2 is $y_{2,t} = A^\alpha f(k_{2,t})$, where $y_{i,t}$ and $k_{i,t}$ are, respectively, output and capital per capita in economy i in period t for i = 1, 2; A is technology; and α $(0<\alpha<1)$ is a constant. The population of each economy is $\frac{L}{2}$; thus, the total for both is L, which is sufficiently large. Firms operate in both economies. The current account balance in economy 1 is τ_t and that in economy 2 is $-\tau_t$. The production functions are specified as

$$y_{i,t} = A^\alpha k_{i,t}^{1-\alpha} \quad ;$$

thus, $Y_{i,t} = K_{i,t}^{1-\alpha}(AL)^{\alpha}\ (i=1,2)$. Because A is given exogenously, this model is an exogenous technology model (Ramsey growth model). The examination of sustainable heterogeneity based on an endogenous growth model is shown in Harashima (2014a).

Because both economies are fully open, returns on investments in each economy are kept equal through arbitration, such that

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} = \frac{\partial y_{2,t}}{\partial k_{2,t}} \quad . \tag{A2}$$

Because equation (A2) always holds through arbitration, equations $k_{1,t} = k_{2,t}$, $\dot{k}_{1,t} = \dot{k}_{2,t}$, $y_{1,t} = y_{2,t}$, and $\dot{y}_{1,t} = \dot{y}_{2,t}$ also hold.

The accumulated current account balance $\int_0^t \tau_s ds$ mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Because $\frac{\partial y_{1,t}}{\partial k_{1,t}}\left(= \frac{\partial y_{2,t}}{\partial k_{2,t}}\right)$ are returns on investments, $\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds$ and $\frac{\partial y_{2,t}}{\partial k_{2,t}}\int_0^t \tau_s ds$ represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}}\int_0^t \tau_s ds$$

is the balance on goods and services of economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}}\int_0^t \tau_s ds - \tau_t$$

is that of economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies, such that

$$\tau_t = \kappa(k_{1,t}, k_{2,t}) \quad .$$

The government (or an international supranational organization) intervenes in the activities of economies 1 and 2 by transferring money from economy 1 to economy 2. The amount of transfer in period t is g_t, and it is assumed that g_t depends on capital inputs, such that

$$g_t = \bar{g}k_{1,t} ,$$

where $\bar{g}$ is a constant. Because $k_{1,t} = k_{2,t}$ and $\dot{k}_{1,t} = \dot{k}_{2,t}$,

$$g_t = \bar{g}k_{1,t} = \bar{g}k_{2,t} \quad .$$

Each household in economy 1 therefore maximizes its expected utility

$$E\int_0^{\infty} u_1(c_{1,t})\exp(-\theta_1 t)dt \quad ,$$

subject to

$$\dot{k}_{1,t} = A^{\alpha}k_{1,t}^{1-\alpha} - c_{1,t} + (1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\int_0^t \tau_s ds - \tau_t - \bar{g}k_{1,t} \quad , \tag{A3}$$

and each household in economy 2 maximizes its expected utility

$$E\int_0^{\infty} u_2(c_{2,t})\exp(-\theta_2 t)dt \quad ,$$

subject to

$$\dot{k}_{2,t} = A^{\alpha}k_{2,t}^{1-\alpha} - c_{2,t} - (1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}\int_0^t \tau_s ds + \tau_t + \bar{g}k_{2,t} \quad , \tag{A4}$$

where $u_{i,t}$ and $c_{i,t}$, respectively, are the utility function and per capita consumption in economy i in period t for i = 1, 2; and E is the expectation operator. Equations (A3) and (A4) implicitly assume that each economy does not have foreign assets or debt in period t = 0.

A1.2.2 Sustainable heterogeneity without government intervention

Heterogeneity is defined as being sustainable if all of the optimality conditions of all heterogeneous households are satisfied indefinitely. First, the natures of the model when the government does not intervene (i.e., $\bar{g}=0$) are examined. The growth rate of consumption in economy 1 is

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1}\left\{(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha} + (1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\frac{\partial\int_0^t \tau_s ds}{\partial k_{1,t}} - \alpha(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha-1}\int_0^t \tau_s ds - \frac{\partial \tau_t}{\partial k_{1,t}} - \theta_1\right\} \quad .$$

Hence,

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \varepsilon^{-1}\lim_{t\to\infty}\left\{(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha} + (1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\frac{\partial\int_0^t \tau_s ds}{\partial k_{1,t}} - \alpha(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha-1}\int_0^t \tau_s ds - \frac{\partial \tau_t}{\partial k_{1,t}} - \theta_1\right\} = 0$$

and thereby

$$\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}[1+(1-\alpha)\Psi] - \Xi - \theta_1 = 0 \quad ,$$

where $\Xi = \lim_{t\to\infty}\frac{\tau_t}{k_{1,t}} = \lim_{t\to\infty}\frac{\tau_t}{k_{2,t}}$ and $\Psi = \lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}} = \lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{2,t}}$. $\lim_{t\to\infty}\frac{\dot{y}_{1,t}}{y_{1,t}} = \lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} = \lim_{t\to\infty}\frac{\dot{k}_{1,t}}{k_{1,t}} = \lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = 0$, and Ψ is constant at steady state because $k_{1,t}$ and τ_t are constant; thus, $\Xi = \lim_{t\to\infty}\frac{\tau_t}{k_{1,t}}$ is constant at steady state. For Ψ to be constant at steady state, it is necessary that

$\lim_{t\to\infty} \tau_t = 0$ and thus $\Xi = 0$. Therefore,

$$\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\left[1+(1-\alpha)\Psi\right]-\theta_1 = 0 \quad , \tag{A5}$$

and

$$\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}\left[1-(1-\alpha)\Psi\right]-\theta_2 = 0 \tag{A6}$$

because

$$\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}} = \varepsilon^{-1}\lim_{t\to\infty}\left\{(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha} - (1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}\frac{\partial\int_0^t \tau_s ds}{\partial k_{2,t}} + \alpha(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha-1}\int_0^t \tau_s ds + \frac{\partial \tau_t}{\partial k_{2,t}} - \theta_2\right\} = 0 \quad .$$

Because $\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\left[1+(1-\alpha)\Psi\right] = \theta_1$, $\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}\left[1-(1-\alpha)\Psi\right] = \theta_2$, and $\frac{\partial y_{1,t}}{\partial k_{1.t}} = \frac{\partial y_{2,t}}{\partial k_{2.t}} = A^{\alpha}k_{1,t}^{-\alpha} = A^{\alpha}k_{2,t}^{-\alpha}$, then

$$\Psi = \frac{\theta_1 - \theta_2}{2(1-\alpha)\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}}} \quad . \tag{A7}$$

By equations (A5) and (A7),

$$\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}} + \lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}}(1-\alpha)\Psi = \theta_1 \quad ;$$

thus,

$$\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}} = \frac{\theta_1+\theta_2}{2} = \lim_{t\to\infty}\frac{\partial y_{2,t}}{\partial k_{2.t}} \quad . \tag{A8}$$

If equation (A8) holds, all of the optimality conditions of both economies are indefinitely satisfied. The state indicated by equation (A8) is called the “multilateral steady state” or “multilateral state” in the following discussion. By procedures similar to those used for the endogenous growth model in Harashima (2014a), the condition of the multilateral steady state for H economies that are identical except for their RTPs is shown as

$$\lim_{t\to\infty}\frac{\partial y_{i,t}}{\partial k_{i.t}} = \frac{\sum_{q=1}^{H}\theta_q}{H} \tag{A9}$$

for any i, where $i = 1, 2, \dots , H$.

Because

$$\Psi = \frac{\theta_1 - \theta_2}{2(1-\alpha)\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1,t}}} = \frac{\theta_1 - \theta_2}{(1-\alpha)(\theta_1 + \theta_2)} < 0$$

by equation (A8), then by $\lim_{t\to\infty}\frac{\int_0^t \tau_s ds}{k_{1,t}} = \Psi < 0$,

$$\lim_{t\to\infty}\int_0^t \tau_s ds < 0 ;$$

that is, economy 1 possesses accumulated debts owed to economy 2 at steady state, and economy 1 has to export goods and services to economy 2 by

$$\left|(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\int_0^t \tau_s ds\right|$$

in every period to pay the debts. Nevertheless, because $\lim_{t\to\infty}\tau_t = 0$ and $\Xi = 0$, the debts do not explode but stabilize at steady state. Because of the debts, the consumption of economy 1 is smaller than that of economy 2 at steady state under the condition of sustainable heterogeneity.

Note that many empirical studies conclude that RTP is negatively correlated with income (e.g., Lawrance, 1991; Samwick, 1998; Ventura, 2003). Suppose that, in addition to the heterogeneity in RTP ($\theta_1 < \theta_2$), the productivity of economy 1 is higher than that of economy 2. At steady state, the consumption of economy 1 would be larger than that of economy 2 as a result of the heterogeneity in productivity. However, as a result of the heterogeneity in RTP, the consumption of economy 1 is smaller than that of economy 2 at steady state under sustainable heterogeneity. Which effect prevails will depend on differences in the degrees of heterogeneity. For example, if the difference in productivity is relatively large whereas that in RTP is relatively small, the effect of the productivity difference will prevail and the consumption of economy 1 will be larger than that of economy 2 at steady state under sustainable heterogeneity.

A1.2.3 Sustainable heterogeneity with government intervention

Sustainable heterogeneity is a very different state from the one Becker (1980) described. The difference emerges because, in a multilateral state, economy 1 behaves by fully considering economy 2's conditions. The multilateral state therefore will not be naturally selected by economy 1, and the path selection may have to be decided politically (see Harashima, 2010). On the other hand, when economy 1 behaves unilaterally, the government may intervene in economic activities so as to achieve, for example, social justice.

In this section, I show that, even if economy 1 behaves unilaterally, sustainable heterogeneity can always be achieved with appropriate government intervention.

A1.2.3.1 The two-economy model

Government intervention is first considered in the two-economy model constructed in Section A1.2.1. If the government intervenes (i.e., $\bar{g} > 0$),

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}} \neq \lim_{t\to\infty}\frac{\dot{\tau}_t}{\tau_t} = \lim_{t\to\infty}\frac{\frac{d\left(\int_0^t \tau_s ds\right)}{dt}}{\int_0^t \tau_s ds} .$$

Because $\bar{g} > 0$, equations (A5) and (A6) are changed to

$$\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\left[1+(1-\alpha)\Psi\right]-\theta_1-\bar{g}=0 \quad , \tag{A10}$$

and

$$\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{2,t}^{-\alpha}\left[1-(1-\alpha)\Psi\right]-\theta_2+\bar{g}=0 \quad . \tag{A11}$$

If economy 1 behaves unilaterally such that equation (A10) is satisfied, then

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=0$$

and

$$\Psi=\frac{(\theta_1+\bar{g})\left[\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}\right]^{-1}-1}{1-\alpha} \quad .$$

At the same time, if economy 2 behaves unilaterally such that equation (A11) is satisfied, then

$$\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=0 \quad .$$

By equations (A10) and (A11)

$$\bar{g}=\frac{\theta_2-\theta_1}{2}+\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}(1-\alpha)\Psi$$

because $k_{1,t}=k_{2,t}$. In addition,

$$\lim_{t\to\infty}(1-\alpha)A^{\alpha}k_{1,t}^{-\alpha}=\frac{\theta_1+\theta_2}{2}=\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}}=\lim_{t\to\infty}\frac{\partial y_{2,t}}{\partial k_{2.t}} \quad .$$

This equation is identical to equation (A8) and is satisfied at the multilateral steady state. Therefore,

$$\bar{g}=\frac{\theta_2-\theta_1}{2}+(1-\alpha)\lim_{t\to\infty}\frac{\partial y_{1,t}}{\partial k_{1.t}}\Psi=\frac{\theta_2-\theta_1}{2}+(1-\alpha)\frac{\theta_1+\theta_2}{2}\lim_{t\to\infty}\frac{\int_0^t\tau_s ds}{k_{1,t}} \quad . \tag{A12}$$

If $\bar{g}$ is set equal to equation (A12), all optimality conditions of both economies 1 and 2 are satisfied even though economy 1 behaves unilaterally.

There are various values of Ψ, depending on the initial consumption economy 1 sets. If economy 1 behaves in such a way as to make $\lim_{t\to\infty}\int_0^t\tau_s ds<0$, and particularly, make $\bar{g}$ = 0 such that

$$\bar{g} = \frac{\theta_2 - \theta_1}{2} + (1-\alpha)\frac{\theta_1 + \theta_2}{2} \lim_{t \to \infty} \frac{\int_0^t \tau_s ds}{k_{1,t}} = 0 \quad ,$$

then

$$\Psi = \frac{\theta_1 - \theta_2}{2(1-\alpha)\lim_{t \to \infty} \frac{\partial y_{1,t}}{\partial k_{1,t}}} \tag{A13}$$

by equation (A12). Equation (A13) is identical to equation (A7); that is, the state where equation (A13) is satisfied is identical to the multilateral state with no government intervention (i.e., $\bar{g}$ = 0). On the other hand, if economy 1 behaves in such a way as to make $\lim_{t \to \infty} \int_0^t \tau_s ds = 0$,

$$\bar{g} = \frac{\theta_2 - \theta_1}{2} > 0 \quad .$$

This condition is identical to that for sustainable heterogeneity with government intervention in the endogenous growth model shown by Harashima (2012). Furthermore, if economy 1 behaves in such a way as to make $\lim_{t \to \infty} \int_0^t \tau_s ds > 0$, $\bar{g}$ is positive and is given by equation (A12).

There are various steady states, depending on the values of $\Psi = \lim_{t \to \infty} \frac{\int_0^t \tau_s ds}{k_{1,t}}$ and the initial consumption set by economy 1. Nevertheless, at any steady state that satisfies equation (A13), all of the optimality conditions of economy 1 are satisfied (by government intervention, all optimality conditions of economy 2 are also satisfied). For economy 1, all steady states are equally optimal. Economy 1 selects one of the steady states (i.e., sets the initial consumption); for example, it may select the one that gives the highest expected utility, the highest steady state consumption, or some values based on other criteria. Note, however, that an overly large positive Ψ requires zero initial consumption and thus a certain upper bound of Ψ will exist.

A1.2.3.2 The multi-economy model

In this section, for simplicity, only the case of $\Psi = \lim_{t \to \infty} \frac{\int_0^t \tau_s ds}{k_{1,t}} = 0$ is considered. It is assumed that there are H economies that are identical except for their RTPs. If H = 2, when sustainable heterogeneity is achieved, economies 1 and 2 consist of a combined economy (economy 1+2) with twice the population and a RTP of $\frac{\theta_1 + \theta_2}{2}$. Suppose there is a third economy with a RTP of θ_3. Because economy 1+2 has twice the population of economy 3, if

$$\bar{g} = \frac{\theta_3 - \frac{\theta_1 + \theta_2}{2}}{3} \quad ,$$

then

$$\lim_{t\to\infty}\frac{\dot{c}_{1,t}}{c_{1,t}}=\lim_{t\to\infty}\frac{\dot{c}_{2,t}}{c_{2,t}}=\lim_{t\to\infty}\frac{\dot{c}_{3,t}}{c_{3,t}}=0 \quad .$$

By iterating similar procedures, if government transfer between economy H and economy 1+2+ · · · + (H – 1) is such that

$$\bar{g}=\frac{\theta_H-\frac{\sum_{q=1}^{H-1}\theta_q}{H-1}}{H} \quad ,$$

then

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=0$$

for any i (= 1, 2, · · ·, H).

A1.3 An alternative definition of the representative household

A1.3.1 The definition

Section A1.2 indicates that, when sustainable heterogeneity is achieved, all heterogeneous households are connected (in the sense that all households behave by considering other households' optimality) and appear to be behaving collectively as a combined supra-household that unites all households, as equations (A8) and (A9) indicate. The supra-household is unique and its behavior is time-consistent. Its actions always and consistently represent those of all households. Considering these natures of households under sustainable heterogeneity, I present the following alternative definition of the representative household: "the behavior of the representative household is defined as the collective behavior of all households under sustainable heterogeneity."

Even if households are heterogeneous, they can be represented by a representative household as defined above. Unlike the representative household defined as the average household, the collective representative household reaches a steady state where all households satisfy all of their optimality conditions in dynamic models. In addition, this representative household has a RTP that is equal to the average RTP as shown in equations (A8) and (A9).[1] Hence, we can assume not only a representative household but also that its RTP is the average rate of all households.

A1.3.2 Universality of sustainable heterogeneity

An important point, however, is that this alternatively defined representative household can be used in dynamic models only if sustainable heterogeneity is achieved, but this condition is not necessarily always naturally satisfied. Sustainable heterogeneity is achieved only if households with lower RTPs behave multilaterally or the government appropriately intervenes. Therefore, the representative household assumption is not necessarily naturally acceptable in dynamic

[1] If sustainable heterogeneity is achieved with the help of the government's intervention, the time preference rate of the representative household will not be exactly equal to the average rate of time preference.

models unless it is confirmed that sustainable heterogeneity is usually achieved in an economy.

Notwithstanding this flaw, the representative household assumption has been widely used in many macroeconomic studies that use dynamic models. Furthermore, these studies have been little criticized for using the inappropriate representative household assumption. In addition, in most economies, the dire state that Becker (1980) predicts has not been observed even though RTPs of households are unquestionably heterogeneous. These facts conversely indicate that sustainable heterogeneity—probably with government interventions—has been usually and universally achieved across economies and time periods. In a sense, these facts are indirect evidence that sustainable heterogeneity usually prevails in economies.

Note that because the representative household's behavior in dynamic models is represented by the collective behavior of all households under sustainable heterogeneity, RH's RTP is not intrinsically known to households, but they do need to have an expected rate. Each household intrinsically knows its own preferences, but it does not intrinsically know the collective preference of all households. Therefore, in dynamic models, it must be assumed that all households do not *ex ante* know RH's RTP, but households estimate it from information on the behaviors of other households and the government.

A2 Need for an expected RTP RH

A2.1 The behavior of household

Achieving sustainable heterogeneity affects the behavior of the individual household because sustainable heterogeneity indicates that each household must consider the other households' optimality (as well as the behavior of the government, if necessary). This feature does not mean that households behave cooperatively with other households. Each household behaves non-cooperatively based on its own RTP, but at the same time, it behaves considering whether the other households' optimality conditions are achieved or not. This consideration affects the actions a household takes in that it affects the choice of a household's initial consumption.

Sustainable heterogeneity indicates that a household's future path of consumption has to be consistent with the future path of sustainable heterogeneity. Thereby, a household sets its initial consumption such that it will proceed on the path that is consistent with the path of sustainable heterogeneity and eventually reach a steady state.

A2.2 Deviation from sustainable heterogeneity

A2.2.1 Political elements

What happens if a household deviates from sustainable heterogeneity? A deviation means that a household sets its initial consumption at a level that is not consistent with sustainable heterogeneity. For less advantaged households (i.e., households with higher RTPs), the only way to satisfy all of their optimality conditions is to set their initial consumption consistent with sustainable heterogeneity. Therefore, they will not take the initiative to deviate. In contrast, the most advantaged households (i.e., those with the lowest RTP) can satisfy all of their optimality conditions even if they set initial consumption independent of sustainable heterogeneity. The incentive for the most advantaged household to select a multilateral path will be weak because the growth rate of the most advantaged household on the multilateral path is lower than that on the unilateral path.

When economy 1 selects the unilateral path, does economy 2 quietly accept the unfavorable consequences shown in Becker (1980)? From an economic perspective, the optimal response of economy 2 is the one shown in Harashima (2010): economy 2 should behave as a follower and accept the unfavorable consequences. However, if other factors—particularly political ones—are taken into account, the response of economy 2 will be different. Faced with a situation in which all the optimality conditions cannot be satisfied, it is highly likely that economy 2 would politically protest and resist economy 1. It should be emphasized economy 2 is not responsible for its own non-optimality, which is a result of economy 1's unilateral behavior in a heterogeneous population. Economy 2 may overlook the non-optimality if it is temporary, but it will not if it is permanent. As shown in Harashima (2010), the non-optimality is permanent, it is quite likely that economy 2 will seriously resist economy 1 politically.

If economy 1 could achieve its optimality only on the unilateral path, economy 1 would counter the resistance of economy 2, but this is not the case. Because of this, economy 2's demand does not necessarily appear to be unreasonable or selfish. Faced with the protest and resistance by economy 2, economy 1 may compromise or cooperate with economy 2 and select the multilateral path.

A2.2.2 Resistance

The main objective of economy 2 is to force economy 1 to select the multilateral path and to establish sustainable heterogeneity. This objective may be achieved through cooperative measures, non-violent civil disobedience (e.g., trade restrictions), or other more violent means.

Restricting or abolishing trade between the two economies will cost economy 1 because it necessitates a restructuring of the division of labor, and the restructuring will not be confined to a small scale. Large-scale adjustments will develop that involve all levels of divided labor, because they are all correlated with each other. For example, if an important industry had previously existed only in one economy, owing to a division of labor, and trade between the two economies was no longer permitted, the other economy would have to establish this industry while also maintaining other industries. As a result, economy 1 would incur non-negligible costs. More developed economies have more complicated and sophisticated divisions of labor, and restructuring costs from the disruption of trade will be much higher in developed economies. In addition, more resources will need to be allocated to the generation of technology because technology will also no longer be traded. Finally, all of the conventional benefits of trade will be lost. Trade is beneficial because of the heterogeneous endowment of resources, as the Heckscher-Ohlin theorem shows. Because goods and services are assumed to be uniform in the models presented in this paper, the benefits of trade are implicit in the models. However, in the real word, resources such as oil and other raw materials are unevenly distributed, so a disruption or restriction of trade will substantially damage economic activities on both national and international levels.

The damage done by trade restrictions has an upper limit, however, because the restructuring of the division of labor, additional resource allocation to innovation, and loss of trade benefits are all finite. Therefore, in some cases, particularly if economies are not sufficiently developed and division of labor is not complex, the damage caused will be relatively small. Hence, a disruption of trade (non-violent civil disobedience in the national models) may not be sufficiently effective as a means of resistance under some these conditions.

In some cases, harassment, sabotage, intimidation, and violence may be used, whether legal or illegal. In extreme cases, war or revolution could ensue. In such cases, economy 1 will be substantially damaged in many ways and be unable to achieve optimality. The resistance and resulting damages will continue until sustainability is established.

In any case, the objective of economy 2's resistance conversely implies that establishing sustainability eliminates the risk and cost of political and social instability. The resistance of economy 2 will lower the desire of economy 1 to select the unilateral path.

A2.2.3 United economies

An important countermeasure to the fragility of sustainable heterogeneity for less advantaged economies is the formation of a union of economies. If economies other than economy 1 are united by commonly selecting the multilateral path within them, their power to resist economy 1 will be substantially enhanced. Consider the multi-economy model shown in Harashima (2010). If the economies do not form a union, the power to resist the unilateral actions of economy 1 is divided and limited to the power of each individual economy. However, if the economies are united, the power to resist economy 1 increases. If a sufficient number of economies unite, the multilateral path will almost certainly be selected by economy 1.

To maintain the union, any economy in the union should have the explicit and resolved intention of selecting the multilateral path within the union, even if it is relatively more advantaged within the union. To demand that relatively more advantaged economies select the multilateral path, less advantaged economies themselves must also select the multilateral path in any case. Otherwise, less advantaged economies will be divided and ruled by more advantaged economies. For all heterogeneous people to happily coexist, all of them should behave multilaterally. At the same time, Harashima (2010) indicates that the more advantaged an economy is, the more modestly it should behave, i.e., the more it should restrain itself from accumulating extra capitals.

In general, therefore, the most advantaged (the lowest RTP) household will be forced to set its initial consumption consistent with sustainable heterogeneity.

A2.3 Need for an expected RTP RH

Because all households need to set their initial consumption consistent with sustainable heterogeneity to achieve it, households must calculate the path of sustainable heterogeneity before setting their initial consumption levels. To calculate this level, each household first must know the value of RTP RH. However, although a household naturally knows the value of its own RTP, it does not intrinsically know the value of RTP RH. To know this, a household would have to know the values of all of the other households' RTPs. Hence, the expected value of RTP RH must somehow be generated utilizing all other relevant available information. The necessity of an expected RTP RH is critically important because RTP plays a crucial role as the discount factor in dynamic models.

Note that, if we assume that RTP is identical for all households, an expected RTP RH is no longer needed because any household's own RTP is equal to the RTP RH. This solution is still problematic, however, because the assumption is not merely expedient for the sake of simplicity; rather, it is a critical requirement to eliminate the need for an expected RTP RH. Therefore, any rationale for assuming identical RTPs should be validated; that is, it should be demonstrated that identical RTPs do exist and are universally observed. However, RTP is unquestionably not identical among households. Therefore, households must use expected values of RTP RH.

A3 The RTP model

A3.1 Need to know the structural model

If RTP RH is a constant parameter, as has been long and widely assumed, the need for an expected RTP RH would not be a serious problem. The historical mean of an unchanging RTP RH could be estimated relatively precisely based on long-term data of various economic indicators even if the structural model remained unknown. The RTP RH could be specified as the RTP that is most consistent with long-term trends of the indicators.

Although RTP has been treated as a constant parameter in many studies, this feature has not been demonstrated either empirically or theoretically. Rather, the assumption is merely expedient for the sake of simplicity. There is another practical reason for this treatment: models with a permanently constant RTP exhibit excellent tractability (see Samuelson, 1937). However, some have argued that it is natural to view RTP as temporally variable, and the concept of a temporally varying RTP has a long history (e.g., Böhm-Bawerk, 1889; Fisher, 1930). More recently, Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant RTPs by nature and that economic and social factors affect the formation of RTPs. Their arguments indicate that many incidents can affect and change RTP. Models of endogenous RTP have been presented, the most familiar of which is Uzawa's (1968) model.

If the RTP RH is temporally variable, its future stream must be expected by households, and a rational expectation is a model-consistent expectation. To generate rational expectations of RTP RH, therefore, the structural model of the RTP RH (i.e., equations that fundamentally describe how it is endogenously formed) needs to be known.

A3.2 Endogenous RTP models

A3.2.1 Uzawa's (1968) model

The most well-known endogenous RTP model is that of Uzawa (1968). It has been applied in many analyses (e.g., Epstein and Hynes, 1983; Lucas and Stokey, 1984; Epstein, 1987; Obstfeld, 1990). However, Uzawa's model has not necessarily been regarded as a realistic expression of the endogeneity of RTP because it has a serious drawback in that impatience increases as income, consumption, and utility increase. The basic structure of Uzawa's model is

$$\theta_t = \theta^*\left[u(c_t)\right],$$
$$0 < \frac{d\theta_t}{du(c_t)},$$

in which RTP in period t (θ_t) is temporally variable and an increasing function of present utility $u(c_t)$ where c_t is consumption in period t. The condition $0 < \frac{d\theta_t}{du(c_t)}$ is necessary for the model to be stable. This property is quite controversial and difficult to accept *a priori* because many empirical studies have indicated that RTP is negatively correlated with permanent income (e.g., Lawrance, 1991); thus, many economists are critical of Uzawa's model. Epstein (1987), however, discussed the plausibility of increasing impatience and offered some counter-arguments. However, his view is in the minority, and most economists support arguments in favor of a decreasing RTP, such that $\frac{d\theta_t}{du(c_t)} < 0$. Hence, although Uzawa's model attracted some attention, the analysis of the endogeneity of RTP has progressed very little. Although Uzawa's model may be flawed, it does not mean that the conjecture that RTP is influenced by future income, consumption, and utility is fallacious. Rather, it means that an appropriate model in which RTP is negatively correlated with income, consumption, and utility has not been presented.

A3.2.2 Size effect on impatience

The problem of $0 < \frac{d\theta_t}{du(c_t)}$ in Uzawa's model arises because distant future levels of consumption have little influence on factors that form RTP; that is, RTP is formed only with the information on present consumption, and it must be revised every period in accordance with consumption growth. However, there is no *a priori* reason why information on distant future activities should be far less important than the information on the present and near future activities. Fisher (1930) argued that

> [O]ur first step, then, is to show how a person's impatience depends on the *size* of his income, assuming the other three conditions to remain constant; for, evidently, it is possible that two incomes may have the same time shape, composition and risk, and yet differ in size, one being, say, twice the other in every period of time.
>
> In general, it may be said that, other things being equal, the smaller the

income, the higher the preference for the present over the future income. It is true of course that a permanently small income implies a keen appreciation of wants as well as of immediate wants. … But it increases the want for immediate income *even more* than it increases the want for future income. (p. 72)

According to Fisher's (1930) view, a force that influences RTP is a psychological response derived from the perception of the "size of the entire income or utility stream." This view indicates that it is necessary to probe how people perceive the size of the entire income or utility stream.

Little effort has been directed toward probing the nature of the size of the utility or income stream on RTP, although numerous psychological experiments have been performed with regard to the anomalies of the expected utility model with a constant RTP (e.g., Frederick et al., 2002). Analyses using endogenous RTP models so far have merely introduced the *a priori* assumption of endogeneity of RTP without explaining the reasoning for doing so in detail. Hence, even now, Fisher's (1930) insights are very useful for the examination of the size effect. An important point in Fisher's quote is that the size of the infinite utility stream is perceived as "permanently" high or low. The size difference among the utility streams may be perceived as a permanently continuing difference of utilities among different utility streams. Anticipation of a permanently higher utility may enhance an emotional sense of well-being because people feel they are in a long-lasting secure situation, which will generate a positive psychological response and make people more patient. If that is true, distant future utilities should be taken into account equally with present utility. Otherwise, it is impossible to distinguish whether the difference of utilities will continue permanently.

From this point of view, the specification that only the present utility influences the formation of RTP, as is the case of Uzawa's model, is inadequate. Instead, a simple measure of the size where present and future utilities are summed with equal weight will be a more appropriate measure of the size of a utility stream.[2]

A3.3 Model of RTP[3]

A3.3.1 The model

The representative household solves the maximization problem as shown in Section A1.1.3. Taking the arguments in Section A3.2 into account, the "size" of the infinite utility stream can be defined as follows.

Definition 1: The size of the utility stream W for a given technology A is

$$W = \lim_{T \to \infty} E \int_0^T \rho(t) u(c_t) dt \ ,$$

where E is the expectation operator, and

$$\rho(t) = \frac{1}{T} \quad \text{if} \quad 0 \le t \le T$$
$$\rho(t) = 0 \qquad \text{otherwise.}$$

$\rho(t)$ indicates weights and has the same value in any period. Thus, the weights for the

[2] Das (2003) showed another stable endogenous time preference model with decreasing impatience. Her model is stable, although the rate of time preference is decreasing because endogenous impatience is almost constant. In this sense, the situation her model describes is very special.

[3] The idea of this type of endogenous time preference model was originally presented in Harashima (2004).

evaluation of future utilities are distributed evenly over time, as discussed in Section A3.2.

To this point, technology A has been assumed to be constant. If A is temporally variable (A_t) and grows at a constant rate and the economy is on a balanced growth path such that A_t, y_t, k_t, and c_t grow at the same rate, then the definition of W needs to be modified because any stream of c_t and $u(c_t)$ grows to infinity. It is then impossible to distinguish the sizes of the utility stream by simply summing up c_t as $T \to \infty$ as shown in Definition 1. Because balanced growth is possible only when technological progress is Harrod neutral, I assume a Harrod neutral production function such that

$$y_t = \omega A_t^{\varpi} k_t^{1-\varpi},$$

where $\varpi(0 < \varpi < 1)$ and $\omega(0 < \omega)$ are constants. To distinguish the sizes of utility stream, the following value is set as the standard stream of utility,

$$u\left(\tilde{c}\, e^{\psi t}\right),$$

where $\tilde{c}\,(0 < \tilde{c})$ is a constant and $\psi\,(0 < \psi)$ is a constant rate of growth. Streams of utility can be compared with this standard stream. If a constant relative risk aversion utility function is assumed, a stream of utility can be compared with the standard stream of utility as follows:

$$\frac{u(c_t)}{u\left(\tilde{c}\, e^{\psi t}\right)} = \frac{c_t^{1-\gamma}}{\left(\tilde{c}\, e^{\psi t}\right)^{1-\gamma}} = \frac{1-\gamma}{\tilde{c}^{1-\gamma}} u\left(\frac{c_t}{e^{\psi t}}\right).$$

By using this ratio, a given stream of utility can be distinguished from the standard stream of utility. That is, the size of a utility stream W for a given stream of technology A_t that grows at the same rate ψ as y_t, k_t, and c_t can be alternatively defined as

$$W = \lim_{T \to \infty} E \int_0^T \rho(t) u\left(\frac{c_t}{e^{\psi t}}\right) dt.$$

Clearly, if $\psi = 0$, then the size (W) degenerates into the one shown in Definition 1.

If there is a steady state such that

$$\lim_{t \to \infty} E[u(c_t)] = E\left[u\left(c^*\right)\right],$$

or for the case of expected balanced growth,

$$\lim_{t \to \infty} E\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right] = E\left[u\left(c^*\right)\right],$$

where c^* is a constant and indicates steady-state consumption, then

$$W = E\left[u\left(c^*\right)\right]$$

for the following reason. Because $\lim_{t \to \infty} E[u(c_t)] = E\left[u\left(c^*\right)\right]$ (or $\lim_{t \to \infty} E\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right] = E\left[u\left(c^*\right)\right]$), then

$$\lim_{T\to\infty}\int_0^T \rho(t)\{E[u(c^*)] - E[u(c_t)]\}\,dt = E[u(c^*)] - W$$

$$\left(\text{or } \lim_{T\to\infty}\int_0^T \rho(t)\left\{E[u(c^*)] - E\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right]\right\}dt = E[u(c^*)] - W\right).$$

In addition,

$$\lim_{T\to\infty}\int_0^T \rho(t)\{E[u(c^*)] - E[u(c_t)]\}\,dt = 0$$

$$\left(\text{or } \lim_{T\to\infty}\int_0^T \rho(t)\left\{E[u(c^*)] - E\left[u\left(\frac{c_t}{e^{\psi t}}\right)\right]\right\}dt = 0\right).$$

Hence, $W = E[u(c^*)]$; that is, RTP is determined by steady-state consumption (c*).

The RTP model presented in this paper is constructed on the basis of this measure of W. An essential property that must be incorporated into the model is that RTP is sensitive to, and a function of, W such that

$$\theta = \theta^{**}(W),$$

where $\theta^{**}(W)$ is monotonically continuous and continuously differentiable. Because W is a sum of utilities, this property simply reflects the core idea of an endogenous RTP. However, this property is new in the sense that RTP is sensitive not only to the present utility but also to the entire stream of utility, that is, the size of the utility stream represented by the utility of steady-state consumption. This property is intuitively acceptable because it is likely that people set their principles or parameters for their behaviors considering the final consequences of their behavior (i.e., the steady state; see, e.g., Barsky and Sims, 2012).

Another essential property that must be incorporated into the model is

$$\frac{d\theta}{dW} < 0.$$

Because $W = E[u(c^*)]$ and $0 < \frac{du(c_t)}{dc_t}$, RTP is inversely proportionate to c*. This property is consistent with the findings in many empirical studies, which have shown that RTP is negatively correlated with permanent income (e.g., Lawrance, 1991).

In summary, the basic structure of the model is:

$$\theta = \theta^{**}(W) = \theta^{**}\{E[u(c^*)]\} ,$$
$$\frac{d\theta}{dW} = \frac{d\theta}{dE[u(c^*)]} < 0 \quad . \tag{A14}$$

This model is deceptively similar to Uzawa's endogenous RTP model and simply replaces c_t with c* and $0 < \frac{d\theta_t}{du(c_t)}$ with $\frac{d\theta}{dE[u(c^*)]} < 0$. However, the two models are completely different because of the opposite characteristics of $0 < \frac{d\theta_t}{du(c_t)}$ and $\frac{d\theta}{dE[u(c^*)]} < 0$.

A3.3.2 Nature of the model

The model can be regarded as successful only if it exhibits stability. In Uzawa's model, the economy becomes unstable if $0 < \frac{d\theta_t}{du(c_t)}$ is replaced with $\frac{d\theta_t}{du(c_t)} < 0$. In this section, I examine the stability of the model.

A3.3.2.1 Equilibrium RTP

In Ramsey-type models, such as shown in Section A1.1.3, if a constant RTP is given, the value of the marginal product of capital (i.e., the value of the real interest rate) converges to that of the given RTP as the economy approaches the steady state. Hence, when a RTP is specified at a certain value, the corresponding expected steady-state consumption is uniquely determined. Given fixed values of other exogenous parameters, any predetermined RTP has unique values of expected consumption and utility at steady state. There is a one-to-one correspondence between the expected utilities at steady state and the RTPs; therefore, the expected utility at steady state can be expressed as a function of RTP. Let c_x^* be a set of steady-state consumption levels, given a set of RTPs (θ_X) and other fixed exogenous parameters. The concept of $\theta \rightarrow W$ discussed above can be described as

$$g(\theta) = E[u(c^*)](=W) \ , \tag{A15}$$

where $c^* \in c_x^*$ and $\theta \in \theta_x$. On the other hand, RTP is a continuous function of steady-state consumption as shown in equation (A14) such that $\theta = \theta^{**}(W) = \theta^{**}\{E[u(c^*)]\}$. The reverse function is

$$h(\theta) = E[u(c^*)](=W). \tag{A16}$$

The equilibrium RTP is determined by the point of intersection of the two functions, $g(\theta)$ and $h(\theta)$, as shown in Figure A1. Figure A2 shows the special but conventionally assumed case for $h(\theta)$ in which θ is not sensitive to W, and RTP is constant. There exists a point of intersection because both $g(\theta)$ and $h(\theta)$ are monotonically continuous for $\theta > 0$. $h(\theta)$ is monotonically continuous because $\theta^{**}(W)$ is monotonically continuous. $g(\theta)$ is monotonically continuous because, as a result of utility maximization, $c^* = f(k^*)$ and $\theta = \frac{df(k^*)}{dk^*}$, where k^* is capital input per capita at steady state such that $k^* = \lim_{t\to\infty}(k_t)$. Because $f(k^*)$ and $\frac{df(k^*)}{dk^*}$ are monotonically continuous for $k^* > 0$, c^* is a monotonically continuous function of θ for $\theta > 0$. Here, because u is monotonically continuous, then $E[u(c^*)] = g(\theta)$ is also monotonically continuous for $\theta > 0$.

The function $g(\theta) = E[u(c^*)] = W$ is a decreasing function of θ because higher RTP results in lower steady state consumption. The function $h(\theta) = E[u(c^*)] = W$ is also a decreasing function of θ because $\frac{d\theta}{dW} < 0$. Thus, both $g(\theta)$ and $h(\theta)$ are decreasing, but the slope of $h(\theta)$ is steeper than that of $g(\theta)$ as shown in Figure A1. This is true because $g(\theta) = W$ is the consequence of a Ramsey-type model as shown in Section A1.1.3; thus, if $\theta \to \infty$, then $g(\theta) = W \to 0$ because $\theta = i_t \to \infty$ and $k_t \to 0$, and if $\theta \to 0$, then $g(\theta) = W \to \infty$ because $\theta = i_t \to 0$ and $k_t \to \infty$. The function $h(\theta) = W$ indicates the endogeneity of RTP, and because RTP is usually neither zero nor infinity, then even if $h(\theta) = W \to 0$, $\theta < \infty$, and $h(\theta) = W \to \infty$, $0 < \theta$. Hence, the locus $h(\theta) = W$ cuts the locus $g(\theta) = W$ downward from the top, as shown in Figure A1. Hence, the locus $h(\theta) = W$ is more vertical than $g(\theta) = W$, and thereby a permanently constant RTP, as shown in Figure A2, has probably been used as an approximation of the locus $h(\theta) = W$ for simplicity.

A3.3.2.2 Stability of the model

RTP is constant unless a shock that changes the expected c^* occurs because W does not depend on t but on the expected c^*. Thus, the same RTP and steady state continue until such a shock hits the economy. Therefore, the endogeneity of RTP only matters when a shock occurs. This constancy is the key for the stability of the model. Once the RTP corresponding to the intersection (Fig. 1) is determined, it is constant and the economy converges at a unique steady state unless a shock that changes the expected c^* occurs. The shock is exogenous to the model, and the economy does not explode endogenously but stabilizes at the steady state. Hence, the property $\frac{d\theta}{dW} < 0$ in the model, which is consistent with empirical findings, does not cause instability.

The model is therefore acceptable as a model of endogenous RTP. Furthermore, because RTP is endogenously determined, the assumption of irrationality is not necessary for the determination of RTP. Nevertheless, a shock on RTP can be initiated by a shock on the expected c^*; thus, even if the so-called animal spirits are directly irrelevant to determination of RTP, they may be relevant in the generation of shocks on the expected c^*.

A4 Frequent RTP shocks

A4.1 Difficulty in knowing RTP RH

To estimate the parameter values of equation (A16) in the structural model of RTP RH, it is necessary to obtain a sufficiently large amount of data on the value of RTP RH. To obtain these data, a household must know the RTPs of all the other households. Although a household knows its own RTP, it has almost no information about the RTPs of all the other households much less time-series data on each household's RTP. Because of the lack of available data, a household cannot estimate the parameter values in equation (A16) in the structural model of RTP RH even if it knows the functional forms of equations in the structural model.

We can easily generate data on aggregate consumption, investment, production, inflation, trade, and other factors at a relatively low cost, but we cannot directly observe the value of RTP RH. Nonetheless, many estimates of RTP have been reported, but they are not based on a structural model of RTP. Most are the results of experimental studies or indirect estimates based on other models (e.g., Ramsey growth models) on the assumption that RTP is constant. Experiments can give us some information on the RTPs of test subjects, but we should not naively use these estimates as the RTP RH in the calculation of the future path of economy because they vary widely according to the experimental environments. Furthermore, most of the indirect estimates were calculated on the assumption that RTP is constant, which as discussed previously, is most likely not the case. The basic problem is that no credible estimation method of RTP RH has been established.

A4.2 Expectations based on beliefs

The lack of observable data on RTP RH will significantly hinder households from generating rational expectations of the future path of economy. How do households rationally expect their future streams of consumption and production and calculate their optimal paths without information on RTP RH, which is indispensable as the discount factor? The historical mean of RTP RH estimated by long-term data is not consistent with a rational expectation of the future stream because RTP is not constant. Without a reliable method for estimating the parameters of the structural model, it is impossible for households to generate rational expectations of the future path of the economy.

An alternative way of estimating expected values of RTP RH is needed, but even if an alternative method is utilized, households still have to behave as rationally as possible even in an environment of significantly incomplete information. In this situation, household may have to use the concept of bounded rationality to make decisions. It is possible that the only alternative for a household is to use its "belief" about the RTP RH. The use of a belief does not mean that households deviate from rationality; rather, it is the most rational behavior they can use in an environment where insufficient information is available.

Such a belief is defined in this paper as the range of values of RTP RH within which a household believes that the true RTP RH exists. Households utilize the belief in place of equation (A16). More specifically, suppose that household i ($i \in N$) believes that the RTP RH in the future is situated in the range λ_i, where the subjective probability density at any point on λ_i is identical (i.e., its distribution shape is uniform). Because households have no information about the shape of the distribution, they assume that it is uniform. This supposition means that household i believes that λ_i is stationary. Let $\overline{\lambda}_i$ be the mean of λ_i. Suppose that household i calculates its optimal future path on the belief that the mean of future values of RTP RH is $\overline{\lambda}_i$. By equation (A15), W can be calculated based on $\overline{\lambda}_i$, and the expected future path of economy can be calculated.

Households can equally access all relevant information. Therefore, if the belief of a household is very different from those of the majority, the household will soon perceive that its belief is different, through observing the behavior of majority. The household will change its belief to the almost same as those of the majority because otherwise it cannot achieve optimality as expected on the assumption that sustainable heterogeneity is achieved. Hence, it is likely that households' beliefs become similar, and thereby, it is assumed for simplicity that households' beliefs are identical.

Note that households do not cooperatively and collectively expect the future path of economy (i.e., the representative household's future path), but each household independently and individually generates its own expectations based on its belief in RTP RH. The household thereby creates its own expected future path considering the expected representative household's future path. The aggregates are the sum of all household's independent and individual activities, but if sustainable heterogeneity is achieved, the aggregates appear to be the same as the results of the representative household's activities.

A4.3 Refining beliefs

A household knows that its expectation is based on its beliefs and not the structural model. Therefore, it will always want to refine the belief, that is, raise the probability that the belief is the correct value, by exploiting all currently available relevant information. Let a set of currently available economic indicators be $\boldsymbol{I}_t$ (e.g., the observed data on consumption, production, inventory, etc.). These data may provide some useful information on the past RTP

RH, and a household may refine its belief based on this information. These data and equation (A15) can be used to generate estimates of past values of RTP RH. However, $\boldsymbol{I}_t$ includes noise, and data in $\boldsymbol{I}_t$ will usually be somewhat inconsistent between the elements of $\boldsymbol{I}_t$. In addition, because equation (A15) indicates the steady state values that are achieved after a long-period transition, the short-term past data included in $\boldsymbol{I}_t$ are basically insufficient to obtain a credible estimate. Therefore, the estimate of the past values of RTP RH based on $\boldsymbol{I}_t$ and equation (A15) will usually have a large confidence interval. Let $\overline{\mu}_I$ be the estimated past RTP RH and μ_I be its confidence interval of, for example, 95%. Because households can equally access all relevant information, assume for simplicity that μ_I and $\overline{\mu}_I$ are identical for all households.

Although a household knows that $\overline{\mu}_I$ is not a credible estimate, has a large confidence interval, and is merely an estimate (usually a point estimate) of a past value, it will strive to utilize the information derived from $\overline{\mu}_I$ to refine its beliefs in the future value of RTP RH. Usually $\overline{\mu}_I$ will not be equal to $\overline{\lambda}_i$, but the ranges of λ_i and μ_I may partly overlap. Household i may utilize the information from this partial overlap to refine its belief (i.e., information of how λ_i is different from μ_I). $\overline{\mu}_I \neq \overline{\lambda}_i$ indicates that the belief $\overline{\lambda}_i$ is wrong, $\overline{\mu}_I$ is wrong, both are wrong, or both are right if the true past RTP RH is $\overline{\mu}_I$ but the true future RTP RH is $\overline{\lambda}_i$. The belief $\overline{\lambda}_i$ may be wrong because the RTP RH will change in the near future, and $\overline{\mu}_I$ may be wrong because the RTP RH changed during the period in which the data were obtained. In addition, a household knows that μ_I is the result of all households' activities based on their beliefs, not on the true value of RTP RH. These uncertainties arise because households cannot know the parameters of the structural model. Without using the structural model, household i cannot judge whether $\overline{\lambda}_i$ is wrong, $\overline{\mu}_I$ is wrong, both are wrong, or both are right. As a result, household i will not easily adjust its belief from $\overline{\lambda}_i$ to $\overline{\mu}_I$.

However, it is still likely that information about the difference between λ_i and μ_I can be used to refine the belief. To extract the useful information, the following rules may be used:

Rule 1: if $\overline{\mu}_I$ is included in λ_i, the belief is not adjusted; otherwise, the belief is adjusted from $\overline{\lambda}_i$ to $\overline{\mu}_I$.
Rule 2: if $\overline{\lambda}_i$ is included in μ_I, the belief is not adjusted; otherwise, the belief is adjusted from $\overline{\lambda}_i$ to $\overline{\mu}_I$.
Rule 3: if λ_i and μ_I overlap at or above a specified ratio, the belief is not adjusted; otherwise, the belief is adjusted from $\overline{\lambda}_i$ to $\overline{\mu}_I$.

The above rules may be seen as a type of adaptive expectation because μ_I indicates the past RTP RH. However, in the situation where the parameters of the structural model of the RTP RH are unknown, it may be seen as rational to utilize the information contained in μ_I by adopting one of these rules.

A4.4 Changing beliefs

However, it does not seem likely that a household will refine its belief following one of the rules shown above because the rules are basically backward looking and will not be adopted as a tool for refining the belief if a household is convinced that the RTP RH is temporally variable. The belief will only be changed if forward-looking information is available, that is, when a household becomes aware of information about the future RTP RH in μ_I. For example, the difference between λ_i and μ_I may reflect an unexpected and large positive technology shock that occurred after the formation of belief λ_i. Because the effects of the technology shock will persist for long periods in the future, household i will most likely change its belief. In this case, a household will not simply refine its belief from $\overline{\lambda}_i$ to $\overline{\mu}_I$; it will change to another value that is formed as an entirely new belief.

Whether a household changes its belief or not, therefore, will depend not simply on μ_I but on the information the household can extract from μ_I about the future path of the economy. Hence, in some cases, a household will change its belief when new values of μ_I are obtained, but in other cases, it will not, depending on how the household interprets the information contained in μ_I.

A4.5 Heuristics

When a household interprets μ_I, it may also use heuristic methods, for example, a simplified linear reduced form model of RTP RH. Studies of the use of heuristics and bounded rationality in this context would be useful for better understanding the interpretation mechanism of μ_I. There are many possible simplified linear reduced form models of RH's RTP that could be used as heuristic methods although most of them may be ad hoc. Even though such reduced form models are far less credible than a structural model, they may be utilized as a heuristic method of interpreting μ_I by households. Although these types of models may often result in misleading conclusions, they may sometimes provide useful information. For example, if a linear correlation between RTP RH and a financial indicator exists, even if it is weak or temporary, changes in the financial indicator may contain useful information about changes in the RTP RH. Therefore, if a household believes that this correlation exists, it will use this information to interpret μ_I.

A4.6 Frequent RTP shocks

Households must have expected values of RTP RH for sustainable heterogeneity, but as previously discussed, the expectations are not based on the structural model but rather on a belief that is not guaranteed to generate the correct value. In addition, the belief can be influenced by heuristic considerations. These features indicate that the expected values of RTP RH will fluctuate more frequently than the intrinsic RTP RH.

Households' expectations of RTP RH will change when the intrinsic RTP RH shifts, for example, when new information about shocks on the factors that determine equation (A15) becomes available. For a given θ, $E[u(c^*)]$ changes if the expectation of future productivity changes. Productivity at the macro level will be influenced by scientific technology, financial technology, social infrastructure, and other factors. If expectations about these factors in the future changes, the expected future productivity and $E[u(c^*)]$ will also change. In addition, even if intrinsic RTP RH does not change, the expected RTP RH will change if a household's belief is altered because of new information contained in μ_I. Hence, the expected RTP RH can change independently of intrinsic changes in RTP RH. Therefore, even if intrinsic changes in RTP RH occur infrequently, changes in the expected RTP RH may occur more frequently.

A household's expected RTP RH can potentially change every time new information on μ_I becomes available if it contains the information that makes beliefs change. Information concerning factors that affect the expected RTP RH will become available frequently, and at least some of the information may be both very important and unexpected. In addition, there will be many disturbances in the fundamental factors that affect equation (A15), and many of these disturbances will also cause μ_I to change. As discussed previously, a household may interpret these changes in μ_I as a change in the true RTP RH. Therefore, it is likely that households' expected RTP RH change more frequently than the intrinsic RTP RH, and thereby, that time preference shocks also occur more frequently than previously thought.

Even a small piece of additional information about the belief can significantly change the path of the economy. For example, if many households believe a rumor (whether it is true or not) related to information about the interpretation of μ_I and respond similarly to it, their expectations will be changed in the same direction by the rumor. If all households respond similarly to an untrue rumor and change their expectations equally to an untrue value, the economy will proceed based on the incorrect expectation of RTP RH. The $\bar{\mu}_I$ that is observed a few periods later will follow these wrongly expected values of RTP RH. Upon obtaining new data of $\bar{\mu}_I$ that are consistent with these wrongly expected values, households will judge that their (incorrect) changes were in fact correct. As a result, the incorrect expectations become self-fulfilling. This spurious situation may reach an impasse at some point in the future because the expectations are based not on a structural model but on the (incorrect) beliefs. Households will not anticipate the impasse until the economy reaches it because they believe that the wrongly expected RTP RH (i.e., the currently held belief) is true.

APPENDIX B

The NASH Equilibrium of a Pareto Inefficient Path

B1 Model with non-cooperative households [4]

B1.1 The shock

The model describes the utility maximization of households after an upward time preference shock. This shock was chosen because it is one of the few shocks that result in a Nash equilibrium of a Pareto inefficient path. Another important reason for selecting an upward time preference shock is that it shifts the steady state to lower levels of production and consumption than before the shock, which is consistent with the phenomena actually observed in a recession.

Although the rate of time preference is a deep parameter, it has not been regarded as a source of shocks for economic fluctuations, possibly because the rate of time preference is thought to be constant and not to shift suddenly. There is also a practical reason, however. Models with a permanently constant rate of time preference exhibit excellent tractability (see Samuelson, 1937). However, the rate of time preference has been naturally assumed and actually observed to be time-variable. The concept of a time-varying rate of time preference has a long history (e.g., Böhm-Bawerk, 1889; Fisher, 1930). More recently, Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant rates of time preference by nature and that economic and social factors affect the formation of time preference rates. Their arguments indicate that many incidents can affect and change the rate of time preference throughout a person's life. For example, Parkin (1988) examined business cycles in the United States, explicitly considering the time-variability of the time preference rate, and showed that the rate of time preference was as volatile as technology and leisure preference.

B1.2 Households

Households are not intrinsically cooperative. Except in a strict communist economy, households do not coordinate themselves to behave as a single entity when consuming goods and services. The model in this paper assumes non-cooperative, identical, and infinitely long living households and that the number of households is sufficiently large. Each of them equally maximizes the expected utility

$$E_0 \int_0^\infty \exp(-\theta t) u(c_t) dt \quad ,$$

subject to

$$\frac{dk_t}{dt} = f(A, k_t) - \delta k_t - c_t \quad ,$$

where y_t, c_t, and k_t are production, consumption, and capital per capita in period t, respectively; A is technology and constant; u is the utility function; $y_t = f(A, k_t)$ is the production function; $\theta(>0)$ is the rate of time preference; δ is the rate of depreciation; and E_0 is the expectations operator conditioned on the agents' period 0 information set. y_t, c_t, and k_t are monotonically continuous and differentiable in t, and u and f are monotonically continuous functions of c_t and k_t, respectively. All households initially have an identical amount of financial

[4] The model in Section B1 is based on the model by Harashima (2012). See also Harashima (2004, 2013b).

assets equal to k_t, and all households gain the identical amount of income $y_t = f(A, k_t)$ in each period. It is assumed that $\frac{du(c_t)}{dc_t} > 0$ and $\frac{d^2u(c_t)}{dc_t^2} < 0$; thus, households are risk averse. For simplicity, the utility function is specified to be the constant relative risk aversion utility function

$$u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma} \quad \text{if} \quad \gamma \neq 1$$
$$u(c_t) = \ln(c_t) \quad \text{if} \quad \gamma = 1 \quad ,$$

where γ is a constant and $0 < \gamma < \infty$. In addition, $\frac{\partial f(A, k_t)}{\partial k_t} > 0$ and $\frac{\partial^2 f(k_t)}{\partial k_t^2} < 0$. Both technology ($A$) and labor supply are assumed to be constant.

The effects of an upward shift in time preference are shown in Figure B1. Suppose first that the economy is at steady state before the shock. After the upward time preference shock, the vertical line $\frac{dc_t}{dt} = 0$ moves to the left (from the solid vertical line to the dashed vertical line in Fig. 1). To keep Pareto efficiency, consumption needs to jump immediately from the steady state before the shock (the prior steady state) to point Z. After the jump, consumption proceeds on the Pareto efficient saddle path after the shock (the posterior Pareto efficient saddle path) from point Z to the lower steady state after the shock (the posterior steady state). Nevertheless, this discontinuous jump to Z may be uncomfortable for risk-averse households that wish to smooth consumption and not to experience substantial fluctuations. Households may instead take a shortcut and, for example, proceed on a path on which consumption is reduced continuously from the prior steady state to the posterior steady state (the bold dashed line in Fig. 1), but this shortcut is not Pareto efficient.

Choosing a Pareto inefficient consumption path must be consistent with each household's maximization of its expected utility. To examine the possibility of the rational choice of a Pareto inefficient path, the expected utilities between the two options need be compared. For this comparison, I assume that there are two options for each non-cooperative household with regard to consumption just after an upward shift in time preference. The first is a jump option, *J*, in which a household's consumption jumps to Z and then proceeds on the posterior Pareto efficient saddle path to the posterior steady state. The second is a non-jump option, *NJ*, in which a household's consumption does not jump but instead gradually decreases from the prior steady state to the posterior steady state, as shown by the bold dashed line in Figure B1. The household that chooses the *NJ* option reaches the posterior steady state in period $s(\geq 0)$. The difference in consumption between the two options in each period t is $b_t\ (\geq 0)$. Thus, b_0 indicates the difference between Z and the prior steady state. b_t diminishes continuously and becomes zero in period s. The *NJ* path of consumption (c_t) after the shock is monotonically continuous and differentiable in t and $\frac{dc_t}{dt} < 0$ if $0 \leq t < s$. In addition,

$$\bar{c} < c_t < \hat{c}_t \quad \text{if} \quad 0 \leq t < s$$
$$c_t = \bar{c} \quad \text{if} \quad 0 \leq s \leq t \quad ,$$

where $\hat{c}_t$ is consumption when proceeding on the posterior Pareto efficient saddle path and $\bar{c}$ is consumption in the posterior steady state. Therefore,

$$b_t = \hat{c}_t - c_t > 0 \quad \text{if } 0 \le t < s$$
$$b_t = 0 \quad \text{if } 0 \le s \le t \quad .$$

It is also assumed that, when a household chooses a different option from the one the other households choose, the difference in the accumulation of financial assets resulting from the difference in consumption (b_t) before period s between that household and the other households is reflected in consumption after period s. That is, the difference in the return on financial assets is added to (or subtracted from) the household's consumption in each period after period s. The exact functional form of the addition (or subtraction) is shown in Section B1.4.

B1.3 Firms

Unutilized products (b_t) are eliminated quickly in each period by firms because holding b_t for a long period is a cost to firms. Elimination of b_t is accomplished by discarding the goods or preemptively suspending production, thereby leaving some capital and labor inputs idle. However, in the next period, unutilized products are generated again because the economy is not proceeding on the Pareto efficient saddle path. Unutilized products are therefore successively generated and eliminated. Faced with these unutilized products, firms dispose of the excess capital used to generate b_t. Disposing of the excess capital is rational for firms because the excess capital is an unnecessary cost, but this means that parts of the firms are liquidated, which takes time and thus disposing of the excess capital will also take time. If the economy proceeds on the *NJ* path (that is, if all households choose the *NJ* option), firms dispose of all of the remaining excess capital that generates b_t and adjust their capital to the posterior steady-state level in period s, which also corresponds to households reaching the posterior steady state. Thus, if the economy proceeds on the *NJ* path, capital k_t is

$$\bar{k} < k_t \le \hat{k}_t \quad \text{if } 0 \le t < s$$
$$k_t = \bar{k} \quad \text{if } 0 \le s \le t \quad ,$$

where $\hat{k}_t$ is capital per capita when proceeding on the posterior Pareto efficient saddle path and $\bar{k}$ is capital per capita in the posterior steady state.

The real interest rate i_t is

$$i_t = \frac{\partial f(A, k_t)}{\partial k_t} \quad .$$

Because the real interest rate equals the rate of time preference at steady state, if the economy proceeds on the *NJ* path,

$$\tilde{\theta} \le i_t < \theta \quad \text{if } 0 \le t < s$$
$$i_t = \theta \quad \text{if } 0 \le s \le t \quad ,$$

where $\tilde{\theta}$ is the rate of time preference before the shock and θ is the rate of time preference after the shock. i_t is monotonically continuous and differentiable in t if $0 \le t < s$.

B1.4 Expected utility after the shock

The expected utility of a household after the shock depends on its choice of the *J* or *NJ* path. Let *Jalone* indicate that the household chooses option *J*, but the other households choose option *NJ*; *NJalone* indicate that the household chooses option *NJ*, but the other households choose option *J*; *Jtogether* indicate that all households choose option *J*; and *NJtogether* indicate that all households choose option *NJ*. Let p ($0 \le p \le 1$) be the subjective probability of a household that the other households choose the *J* option (e.g., $p = 0$ indicates that all the other households choose option *NJ*). With p, the expected utility of a household when it chooses option *J* is

$$E_0(J) = pE_0(Jtogether) + (1-p)E_0(Jalone) \ , \tag{B1}$$

and when it chooses option *NJ* is

$$E_0(NJ) = pE_0(NJalone) + (1-p)E_0(NJtogether) \ , \tag{B2}$$

where $E_0(Jalone)$, $E_0(NJalone)$, $E_0(Jtogether)$, and $E_0(NJtogether)$ are the expected utilities of the household when choosing *Jalone*, *NJalone*, *Jtogether*, and *NJtogether*, respectively. Given the properties of *J* and *NJ* shown in Sections B1.2 and B1.3,

$$\begin{aligned} E_0(J) = {} & pE_0\left[\int_0^s \exp(-\theta t)u(c_t + b_t)dt + \int_s^\infty \exp(-\theta t)u(\hat{c}_t)dt\right] \\ & + (1-p)E_0\left[\int_0^s \exp(-\theta t)u(c_t + b_t)dt + \int_s^\infty \exp(-\theta t)u(\bar{c} - \bar{a})dt\right] \ , \end{aligned} \tag{B3}$$

and

$$\begin{aligned} E_0(NJ) = {} & pE_0\left[\int_0^s \exp(-\theta t)u(c_t)dt + \int_s^\infty \exp(-\theta t)u(\hat{c}_t + a_t)dt\right] \\ & + (1-p)E_0\left[\int_0^s \exp(-\theta t)u(c_t)dt + \int_s^\infty \exp(-\theta t)u(\bar{c})dt\right] \ , \end{aligned} \tag{B4}$$

where

$$\bar{a} = \theta \int_0^s b_r \exp \int_r^s i_q \, dq \, dr \ , \tag{B5}$$

and

$$a_t = i_t \int_0^s b_r \exp \int_r^s i_q \, dq \, dr \ , \tag{B6}$$

and the shock occurred in period t = 0. Figure B2 shows the paths of *Jalone* and *NJalone*. Because there is a sufficiently large number of households and the effect of an individual household on the whole economy is negligible, in the case of *Jalone*, the economy almost proceeds on the *NJ* path. Similarly, in the case of *NJalone*, it almost proceeds on the *J* path. If the other households choose the *NJ* option (*Jalone* or *NJtogether*), consumption after s is constant as $\bar{c}$ and capital is adjusted to $\bar{k}$ by firms in period s. In addition, a_t and i_t are constant after s such that a_t equals $\bar{a}$ and i_s equals θ, because the economy is at the posterior steady state. Nevertheless, during the transition period before s, the value of i_t changes from the value of the prior time preference rate to that of the posterior rate. If the other households choose option J (*NJalone* or *Jtogether*), however, consumption after s is $\hat{c}_t$ and capital is not adjusted to $\bar{k}$ by firms in period s and remains at $\hat{k}_t$.

As mentioned in Section B1.2, the difference in the returns on financial assets for the household from the returns for each of the other households is added to (or subtracted from) its consumption in each period after period s. This is described by a_t and $\bar{a}$ in equations (B3) and (B4), and equations (B5) and (B6) indicate that the accumulated difference in financial assets resulting from b_t increases by compound interest between the period r to s. That is, if the household takes the *NJalone* path, it accumulates more financial assets than each of the other J households, and instead of immediately consuming these extra accumulated financial assets after period s, the household consumes the returns on them in every subsequent period. If the household takes the *Jalone* path, however, its consumption after s is $\bar{c} - \bar{a}$, as shown in equation (B3). $\bar{a}$ is subtracted because the income of each household, $y_t = f(A, k_t)$, including the *Jalone* household, decreases equally by b_t. Each of the other *NJ* households decreases consumption by b_t at the same time, which compensates for the decrease in income; thus, its financial assets (i.e., capital per capita; k_t) are kept equal to $\hat{k}_t$. The *Jalone* household, however, does not decrease its consumption, and its financial assets become smaller than those of each of the other *NJ* households, which results in the subtraction of $\bar{a}$ after period s.

B2 Pareto inefficient transition path [5]

B2.1 Rational Pareto inefficient path

B2.1.1 Rational choice of a Pareto inefficient path

Before examining the economy with non-cooperative households, I first show that, if households are cooperative, only option J is chosen as the path after the shock because it gives a higher expected utility than option *NJ*. Because there is no possibility of *Jalone* and *NJalone* if households are cooperative, then $E_0(J) = E_0(Jtogether)$ and $E_0(NJ) = E_0(NJtogether)$. Therefore,

$$\begin{aligned}
&E_0(J) - E_0(NJ) \\
&= E_0\left[\int_0^s \exp(-\theta t) u(c_t + b_t) dt + \int_s^\infty \exp(-\theta t) u(\hat{c}_t) dt\right] - E_0\left[\int_0^s \exp(-\theta t) u(c_t) dt + \int_s^\infty \exp(-\theta t) u(\bar{c}) dt\right] \\
&= E_0\left\{\int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)] dt + \int_s^\infty \exp(-\theta t)[u(\hat{c}_t) - u(\bar{c})] dt\right\} > 0
\end{aligned}$$

because $c_t < c_t + b_t$ and $\bar{c} < \hat{c}_t$.

Next, I examine the economy with non-cooperative households. First, the special case with a utility function with a sufficiently small γ is examined.

Lemma 1: If $\gamma\ (0 < \gamma < \infty)$ is sufficiently small, then $E_0(Jalone) - E_0(NJtogether) > 0$.

Proof: $\lim_{\gamma \to 0}[E_0(Jalone) - E_0(NJtogether)]$

$$\begin{aligned}
&= E_0 \int_0^s \exp(-\theta t) \lim_{\gamma \to 0}[u(c_t + b_t) - u(c_t)] dt + E_0 \int_s^\infty \exp(-\theta t) \lim_{\gamma \to 0}[u(\bar{c} - \bar{a}) - u(\bar{c})] dt \\
&= E_0 \int_0^s \exp(-\theta t) b_t\, dt - E_0 \int_s^\infty \exp(-\theta t) \bar{a}\, dt
\end{aligned}$$

[5] The idea of a rationally chosen Pareto inefficient path was originally presented by Harashima (2004).

$$= E_0 \int_0^s \exp(-\theta t) b_t dt - E_0 \theta \left[\int_0^s \left(b_r \exp \int_r^s i_q \, dq \right) dr \right] \int_s^\infty \exp(-\theta t) dt$$
$$= E_0 \int_0^s \exp(-\theta t) b_t dt - E_0 \exp(-\theta s) \int_0^s \left(b_r \exp \int_r^s i_q \, dq \right) dr$$
$$= E_0 \exp(-\theta s) \int_0^s b_t \left\{ \exp[\theta(s-t)] - \exp \int_t^s i_q \, dq \right\} dt \;\; > 0 \,,$$

because, if $0 \le t < s$, then $i_t < \theta$ and $\exp[\theta(s-t)] > \exp \int_t^s i_q \, dq$. Hence, because $\exp[\theta(s-t)]$ $> \exp \int_t^s i_q \, dq$, $E_0(Jalone) - E_0(NJtogether) > 0$ for sufficiently small γ. ■

Second, the opposite special case (i.e., a utility function with a sufficiently large γ) is examined.

Lemma 2: If $\gamma(0 < \gamma < \infty)$ is sufficiently large and if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$, then $E_0(Jalone) -$ $E_0(NJtogether) < 0$.

Proof: Because $0 < b_t$, then

$$\lim_{\gamma \to \infty} \frac{1-\gamma}{\bar{c}^{\,1-\gamma}} [u(c_t + b_t) - u(c_t)] = \lim_{\gamma \to \infty} \left[\left(\frac{c_t + b_t}{\bar{c}} \right)^{1-\gamma} - \left(\frac{c_t}{\bar{c}} \right)^{1-\gamma} \right] = 0$$

for any period $t(< s)$. On the other hand, because $0 < \bar{a}$, then for any period $t(< s)$, if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$,

$$\lim_{\gamma \to \infty} \frac{1-\gamma}{\bar{c}^{\,1-\gamma}} [u(\bar{c} - \bar{a}) - u(\bar{c})] = \lim_{\gamma \to \infty} \left[\left(1 - \frac{\bar{a}}{\bar{c}} \right)^{1-\gamma} - 1 \right] = \infty \;\; .$$

Thus,

$$\begin{aligned} &\lim_{\gamma \to \infty} \frac{1-\gamma}{\bar{c}^{\,1-\gamma}} [E_0(Jalone) - E_0(NJtogether)] \\ &= \lim_{\gamma \to \infty} \frac{1-\gamma}{\bar{c}^{\,1-\gamma}} \int_0^s \exp(-\theta t) \lim_{\gamma \to \infty} [u(c_t + b_t) - u(c_t)] dt \\ &+ \lim_{\gamma \to \infty} \frac{1-\gamma}{\bar{c}^{\,1-\gamma}} \int_s^\infty \exp(-\theta t) \lim_{\gamma \to \infty} [u(\bar{c} - \bar{a}) - u(\bar{c})] dt \\ &= 0 + \infty > 0 \;\; . \end{aligned}$$

Because $\frac{1-\gamma}{\bar{c}^{\,1-\gamma}} < 0$ for any $\gamma(1 < \gamma < \infty)$, then if $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$, $E_0(Jalone) - E_0(NJtogether)$ < 0 for sufficiently large $\gamma(< \infty)$. ■

The condition $0 < \lim_{\gamma \to \infty} \frac{\bar{a}}{\bar{c}} < 1$ indicates that path *NJ* from c_0 to $\bar{c}$ deviates sufficiently from the posterior Pareto efficient saddle path and reaches the posterior steady state $\bar{c}$ not taking much time. Because steady states are irrelevant to the degree of risk aversion (γ), both c_0 and $\bar{c}$ are irrelevant to γ.

By Lemmas 1 and 2, it can be proved that $E_0(Jalone) - E_0(NJtogether) < 0$ is possible.

Lemma 3: If $0 < \lim_{\gamma\to\infty} \frac{\bar{a}}{\bar{c}} < 1$, then there is a $\gamma^*(0 < \gamma^* < \infty)$ such that if $\gamma^* < \gamma < \infty$, $E_0(Jalone) - E_0(NJtogether) < 0$.

Proof: If $\gamma(> 0)$ is sufficiently small, then $E_0(Jalone) - E_0(NJtogether) > 0$ by Lemma 1, and if $\gamma(< \infty)$ is sufficiently large and if $0 < \lim_{\gamma\to\infty} \frac{\bar{a}}{\bar{c}} < 1$, then $E_0(Jalone) - E_0(NJtogether)$ < 0 by Lemma 2. Hence, if $0 < \lim_{\gamma\to\infty} \frac{\bar{a}}{\bar{c}} < 1$, there is a certain $\gamma^*(0 < \gamma^* < \infty)$ such that, if $\gamma^* < \gamma < \infty$, then $E_0(Jalone) - E_0(NJtogether) < 0$. ■

However, $E_0(Jtogether) - E_0(NJalone) > 0$ because both *Jtogether* and *NJalone* indicate that all the other households choose option *J*; thus, the values of i_t and k_t are the same as those when all households proceed on the posterior Pareto efficient saddle path. Faced with these i_t and k_t, deviating alone from the Pareto efficient path (*NJalone*) gives a lower expected utility than *Jtogether* to the *NJ* household. Both *Jalone* and *NJtogether* indicate that all the other households choose option *NJ* and i_t and k_t are not those of the Pareto efficient path. Hence, the sign of $E_0(Jalone) - E_0(NJtogether)$ varies depending on the conditions, as Lemma 3 indicates.

By Lemma 3 and the property $E_0(Jtogether) - E_0(NJalone) > 0$, the possibility of the choice of a Pareto inefficient transition path, that is, $E_0(J) - E_0(NJ) < 0$, is shown.

Proposition 1: If $0 < \lim_{\gamma\to\infty} \frac{\bar{a}}{\bar{c}} < 1$ and $\gamma^* < \gamma < \infty$, then there is a $p^*(0 \le p^* \le 1)$ such that if $p = p^*$, $E_0(J) - E_0(NJ) = 0$, and if $p < p^*$, $E_0(J) - E_0(NJ) < 0$.

Proof: By Lemma 3, if $\gamma^* < \gamma < \infty$, then $E_0(Jalone) - E_0(NJtogether) < 0$ and $E_0(Jtogether)$ $- E_0(NJalone) > 0$. By equations (B1) and (B2),

$$E_0(J) - E_0(NJ) = p[E_0(Jtogether) - E_0(NJalone)] + (1-p)[E_0(Jalone) - E_0(NJtogether)] \ .$$

Thus, if $0 < \lim_{\gamma\to\infty} \frac{\bar{a}}{\bar{c}} < 1$ and $\gamma^* < \gamma < \infty$, $\lim_{p\to 0} [E_0(J) - E_0(NJ)] = E_0(Jalone) - E_0(NJtogether) < 0$ and $\lim_{p\to 1} [E_0(J) - E_0(NJ)] = E_0(Jtogether) - E_0(NJalone) > 0$. Hence, by the intermediate value theorem, there is $p^*(0 \le p^* \le 1)$ such that if $p = p^*$, $E_0(J) - E_0(NJ) = 0$ and if $p < p^*$, $E_0(J) - E_0(NJ) < 0$. ■

Proposition 1 indicates that, if $0 < \lim_{\gamma\to\infty} \frac{\bar{a}}{\bar{c}} < 1$, $\gamma^* < \gamma < \infty$, and $p < p^*$, then the choice of option *NJ* gives the higher expected utility than that of option *J* to a household; that is, a household may make the rational choice of taking a Pareto inefficient transition path. The lemmas and proposition require no friction, so a Pareto inefficient transition path can be chosen even in a frictionless economy. This result is very important because it offers counter-evidence against the conjecture that households never rationally choose a Pareto inefficient transition path in a frictionless economy.

B2.1.2 Conditions for a rational Pareto inefficient path

The proposition requires several conditions. Among them, $\gamma^* < \gamma < \infty$ may appear rather strict. If γ^* is very large, path *NJ* will rarely be chosen. However, if path *NJ* is such that consumption is reduced sharply after the shock, the *NJ* option yields a higher expected utility than the *J* option even though γ is very small. For example, for any $\gamma(0 < \gamma < \infty)$,

$$\lim_{s \to 0} \frac{1}{s}[E_0(Jalone) - E_0(NJtogether)]$$
$$= \lim_{s \to 0} \frac{1}{s} \int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)]dt + \lim_{s \to 0} \frac{1}{s} \int_s^\infty \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\bar{c})]dt$$
$$= u(c_0 + b_0) - u(c_0) - \frac{1}{\theta} \lim_{s \to 0} \frac{u(\bar{c}) - u(\bar{c} - s\theta b_0)}{s} = u(c_0 + b_0) - u(c_0) - b_0 \frac{du(\bar{c})}{d\bar{c}}$$
$$= \frac{(c_0 + b_0)^{1-\gamma} - c_0^{1-\gamma}}{1-\gamma} - b_0 \bar{c}^{-\gamma} = \bar{c}^{-\gamma} \left\{ \bar{c}^{\gamma} \left[\frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] - b_0 \right\} < 0 \quad ,$$

because $\lim_{\gamma \to 1} \bar{c}^{\gamma} \left[\frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] = \bar{c}[\ln(c_0 + b_0) - \ln(c_0)] = \bar{c} \ln\left(1 + \frac{b_0}{c_0}\right) < b_0$ and $\lim_{\gamma \to \infty} \bar{c}^{\gamma} \left[\frac{(c_0 + b_0)^{1-\gamma}}{1-\gamma} - \frac{c_0^{1-\gamma}}{1-\gamma} \right] = \lim_{\gamma \to \infty} \bar{c}^{\gamma} c_0^{1-\gamma} \left[\frac{\left(1 + \frac{b_0}{c_0}\right)^{1-\gamma} - 1}{1-\gamma} \right] = 0$ because $\bar{c} < c_0$. That is, for each combination of path *NJ* and γ, there is $s^*(>0)$ such that, if $s < s^*$, then $E_0(Jalone) - E_0(NJtogether) < 0$.

Consider an example in which path *NJ* is such that b_t is constant and $b_t = \bar{b}$ before s (Figure B3); thus, $E_0 \int_0^s b_t = s\bar{b}$. In this *NJ* path, consumption is reduced more sharply than it is in the case shown in Figure B2. In this case, because $\bar{a} > E_0 \theta \int_0^s b_t = \theta s \bar{b}$, $0 < \gamma$, and $c_s < c_t$ for $t < s$, then $E_0 \int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)]dt < E_0 \int_0^s \exp(-\theta t) dt [u(c_s + \bar{b}) - u(c_s)] =$ $E_0 \frac{1 - \exp(-\theta s)}{\theta} [u(c_s + \bar{b}) - u(c_s)]$, and in addition, $E_0 \int_s^\infty \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\bar{c})]dt =$ $E_0 \int_s^\infty \exp(-\theta t) dt [u(\bar{c} - \bar{a}) - u(\bar{c})] = E_0 \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \bar{a}) - u(\bar{c})] < E_0 \frac{\exp(-\theta s)}{\theta} [u(\bar{c} - \theta s \bar{b}) - u(\bar{c})]$. Hence,

$$E_0(Jalone) - E_0(NJtogether)$$
$$= E_0 \int_0^s \exp(-\theta t)[u(c_t + b_t) - u(c_t)]dt + E_0 \int_s^\infty \exp(-\theta t)[u(\bar{c} - \bar{a}) - u(\bar{c})]dt$$

$$< E_0 \frac{1-\exp(-\theta s)}{\theta}\left[u\left(c_s+\bar{b}\right)-u(c_s)\right]+E_0 \frac{\exp(-\theta s)}{\theta}\left[u\left(\bar{c}-\theta s\bar{b}\right)-u(\bar{c})\right]$$
$$= E_0 \frac{1-\exp(-\theta s)}{\theta}\left\{\left[u\left(c_s+\bar{b}\right)-u(c_s)\right]-\frac{\exp(-\theta s)}{1-\exp(-\theta s)}\left[u(\bar{c})-u\left(\bar{c}-\theta s\bar{b}\right)\right]\right\} .$$

As γ increases, the ratio $\frac{u\left(c_s+\bar{b}\right)-u(c_s)}{u(\bar{c})-u\left(\bar{c}-\theta s\bar{b}\right)}$ decreases; thus, larger values of s can satisfy $E_0(Jalone)-E_0(NJtogether)<0$. For example, suppose that $\bar{c}$ = 10, c_s = 10.2, $\bar{b}$ = 0.3, and θ = 0.05. If $\gamma=1$, then s^* = 1.5 at the minimum, and if $\gamma=5$, then s^* = 6.8 at the minimum. This result implies that, if option *NJ* is such that consumption is reduced relatively sharply after the shock (e.g., $b_t=\bar{b}$) and $p<p^*$, option *NJ* will usually be chosen. Choosing option *NJ* is not a special case observed only if γ is very large, but option *NJ* can normally be chosen when the value of γ is within usually observed values. Conditions for generating a rational Pareto inefficient transition path therefore are not strict. In a recession, consumption usually declines sharply after the shock, which suggests that households have chosen the *NJ* option.

B3 Nash equilibrium

B3.1 A Nash equilibrium consisting of *NJ* strategies

A household strategically determines whether to choose the *J* or *NJ* option, considering other households' choices. All households know that each of them forms expectations about the future values of its utility and makes a decision in the same manner. Since all households are identical, the best response of each household is identical. Suppose that there are $H\,(\in N)$ identical households in the economy where *H* is sufficiently large (as assumed in Section B1). Let $q_\eta\,(0\le q_\eta\le 1)$ be the probability that a household $\eta\,(\in H)$ chooses option *J*. The average utility of the other households almost equals that of all households because *H* is sufficiently large. Hence, the average expected utilities of the other households that choose the *J* and *NJ* options are E_0(*Jtogether*) and E_0(*NJtogether*), respectively. Hence, the payoff matrix of the *H*-dimensional symmetric mixed strategy game can be described as shown in Table B1. Each identical household determines its behavior on the basis of this payoff matrix.

In this mixed strategy game, the strategy profiles

$$(q_1,q_2,\ldots,q_H) = \{(1,1,\ldots,1), (p^*,p^*,\ldots,p^*), (0,0,\ldots,0)\}$$

are Nash equilibria for the following reason. By Proposition 1, the best response of household η is *J* (i.e., q_η = 1) if $p>p^*$, indifferent between *J* and *NJ* (i.e., any $q_\eta\in[0,1]$) if $p=p^*$, and *NJ* (i.e., q_η = 0) if $p<p^*$. Because all households are identical, the best-response correspondence of each household is identical such that q_η = 1 if $p>p^*$, [0,1] if $p=p^*$, and 0 if $p<p^*$ for any household $\eta\in H$. Hence, the mixed strategy profiles (1, 1,…,1), ($p^*,p^*,\ldots,p^*$), and (0,0,…,0) are the intersections of the graph of the best-response correspondences of all households. The Pareto efficient saddle path solution (1,1,…,1) (i.e., *Jtogether*) is a pure strategy Nash equilibrium, but a Pareto inefficient transition path (0,0,…,0) (i.e., *NJtogether*) is also a pure strategy Nash equilibrium. In addition, there is a mixed strategy Nash equilibrium ($p^*,p^*,\ldots,p^*$).

B3.2 Selection of equilibrium

Determining which Nash equilibrium, either *NJtogether* (0,0,…,0) or *Jtogether* (1,1,…,1), is dominant requires refinements of the Nash equilibrium, which necessitate additional criteria. Here, if households have a risk-averse preference in the sense that they avert the worst scenario when its probability is not known, households suppose a very low p and select the *NJtogether* (0,0,…,0) equilibrium. Because

$$
\begin{aligned}
& E_0(Jalone) - E_0(NJalone) \\
&= E_0\left\{\int_0^s \exp(-\theta t)\left[u(c_t + b_t) - u(c_t)\right]dt + \int_s^\infty \exp(-\theta t)\left[u(\bar{c} - \bar{a}) - u(\hat{c}_t + a_t)\right]dt\right\} \\
&< E_0\left\{\int_0^s \exp(-\theta t)\left[u(c_t + b_t) - u(c_t)\right]dt + \int_s^\infty \exp(-\theta t)\left[u(\bar{c} - \bar{a}) - u(\bar{c})\right]dt\right\} \\
&= E_0\,(Jalone) - E_0\,(NJtogether) < 0 \ ,
\end{aligned}
\tag{B7}
$$

by Lemma 3, *Jalone* is the worst choice in terms of the amount of payoff, followed by *NJtogether*, and *NJalone*, and *Jtogether* is the best. The outcomes of choosing option *J* are more dispersed than those of option *NJ*. If households have a risk-averse preference in the above-mentioned sense and avert the worst scenario when they have no information on its probability, a household will prefer the less dispersed option (*NJ*), fearing the worst situation that the household alone substantially increases consumption while the other households substantially decrease consumption after the shock. This behavior is rational because it is consistent with preferences. Because all households are identical and know inequality (B7), all households will equally suppose that they all prefer the less dispersed *NJ* option; therefore, all of them will suppose a very low p, particularly $p = 0$, and select the *NJtogether* (0,0,…,0) equilibrium, which is the Nash equilibrium of a Pareto inefficient path. Thereby, unlike most multiple equilibria models, the problem of indeterminacy does not arise, and "animal spirits" (e.g., pessimism or optimism) are unnecessary to explain the selection.

B4 Amplified generation of unutilized resources

A Nash equilibrium of a Pareto inefficient path successively generates unutilized products (b_t). They are left unused, discarded, or preemptively not produced during the path. Unused or discarded goods and services indicate a decline in sales and an increase in inventory for firms. Preemptively suspended production results in an increase in unemployment and idle capital. As a result, profits decline and some parts of firms need to be liquidated, which is unnecessary if the economy proceeds on the *J* path (i.e., the posterior Pareto efficient path). If the liquidation is implemented immediately after the shock, b_t will no longer be generated, but such a liquidation would generate a tremendous shock. The process of the liquidation, however, will take time because of various frictions, and excess capital that generates b_t will remain for a long period. During the period when capital is not reduced to the posterior steady-state level, unutilized products are successively generated. In a period, b_t is generated and eliminated, but in the next period, another, new, b_t is generated and eliminated. This cycle is repeated in every period throughout the transition path, and it implies that demand is lower than supply in every period. This phenomenon may be interpreted as a general glut or a persisting disequilibrium by some definitions of equilibrium.

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Figure A1: Endogenous time preference

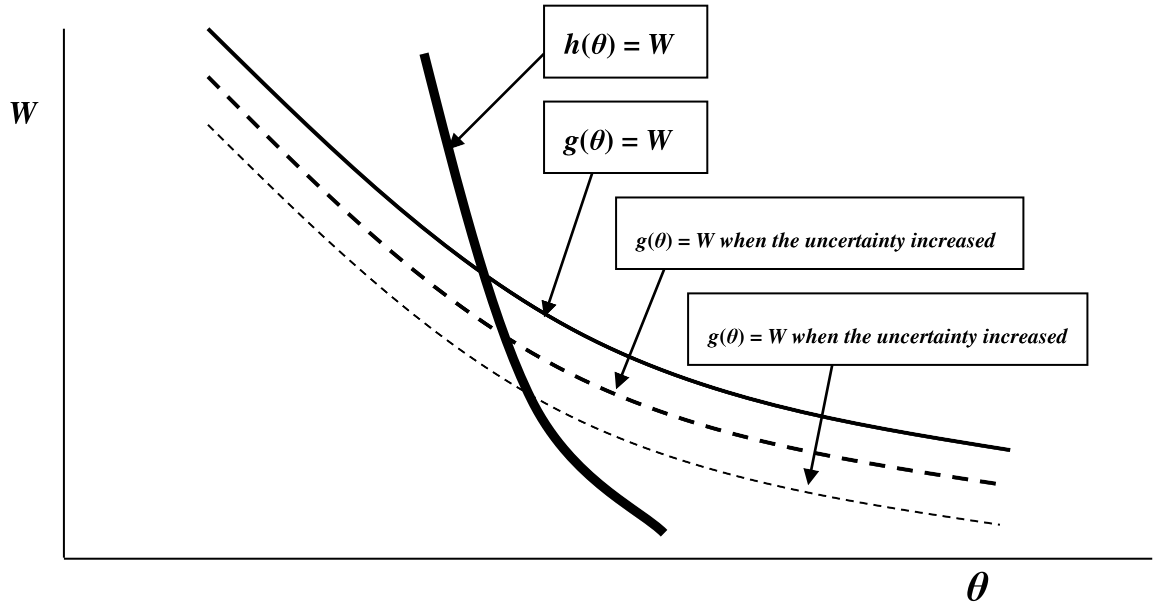

Figure A2: Permanently constant time preference

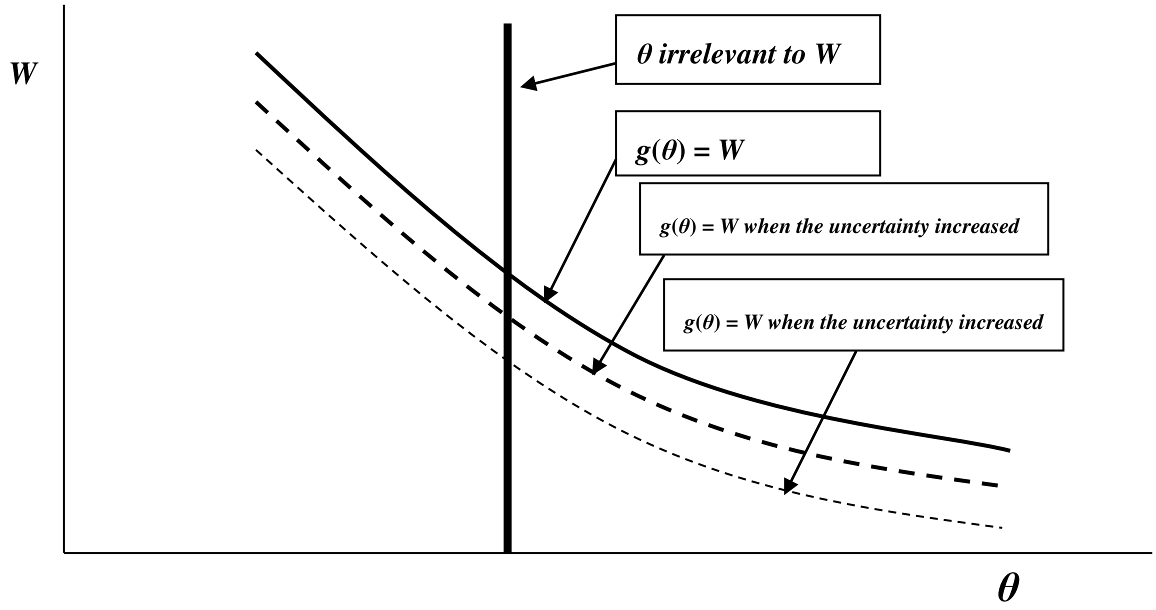

Figure B1: A time preference shock

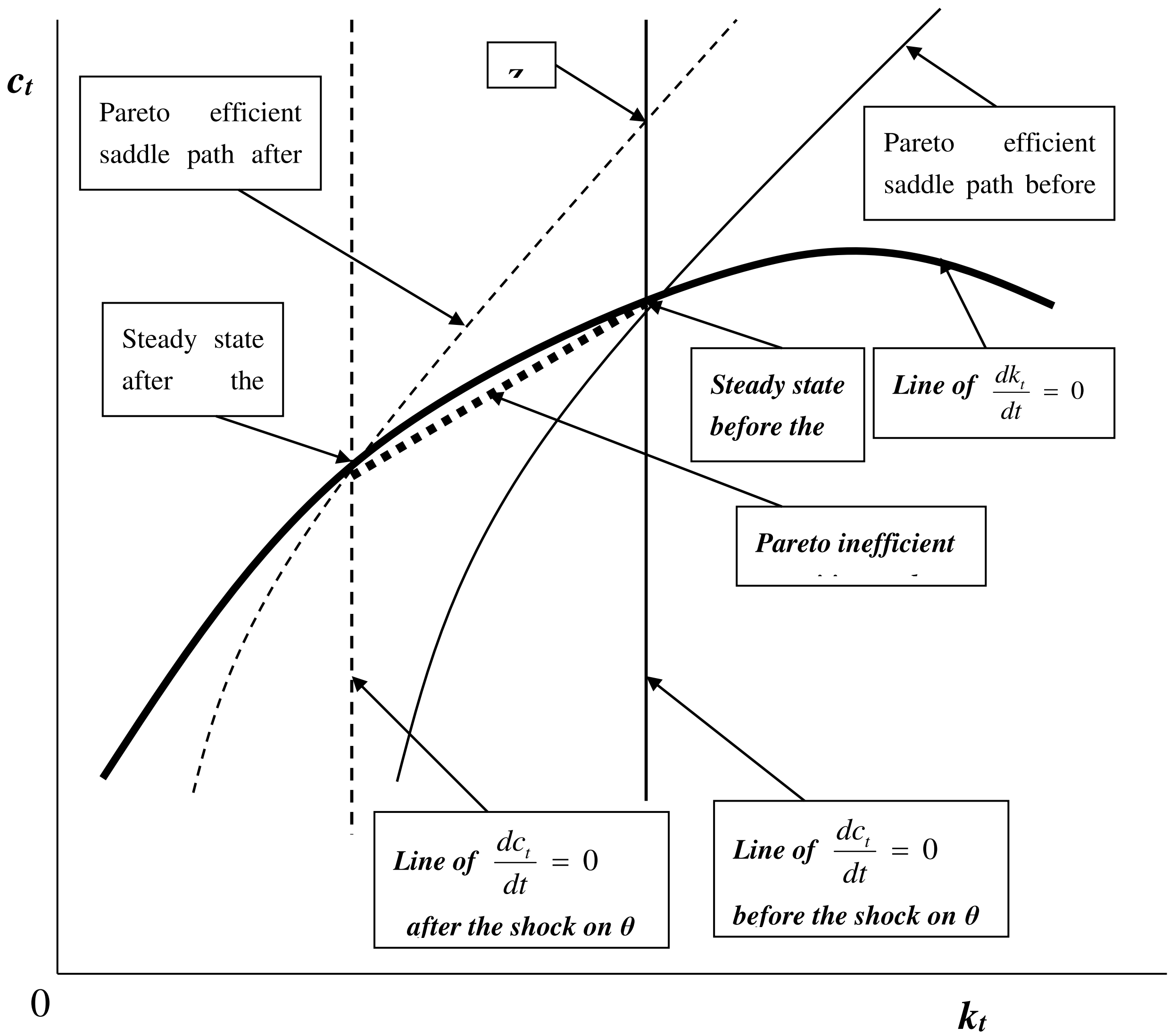

Figure B2: The paths of *Jalone* and *NJalone*

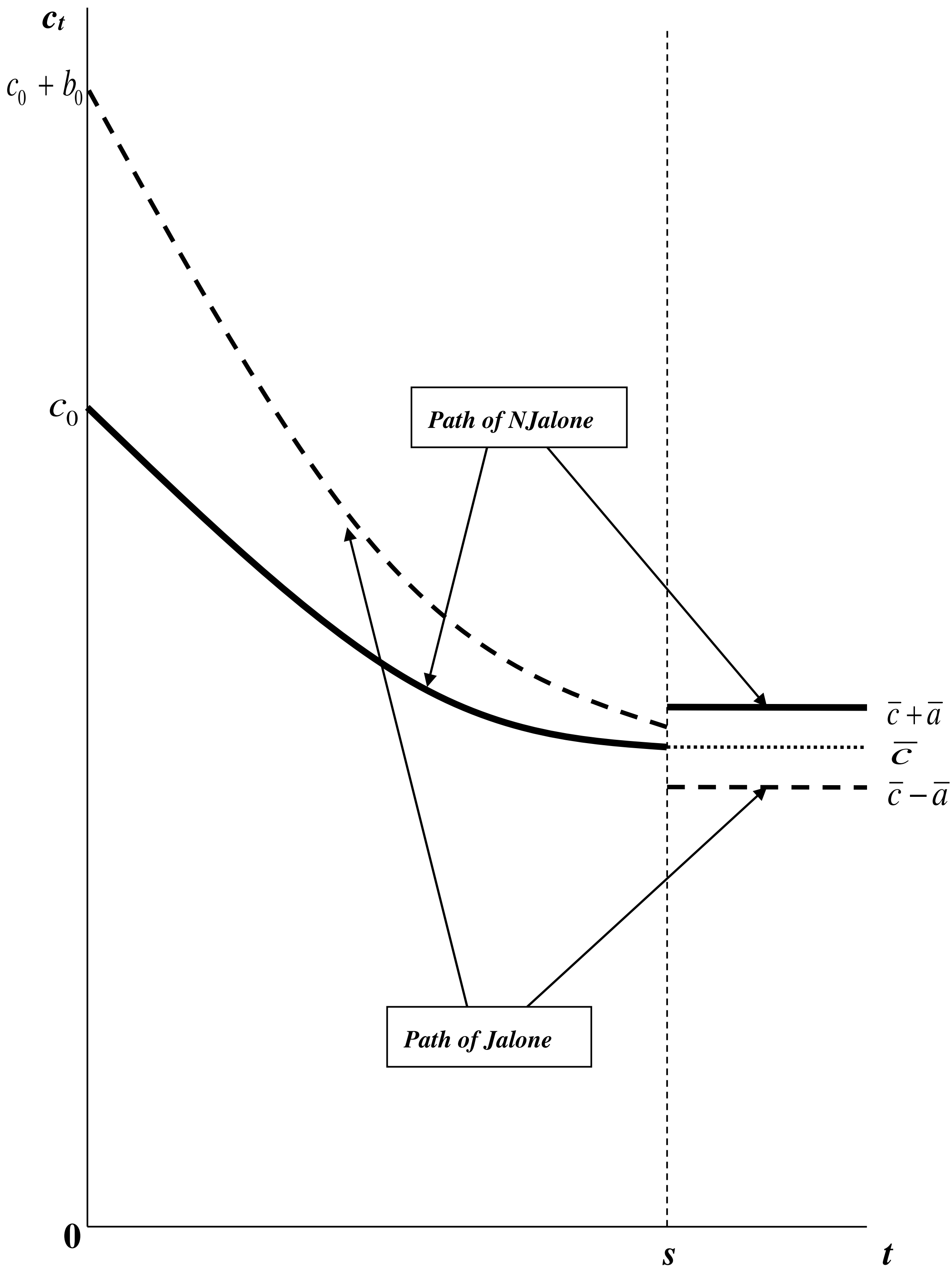

Figure B3: A Pareto inefficient transition path

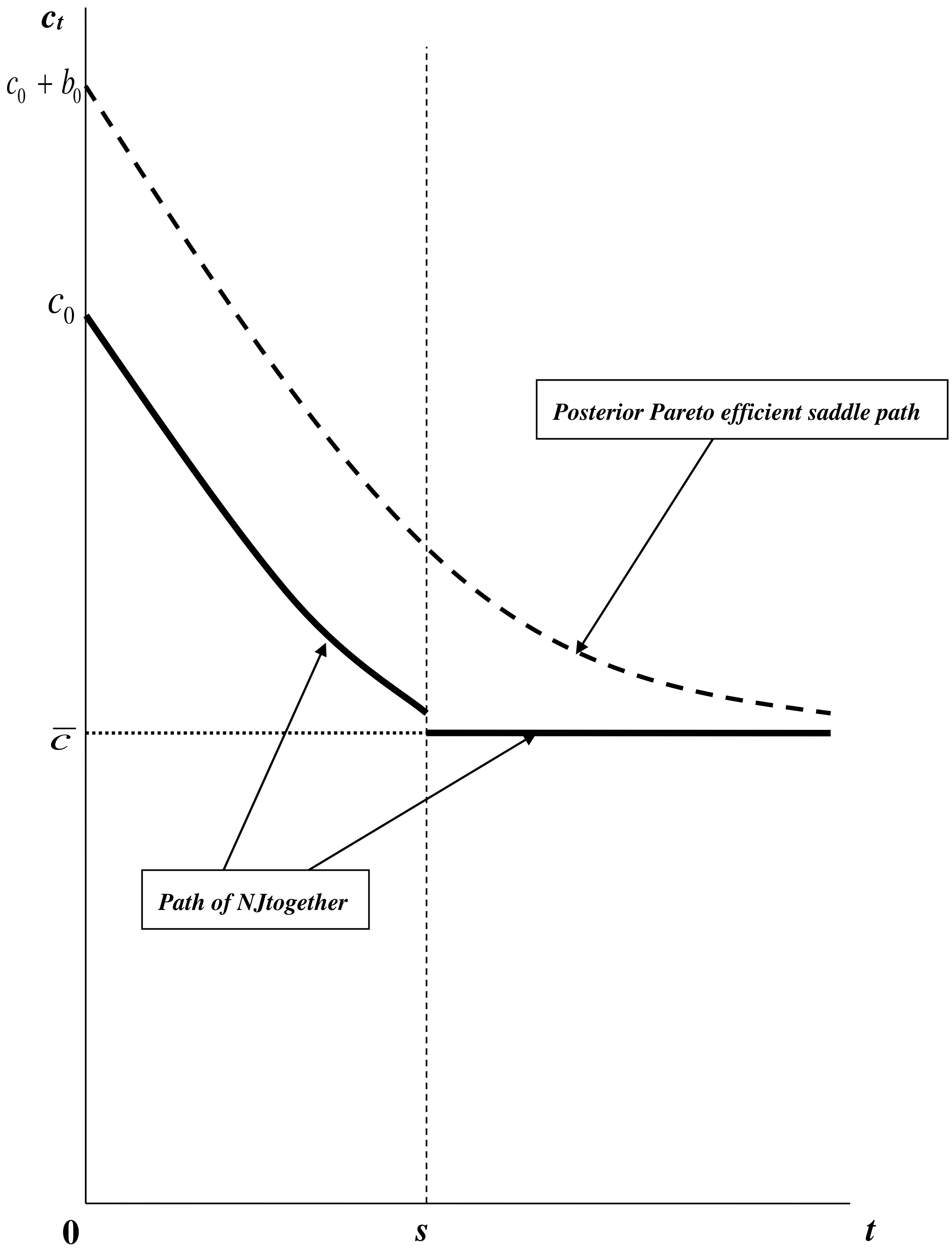

Table B1 The payoff matrix

		Any other household	
		J	*NJ*
Household A	*J*	E_0(*Jtogether*), E_0(*Jtogether*)	E_0(*Jalone*), E_0(*NJtogether*)
	NJ	E_0(*NJalone*), E_0(*Jtogether*)	E_0(*NJtogether*), E_0(*NJtogether*)