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Olajide, Victor

University of Benin, Ugbowo

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By

OLAJIDE, Victor Chukwujike

Graduate Student, University of Benin, Ugbowo, Edo State

Email: olajidechukwujike@gmail.com; Phone No.: 08069170234

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Abstract

Regions in a federal system, as in the case Nigeria, tend to have economic characteristics that diverge from each other and while monetary and fiscal policies appear to be strongly nationally oriented their importance and relevance appear to be regionally determined. However, also important, and more important for this study, is the view that regional macroeconomic policies – that is, the fiscal policies of the state or regional governments and the implications of national monetary policies for regional economies – do not only impact on the source region but can also transcend the borders of the source region to other neighboring regions and cause adjustments or distortions that may have important macroeconomic implications for the regions in a federating system.

Towards this end, this article investigates, using the aggregate supply, aggregate demand and balance of payment (AS/AD/BP) framework with special assumptions that capture the characteristics of the regions in a federating system typical of Nigeria, the macroeconomic interconnectedness of regions in a federating unit and considers the macroeconomic spillover effects of the fiscal policies of regional governments as well as the regional implications of national monetary policies.

1. BACKGROUND

Federal systems present a picture of regional economies and a national economy running concurrently with regional fiscal and national monetary policies being enacted and their impacts playing in out. Such a picture is expected to present an interesting mix of implications of regional fiscal and national monetary policies which often may be conflicting or in agreement with respect to the response of regional macroeconomic variables. This is undoubtedly the situation in Nigeria and more interesting is the observed fact that some regions in the federating system are more developed that the others – as reflected in the concentration of most of the financial institutions in Lagos state than in some other state like Taraba or Borno – and this sophistication in developed regions could be reflected in the consumers and investors having access to loan facilities and having a hoard of real money balances. In addition, it is also observed that some of these relatively more developed regions in federal countries like Nigeria are relatively more open to international trade whilst less developed regions engage in interregional trade with the relative more open regional economies. Also worthy of note is the relative strength of labour unions in developed regions in a federal economy unlike in regions that are not as developed.

The duality of federal economies is not a new phenomenon in literature as it is reflected in the concept of ‘first city bias’, as seen in Todaro and Smith (2012, pp325), where lopsided development in the regions of a developing country results from the initial socio-economic and political contact of the colonial powers with these developing countries. However given this phenomenon there is a paucity of studies that have adequately addressed the macroeconomic spillover effects of regional fiscal policies and the regional macroeconomic response to national monetary policies in a dualistic federal economic systemin developing countries like Nigeria, unlike in advanced economies and this is reflected in the literature review.

Towards the end of examine the theoretical ramifications of the effect of regional fiscal and national monetary policies on regions in a federating system, this article avails the Keynesian Aggregate Supply-Aggregate Demand-Balance of Payment (AS-AD-BP) framework and examines a hypothetical case of two regions in a federal economy with each region's fiscal policies and the national monetary policy impact on the regional macroeconomic variables being examined.

This article is composite of four sections. The following section highlights the literature review and this is followed by the section that contains the derivation of the general equilibrium AS-AD-BP model. The last section discusses the implications of the a priori findings of the third section and details of the AS-AD-BP analysis are presented in the appendix to this article.

2. LITERATURE REVIEW

The analysis of the regional impact of fiscal and monetary policies and how they can engenderinter-regional spillovers that could be reflected in the convergence of the regions in the country, a redistribution of resources between regions in a dualistic national economy and response of macroeconomic variables in these regions is evident in relatively recent studies like: Barro and Sala-i-Martin (1990); Obstfeld and Peri (1998); Rey and Montouri (1999); Bouvet (2007) and Porsse et al (2007). However similar studies on developing countries, like Nigeria, are scant as most authors, such as: Omitogun and Ayinla (2007); Appah (2010); Babalola and Aminu (2011); Osuala and Ebieri (2014) have only examined the effect of federal fiscal policies on the aggregate output with no consideration of the differential effect of such policies on the regions of the federation or the effect of regional policies on the regional macroeconomic climate – with the exception of studies like Wantchekon and Asadurian (2002) that investigates the case of transfer dependence and regional disparities in Nigeria. Whilst the limitation imposed by the availability of data could readily be a reason for the paucity of research in this area, the immense benefit of the results of such a study to policy making at the regional level is profoundly relevant for arming regional governments with information that would enable them tailor fiscal policy operations to enable them maximize the social benefits.

3. GENERAL MACROECONOMIC EQUILIBRIUM OF TWO REGIONS IN A FEDERATING SYSTEM

In this section of this article a theoretical examination of the macroeconomic equilibrium of the regions in a federal system is considered using the AS-AD-BP framework. The statement of assumptions underlying the model is presented, with these assumptions based on some of the empirical facts derived from the regional economies in Nigeria and the sequel to the statement of assumptions the mathematical model is presented and the necessary propositions on the role of monetary and fiscal policy are outlined.

3.1 STATEMENT OF ASSUMPTIONS

The macroeconomic model designed in this study is based on certain assumptions that capture the features of a national economy made up of regions. There are two regions, regions 1 and 2, in the hypothetical national economy. The region 2 is assumed to be relatively less sophisticated than the region 1 as the aggregate consumption function is not a function of interest rate or real money balances as in the case of the region 1. The aggregate investment function for the region 1 is augmented with real money balances while the region 2 is not and in both regions government expenditure is assumed to be constant. Still on the Keynesian identity for both regions, region 1 is assumed to be a small open economy that records real and capital flows with respect to the global economy and the region 2. However the region 2 is assumed to be closed to international trade but open to real and capital flows from the region 1.

Labour in the region 1 is assumed to be organized into unions and hence wages in the region 1 are, in addition to other factors, subject to expected prices but in the region 2 this is not the case as expected prices are not factored into the wage fixing decisions. Labour is assumed to be mobile between the regions and hence the employment in each region is assumed to partly growth steadily over time and partly a function of the wage differentials between the two regions.

Interest rates in both regions are directly affected by the monetary policy rate which is itself based on a rule that avails the aggregate quantities of the first difference in the price level and the output level. It must be noted that the aggregate price level is the weighted average of the prices of the regions 1 and 2 as well as the aggregate output. The monetary policy rule is assumed not to discriminate in its fixing of the policy rate in response to the peculiarities of the regions 1 and 2 but rather it fixes a single policy rate based on aggregate quantities of price and output.

3.2 THEORETICAL ANALYSIS

3.2.1 Real Sector Equilibrium in the Regions 1 and 2

Assuming a country with two regions and a central government, a cursory examination of the regions and the central government is conducted. The first region is assumed to be a small open economy and hence its national output Y_{1t} equals the sum of its consumption expenditure C_{1t}, investment expenditure I_{1t} and net exports $X_{1t} - IM_{1t}$ which has components of international and inter-regional trade and hence:

$$Y_{1t} = C_{1t} + I_{1t} + G_{1t} + X_{1t} - IM_{1t} \ldots 3.1$$

Consumption C_{1t} in the first region is assumed to be influenced by current income Y_{1t}, interest rate r_{1t}, as financial institutions are assumed to be relatively functional, and real cash balances M_{1t}/P_{1t}as agents in the first region are assumed to hold real cash balances and hence the consumption function is given as:

$$C_{1t} = C_1\left(Y_{1dt}, r_{1t}, \frac{M_{1t}}{P_{1t}}\right) \text{where } C_{1yd} > 0; C_{1r} < 0, C_{1m} > 0 \ldots 3.2$$

Disposable income Y_{1dt} is given as:

$$Y_{1dt} = Y_{1t} - T_1(Y_{1t})\text{where } 0 < T_y = t_1 < 1 \ldots 3.3$$

And hence the consumption function is given as:

$$C_{1t} = C_1\left(Y_{1t} - T_1(Y_{1t}), r_{1t}, \frac{M_{1t}}{P_{1t}}\right) \ldots 3.4$$

Investment I_{1t} in the first region is partly dependent on the level of income Y_{1t}, the interest rate r_{1t} and real money balances M_{1t}/P_{1t} and hence the investment function is given as:

$$I_{1t} = I_1\left(Y_{1t}, r_{1t}, \frac{M_{1t}}{P_{1t}}\right) \text{where } I_{1y} > 0, I_{1r} < 0, I_{1m} > 0, I_{1p} < 0 \ldots 3.5$$

Government consumption expenditure in the region 1, G_{1t} is assumed to be exogenously given as $\bar{G}_1$and hence:

$G_{1t} = \bar{G}_1 \ldots 3.6$

Exports X_{1t} in the first region is assumed to be a function of real exchange rate $ex_t.P_{ft}/P_{1t}$ while imports IM_{1t} is assumed to be a function of both real exchange rate $ex_t P_{ft}/P_{1t}$ and income Y_{1t} and hence:

$$X_t = X\left(ex_t, \frac{P_f}{P_{1t}}, \frac{P_{2t}}{P_{1t}}\right) \text{where } X_{ex} > 0, X_{p_{1t}} < 0, X_{p_{2t}} > 0 \ldots 3.7$$

$$IM_t = IM\left(ex_t, \frac{P_f}{P_{1t}}, \frac{P_{2t}}{P_{1t}}, Y_{1t}\right) \text{where } IM_{ex} < 0; IM_{p_{1t}} > 0; IM_{p_{2t}} > 0; IM_y > 0 \ldots 3.8$$

On substituting the equations 3.2, 3.3, 3.4, 3.5, 3.6, 3.7 and 3.8 into the equation 3.1 then the following results:

$$Y_{1t} = C_1\left(Y_{1t} - T_1(Y_{1t}), r_{1t}, \frac{M_{1t}}{P_{1t}}\right) + I_1\left(Y_{1t}, r_{1t}, \frac{M_{1t}}{P_{1t}}\right) + \bar{G}_1 + X\left(ex_t, \frac{P_f}{P_{1t}}, \frac{P_{2t}}{P_{1t}}\right) - IM\left(ex_t, \frac{P_f}{P_{1t}}, \frac{P_{2t}}{P_{1t}}, Y_{1t}\right) \ldots 3.9$$

In the region 2, households finance consumption from current disposable income Y_{2dt} alone as households do not have access to lending facilities and do not have wealth holdings, investments I_{2t} are assumed to be dependent on income Y_{2t} and interest rate r_{2t}, while government spending G_{2t} for the region 2 is assumed to be exogenously given as $\bar{G}_2$. In addition it is assumed that the region 2 is not open to international trade, unlike the region 1, but it carries out inter-regional trade with the region 1 such that what would be termed 'net exports' is actually the net trade with the region 1. The aggregate expenditure Y_{2t} is defined in the following manner.

$$Y_{2t} = C_{2t} + I_{2t} + G_{2t} + X_{2t} - IM_{2t} \ldots 3.10$$

In the light of the aforementioned assumptions the consumption, disposable income function and investment functions of the region 2 is given as:

$$C_{2t} = C_2(Y_{dt}) \text{where } C_y > 0 \ldots 3.11$$

$$Y_{2dt} = Y_{2t} - T_2(Y_t) \text{where } T_{2y} > 0 \ldots 3.12$$

$$I_{2t} = I_2(Y_{2t}, r_{2t}) \text{where } I_{2y} > 0, I_{2r} < 0 \ldots 3.13$$

and

$$G_{2t} = \bar{G}_2 \ldots 3.14$$

Net exports for the region 2, defined as net inter-regional trade between the region 1 and the region 2, is given as:

$$X\left(\frac{P_{2t}}{P_{1t}}\right) - IM\left(\frac{P_{2t}}{P_{1t}}, Y_{2t}\right) \text{where } X_{p_2} < 0; X_{p_1} < 0; IM_{p_2} > 0; IM_{p_1} > 0; IM_{y_2} > 0$$

On substituting the values of the equations 3.11, 3.12, 3.13 and 3.14 into the equations 3.10, differentiating totally and re-arranging the terms the following results:

$$Y_{2t} = C_2\big(Y_{2t} - T_2(Y_t)\big) + I_2(Y_{2t}, r_{2t}) + \bar{G}_2 + X\left(\frac{P_{2t}}{P_{1t}}\right) - IM\left(\frac{P_{2t}}{P_{1t}}, Y_{2t}\right) \ldots 3.15$$

3.3.2 Monetary Equilibrium in the Regions 1 and 2

On turning to the monetary equilibrium in the 2 regions, it is assumed that households in the region 1 have both speculative and transactionary motives for holding money while the region 2, much less sophisticated, has households holding money for transactionary purposes and hence the following liquidity preference functions are defined for the regions 1 and 2 respectively:

$L_{1t} = L_1(Y_{1t}, r_{1t})$where $L_{1y} > 0, L_{1r} < 0 \dots 3.16$

$L_{2t} = L_2(Y_{2t})$ where $L_{2y} > 0 \dots 3.17$

and given the supply of money for both regions which are exogenously given: M_{1t}/P_{1t} and M_{2t}/P_{2t} the equilibrium in the money sectors for both regions are given as:

$$\frac{M_{1t}}{P_{1t}} = L_1(Y_{1t}, r_{1t}) \dots 3.18$$

$$\frac{M_{2t}}{P_{2t}} = L_2(Y_{2t}) \dots 3.19$$

3.3.3 Balance of Payment Equilibrium

The region 1 is not only assumed to have trade ties with the region 2 but it also has financial ties and hence with this information the balance of payment identity corresponding to the region 1 is depicted equating the sum of the net exports for region 1, $X_{1t} - IM_{1t}$, capital inflows KA_1 from the foreign sector and capital inflows KA_2 from the region 2 to the reserves RES_t and therefore the following results:

$$X\left(ex_t, \frac{P_f}{P_{1t}}, \frac{P_{2t}}{P_{1t}}\right) - IM\left(ex_t, \frac{P_f}{P_{1t}}, \frac{P_{2t}}{P_{1t}}, Y_{1t}\right) + KA_1(r_{1t} - r_f) + KA_2(r_{1t} - r_{2t})$$
$$= RES_{1t}\text{where } K_{1r_1} > 0, K_{2r_2} < 0, K_{1r_f} < 0 \dots 3.20$$

In the region 2, net exports: $X_{2t} - IM_{2t}$, which is specially defined as the net trade between it and the region 1, is summed with the negative of the capital inflow function of the region 1, $-KA_2$ to obtain the reserves for the region 2 which is given as:

$$X\left(\frac{P_{2t}}{P_{1t}}\right) - IM\left(\frac{P_{2t}}{P_{1t}}, Y_{1t}\right) - KA_2(r_{1t} - r_{2t}) = RES_{2t}\text{where } K_{1r_1} > 0, K_{2r_2} < 0, K_{1r_f} < 0 \dots 3.21$$

3.3.4 Equilibrium Productivity and Employment in the Regions 1 and 2

Given the production functions for the regions 1 and 2, where capital is held constant, signaling that this analysis is in the short run where capital is fixed in both regions:

$$Y_{1t} = Y(N_{1t}, \overline{K}_1) \text{where } Y_{1N_1} > 0, Y_{1N_1N_1} < 0 \ \dots 3.22$$

$$Y_{2t} = Y(N_{2t}, \overline{K}_2) \text{where } Y_{2N_2} > 0, Y_{2N_2N_2} < 0 \ \dots 3.23$$

However, going by the marginal productivity theory, firms would continue to demand for labour until the marginal physical product of labour equals the real wage which is assumed to be given, assuming that there is perfect competition in the labour and goods markets in both regions. Hence the following results:

$$Y_{1N_1} = \frac{W_{1t}}{P_{1t}} \dots 3.24$$

$$Y_{2N_2} = \frac{W_{2t}}{P_{2t}} \dots 3.25$$

On the supply of employment, the real wages W_{1t}/P_{1t} in the region 1 is assumed to be positive function of the supply of employment N_{1t} and expected prices P_{1et} and there is a equi-proportional positive relationship between the real wages W_{1t}/P_{1t} and the ratio of the expected price P_{1et} to the actual price P_{1t}:

$$\frac{W_{1t}}{P_{1t}} = \frac{P_{1et}}{P_{1t}}. W_1(N_{1t}) \text{where } W_{1N_1} > 0 \ \dots 3.26$$

In the region 2, employment is mostly informal and thus the wage bill W_{2t} in this region is assumed to be a function of employment N_{2t} only as labour in the region 2 is not organized into labour unions and do not have the power to revise their real or nominal wages in the event of an inflationary trend, unlike in the region 1 and hence:

$$W_{2t} = W_2(N_{2t}) \text{where } W_{2N_2} > 0 \quad \ldots 3.27$$

At equilibrium it is expected that:

$$W_1 = Y_{1N_1} . \frac{P_{1t}}{P_{1et}} \ldots 3.28$$

$$W_2 = Y_{2N_2} P_{2t} \ldots 3.29$$

In conclusion, of the equilibrium in the productivity and employment sectors of the regions 1 and 2, it is assumed that labour, and hence employment N_{1t}, in the region 1 is partly due to the natural growth rate of the labour force participation in regions 1 which is assumed to be identical to the population growth rate ϕ_{pg1} and partly due to the region 2 to region 1 migration in response to real wage differentials $W_{1t}/P_{1t} - W_{2t}/P_{2t}$ and hence the following results:

$$N_{1t} = e^{\phi_{pg1}t} N_1 \left(\frac{W_{1t}}{P_{1t}} - \frac{W_{2t}}{P_{2t}}\right) \text{where } 0 < \phi_{pg1} < 1, N_{1W_1} > 0, N_{1P_1} < 0, N_{1W_2} < 0, N_{1P_2} > 0 \ldots 3.30$$

In the region 2, a symmetrical effect occurs and the expected relationships in the equations above are reversed the yield the below equation:

$$N_{2t} = e^{\phi_{pg2}t} N_2 \left(\frac{W_{1t}}{P_{1t}} - \frac{W_{2t}}{P_{2t}}\right) \text{where } 0 < \phi_{pg2} < 1, N_{2W_1} < 0, N_{1P_1} > 0, N_{1W_2} > 0, N_{1P_2} < 0 \ldots 3.31$$

3.3.5 Monetary Policy Rule

The monetary authority, which operates at the national level, is assumed to carry out its operations in strict accordance with the objectives of maintaining the internal and external stability and this it does by commitment to the monetary policy rule which stipulates that the monetary authority sets the policy rate r_t equal to the sum of the long run real interest rate r^*, the term in inflation rate: $\tau_p dP_t/P_t$, income term $\tau_y Y_t$

$$r_t = \tau_0 + r^* + \tau_p \left(\frac{dP_t}{P_t}\right) + \tau_y Y_t \ldots 3.32$$

where $\tau_0 = \tau_p - \tau_p d\bar{P}/\bar{P} - \tau_y \bar{Y} > 0;\ \tau_p > 1;\ 0 < \tau_y < 1$

However implicit in the monetary rule of 3.32 is the assumption that the monetary policy rule does not provide for discriminatory impact coefficients for the regions 1 and 2 and hence builds its policy response to aggregate price changes and aggregate income changes. This hence requires that an aggregate condition be designed for the general price level P_t and the national output Y_t and these are defined below:

$$P_t = \phi_p P_{1t} + (1 - \phi_p) P_{2t} \ldots 3.33$$

and

$$Y_t = Y_{1t} + Y_{2t} \ldots 3.34$$

Though monetary policy is assumed to respond to aggregate quantities, and not respond in a discriminating manner to the peculiarities of the regions 1 and 2, the effects of monetary policy is actually discriminating as it is assumed that there are differentials in responses of commercial banks, the sole assumed vehicle of monetary policy, with respect to the policies of the monetary authority. With the financial system in the region 1 more sophisticated that those of the region 2 it is expected that given the trajectory of commercial banks lending in the region 1, monetary policy rate r_t would have a weaker influence ϕ_{r1} on region 1 interest rate r_{1t} and hence liquidity as commercial banks would avail its reserves and reallocate liquidity from the region 1 to act a buffer and by so doing raise the interest rate of the region 2 by the term $\phi_{r21}r_{1t}$. Unlike the region 1, the region 2 is open to the greater influence of monetary policy, denoted by the term $\phi_{r2}r_t$, through the commercial banks due to preferential treatment given by the commercial banks to the region 1. The region 1 interest rate equation is augmented with the parity condition $(ex_{et}/ex)(1+r_f)$ for liquidity flows from the foreign sector. Hence in the light of the aforementioned the following results:

$$r_{1t} = \phi_{10} + \phi_{r1}r_t + r_f - ex_t \ldots 3.35$$

$$r_{2t} = \phi_{20} + \phi_{r2}r_t + \phi_{r21}r_{1t} \ldots 3.36$$

where$\phi_{r2} > \phi_{r1}, \phi_{r1} > 0, \phi_{r2} > 0, \phi_{r21} > 0; \phi_{10} < 0; \phi_{20} < 0$

The main equations that shall form the basis of tracing the impact of regional governments and national monetary policies are:

$$Y_{1t} = C_1\left(Y_{1t} - T_1(Y_{1t}), r_{1t}, \frac{M_{1t}}{P_{1t}}\right) + I_1\left(Y_{1t}, r_{1t}, \frac{M_{1t}}{P_{1t}}\right) + \bar{G}_1 + X\left(ex_t, \frac{P_f}{P_{1t}}, \frac{P_{2t}}{P_{1t}}\right) - IM\left(ex_t, \frac{P_f}{P_{1t}}, \frac{P_{2t}}{P_{1t}}, Y_{1t}\right) = 0 \ldots 3.9$$

$$Y_{2t} = C_2\big(Y_{2t} - T_2(Y_t)\big) + I_2(Y_{2t}, r_{2t}) + \bar{G}_2 + X\left(\frac{P_{2t}}{P_{1t}}\right) - IM\left(\frac{P_{2t}}{P_{1t}}, Y_{2t}\right) \ldots 3.15$$

$$\frac{M_{1t}}{P_{1t}} = L_1(Y_{1t}, r_{1t}) \ldots 3.18$$

$$\frac{M_{2t}}{P_{2t}} = L_2(Y_{2t}) \dots 3.19$$

$$RES_{1t} - X\left(ex_t, \frac{P_f}{P_{1t}}, \frac{P_{2t}}{P_{1t}}\right) + IM\left(ex_t, \frac{P_f}{P_{1t}}, \frac{P_{2t}}{P_{1t}}, Y_{1t}\right) - KA_1(r_{1t} - r_f) - KA_2(r_{1t} - r_{2t})$$

$$= 0 \ \dots 3.20$$

$$RES_{2t} - X\left(\frac{P_{2t}}{P_{1t}}\right) + IM\left(\frac{P_{2t}}{P_{1t}}, Y_{1t}\right) + KA_2(r_{1t} - r_{2t}) = 0 \ \dots 3.21$$

$$W_{1t} = Y_{1N_{1t}} . \frac{P_{1t}}{P_{1et}} \dots 3.28$$

$$W_{2t} = Y_{2N_{2t}} P_{2t} \dots 3.29$$

$$N_{1t} = e^{\phi_{pg\,1}t} N_1\left(\frac{W_{1t}}{P_{1t}} - \frac{W_{2t}}{P_{2t}}\right) \dots 3.30$$

$$N_{2t} = e^{\phi_{pg\,2}t} N_2\left(\frac{W_{1t}}{P_{1t}} - \frac{W_{2t}}{P_{2t}}\right) \dots 3.31$$

$$r_t = \tau_0 + r^* + \tau_p(P_t - P_{t-1}) + \tau_y Y_t \dots 3.32$$

$$P_t = \phi_p P_{1t} + (1 - \phi_p) P_{2t} \dots 3.33$$

$$Y_t = Y_{1t} + Y_{2t} \dots 3.34$$

$$r_{1t} = \phi_{10} + \phi_{r1} r_t + r_f - ex_t \dots 3.35$$

$$r_{2t} = \phi_{20} + \phi_{r2} r_t + \phi_{r21} r_{1t} \dots 3.36$$

which is a set of fifteen equations in fifteen unknowns: $Y_{1t}, M_{1t}, r_{1t}, N_{1t}, W_{1t}, P_{1t}, Y_{2t}, M_{2t}, r_{2t}, N_{2t}, W_{2t}, P_{2t}, r_t, P_t, Y_t$

3.3 STATEMENT OF THE THEORETICAL PROPOSITIONS

The system of equations above provide a guide to the analysis of the regional macroeconomic impacts of fiscal policies, reflected in an increase in government spending, and the national monetary policy reflected in changes in the policy instrument – the monetary policy rate, fixed by a variant of the Taylor (1993) rule.

With respect to the regions 1 and 2, the impact of government spending in both the regions and the national monetary policy are revealed in the following propositions based on the solution to the system of equations presented above:

i. *Region 1's fiscal expansion in the government spending could have an expansionary and contractionary growth impact if the respective conditions:* $1 - C_{1y_1}(1 - T_{1y_1}) - I_{1y_1} - L_{1y_1}(C_{1m} + I_{1m}) + 2IM_{1y_1} \lesseqgtr 0$ *and* $1 - C_{1y_1}(1 - T_{1y_1}) - I_{1y_1} - L_{1y_1}(C_{1m} + I_{1m}) + 2IM_{1y_1} \gtreqless 0$ *so strongly as to ensure that* $|A|_1 < 0$.*Region 2's fiscal expansion in government spending has an inter-regional effect on region 1's growth that is complexly dependent on the sizes of its marginal propensities to consume and invest out of income and money supply on the one hand and the size of the response of region 1's exports to international and inter-regional terms of trade. Hence if:*$1 - C_{1y_1}(1 - T_{1y_1}) - I_{1y_1} - L_{1y_1}(C_{1m} + I_{1m}) + 2IM_{1y_1} \lesseqgtr 0$;
$1 - C_{2y_2}(1 - T_{2y_2}) - I_{2y_2} \lesseqgtr 0 and X_{1\frac{p_f}{p_1}} P_f + X_{1\frac{p_2}{p_1}} P_{2t} \lesseqgtr P_{1t}^3 + IM_{1\frac{p_f}{p_1}}$ *then region 1 is expected to record output growth and retardation if otherwise. On the contrary the expansion in the region 2's output due to an expansion in government expenditure is contingent on the violation of the condition* $1 - C_{2y_2}(1 - T_{2y_2}) - I_{2y_2} > 0$ *and hence it*

is seen that the benefit to the regions from the government expenditure operations of the region 2 is mutually exclusive. However the region 2 is better off if region 1 expands its government expenditure given the conditions $1 - C_{1y_1}(1 - T_{1y_1}) - I_{1y_1} - L_{1y_1}(C_{1m} + I_{1m}) + 2IM_{1y_1} \lesseqgtr 0; 1 - C_{2y_2}(1 - T_{2y_2}) - I_{2y_2} \lesseqgtr 0$ *and hence the region 2 benefits from the positive external effects of the expansion in government spending in the region 1..*

The positive impact of government spending on output growth is contingent on how large the marginal propensity to import IM_{1y_1} for the region 1 relative to the sizes of the marginal propensities of consumption and investment given the tax rate. The violation of this condition, albeit strongly, will result in a contraction in the national output.

Inter-regional positive growth spillovers are the result if the affected region in question has a net export component that is quite large enough in relation to the total final expenditure components and hence in a national economic configuration made up of regional economies with inherent imbalances in the magnitudes of growth it is expected that regions will seek to expand its net export operations to regions with comparatively high demand for its exports and this would ensure that fiscal expansion in the export demanding region would accelerate growth in the exporting region and this is in addition to the growth prospects from exports to other countries of the world of goods and services where comparative advantages exist.

ii. *Fiscal expansion in the region 1 is expected to expand money supply in the same region provided* $1 - C_{1y_1}(1 - T_{1y_1}) - I_{1y_1} - L_{1y_1}(C_{1m} + I_{1m}) + 2IM_{1y_1} > 0$ *holds otherwise money supply contracts. However fiscal expansion in the region 2 given the conditions:* $C_{1m}L_{1y_1} + I_{1m}L_{1y_1} + C_{1y_1}(1 - T_{1y_1}) + I_{1y_1} \lesseqgtr 1$ *and* $1 - C_{2y_2}(1 -$

$T_{2y_2}) - I_{2y_2} \lesseqgtr 0$ *would also trigger an expansion in the money supply of the region 1 but the violation of either of these conditions would result in a contraction of money supply in the event of an expansion in extra-regional government spending. Region 2's fiscal expansion by way of an expansion in government expenditure has the effect of increasing money supply if* $\frac{M_{2t}}{P_{1t}} IM_{2y_2} + L_{2y_2}\left(X_{2\frac{p2}{p1}} - IM_{2\frac{p2}{p1}}\right) \lesseqgtr 0$ *and* $1 - C_{2y_2}\left(1 - T_{2y_2}\right) - I_{2y_2} \lesseqgtr 0$ *else government expenditure expansion may result in monetary contraction. However given the violation of the later condition* $\frac{M_{2t}}{P_{1t}} IM_{2y_2} + L_{2y_2}\left(X_{2\frac{p2}{p1}} - IM_{2\frac{p2}{p1}}\right) > 0$ *it is expected that the expansion in the government spending of the region 1 is expected to have positive monetary expansionary effects on the region 2. Thus if* $1 - C_{2y_2}\left(1 - T_{2y_2}\right) - I_{2y_2} > 0$ *then the region 2 could record monetary expansion resulting from own government spending and extra-regional government spending.*

Government expenditure is compatible with increasing quantum of money if the ratio of the marginal propensity to import to the demand for money sensitivity to income exceeds the combined marginal propensities to consume and import out of real money balances. The violation of this condition results in a contraction of money supply. The attainment of the two conditions for the expansionary monetary effect of an increase in extra-regional government expenditure is unlikely as the fact that the derivatives all lie between zero and unity renders it possible that the attainment of one of the conditions could imply the invalidation of the other condition and hence extra-regional government expenditure could stem the growth of money supply in a given region. There are conditions under which regional governments can have monetary expansion owing to own government spending as well as regional government spending and this is expected to call for a more discriminatory monetary policy formulation to stem the likely inflationary effects from government spending on the aggregate.

iii. *Fiscal expenditure growth is expected to be inflationary in the region 1 given the condition:* $1 - C_{1y_1}(1 - T_{1y_1}) - I_{1y_1} - L_{1y_1}(C_{1m} + I_{1m}) + 2IM_{1y_1} > 0$ *and the given that same condition if* $1 - C_{2y_2}(1 - T_{2y_2}) - I_{2y_2} > 0$ *then it is expected that region 2 growth in government expenditure would result in a deflationary trend in the region 1. The attainment of the condition* $1 - C_{2y_2}(1 - T_{2y_2}) - I_{2y_2} > 0$ *which is in conjunction with the former condition is expected to engender a deflationary trend in the region 1, will result in an inflationary trend in the region 2 and this is in consonance with the effect of region2's government spending on monetary expansion. In addition to the later condition, if* $1 - C_{1y_1}(1 - T_{1y_1}) - I_{1y_1} - L_{1y_1}(C_{1m} + I_{1m}) + 2IM_{1y_1} > 0$ *then it is expected that region 1 government spending could engender an inflationary trend in the region 2.*

While government expenditure expansion may be inflationary in the region where fiscal expansion is recorded, dependent on the relative size of propensity to consume and invest out of income to those of real money balances, extra regional fiscal expansion reflected in government expenditure expansion has the effect of engendering a deflationary trend. However if the size of the propensity to consume and invest out of real money balances exceeds that of the propensities to consume out of income then it is expected that extra-regional expenditure could also be inflationary.

iv. *Interest rate changes in both the regions 1 and 2 are expected to contain extra-regional effects since the policy rate does not respond in a discriminating manner but* *rather avails the aggregate price and output information towards the formulation of the policy rate which impinges on the domestic interest rate.*

Monetary policy in the economy is based on aggregate targets and a policy instrument is availed for the two regions and determines the interest rates in the both regions. Hence expansions or contractions in the output and price, irrespective of the regions responsible for the output and price changes, will induce policy rate changes that would impact on regional interest rates.

4. DISCUSSION OF THE RESULTS

The reality of regional fiscal policy is the possibility that it could either be beneficial or adverse for the implementing region and there is the possibility of also beneficial and adverse spillover effects of extra-regional fiscal policies. In addition, in a country where monetary policy is based on information on aggregates and not tailored to suit the peculiarities of the regions, it is expected that monetary policy could produce impacts in a region where its macroeconomic variables have remained relatively stable – as a result of its inter-connection with regions where macroeconomic dynamics are heavily represented in the aggregate quantities upon which monetary policy is framed.

It is thus pertinent that the analytical framework advanced in this study be examined empirically to quantify the magnitude of these spillover and regional effects of regional policies and highlight ways in which relatively less developed regions can get up to speed in its course of development and trigger a convergence path for the entire regions in the national economy.

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Appendix:

Analytical Framework

On considering the real, money and external sectors of the region 1, the following subsystem of equations are differentiated totally and arranged below:

$$\left(1 - C_{1y_1}\left(1 - T_{1y_1}\right) - I_{1y_1} + IM_{1y_1}\right)dY_{1t} - \frac{C_{1m} + I_{1m}}{P_{1t}}dM_{1t}$$
$$+ \left((C_{1m} + I_{1m})\frac{M_{1t}}{P_{1t}^2} + \left(X_{1\frac{p_f}{p_1}} - IM_{1\frac{p_f}{p_1}}\right)\frac{P_f}{P_{1t}^2} + X_{1\frac{p_2}{p_1}}\frac{P_{2t}}{P_{1t}^2}\right)dP_{1t}$$
$$= d\bar{G}_1 + (X_{1ex} - IM_{1ex})dex + \left(X_{1\frac{p_2}{p_1}} - IM_{1\frac{p_f}{p_1}}\right)\frac{dP_{2t}}{P_{1t}}$$
$$+ \left(C_{1r_1} + I_{1r_1}\right)\left(d\phi_{10} + \phi_{r1}dr_t + dr_f - dex_t\right) \dots 1$$

$$L_{1y_1}dY_{1t} - \frac{dM_{1t}}{P_{1t}} + \frac{M_{1t}}{P_{1t}^2}dP_{1t} = -L_{1r_1}\left(d\phi_{10} + \phi_{r1}dr_t + dr_f - dex_t\right) \dots 2$$

$$IM_{1y_1}dY_{1t} - \left(X_{1\frac{p_f}{p_1}} - IM_{1\frac{p_f}{p_1}}\right)\frac{P_f}{P_{1t}^2}dP_{1t}$$
$$= \left(KA_{1r_1} + KA_{2r_1}\right)\left(d\phi_{10} + \phi_{r1}dr_t + dr_f - dex_t\right) + (X_{1ex} - IM_{1ex})dex$$
$$+ \left(X_{1\frac{p_2}{p_1}} - IM_{1\frac{p_f}{p_1}}\right)\frac{dP_{2t}}{P_{1t}} - KA_{1r_f}dr_f - KA_{2r_2}dr_{2t} - dRES_{1t} \quad \dots 3$$

$$dr_{1t} = d\phi_{10} + \phi_{r1}dr_t + dr_f - dex_t \dots 4$$

$$dr_{2t} = d\phi_{20} + \phi_{r2}dr_t + \phi_{r21}dr_{1t} \dots 5$$

and on solving the sub-system for the derivatives and obtaining the partial derivatives of the endogenous variables with respect to prices P_{1t}, P_{2t} and government expenditure G_{1t}:

$$\frac{\delta Y_{1t}}{\delta P_{2t}} = \left(X_{1\frac{p_2}{p_1}} - IM_{1\frac{p_f}{p_1}}\right)\left(\frac{A_{1m}.A_{2p} - A_{2m}A_{1p}}{P_{1t}.|A|_1} + \frac{A_{2m}A_{3p}}{P_{1t}.|A|_1}\right) \dots 6$$

$$\frac{\delta Y_{1t}}{\delta \bar{G}_1} = \frac{A_{3p}A_{2m}}{|A|_1} \dots 7$$

$$\frac{\delta M_{1t}}{\delta P_{2t}} = \left(X_{1\frac{p_2}{p_1}} - IM_{1\frac{p_f}{p_1}}\right)\left(\frac{A_{1p}A_{2y}}{P_{1t}|A|_1} + \frac{\left(A_{2p}A_{3y} - A_{3p}A_{2y}\right)}{P_{1t}|A|_1} - \frac{A_{1y}A_{2p}}{P_{1t}|A|_1}\right) \dots 8$$

$$\frac{\delta M_{1t}}{\delta \bar{G}_1} = \frac{A_{2p}A_{3y} - A_{3p}A_{2y}}{|A|_1} \ldots 9$$

$$\frac{\delta P_{1t}}{\delta P_{2t}} = \left(X_{1\frac{p2}{p1}} - IM_{1\frac{pf}{p1}}\right)\left(\frac{\left(A_{1y}A_{2m} - A_{2y}A_{1m}\right)}{P_{1t}|A|_1} - \frac{A_{3y}A_{2m}}{P_{1t}|A|_1}\right) \ldots 10$$

$$\frac{\delta P_{1t}}{\delta \bar{G}_1} = -\frac{A_{3y}A_{2m}}{|A|_1} \ldots 11$$

$$\frac{\delta P_{1t}}{\delta \bar{G}_2} = \frac{\delta P_{1t}}{\delta P_{2t}} . \frac{\delta P_{2t}}{\delta \bar{G}_2} \ldots 12$$

$$\frac{\delta Y_{1t}}{\delta \bar{G}_2} = \frac{\delta Y_{1t}}{\delta P_{2t}}\frac{\delta P_{2t}}{\delta \bar{G}_2} = -\left(\frac{A_{6y}A_{5m}}{|A|_2}\right)\left(X_{1\frac{p2}{p1}} - IM_{1\frac{pf}{p1}}\right)\left(\frac{A_{1m}.A_{2p} - A_{2m}A_{1p}}{P_{1t}.|A|_1} + \frac{A_{2m}A_{3p}}{P_{1t}.|A|_1}\right) \ldots 13$$

$$\frac{\delta r_{1t}}{\delta G_{it}} = \tau_p \phi_{r1}\phi_p \frac{\delta P_{1t}}{\delta G_{it}} + \tau_p \phi_{r1}\left(1 - \phi_p\right)\frac{\delta P_{1t}}{\delta G_{it}} + \phi_{r1}\tau_y \frac{\delta Y_{1t}}{\delta G_{it}} + \phi_{r1}\tau_y \frac{\delta Y_{2t}}{\delta G_{it}} \ \forall\ i = \{1,2\}$$

where

$$\begin{aligned} A_{1y} &= \left(1 - C_{1y_1}\left(1 - T_{1y_1}\right) - I_{1y_1} + IM_{1y_1}\right) > 0; A_{1m} = -\frac{C_{1m} + I_{1m}}{P_{1t}} < 0; A_{1p} \\ &= \left(\left(C_{1m} + I_{1m}\right)\frac{M_{1t}}{P_{1t}^2} + \left(X_{1\frac{pf}{p1}} - IM_{1\frac{pf}{p1}}\right)\frac{P_f}{P_{1t}^2} + X_{1\frac{p2}{p1}}\frac{P_{2t}}{P_{1t}^2}\right) > 0; dB_1 \\ &= \left(X_{1ex} - IM_{1ex}\right)dex + \left(C_{1r_1} + I_{1r_1}\right)\left(d\phi_{10} + \phi_{r1}dr_t + dr_f - dex_t\right) \gtreqless 0 \end{aligned}$$

$$A_{2y} = L_{1y_1} > 0; A_{2m} = -\frac{1}{P_{1t}} < 0; A_{2p} = \frac{M_{1t}}{P_{1t}^2} > 0$$

$$A_{3y} = IM_{1y_1} > 0; A_{3p} = -\left(X_{1\frac{pf}{p1}} - IM_{1\frac{pf}{p1}}\right)\frac{P_f}{P_{1t}^2} < 0;$$

$$\begin{aligned} dB_3 = {} & \left(KA_{1r_1} + KA_{2r_1} - KA_{2r_2}\phi_{r21}\right)d\phi_{10} - KA_{2r_2}d\phi_{20} \\ & + \left(KA_{1r_1}\phi_{r1} + KA_{2r_1}\phi_{r1} - KA_{2r_2}\phi_{r2} - KA_{2r_2}\phi_{r21}\phi_{r1}\right)dr_t \\ & + \left(KA_{1r_1} + KA_{2r_1} - KA_{1r_f} - KA_{2r_2}\phi_{r21}\right)dr_f \\ & + \left(X_{1ex} - IM_{1ex} - KA_{1r_1} - KA_{2r_1} + KA_{2r_2}\phi_{r21}\right)dex_t - dRES_{1t} \gtreqless 0 \end{aligned}$$

$$dB_2 = -L_{1r_1}\left(d\phi_{10} + \phi_{r1}dr_t + dr_f - dex_t\right) \gtreqless 0$$

$$|A|_1 = A_{3y}A_{1m}A_{2p} + A_{3p}A_{1y}A_{2m} - A_{3y}A_{2m}A_{1p} - A_{3p}A_{2y}A_{1m}$$

On turning to the subsystem of equations dealing with the real, money and external sectors of the region 2

$$\left(1 - C_{2y_2}\left(1 - T_{2y_2}\right) - I_{2y_2} + IM_{2y_2}\right)dY_{2t} + \left(X_{2\frac{p2}{p1}}\frac{P_{2t}}{P_{1t}^2} - IM_{2\frac{pf}{p1}}\frac{P_{2t}}{P_{1t}^2}\right)dP_{1t}$$
$$= d\bar{G}_2 + I_{2r_2}(d\phi_{20} + \phi_{r2}dr_t + \phi_{r21}dr_{1t}) + \left(X_{2\frac{p2}{p1}} - IM_{2\frac{p2}{p1}}\right)\frac{dP_{2t}}{P_{1t}} \ldots 13$$

$$L_{2y_2}dY_{2t} - \frac{dM_{2t}}{P_{2t}} + \frac{M_{2t}}{P_{2t}^2}dP_{2t} = -L_{2r_2}(d\phi_{20} + \phi_{r2}dr_t + \phi_{r21}dr_{1t}) \ldots 14$$

$$IM_{2y_2}dY_{2t} - \left(X_{2\frac{p2}{p1}} - IM_{2\frac{p2}{p1}}\right)\frac{dP_{2t}}{P_{1t}}$$
$$= KA_{2r_2}(d\phi_{20} + \phi_{r2}dr_t + \phi_{r21}dr_{1t}) + \left(X_{2\frac{pf}{p1}} - IM_{2\frac{p2}{p1}}\right)\frac{P_f}{P_{1t}^2}dP_{1t} - KA_{2r_1}dr_{1t}$$
$$- dRES_{2t} \;\ldots 15$$

$$dr_{1t} = d\phi_{10} + \phi_{r1}dr_t + dr_f - dex_t \ldots 4$$

$$dr_{2t} = d\phi_{20} + \phi_{r2}dr_t + \phi_{r21}dr_{1t} \ldots 5$$

On solving for the endogenous derivatives in the sub-system of equations corresponding to the region 2 and obtaining the partial derivatives with respect to prices P_{1t}, P_{2t} and government expenditure G_{2t} the following results:

$$\frac{\delta Y_{2t}}{\delta P_{1t}} = \left(X_{2\frac{p2}{p1}} - IM_{2\frac{pf}{p1}}\right)\left(\frac{A_{5m}A_{6p}P_{2t}}{|A|_2 P_{1t}^2} + \frac{A_{4p}A_{5m}P_f}{|A|_2 P_{1t}^2}\right)$$

$$\frac{\delta Y_{2t}}{\delta \bar{G}_1} = \frac{A_{5m}A_{6p}}{|A|_2}$$

$$\frac{\delta M_{2t}}{\delta P_{1t}} = \left(X_{2\frac{pf}{p1}} - IM_{2\frac{pf}{p1}}\right)\left(\frac{\left(A_{4p}A_{5y} - A_{4y}A_{5p}\right)P_f}{P_{1t}^2|A|_2} + \frac{\left(A_{5y}A_{6p} - A_{5p}A_{6y}\right)P_{2t}}{P_{1t}^2|A|_2}\right)$$

$$\frac{\delta M_{2t}}{\delta \bar{G}_2} = \frac{\left(A_{5p}A_{6y} - A_{5y}A_{6p}\right)}{|A|_2}$$

$$\frac{\delta P_{2t}}{\delta P_{1t}} = \left(\frac{A_{4y}A_{5m}P_f}{P_{1t}^2|A|_2} + \frac{A_{6y}A_{5m}P_{2t}}{P_{1t}^2|A|_2}\right)\left(X_{2\frac{pf}{p1}} - IM_{2\frac{pf}{p1}}\right)$$

$$\frac{\delta P_{2t}}{\delta \bar{G}_2} = -\frac{A_{6y}A_{5m}}{|A|_2}$$

$$\frac{\delta r_{2t}}{\delta G_{it}} = \tau_p \phi_{r2} \phi_p \frac{\delta P_{1t}}{\delta G_{it}} + \tau_p \phi_{r2} (1 - \phi_p) \frac{\delta P_{2t}}{\delta G_{it}} + \phi_{r2} \tau_y \frac{\delta Y_{1t}}{\delta G_{it}} + \phi_{r2} \tau_y \frac{\delta Y_{2t}}{\delta G_{it}} \ \forall \ i = \{1,2\}$$

where:

$$A_{4y} = \left(1 - C_{2y_2}\left(1 - T_{2y_2}\right) - I_{2y_2} + IM_{2y_2}\right) > 0; A_{4p} = -\left(X_{2\frac{p2}{p1}} - IM_{2\frac{pf}{p1}}\right)\frac{1}{P_{1t}} < 0; dB_4$$
$$= I_{2r_2} d\phi_{20} + I_{2r_2} \phi_{r21} d\phi_{10} + I_{2r_2} (\phi_{r2} + \phi_{r21} \phi_{r1}) dr_t + I_{2r_2} \phi_{r21} dr_f$$
$$- I_{2r_2} \phi_{r21} dex_t \gtreqless 0$$

$$A_{5y} = L_{2y_2} > 0; A_{5m} = -\frac{1}{P_{2t}} < 0; A_{5p} = \frac{M_{2t}}{P_{2t}^2} > 0; dB_5$$
$$= -L_{2r_2}\left(d\phi_{20} + \phi_{r21} d\phi_{10} + (\phi_{r2} + \phi_{r21}\phi_{r1}) dr_t + \phi_{r21} dr_f - \phi_{r21} dex_t\right) \gtreqless 0$$

$$A_{6y} = IM_{2y_2} > 0; dB_6$$
$$= KA_{2r_2} d\phi_{20} + KA_{2r_2} \phi_{r21} d\phi_{10} + KA_{2r_2} (\phi_{r2} + \phi_{r21}\phi_{r1}) dr_t + KA_{2r_2} \phi_{r21} dr_f$$
$$- KA_{2r_2} \phi_{r21} dex_t - KA_{2r_1} d\phi_{10} - KA_{2r_1} \phi_{r1} dr_t - KA_{2r_1} dr_f + KA_{2r_1} dex_t$$
$$- dRES_{2t} \gtreqless 0$$

$$A_{6p} = -\left(X_{2\frac{p2}{p1}} - IM_{2\frac{pf}{p1}}\right)\frac{1}{P_{1t}} > 0; |A|_2 = A_{4y} A_{5m} A_{6p} - A_{4p} A_{5m} A_{6y}$$

On turning to the productivity and employment sectors of the region 1 and region 2, the total derivatives are:

$$dW_{1t} = Y_{1N_1N_1} dN_{1t} + \frac{dP_{1t}}{P_{1et}} - \frac{P_{1t}}{P_{1et}^2} dP_{1et} \ \ldots 16$$

$$dW_{2t} = Y_{2N_2N_2} dN_{2t} + dP_{2t} \ \ldots 17$$

$$dN_{1t} = \phi_{pg1} N_{1t} + N_{1\frac{w1}{p1}} . \frac{dW_{1t}}{P_{1t}} - N_{1\frac{w1}{p1}} . \frac{W_{1t}}{P_{1t}^2} . dP_{1t} + N_{1\frac{w2}{p2}} . \frac{dW_{2t}}{P_{2t}} - N_{1\frac{w2}{p2}} . \frac{W_{2t}}{P_{2t}^2} . dP_{2t} \ \ldots 18$$

$$dN_{2t} = \phi_{pg2} N_{2t} + N_{2\frac{w1}{p1}} . \frac{dW_{1t}}{P_{1t}} - N_{2\frac{w1}{p1}} . \frac{W_{1t}}{P_{1t}^2} . dP_{1t} + N_{2\frac{w2}{p2}} . \frac{dW_{2t}}{P_{2t}} - N_{2\frac{w2}{p2}} . \frac{W_{2t}}{P_{2t}^2} . dP_{2t} \ \ldots 19$$

On solving the system of equations and obtaining the partial derivatives with respect to prices P_{1t}, P_{2t} the following results:

$$\frac{\delta N_{2t}}{\delta P_{1t}} = \left\{\left(\frac{E_{1N_{1t}}}{E_{1N_{1t}} E_{2N_{2t}} - E_{2N_{1t}} E_{1N_{2t}}}\right)\frac{N_{1t}}{N_1} N_{1\frac{w1}{p1}} - \left(\frac{E_{2N_{1t}}}{E_{1N_{1t}} E_{2N_{2t}} - E_{2N_{1t}} E_{1N_{2t}}}\right)\frac{N_{2t}}{N_2} N_{2\frac{w1}{p1}}\right\}\left(\frac{1}{P_{1t} P_{1et}}\right.$$
$$\left. - \frac{W_{1t}}{P_{1t}^2}\right)$$

$$\frac{\delta N_{2t}}{\delta P_{2t}} = \left\{\left(\frac{E_{1N_{1t}}}{E_{1N_{1t}}E_{2N_{2t}} - E_{2N_{1t}}E_{1N_{2t}}}\right)\frac{N_{2t}}{N_2}N_{2\frac{w2}{p2}} - \left(\frac{E_{2N_{1t}}}{E_{1N_{1t}}E_{2N_{2t}} - E_{2N_{1t}}E_{1N_{2t}}}\right)\frac{N_{1t}}{N_1}N_{1\frac{w2}{p2}}\right\}\left(\frac{1}{P_{2t}} - \frac{W_{2t}}{P_{2t}^2}\right)$$

$$\frac{\delta N_{1t}}{\delta P_{1t}} = \left\{\left(\frac{E_{2N_{2t}}}{E_{1N_{1t}}E_{2N_{2t}} - E_{2N_{1t}}E_{1N_{2t}}}\right)\frac{N_{1t}}{N_1}N_{1\frac{w1}{p1}} - \left(\frac{E_{1N_{2t}}}{E_{1N_{1t}}E_{2N_{2t}} - E_{2N_{1t}}E_{1N_{2t}}}\right)\frac{N_{2t}}{N_2}N_{2\frac{w1}{p1}}\right\}\left(\frac{1}{P_{1t}P_{1et}} - \frac{W_{1t}}{P_{1t}^2}\right)$$

$$\frac{\delta N_{1t}}{\delta P_{2t}} = \left\{\left(\frac{E_{2N_{2t}}}{E_{1N_{1t}}E_{2N_{2t}} - E_{2N_{1t}}E_{1N_{2t}}}\right)\frac{N_{1t}}{N_1}N_{1\frac{w2}{p2}} - \left(\frac{E_{1N_{2t}}}{E_{1N_{1t}}E_{2N_{2t}} - E_{2N_{1t}}E_{1N_{2t}}}\right)\frac{N_{2t}}{N_2}N_{2\frac{w2}{p2}}\right\}\left(\frac{1}{P_{2t}} - \frac{W_{2t}}{P_{2t}^2}\right)$$

Where

$$E_{1N_{1t}} = 1 - N_{1\frac{w1}{p1}}.\frac{Y_{1N_1N_1}}{P_{1t}} > 0; E_{1N_{2t}} = -N_{1\frac{w2}{p2}}.\frac{Y_{2N_2N_2}}{P_{2t}} < 0;$$

$$E_{2N_{1t}} = -N_{2\frac{w1}{p1}}.\frac{Y_{1N_1N_1}}{P_{1t}} < 0; E_{2N_{2t}} = 1 - N_{2\frac{w2}{p2}}.\frac{Y_{2N_2N_2}}{P_{2t}} > 0;$$

and hence in the case of the respective wages of the regions 1 and 2 the following are expected:

$$\frac{\delta W_{1t}}{\delta P_{1t}} = Y_{1N_1N_1}\frac{\delta N_{1t}}{\delta P_{1t}} + \frac{1}{P_{1et}}$$

$$\frac{\delta W_{1t}}{\delta P_{2t}} = Y_{1N_1N_1}\frac{\delta N_{1t}}{\delta P_{2t}}$$

and

$$\frac{\delta W_{2t}}{\delta P_{1t}} = Y_{2N_2N_2}\frac{\delta N_{2t}}{\delta P_{1t}}$$

$$\frac{\delta W_{2t}}{\delta P_{2t}} = Y_{2N_2N_2}\frac{\delta N_{2t}}{\delta P_{2t}} + 1$$

On substituting the aggregate price level equation and the aggregate output equation into the monetary policy rule, the total derivative is given as:

$$dr_t = d\tau_0 + dr^* + \tau_p\left(\phi_p dP_{1t} + \left(1 - \phi_p\right)dP_{2t}\right) + \tau_p\left(\phi_p dP_{1t-1} + \left(1 - \phi_p\right)dP_{2t-1}\right) + \tau_y dY_{1t} + \tau_y dY_{2t} \ldots 20$$

On the basis of the monetary rule the following ensues:

$$\frac{\delta r_t}{\delta P_{it}} = \frac{\phi_p^* \tau_p}{P_{it}} \ \forall\, i = \{1,2\};\ \phi_p^* = \{\phi_p, 1 - \phi_p\}$$

$$\frac{\delta r_t}{\delta Y_{it}} = \tau_y \ \forall\, i = \{1,2\}$$