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The Demographic and Educational Transitions and the Sustainability of the Spanish Public Pension System*

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Abstract

We use a calibrated overlapping generations model economy to quantify the consequences of the demographic and the educational transitions for the viability of the Spanish public pension system. The households in our model economy differ in their education and in the random market value of their time, they understand the link between payroll taxes and public pensions, and they choose when to retire from the labor force. We find that the demographic transition makes the public pension system in our model economy unsustainable. The pension system starts running a deficit in the year 2016, the pension fund is depleted in the year 2029, and the accumulated value of the pension system debt reaches a shocking 277 percent of the model economy output by the year 2060.

Keywords: Computable General Equilibrium, Social Security, Ageing, Education

JEL Classification: C68, H55, J11, I20

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1 Introduction

• Some Facts. The financial viability of pay-as-you-go pension systems is being questioned in many countries for two main reasons: the aging of their populations and the early retirement of their workers. Consequently, in the next few decades, the retiree to worker ratios of developed economies will increase significantly and the financial viability of their current unfunded pension systems is seriously at risk.

More specifically, in 1997 in Spain there were 23 retirees for every hundred working-age people. According to the projections of the Spanish Instituto Nacional de Estadística, by the year 2050 this number will have increased to no less than 56. This change is partly due to a very large reduction in Spanish birth-rates. Between 1957 and 1977 the average number of children per fertile woman was 2.8. Since 1980 this number has decreased continuously, and in 1998 it was only 1.16. As we show in this article, these demographic changes make the current pay-as-you-go Spanish public pension system completely unsustainable.

In some countries there is another trend which affects the financial sustainability of unfunded pensions systems: the tendency of workers to become more educated. In 1977 only about nine percent of Spanish working-age people had completed high school and only about three percent had completed college. Twenty years later, those shares were 24 percent and 13 percent. By the year 2050 they are projected to be 38 percent and 24 percent (see Meseguer, 2001). This educational transition is also important for the sustainability of the Spanish pay-as-you-go pension system. First, more educated people pay higher payroll taxes during their working lives and they contribute to sustain the system but, later when their retire, their pension entitlements are higher, and they make the pension system less sustainable.

• Questions and Answers. The purpose of this article is to quantify the consequences of the Spanish demographic and educational transitions for the sustainability of the Spanish public pension system. To answer this question we construct a fully detailed overlapping generations model of the Spanish economy and we carry out the following exercise: First we simulate the model economy under the counterfactual assumption that after 1997 both the retiree-to-worker ratios and the educational shares of workers remain constant. We find that if this had been the case, by the year 2060 the model economy public pension system would have had a small deficit of 0.2 percent of output, and that the value of the accumulated pension fund would be 33.6 percent of output. Next, we keep the retiree-to-worker ratios constant, but we simulate the Spanish educational transition. It turns out that the educational transition improves the viability of the current public pension system. By the year 2060 the pension system would have had a surplus of 1.0 percent of output, and the value of the pension fund would have been 150.0 percent of the model economy output. Finally, we simulate both the demographic and the educational transitions and we find that current Spanish pension system is completely
unsustainable. By the year 2060 the pension system deficit will have increased to 7.1 percent of output, and that the accumulated value of the pension system debt will reach a shocking 277.1 percent of the model economy output.

- **The Model Economy.** Our overlapping generations model economy combines various features of similar models described elsewhere in the literature. First, our model economy is populated by natives and immigrants as in Sánchez-Martín (2003). Second, our households face stochastic lifetimes as in Hubbard and Judd (1987). Third, they differ in their education levels as in Cub addu (1998). Fourth, they face an uninsurable idiosyncratic shock to their endowments of efficiency labor units as in Comess and Krueger (1999). Fifth, our households understand the link between the payroll taxes that they pay and the pensions to which they are entitled as in Hugget and Ventura (1999). Sixth, they decide optimally when to retire as in Sánchez-Martín (2003). Finally, our households face the possibility of becoming disabled and receiving a disability pension. Rust and Phelan (1997) introduce this feature in a partial equilibrium model.

We also model the current Spanish public pension system in very much detail. Specifically the model economy pension system incorporates the Spanish payroll tax cap, the minimum and maximum pensions, the pension replacement rate, the penalties for early retirement, and the pension fund. In addition, the government in our model economy taxes labor income, capital income and consumption, it finances public consumption and transfers other than pensions and it services a stock of public debt.

Other important features of our model economy are the following: we calibrate the random component of the efficiency labor units endowment process so that our model economy replicates the Lorenz curves of the Spanish earnings and income distributions as reported in Budría and Díaz-Giménez (2006). Therefore, the processes on income and earnings of our model economy are consistent with both the aggregate and the distributional properties of Spanish data. Finally, our model economy replicates in very much detail the main features of the retirement behavior of Spanish households, such as the average retirement age, the participation rates by educational types of workers in the 60 to 64 age cohort, and the conditional probabilities of retirement.

- **Literature Review.** The consequences of the Spanish demographic transition for the viability of the public pension system has been studied by a large body of previous literature. Here, we summarize the findings of de Miguel and Montero (2004), of Rojas (1999), and of Sánchez-Martín (2003). These articles share the feature that they make use of multiperiod overlapping generations models, just as we do. For a summary of the findings of alternative modeling approaches, we refer the reader to Jimeno (2000) and Conde-Ruiz and Meseguer (2004).

De Miguel and Montero (2004) study an overlapping generations model economy populated
by representative households who face a survival risk. Unlike ours, their model economy omits most of the institutional features of the Spanish public pension system. Their initial steady state is 1995 and they simulate the Spanish demographic transition under two different government policies. First, the retirement pension is kept constant at its 1995 value, and the payroll tax is adjusted to balance the pension system budget. They find that the payroll tax must be increased from 11.6 percent in 1995 to 19.2 percent in 2050. Second, they assume that the payroll tax is kept constant at its 1995 value and that the retirement pension is adjusted to keep the pension system in balance. In this case, they find that the ratio of the average pension to average earnings must be reduced from 40.0 percent in 1995 to 22.2 percent in 2050.\footnote{Arjona (2000) studies a very similar model economy and he finds that, by the end of the Spanish demographic transition, the average pension must be reduced to 34 percent of its 1995 value to preserve the balance of the pension system.}

Rojas (1999) introduces credit constraints, a maximum retirement pension, and models two roles for the government. First, it runs a balanced pay-as-you-go pension system where the payroll tax is adjusted each period and, second, the government consumes a constant proportion of output each period. This government consumption is financed with a proportional tax on capital and labor income. He simulates the Spanish demographic transition, and he finds that the payroll tax must increase from 16.5 percent in 1995 to 39.9 percent in 2050 to preserve the balance of the system.

Sánchez-Martín (2003) studies the consequences of the demographic transition in a model economy whose households differ in their education levels and decide optimally when to retire from the labor force. In his model economy the government runs a pay-as-you-go pension system with a minimum retirement pension and it consumes a constant share of output each period. These government outlays are financed with a proportional payroll tax, a confiscatory tax on accidental bequests and a lump-sum tax that is adjusted to balance the consolidated government and pension system budgets. He simulates the Spanish demographic transition starting from 1995, and he finds that by the year 2050 the pension system deficit will be approximately nine percent of the model economy output. The main differences between Sánchez-Martín (2003) and this article are that Sánchez-Martín abstracts from the educational transition, that he does not model maximum pensions, disability pensions, or the pension fund, and that his payroll tax is uncapped. Moreover, his model economy does not assume the Spanish pension replacement rate, and it abstracts from consumption taxes, capital and labor income taxes, public transfers and public debt.
2 The facts

Aging. During the last thirty years Spanish demography has experienced large changes. According to the Instituto Nacional de Estadística (INE), between 1957 and 1977, the average number of children born per woman in Spain was 2.8. However, since 1978 this rate has decreased continuously and it has reached a minimum value of 1.16 in 1998 (see Panel A of Figure 1). Partly as a result of this change in fertility, the old-age dependency ratio of the Spanish economy, which we define as the ratio of the number of people in the 65+ age cohort to the number of people in the 20 64 age cohort, will increase from 26.5 percent in 1997 to a projected 59.9 percent in 2050 under the INE's population Hypothesis 1 (see Panel B of Figure 1).\(^2\) Notice that this ratio is only a rough approximation to the pensioners to payroll tax-payers ratio. This is because not every person in the 20–64 age cohort pays payroll taxes, not every person in the 65+ cohort is a pensioner, and not every pensioner is 65 or older.

Education. Another important change experienced by the Spanish households during the last thirty years is that they have become significantly more educated. According to Mosquera (2002) in 1977 in Spain, only about nine percent of the Spanish working-age people had completed high school and only about three percent had completed college. Twenty years later, in 1997, these shares had increased dramatically to 24 percent and 13 percent. According to Mosquera's projections, these shares will keep on increasing and they will reach 38 percent.

\(^2\)The INE makes two hypothesis about the evolution of the Spanish population. They differ in the net inflow of immigrants between 2007 and 2059 (14.6 million under Hypothesis 1 and 5.8 million under Hypothesis 2), and in the life expectation in year 2059 (80.9 years for men and 87.0 years for women under Hypothesis 1 and 80.7 and 86.3 years under Hypothesis 2).
and 24 percent by the year 2050.

3 The model economy

Our model economy is an overlapping generations economy where each period corresponds to one year. In the economy there are three types of agents: households, firms and a government which we describe in the subsections below.

3.1 The government

The government in this model economy runs a pay-as-you-go pension system, it collects income and consumption taxes and it uses the proceeds of taxation to finance flows of government consumption and transfers other than pensions, and to service a stock of public debt.

3.1.1 The public pension system

In Table 1 we compare the features of the Spanish pension system and those of the pension system in our model economy. These features are the following:

**Payroll taxes.** The pension system is financed with a capped payroll tax on gross labor earnings. This payroll tax is described by function, \( \tau_j(y_t) \), where \( y_t \) denotes gross labor earnings at period \( t \).

**Retirement pensions.** A retiree of age \( j \) is entitled to receive a pension \( b_r \leq b(j) \leq \bar{b}_r \), where \( b_r \) and \( \bar{b}_r \) are the minimum and maximum retirement pensions. The retirement pension, \( b(j) \), is computed according to the following formula:

\[
b(j) = (1 - \lambda_j)\phi \left[ \frac{1}{N_b} \left\{ \sum_{j-N_b}^{j-1} y_i \right\} \right]
\]

(1)

where \( 0 \leq \lambda_j < 1 \) denotes the penalty for early retirement, \( 0 < \phi < 1 \) is the pension system replacement rate, and \( N_b \) denotes the number of years before retirement that are used to compute the pension.

**Pension fund.** The government also operates a pension fund, \( F_t \). For simplicity we assume that this fund is invested in foreign assets, and that these assets obtain an exogenous rate of return, \( r^* \). The fund works as follows: whenever there is a surplus in the pension system, it is invested in the fund, and whenever the public pension system goes into a deficit, the fund is used to finance the deficit until it is exhausted. After the fund is exhausted, the government
Table 1: Payroll taxes and Pensions in Spain and in the Model Economy

<table>
<thead>
<tr>
<th>Payroll Taxes</th>
<th>Spain</th>
<th>Model Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax Rate</td>
<td>Proportional</td>
<td>Proportional</td>
</tr>
<tr>
<td>Maximum Cap</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Tax Exempt Minimum</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pensions</th>
<th>Spain</th>
<th>Model Economy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regulatory Base</td>
<td>Last 15 years prior to retirement</td>
<td>Last 15 years prior to retirement</td>
</tr>
<tr>
<td>Replacement Rate</td>
<td>Dependent on the years of contributions</td>
<td>Independent of the years of contributions</td>
</tr>
<tr>
<td>Maximum pension</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Minimum pension</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Early retirement penalties</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Pension fund</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Disability pension</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Note: The rules that describe the Spanish public pension system are those of the Régimen General de la Seguridad Social.

borrows as much as necessary at the same rate $r^*$ to finance the pension system deficits. Therefore, the law of motion of the pension fund is the following:

$$F_{t+1} = (1 + r^*)F_t + T_{s,t} - P_t$$

(2)

where $T_{s,t}$ denotes aggregate payroll tax revenues and $P_t$ denotes aggregate pensions.

Disability pensions. In addition, the pension system pays a disability pension to disabled households, $b_{d,t}$.

3.1.2 The government budget

Revenues. The government collects tax revenues, $T_t$, using a proportional consumption tax, $\tau_{c,t}$, a proportional tax on labor income net of social security contributions tax, $\tau_{l,t}$, a proportional tax on capital income, $\tau_{k,t}$, and it confiscates unintentional bequests, $F_t$.

Outlays. Each period the government consumes an exogenous proportion of output, $C_t$, makes lump-sum transfers to households other than pensions, $Z_t$, and services a stock of public debt, $D_t$, which is also an exogenous and constant proportion of output.
**Budget constraint.** Let \( r_t \) be the equilibrium interest rate which we define below, then the government budget constraint is the following:

\[
G_t + Z_t = (1+r_t)D_t - T_t - E_t - D_{t+1},
\]

(3)

### 3.2 Households

**Population dynamics.** We assume that our model economy is inhabited by continuum of heterogeneous households, which we normalize each period so that its measure is always one. The households differ in their birthplace, \( l \in L \), in their age, \( j \in J \), in their education levels, \( h \in H \), in their employment status, \( s \in S \), in their assets, \( a \in A \), and in their pension claims, \( b \in D \). Let \( \mu_t(l, j, h, s, a, b) \) be the measure of households of type \((l, j, h, s, a, b)\). For convenience, whenever we integrate the measure of households over some dimension, we drop the corresponding subscript. For instance, \( \mu_t(j, h) = \mu_t(\ldots, j, h, \ldots, \ldots) \) denotes the period \( t \) measure of households of type \((j, h)\).

The households can either be native to the economy and then \( l = n \), or they can be immigrants and then \( l = i \), and we assume that a measure \( \mu_t(i, j, h, s, a, b) \) of immigrants of type \((j, h, s, a, b)\) enters the economy at the beginning of each period \( t \).

Each period both immigrants and natives face a conditional probability of survival from age \( j \) to \( j + 1 \) which we denote by \( \psi_t(j) \), and an age dependent probability of having offspring which we denote by \( f_t(j) \).\(^3\) Finally, we assume that the offspring of immigrants are natives, and that both the offspring and the youngest immigrants enter the economy at age \( j = 20 \).

These assumptions imply that at the beginning of every period the unnormalized measure of households is \( 1 + n_t \), where \( n_t \) is the rate of growth of the population which we compute as follows

\[
n_t = \mu_{t+1}(i) + \sum_j [\psi_t(j) \cdot f_t(j)] \mu_t(j) = 1.
\]

(1)

They also imply that the law of motion of \( \mu_t(j) \) is

\[
\mu_{t+1}(j) = \frac{1}{1+n_t} \left[ \mu_{t+1}(j, 20) + \sum f_t(j) \mu_t(j) \right]
\]

(5)

and

\[
\mu_{t+1}(j+1) = \frac{1}{1+n_t} \left[ \mu_{t+1}(j+1, 20) + \psi_t(j) \mu_t(j) \right]
\]

(6)

for each \( j \geq 20 \).

\(^3\)We assume that immigrants and natives have the same survival probabilities and fertility rates because independent data for these two population groups are not readily available.
Education. In this article we abstract from the education decision and we assume that the education level of both natives and immigrants is determined when they enter the economy. We also assume that there are three educational levels and, consequently, that \( I = \{1, 2, 3\} \). Educational level \( h = 1 \) denotes that the household has not completed high school.\(^4\) Educational level \( h = 2 \) denotes that the household has completed high school but has not completed college. Finally, educational level \( h = 3 \) denotes that the household has completed college.

Employment status. Households in our economy are either workers, which we denote by \( s \in S \), disabled, which we denote by \( s = d \), or retired, which we denote by \( s = r \). Each period, every worker receives an endowment of efficiency labor units. This endowment has two components: a deterministic component that depends on the age and the education of the worker, \( \epsilon(j, h) \), and a stochastic idiosyncratic component, \( \omega \). The process on the stochastic component follows a finite state Markov chain that is independent and identically distributed across workers, and whose conditional transition probability matrix is \( \Gamma_{\omega \omega'} = Pr(\omega_{t+1} = \omega' | \omega_t = \omega) \), where \( \omega \) and \( \omega' \in S = \{1, 2, \ldots, m_s\} \). We assume that each period workers also face an age and education-dependent disability risk. Specifically, a worker of type \( (j, h) \) faces a probability \( \varphi(j, h) \) of being disabled from age \( j + 1 \) onwards.\(^5\) Finally, we assume that our model economy households decide optimally when to retire and that disabled households and retirees receive no endowments of efficiency labor units. All these assumptions imply that \( S = \{S, d, r\} - \{1, 2, \ldots, m_s, d, r\} \).

Preferences. We assume that the households in our model economy have identical preferences that can be described by the following expected utility function:

\[
F = \sum_j \beta^j u(c_j, 1 - l_j) 
\]

(7)

where the function \( u \) is continuous and strictly concave in both arguments, \( 0 < \beta \) is the time discount factor, \( c_j \) is consumption and \( l_j \) is labor. Consequently, \( 1 - l_j \) is the amount of time that the households allocate to non-market activities.

The households' decision problem

Households in our model economy solve the following decision problems:

\(^4\)In this group we include every household that has not completed the compulsory education. Due to the changes in the Spanish educational laws, we define the compulsory studies to be either the Estudios Secundarios Obligatorios, the Graduado Escolar, the Certificado Escolar, or the Bachiller Elemental.

\(^5\)We model disability explicitly because in many cases disability pensions are an additional pathway to early retirement.Boldrin and Jiménez-Martín (2003) also make this point.
Households of ages 20 to age 59. During this period of their life-cycle the households are not allowed to retire and they solve two different decision problems depending on their employment status.

- Workers. Workers of ages 20 to 59 choose the consumption, savings, and hours worked that solve the following decision problem:

\[
V_t(j, h, \omega, a, b) = \max_{c, l, a'} \{ u(c, (1 - \ell)) + \beta \psi_t(j)(1 - \tau_t)(j, h, \omega, a', b') \}
+ \varphi(j, h)V_{t+1}(j + 1, h, d, a', b') \}
\]

subject to

\[
(1 + \tau_c)c + a' = (1 - \tau)(y - \tau_k(y)) + [1 + r(1 - \tau_k)]a + z
\]

where

\[
b' = \begin{cases} 
0 & \text{if } j < 60 - N_b \\
(b + y)/(j - (60 - N_b - 1)) & \text{if } 60 - N_b < j < 60,
\end{cases}
\]

where \( y = w \times e \times \omega \times l \) denotes gross labor earnings, \( w \) denotes the wage rate, and \( z \) denotes per capita government transfers. The law of motion of \( b \) replicates the rules of the Spanish Régimen General de la Seguridad Social. These rules establish that the retirement pension is a function of the average gross labor earnings of the last \( N_b \) years prior to retirement.\(^6\)

Since that the earliest retirement age is 60, we start to compute the pension entitlement when households are \((60 - N_b)\) years old.

- Disabled households. Disabled households aged 20 to 59 do not work, they may be entitled to receive a retirement pension, and they chose the consumption and savings that solve the following decision problem:

\[
V_t(j, h, d, a, b) = \max_{c, l, a'} \{ u(c, (1 - \ell)) + \beta \psi_t(j)V_{t+1}(j + 1, h, d, a', b') \}
\]

subject to

\[
(1 + \tau_c)c + a' = [1 + r(1 - \tau_k)]a + z + b_d,
\]

where \( b' - b \), and where \( b_d \) denotes the disability pension.

Households of ages 60 to 64. During this period of their lives, the model economy households decide whether or not to retire early and they solve two different decision problems depending on their employment status.

\(^6\)This component of the retirement pension formula is known as the Base Regalado.
• Workers. Workers in this age group decide whether or not to retire comparing the solutions of the following decision problems:

\[
V_t(j, h, a, b) = \max_{c, a, l} \{ u(c, 1 - l) + \beta \psi_t(j)[(1 - \tau_t(j, h)) \sum_{\omega' \in S} \Gamma_{\omega, \omega'} V_{t+1}(j + 1, h, \omega', a', b') + \tau_t(j, h) V_{t+1}(j + 1, h, a', b')] \}
\]

subject to

\[
(1 + \tau_t)c + a' = (1 - \eta)[y + \tau_t(y)] + [1 + r(1 - \tau_k)]a + z
\]

where \(b' = [(N_h, 1)b + y]/N_h\), and

\[
V_t(j, h, a, b) = \max_{c, a, l} \{ u(c, 1 - l) + \beta \psi_t(j)[(1 + 1 - \tau_k)]a + z + b(j) \}
\]

subject to

\[
(1 + \tau_t)c + a' = (1 + r(1 - \tau_k))a + z + b(j)
\]

where \(b' = (1 - \lambda_j)b\), and they choose the option that gives them the higher expected lifetime utility.

To gain some intuition about the trade-offs involved in this decision, let us consider the benefits and costs of continuing to work. The benefits are two: the collected earnings and the avoidance of the early retirement penalty. The costs are also two: the forgone leisure, and the forgone pension. There is also another effect: the change in the pension claim, \(b' - b\). This change could be either a benefit or a cost, depending on both worker's current endowment of efficiency labor units, \(c \times \omega\), and the current pension entitlement, \(b\).

Minimum retirement pensions, \(b\) also play an important role in the early retirement decision. Specifically, since every retiree is entitled to receive the minimum retirement pension, it eliminates the incentive to avoid the early retirement penalty for workers with \(b < b\). Consequently, every household who is only entitled to pension \(b \leq b\) chooses to retire at the earliest possible retirement age, which is 60.

• Disabled households. Disabled households decide whether to continue collecting the disability pension, or whether to give up the disability pension and to move into early retirement. To make this decision they compare the solutions of the following problems:

\[
V_t(j, h, a, b) = \max_{c, a, l} \{ u(c, 1 - l) + \beta \psi_t(j)[(1 + 1 - \tau_k)]a + z + b(j) \}
\]

subject to

\[
(1 + \tau_t)c + a' = (1 + r(1 - \tau_k))a + z + b_d
\]
where \( b' - b \), and

\[
V_t(j, h, d, a, b) = \max_{c, a'} \left\{ u(c, 1 - l) + \beta \psi_t(j) V_{t+1}(j + 1, h, r, a', b') \right\}
\]  \hspace{1cm} (17)

subject to

\[
(1 + \tau_t)c + a' = [1 + r(1 - \tau_t)]a + z + b(j)
\]  \hspace{1cm} (18)

where \( b' = (1 - \lambda_t) b \), and they choose the option that gives them the higher expected lifetime utility.

The retirement pensions of these households are either a function of the average gross labor income earned between ages \((60 - N_R)\) and the age in which they became disabled, or the minimum retirement pension if they became disabled before age \((60 - N_R)\).

**Households of ages 65 to 100.** Every household that reaches age 65 is forced to retire and it chooses the sequences of consumption and savings that solve the following decision problem:

\[
V_t(j, h, d, a, b) = \max_{c, a'} \left\{ u(c, 1 - l) + \beta \psi_t(j) V_{t+1}(j + 1, h, r, a', b') \right\}
\]  \hspace{1cm} (19)

subject to

\[
(1 + \tau_t)c + a' = [1 + r(1 - \tau_t)]a + z + b(j)
\]  \hspace{1cm} (20)

Notice that if \( j = 65, s = \omega, d \) or \( r \) and \( s' = r \), and if \( j > 65, s = s' = r \). Moreover, in both cases, \( b = b' = b(j) \).

### 3.3 Firms

We assume that the firms in our economy behave competitively in the product and factor markets, that they maximize profits, and that they have free access to a production technology that can be described by a constant returns to scale production function, \( Y_t = F(K_t, A_t L_t) \), where \( Y_t \) denotes aggregate output, \( K_t \) denotes aggregate capital and \( L_t \) denotes the aggregate labor input. Variable \( A_t \) denotes an exogenous, labor-augmenting productivity factor whose law of motion is given by \( A_t = (1 + \rho) A_{t-1} \), where \( \rho > 0 \). The aggregate capital stock is obtained aggregating the capital owned by every household and the aggregate labor input is obtained aggregating the efficiency labor units supplied by every household. Finally, we assume that the capital stock depreciates geometrically at a constant rate \( 0 < \delta < 1 \).

The profit maximizing behavior of firms implies that factor prices are the factor marginal productivities

\[
r_t = F_K(K_t, A_t L_t) - \delta
\]  \hspace{1cm} (21)
\[ w_t = F_L(K_t, A_t L_t) \]  

\[(22)\]

Notice that in our model economy labor productivity grows for two reasons: first, because \( \rho > 0 \) and, second, because as workers become more educated they also become more productive.

**Definition of equilibrium**

Let \( \ell \in L = \{i, n\}, j \in J = \{20, 21, \ldots, J\}, h \in H = \{1, 2, 3\}, s \in S, a \in A = R_+ \), and \( b \in B = [b_i, b_n] \), and let \( \mu_\ell(\ell, j, h, s, a, b) \) be a probability measure defined on \( \mathcal{F} = L \times J \times H \times S \times A \times B \). Then, given initial conditions \( \mu_0, K_0, F_0 \) and \( D_0 \), a competitive equilibrium for this economy is a sequence of household value functions \( \{V_i(j, h, s, a, b)\}_{i=0}^{\infty} \); a sequence of household policies, \( \{e_i(j, h, s, a, b), b_i(j, h, s, a, b), a_i(j, h, s, a, b)\}_{i=0}^{\infty} \), a sequence of government policies, \( \{\tau_{\ell, i}, b_{\ell, i}, b_{d,t}, \lambda_j, \phi, N, F_{t+1}, \pi_{t, x}, \pi_{t, z}, Z_{t}, D_{t+1}\}_{t=0}^{\infty} \), a sequence of measures, \( \{\mu_t\}_{t=0}^{\infty} \), a vector of factor prices, \( \{\pi_t, w_t\}_{t=0}^{\infty} \), a vector of macroeconomic aggregates, \( \{K_{t+1}, L_t, T_{n,t}, P_t, T_i, Z_i, E_{t+1}\}_{t=0}^{\infty} \), and a number, \( r_t \), such that the following conditions hold:

(i) Factor inputs, tax revenues, accidental bequests, transfers, and pension payments are obtained aggregating over the model economy households as follows:

\[
K_{t+1} = \int k_t' d\mu_t
\]

\[(23)\]

\[
I_t = \int \pi_t w_t d\mu_t
\]

\[(24)\]

\[
T_{n,t} = \int \tau_{n,t}(y_t) d\mu_t
\]

\[(25)\]

\[
P_t = \int (b_t | b_{d,t}) d\mu_t
\]

\[(26)\]

\[
T_i = \int \{\tau_{\ell, i} e_t + \tau_{k, i} r_t e_t + \pi_{t, x} [y_t - \tau_{s, i}(y_t)]\} d\mu_t
\]

\[(27)\]

\[
Z_t = \int z_t d\mu_t
\]

\[(28)\]

\[
E_{t+1} = \int (1 - \psi_t(j))(1 - r_t) a_t' d\mu_t
\]

\[(29)\]

where all the integrals are defined over the state space \( \mathcal{F} \).

(ii) The government policy satisfies the law of motion of the pension system fund described in expression (2) and the government budget constraint described in expression (3).

(iii) Given, \( K_t, L_t, A_t \), and the government policy, the household policy solves the households' decision problems defined in expressions (8) through (20), and factor prices are the factor marginal productivities defined in expressions (21) and (22).

\[\footnote{We drop the first subscript whenever there are no differences between immigrants and natives.}\]
(iv) The goods market clears:

$$\int_{\mathbb{R}} c_t d\mu_t + K_{t+1} + C_t = F(K_t, A_t L_t) + (1 - \delta) K_t.$$  \hspace{1cm} (30)

(v) The law of motion for \( \mu_t \) is:

$$\mu_{t+1} = \int_{\mathbb{R}} Q_t d\mu_t.$$  \hspace{1cm} (31)

Describing formally function \( Q \) is complicated because it specifies the transitions of the measure of households along its six dimensions. An informal description of this function is the following: since the flows of immigrants are exogenous to the model economy, the evolutions of the first dimension of \( \mu, \ell \), is also exogenously given. The evolution of the second dimension, age, is described in expressions (5) and (6). The evolution of the third dimension, education, is implied by the educational shares of immigrants and native new-entrants, both of which are given exogenously. The evolution of the fourth dimension, the employment status, is governed by the conditional transition probability matrix, \( \Gamma_{\omega t'} \), the probability of becoming disabled, the optimal decision to retire early and the compulsory retirement at age 65. We assume that both immigrants and natives enter the economy as able workers, with no assets, and that their shares are given by the invariant distribution of process \( \{\omega\} \). The evolution of the fifth dimension, the asset holdings, is determined by the optimal savings decision. Finally, the evolution of the sixth dimension, the pension entitlements, is determined by the rules of the Spanish public pension system.

4 Calibration

The purpose of this paper is to evaluate the consequences of the demographic and educational transitions of the Spanish economy for the viability of the pension system. To carry out this purpose, we use the following calibration strategy: First, we choose 1997 as our calibration target year. In this year the main demographic, educational and economic statistics of our model economy should replicate as closely as possible the corresponding statistics of the Spanish economy. Then we choose an initial steady state, which we identify with the year 1950.\(^8\)

The educational transition starts in 1951 and the demographic transition starts in 1998, and they both end in 2131, when the age and educational distribution of the population becomes time invariant. The age and education dynamics of our model economy is completely independent from its economic dynamics. In the subsections that follow we discuss these two dynamic behaviors in turn.

\(^8\)The choice of the initial steady-state is somewhat arbitrary. We chose 1950 because it seems a reasonable starting year for the Spanish educational transition, and because it is a round number.
4.1 The population dynamics

In our model economy, the population dynamics is completely determined by the joint age and educational distribution of immigrants and by the survival probabilities and fertility rates of both immigrants and natives.\footnote{Whenever the fertility rates are not available, we use the population growth rates as an alternative way to determine the numbers of native newborns.} This should make our calibration task easy because, in principle, all these numbers can be obtained from demographic observations and projections. Unfortunately, a full set of Spanish data is not readily available, and this forces us to make some additional assumptions.

The Spanish demographic statistics that our model economy replicates are the following: the share of immigrants in the total population of the year 1996, the age distribution of immigrants of the year 1999 and the total flows of immigrants estimated for the years 1998-2001 and projected for the years 2002-2050 expressed as shares of the total population; the survival probabilities of the year 1998; the age distribution of fertility rates of all residents of the year 2004; the old-age dependency ratios reported for the years 1997-2004 and projected for the year 2050; the expected life-times reported the year 1998 and projected for the year 2050.\footnote{The source for all these data is the INE. Of the two hypotheses that the INE considers when making its projections, we chose the high immigration, high life-expectancy hypothesis (Hypothesis 1).}

Education complicates the population dynamics further. Specifically, we calibrate the educational transition in our model economy so that it replicates the educational distribution of working-age people in Spain estimated by Meseguer (2001) for the year 1997 and his projections for the year 2050. In the subsections below we describe the demographic and the educational transitions in detail.

4.1.1 The age distribution dynamics

To specify the model economy’s age distribution dynamics we must first choose the maximum life-time for its households, $\mathcal{J}$. To choose this number we find the maximum age that, given the Spanish survival probabilities for the year 1998, allows our model economy to replicate the Spanish expected life-time conditional on being alive at age 20 for that year. According to the Tablas de Mortalidad published by INE, this number was 79.4 years. In our model economy we choose $\mathcal{J} = 100$ and the expected lifetime is 79.4 years.

Once we have chosen the maximum life-time, the age distribution dynamics in our model economy are the following:
1950–1997: During this period the age distribution of the population in the model economy is time invariant. To compute this distribution we assume that the survival probabilities of all residents do not change and that they take the values reported by the INE for 1998. Given these survival probabilities, we find the constant population growth rate that implies that the old-age dependency ratio of the model economy in 1997 is 26.5 percent which is the value reported by the INE for the Spanish economy.\textsuperscript{11} This population growth rate is \( n_0 = 0.0104 \). The survival probabilities, the population growth rate and the requirement that the shares of the population must add up to one allow us to compute the invariant measure of 20 year-old and, therefore, the invariant age distribution of the total population.

To find the age distributions of immigrants and natives, we do the following: first we assume that the age distribution of the immigrants is time invariant and that it takes the values reported by the INE for 1999;\textsuperscript{12} next, we assume that the immigrants represent a time-invariant share of the total population equal to 1.47 percent, which is the number reported by the INE for the Spanish economy for 1996;\textsuperscript{13} finally, we find the age distribution of the native population by subtracting the age distribution of immigrants from the age distribution of the total population.

Initial share of immigrants. We also target the share of immigrants within the model economy population to be 1.47 percent between 1950 and 1997. The rationale for this choice is the following: According to INE, in 1996 there were 445,530 immigrants in the 20+ cohort in Spain. For the same year, Spanish population in the 20+ cohort was 30,176,449. This way we obtain the figure 1.47.

1998–2050: During this period, the age distribution of the population changes. These changes arise because the flows of immigrants change, and the survival probabilities and the fertility rates of both immigrants and natives also change. We discuss each of these changes in turn.

- Flows of immigrants. The flows of immigrants expressed as shares of the total population are taken directly from the data published by the INE in the Encuesta de Migraciones (1999). They are estimated for the period 1998-2001 and they are projected for the period 2002–2050 using the high immigration hypothesis (Hypothesis 1). As far as the age distribution of the immigrants is concerned, we assume that it does not change and that it takes the value reported by the INE for 1999 (see Footnote 12 above).

\textsuperscript{11}According to the Encuesta de la Población Activa, in 1997 in Spain there were 6,382,809 people in the 65+ cohort and 24,669,372 people in the 20-64 age cohort. The ratio of these two numbers is 26.5 percent which is the old-age dependency ratio that we target.

\textsuperscript{12}Specifically, in the Encuesta de Migraciones (1999) the INE reports the age distribution of immigrants for the 20-29, 30-44, 44-59 and over-59 age cohorts. We replicate these numbers in our model economy and we assume further that the age distribution is uniform within each cohort.

\textsuperscript{13}Notice that to keep the share of immigrants in the total population time-invariant we must assume that the total flow of immigrants grows at the population growth rate.
• Survival probabilities. We assume that the age dependent survival probabilities grow linearly between 1998 and 2050. The values for 1998 are those reported by the INE.\textsuperscript{14} To compute the survival probabilities in 2050 we solve the following system of equations:
\begin{align}
\psi_{2050}(j) &= \psi_{1998}(j) + a_1 \exp a_2 j \quad \text{for each } j = 20, 21, \ldots, 90 \\
\psi_{2050}(70) &= \psi_{1998}(70) + 0.05 \\
E_{2050} &= 83.9
\end{align}
(32)
where \(E\) denotes the expected lifetime and 83.9 is the value projected by the INE for the Spanish economy for the year 2050 under the high expected life-time population hypothesis (Hypothesis 1). Notice that these choices imply that the growth rates of the survival probabilities increase exponentially with age. We make this assumption because we think that most of the growth in the Spanish life-expectancy can be attributed to the increase in the survival probabilities of older people. The values of parameters \(a_1\) and \(a_2\) that solve system (32) are \(a_1 = 0.0006\) and \(a_2 = 0.0901\), and the expected lifetime in the year 2050 in our model economy is 83.9 years.

• Fertility rates. Between 1998 and 2003 the model economy fertility rates are undetermined. Instead, given the survival probabilities and the age distribution of immigrants, we find the numbers of 20 year-old natives that allow our model economy to replicate the old-age dependency ratios reported by the INE for these years for the Spanish economy. In 2004 we take the age dependent fertility rates of our model economy from the values reported by the INE for that same year for the Spanish economy. During the 2005–2050 period, we assume that the fertility rates increase linearly as follows:

\[
 f_t(j) = \begin{cases} 
 (1 + a_3) f_{t-1}(j) & 2005 \leq t \leq 2018 \\
 (1 + a_4) f_{t-1}(j) & 2019 < t < 2050 \\
 f_{t-1}(j) & t > 2050
\end{cases}
\]  
(33)
where the vector \(f_{2004}(j)\) takes the values reported by the INE.\textsuperscript{15} To find the values of \(a_3\) and \(a_4\), we do the following. Since we expect most of the change in Spanish fertility rates to occur in the early part of the period, we arbitrarily assume that from 2019 to 2050 that the yearly increase is 0.5 percent for all ages and, consequently, that \(a_4 = 0.005\). Given this value for \(a_4\), we compute the value for \(a_3\) that implies that the old-age dependency ratio in our model economy in 2050 is 0.50, which is the value projected by the INE for that same year for the Spanish economy. The value that achieves this target is \(a_3 = 0.015\).

2051–2131: During this period, the age distribution of the population is still changing, even though the flows of immigrants, the fertility rates of natives and the survival probabilities no

\textsuperscript{14} The data can be found at www.ine.es/inbase/cgi/um?M-%2Ft%2Fp%39%2Ft%2F1998%2FO--pcaxisN--L--0.

\textsuperscript{15} The data can be found at www.ine.es/inbase/cgi/um/M-%2Ft%2Fp%39%2Ft%2F2018%2Fp--incbaseN--L--1.
longer change. This is because it takes 81 years for the age distribution of the population to become time invariant and, in the meantime, the numbers of 20-year old natives and the total flows of immigrants change, even though the shares of the immigrants in the total population remain invariant.

2132 \( \infty \): In year 2132 the age distribution of the population in our model economy population becomes time invariant.

4.2 Education Dynamics

To specify the education dynamics in our model economy, we also had to deal with the scarcity of Spanish data. As we have already mentioned, our source for these data is Messeguer (2001) who reports that in 1997, 24.0 percent of the working-age people in Spain had completed their high school studies and 13.4 percent had completed college. He also reports that these numbers are projected to be 38.8 percent and 24.1 percent in 2050. Since we have no other data, we assume that these shares evolve linearly between 1997 and 2050. Next, we project the linear trend backwards, and we obtain the shares for 1950 to be 7.7 percent and 2.8 percent.

Formally, the shares of the educational groups in our model economy evolve according to the following equation:

\[
i_{t+1}(h) = i_t(h) + \eta(h)
\]

(34)

Since we have classified the model economy households into three education groups, to characterize the education dynamics we must choose the values of a total of six parameters which we report in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>( h = 1 )</th>
<th>( h = 2 )</th>
<th>( h = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( i_0(h) )</td>
<td>0.8956</td>
<td>0.0765</td>
<td>0.0279</td>
</tr>
<tr>
<td>( \eta(h) )</td>
<td>0.0057</td>
<td>0.0034</td>
<td>0.0022</td>
</tr>
</tbody>
</table>

Table 2: The Educational Transition Function

To obtain the educational shares of the immigrants, we use the \textit{Censo de Población y Vivienda de 2001} published by the INE. It reports that, in the year 2001, 22.2 percent of the immigrants living in Spain at the time had completed high school and that 18.5 percent had completed college. Since we have no other source of data, we assume that these shares are time invariant and that they are uniformly distributed across ages. Consequently, we assume that every year

\footnote{During this period the flow of immigrants is 0.483 percent of the total population which is the value reported by the INE for the year 2050 under population Hypothesis 1.}
22.2 percent of the immigrants of every age have completed high school and that 18.5 have completed college. These assumptions and the demographic transition described above imply that the educational transition in our model economy is the following:

1951–2005: During this period, the educational shares of native 20-year-old change every year and these changes are transmitted gradually to the older population. For instance, the educational shares of 21-year-old natives change in 1952, of 22-year-old in 1953 and so on. Since in any given period we know the age distribution of both immigrants and natives, and the educational distribution of 20-year-old immigrants, computing the educational shares of the 20-year-old natives that are needed to replicate the estimated shares in the total population is straightforward.

2006–2050: Since the educational shares of native 20-year-olds become time invariant in 2005, the shares of native 21-year-olds become invariant in 2006, the shares of native 22-year-olds become invariant in 2007, and so on until the year 2050 when the entire educational distribution of working-age natives is time invariant.\footnote{Recall that in our model economy the working-life lasts for 45 years and retirement lasts for 36 years.}

2051–2131: During this period the educational transition is completed. The flow of immigrants becomes time invariant in 2050. This implies that it takes an additional 45 years for the educational distribution of the total working-age population to become time invariant, and an additional 36 years for the entire educational distribution to become time invariant.

2132–\infty: In 2132, both the demographic and the educational transitions are completed. Consequently, the educational distribution of the total population is time invariant from year 2132 onwards.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Spain</td>
<td>26.5</td>
<td>26.4</td>
<td>26.8</td>
<td>27.1</td>
<td>27.2</td>
<td>27.1</td>
<td>26.9</td>
<td>26.6</td>
<td>59.9</td>
</tr>
<tr>
<td>Model</td>
<td>26.5</td>
<td>26.4</td>
<td>26.8</td>
<td>27.1</td>
<td>27.2</td>
<td>27.1</td>
<td>26.9</td>
<td>26.6</td>
<td>59.3</td>
</tr>
</tbody>
</table>

4.3 The model economy in 1997

Once we have described the population dynamics we must choose specific forms for various functions that describe our model economy and we must choose specific values for their pa-
4.3.1 Functional forms and parameters

**Pensions.** To characterize the public pension system, we must choose the functional form for the social security tax function, \( \tau_s() \), the minimum and maximum retirement pensions, \( \bar{b}_i \) and \( \bar{b}_f \), the number of years of contributions used to compute the retirement pensions, \( N_p \), the pension replacement rate, \( \phi \), the age dependent penalties for early retirement, \( \lambda_j \), the value of the disability pension, \( b_{di} \), the initial value of the pension fund, \( F_0 \), and the exogenous rate of return earned by the pension fund assets, \( r^* \).

The Spanish payroll tax is a capped proportional tax. To replicate these properties we use the following two-parameter function:

\[
\tau_s(y_t) = \alpha_5 - \left[ \alpha_5 (1 + \alpha_6 y_t)^{-\alpha_6} \right]
\]  (35)

Parameter \( \alpha_5 \) determines the payroll tax cap and parameter \( \alpha_6 \) the payroll tax rate. Figure 3 represents this function for our chosen values of \( \alpha_5 \) and \( \alpha_6 \) (see below).

The Spanish *Régimen General de la Seguridad Social*, establishes that the penalties for early retirement are a linear function of the retirement age. To replicate this rule, our choice for the penalty function is the following

\[
\lambda(j) = \begin{cases} 
\lambda_0 - \lambda_1 (j - 60) & \text{if } j < 65 \\
0 & \text{if } j \geq 65
\end{cases}
\]  (36)
**Figure 3:** The model economy payroll tax function

**Government revenues and outlays.** To characterize the government revenues and outlays, we must choose the values of the labor income tax rate, $\tau_l$, of the capital income tax rate, $\tau_k$, of the consumption tax rate, $\tau_c$, and of the time-invariant government consumption, government transfers and government debt shares of output, $G, Z,$ and $D$. Therefore, to characterize the government policy completely we must choose the values of a total of 17 parameters.

**Figure 4:** The deterministic component of the endowment of efficiency labor units process

**Deterministic component of the endowment of efficiency labor units process.** We assume that the deterministic component of the efficiency labor units profiles is governed by
functions of the following form:

\[ \epsilon(j, h) = \alpha_{h0} + \alpha_{h1}j - \alpha_{h2}j^2 \]  

(37)

This functional form captures the concavity workers’ productivity profiles over their life-cycle in a very parsimonious way (see Figure 4). Since we consider three educational levels, to characterize these functions we must choose the values of nine parameters.

**Stochastic component of the endowment of efficiency labor units process.** We assume that the stochastic component of the endowment of efficiency labor units process, \( \{\omega\} \), takes three values, that is, we assume that \( m_\omega = 3 \). We make this choice because we want to keep the process on \( \omega \) as parsimonious as possible, and because it turns out that three states are sufficient to account for the Lorenz curves of the Spanish distributions of income and labor earnings in very much detail. These choices imply that, to characterize the process on \( \omega \), we must choose the values of 12 parameters: its three values and the nine conditional transition probabilities of matrix \( \Gamma_{\omega\omega'} \).

**Figure 5:** The Probability of Becoming Disabled (%)

![Graph showing the probability of becoming disabled by age and education level](image)

**Disability.** We assume that the conditional probabilities of becoming disabled at age \( j + 1 \) are governed by functions of the following form:

\[ \varphi(j, h) = \xi_{h} \psi(r_{j}, \theta_{j}) \]  

(38)

We make this choice because, according to the Boletín de Estadísticas Laborales, the number of disabled people in Spain increases more than proportionally with age, and because the
number of disabled households differs significantly across educational types (see Figure 5). To characterize these functions, we must choose the values of five parameters.\footnote{The data on disability can be found at www.mtax.cs.cstadisticas/DEL/India.htm.}

Preferences. Our choice for the households’ common utility function is:

\[
u(c_j, (1 - l_j)) = [c_j^{\gamma} (1 - l_j)^{(1 - \gamma)}]^{1 / \sigma} / (1 - \sigma)
\]

Therefore, to characterize the household preferences we must choose the values of three parameters, the consumption share, \(\gamma\), the concavity of the utility function, \(\sigma\), and the time discount factor, \(\beta\).

Technology. We choose a standard Cobb-Douglas aggregate production function, \(Y_t = A_0 K^{\delta} L^{1-\theta}\). Consequently, to determine the production technology, we must choose the values of four additional parameters: the capital income share, \(\theta\), the depreciation rate, \(\delta\), the initial value of the labour augmenting productivity factor, \(A_0\), and the productivity growth rate, \(\rho\).

Adding up. To characterize our model economy fully, we must choose the values of a total of 50 parameters. Of these 50 parameters, 17 describe the government policy, 21 describe the endowment of efficiency labor units profiles, 5 describe the disability risk function, 3 describe the household preferences, and the remaining 4 describe the production technology.

4.3.2 Targets

We choose 1997 as our calibration target year. This is because the data on two of our main calibration targets, namely the Lorenz curves of the Spanish income and earnings distributions, are from that year.

Pensions. We start describing our targets for the pension system.

- Social security tax function. In 1997 in Spain, the payroll tax rate paid by households was 28.3 percent and it was levied only on the first 32,330 euros per annum gross labor income. Hence, the maximum contribution was 9,149 euros which correspond to 73 percent of the Spanish per capita GDP. To replicate this number, in our model economy we choose \(a_s = 0.73\bar{y}_t\) in the payroll tax function described in expression (35), where \(\bar{y}_t\) denotes average output in the model economy. To select a value for parameter \(a_s\) in that same expression we require that the revenues levied by the payroll tax in the model economy match the corresponding revenues
in the Spanish economy which in 1997, according to the Boletín de Estadísticas Laborales, amounted 11.08 percent of Spanish GDP.

- **Minimum and maximum retirement pensions.** The Régimen General de la Seguridad Social establishes various minimum retirement pensions that vary with the personal and economic circumstances of the recipient. In 1997, the minimum retirement pensions ranged from 768 to 5,427 euros per year. In our model economy we assume that the minimum retirement pension is 30 percent of the per capita model economy output, so we set \( b_i = 0.30 \bar{y}_t \). Notice that this value corresponds to a minimum retirement pension of 3,477 euros per annum in the Spanish economy.

In 1997 the maximum retirement pension payed by the Régimen General was 23,912 euros. This number is approximately 1.91 times of the Spanish per capita GDP. Therefore, in our model economy we make \( \bar{y}_t - 1.91 \bar{y}_t \).

- **Number of years of contributions.** The Spanish Régimen General de la Seguridad Social considers the last 15 years of contributions prior to retirement to compute the pension. Consequently, the number of years that we target in our model economy is \( N_0 = 15 \).

- **Replacement Rate.** We choose the replacement rate of our model economy (parameter \( \phi \) expression (1)) so that total expenditure in both retirement and disability matches the corresponding number in the Spanish economy which, according to the Boletín de Estadísticas Laborales (2001), in 1997 amounted 10.10 percent of Spanish GDP.

- **Penalties for early retirement.** The Régimen General de la Seguridad Social, establishes that earliest retirement age is 60 and that the penalty for early retirement is 8 percent per year prior to age 65. Consequently, the maximum retirement penalty is 40 percent. These two targets determine the values of \( \lambda_0 \) and \( \lambda_1 \) in expression (36)).

- **Disability pensions.** The Spanish Social Security establishes several kinds of disability pensions. According to the Boletín de Estadísticas Laborales (2001), in 1997 the annual average permanent disability pension was 6,277 euros, which corresponds to around 50 percent of the Spanish per capita GDP in the same year. Consequently, we target \( b_{d,t} = 0.50 \bar{y}_t \).

- **Pension system fund.** The Spanish public pension system fund received its first revenues in the year 2000. According to Balmaseda et al. (2005), from 2000 to the end of 2004 a total of 19,330 million euros were invested in the fund. This amount corresponds to 2.5 percent of Spanish GDP. Since the model economy fund starts in 2005, this is the fund’s initial value that we target. For the rate of return on the fund’s assets we target \( r^f - 0.04 \).
Government revenues and outlays. To calibrate the government sector in our model economy we try to replicate as closely as possible the items of the 1997 Spanish Government Budget described in Table 4. Therefore, our task is to allocate the different revenue and expenditure items reported in that Table to the model economy tax instruments and government outlay items.

Table 4: Tax Revenues and Public Expenditures in 1997

<table>
<thead>
<tr>
<th>Revenues</th>
<th>%GDP</th>
<th>Expenditures</th>
<th>%GDP</th>
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<tbody>
<tr>
<td>Social Contributions</td>
<td>11.08</td>
<td>Consumption</td>
<td>17.53</td>
</tr>
<tr>
<td>Individual Income Taxes</td>
<td>7.35</td>
<td>Cross Investment</td>
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<tr>
<td>Production Taxes</td>
<td>5.42</td>
<td>Pensions</td>
<td>10.10</td>
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<tr>
<td>Sales and Gross Receipts</td>
<td>5.03</td>
<td>Debt Services</td>
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</tr>
<tr>
<td>Corporate Profit Taxes</td>
<td>2.75</td>
<td>Other Transfers</td>
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<tr>
<td>Estate Taxes</td>
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<td>Other Expenditures</td>
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<tr>
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<td>Other Revenues</td>
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<tr>
<td>Total Revenues</td>
<td>38.62</td>
<td>Total Expenditures</td>
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</tr>
<tr>
<td>Deficit</td>
<td>3.09</td>
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</tr>
</tbody>
</table>

Source: National Accounting reports (INE), and Boletín de Estadísticas Laborales 2001

- Labor income tax. We choose the model economy proportional labor income tax rate so that the revenues obtained from this tax instrument in the benchmark model economy match the labor income tax revenues in the Spanish economy. According to the Spanish Dirección General de Tributos, labor income tax revenues amounted to 79.22 percent of the individual income tax revenues in 1997.\(^{19}\) Since the total individual income tax revenues amounted to 7.35 percent of Spanish GDP that year, the 79.22 percent of the individual income tax revenues is equivalent to 5.82 percent of the Spanish GDP. In addition, we require that this tax instrument collects another 3.0 points in our model economy. The rationale for this choice comes from the fact that social contributions, apart from those collected by the Spanish public pension system, represented 3 percent of the Spanish GDP in 1997. Consequently, we choose the model economy labor income tax rate so that it levies 8.82 percent of the model economy output.

- Capital income tax. We choose the model economy proportional capital income tax rate so that it replicates the Spanish average capital income tax. According to Boscá et al. (1999) this number is 18.7 percent. Therefore, we target \(\tau_K = 0.187\).

- Consumption taxes. We choose the proportional consumption tax rate, \(\tau_c\), so that the

\(^{19}\)The data on income tax revenues is available at www.meh.es/Portals/Topics/Impuestos.
government in the model economy balances its budget as described in equations (3).\textsuperscript{20}

- **Other transfers.** We target a value for the model economy’s aggregate transfers to output ratio, $Z/Y$, of 5.41 percent. This value corresponds to the 1997 Spanish GDP share of transfers other than retirement and disability pensions.

- **Public Debt.** According to the Instituto de Estudios Fiscales (2004) the 1997 ratio of Spanish Public Debt to GDP was 66.7 percent. Consequently, this is the number that we choose for the time invariant public debt to output ratio of our model economy.

- **Government Consumption.** We want our model economy to replicate the total share of government outlays in the Spanish GDP. In 1997 this number was 41.71 percent. Hence, we target the ratio of government expenditures to output in the model economy to be the difference between this number and the sum of the rest of the government outlay items.

The various choices described above give us a total of 17 targets.

**Endowment of efficiency labor units process.** We want the deterministic component of the efficiency units profiles of the educational groups in our model economy, $\psi(j, h)$, to approximate the corresponding profiles reported by the INE in the Encuesta de Salarios en la Industria y los Servicios (2000) for the Spanish economy. Since we approximate these empirical profiles with quadratic functions, the data allows us to determine the values of the nine ($\alpha_{h,0}$, $\alpha_{h,1}$, $\alpha_{h,2}$) parameters of equation (37) and, hence, we have 9 additional targets.

**Disability.** According to the INE, in 2002, in Spain, 80.9 percent of the total number of people who claimed to be disabled had not completed high school, 10.4 percent had completed high school, and the remaining 8.7 percent had completed college. We use these shares to determine the values for $\xi_b$ of equation (37). Moreover, according to the Boletín de Estadísticas Laborales, in 2001, 3.72 percent of the people in Spain in the 20-64 age cohort were receiving a permanent disability pension. To replicate this number, we set $\phi_0 = 0.0014$ and $\phi_1 = 0.0382$ in that same equation. These choices give us 4 targets.

**Preferences.** According to Encuesta sobre el tiempo de trabajo published by the INE, the average number of hours worked per worker in 1996 in Spain was 1,648.\textsuperscript{21} If we consider the endowment of disposable time to be 14 hours per day, the total amount of disposable time is 5,110 hours per year. Dividing 1,648 by 5,110 we obtain 32.2 percent which is the share of

---

\textsuperscript{20}Recall that in our model economy the government confiscates unintentional bequests which are an additional source of government revenue.

\textsuperscript{21}This data is available at www.ine.es/inbase/cgi/un?M = %2Ft22%2Fp186&0 = inbase&N = &L =.
disposable time allocated to working in the market that we target. Next, for the coefficient of the curvature of the utility function we choose a value of $\sigma = 2$. This choice is pretty much standard in the literature. These restrictions on preferences give us 2 additional targets.

**Technology.** Zabalza (1996) reports that 0.375 is the capital income share for the Spanish economy, and this is the value that we target for the capital income share of our model economy. Balmaseda et al. (2005), report that the average labor productivity growth rate in Spain for the period 1988–2004 was 0.6 percent, and this is our target for the growth rate of total factor productivity in our model economy. These choices give us another 2 targets.

**Macroeconomic aggregates.** We still have to choose the targets for the model economy capital to output and investment to output ratios. According to BBVA database, in 1997 the value of the Spanish private capital stock was 631,430 million 1986 euros.\textsuperscript{22} According to INE, in 1997 the Spanish Gross Domestic Product was 263,792 million 1986 euros. Dividing these two numbers, we obtain 2.38, which is our target value for the model economy capital to output ratio. For the investment to output ratio we target a value of $1/Y = 18.80$ percent. This is the value reported by the INE for gross private investment in 1997. These choices give us 2 additional targets.

**The distributions of earnings and income.** We target the two Gini indexes and six points of the Lorenz curves of the Spanish distributions of earnings and income as reported by Budría and Díaz-Ciménez (2006) for 1997 (see Table 9). Therefore, we have 8 additional targets.

**Normalization conditions.** Altogether we have six normalization conditions. First, since the transition probability matrix on the stochastic component of the endowment of efficiency labor units is a Markov matrix, its rows must add up to one. This property imposes three normalization conditions. Second, we normalize the first realization of this process to be $\omega(1) = 1$. Third, we choose the initial value of the total factor productivity to be $A_0 = 1$. Finally, we require that $\sum_{t=1}^{3} \xi_t = 1$ in expression (38). Therefore, the normalization conditions give us 6 additional targets.

**Adding up.** Notice that we have specified a total of 50 targets. Of these 50 targets, 17 are related to the government policy, 9 to the deterministic component of the endowment of efficiency labor units process, 4 to the disability risk function, 2 are related to the household

\textsuperscript{22}This data can be found at http://w3.grouppbva.com/TLFR/TLFindex.htm.
Table 5: Values for the Model Economy Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Public Pension System</strong></td>
<td></td>
</tr>
<tr>
<td>Payroll tax cap</td>
<td>$\kappa_5$ 1.5267</td>
</tr>
<tr>
<td>Payroll tax rate</td>
<td>$\kappa_6$ 0.0726</td>
</tr>
<tr>
<td>Maximum early retirement penalty</td>
<td>$\lambda_0$ 0.4000</td>
</tr>
<tr>
<td>Yearly early retirement penalty</td>
<td>$\lambda_1$ 0.0800</td>
</tr>
<tr>
<td>Minimum retirement pension</td>
<td>$b_2$ 0.6219</td>
</tr>
<tr>
<td>Maximum retirement pension</td>
<td>$b_5$ 3.9785</td>
</tr>
<tr>
<td>Replacement rate</td>
<td>$\phi$ 0.5051</td>
</tr>
<tr>
<td>Number of years of contributions</td>
<td>$N_b$ 15</td>
</tr>
<tr>
<td>Disability pension</td>
<td>$b_{d,t}$ 1.0475</td>
</tr>
<tr>
<td>Initial value of the pension fund</td>
<td>$F_0/Y$ 0.0250</td>
</tr>
<tr>
<td>Pension fund rate of return</td>
<td>$r^*$ 0.0400</td>
</tr>
<tr>
<td><strong>Government Revenues and Outlays</strong></td>
<td></td>
</tr>
<tr>
<td>Labor income tax rate</td>
<td>$\tau_l$ 0.1713</td>
</tr>
<tr>
<td>Capital income tax rate</td>
<td>$\tau_k$ 0.1870</td>
</tr>
<tr>
<td>Consumption tax rate</td>
<td>$\tau_c$ 0.2480</td>
</tr>
<tr>
<td>Government consumption</td>
<td>$G/Y$ 0.2059</td>
</tr>
<tr>
<td>Government transfers</td>
<td>$Z_t/Y$ 0.0541</td>
</tr>
<tr>
<td>Government debt</td>
<td>$D/Y$ 0.6670</td>
</tr>
<tr>
<td><strong>Preferences</strong></td>
<td></td>
</tr>
<tr>
<td>Time Discount Factor</td>
<td>$\beta$ 0.9798</td>
</tr>
<tr>
<td>Consumption Share</td>
<td>$\gamma$ 0.3730</td>
</tr>
<tr>
<td>Relative Risk Aversion</td>
<td>$\sigma$ 2.0000</td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td></td>
</tr>
<tr>
<td>Labor Share</td>
<td>$\theta$ 0.3750</td>
</tr>
<tr>
<td>Capital Depreciation Rate</td>
<td>$\delta$ 0.0782</td>
</tr>
<tr>
<td>Global factor productivity</td>
<td>$A_0$ 1.0000</td>
</tr>
<tr>
<td>Productivity Growth Rate</td>
<td>$\rho$ 0.0060</td>
</tr>
<tr>
<td>Probability of becoming disabled</td>
<td></td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>0.8000</td>
</tr>
<tr>
<td>$\xi_2$</td>
<td>0.1010</td>
</tr>
<tr>
<td>$\xi_3$</td>
<td>0.0870</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>0.0014</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.0382</td>
</tr>
</tbody>
</table>
preferences, 2 to the production technology, 2 are macroeconomic aggregates, 8 target distributional statistics and the remaining 6 are normalization conditions. The 50 parameters and 50 targets define a full rank system of 50 equations in 50 unknowns.

4.3.3 Choices

We obtain values of some of the model parameters directly because they are determined uniquely by one of our targets. In this fashion, we choose \( \sigma = 2, \rho = 0.006, \) and \( \theta = 0.375. \) We obtain the values for parameters \( \lambda_0 \) and \( \lambda_1 \) of the early retirement penalty function described in expression (36) from the rules of the Régimen General de la Seguridad Social. We obtain the number of years of contributions that are taken into account to compute the retirement pensions, \( N_h = 15 \), from the same source.

Similarly, the quadratic approximations to the empirical productivity profiles, allow us to obtain the nine values for parameters \( (\alpha_{h,1}, \alpha_{h,2}, \alpha_{h,3}) \) in expression (34). We obtain the value for the capital income tax rate \( \tau_r = 18.7 \) per cent from Bosca et al. (1999). The values of the three parameters \( \xi_{h,0}, \xi_{h,1} \) of \( \psi_0, \) and \( \psi_1 \) of expression (38) were obtained directly from the INE. We arbitrarily chose \( \lambda_0 = 1 \) and \( r^* = 0.04. \)

We chose the initial value of the pension fund to be 2.5 percent of the model economy output directly from Balmaseda et al. (2005). Finally, the normalization of the endowment of efficiency labor units implies that \( \omega(1) = 1.0. \)

**Table 6: The Deterministic Component of the Endowment Process**

<table>
<thead>
<tr>
<th></th>
<th>( h - 1 )</th>
<th>( h - 2 )</th>
<th>( h - 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{h,0} )</td>
<td>0.8523</td>
<td>0.6260</td>
<td>0.3950</td>
</tr>
<tr>
<td>( \alpha_{h,1} )</td>
<td>0.0821</td>
<td>0.1800</td>
<td>0.3010</td>
</tr>
<tr>
<td>( \alpha_{h,2} )</td>
<td>-0.0011</td>
<td>-0.0029</td>
<td>-0.0046</td>
</tr>
</tbody>
</table>

The choices enumerated so far allow us to determine the values of 25 out of the 50 model economy parameters. To determine the values of the remaining 25 parameters we use the procedure described in Castañeda, Díaz-Giménez and Ríos-Bull (2001), and we solve the system of 25 non-linear equations in 25 unknowns obtained from imposing that the relevant statistics of the model economy should be equal to the corresponding targets. Solutions for these systems are not guaranteed to exist and, when they do exist, they are not guaranteed to be unique.

---

23 We also ran simulations \( r^* = 0.01, r^* = 0.02 \) and \( r^* = 0.03. \) The only results that vary with \( r^* \) are the values of the pension fund and these changes do not change in any way the conclusions of this article.

24 Actually we solved a smaller system of 13 non-linear equations in 13 unknowns because our guesses for the values of aggregate capital and aggregate labor uniquely determine the values of \( a_1, b_1, b_2, x, z, D, \) and \( \tau, \) because the value of \( C \) is determined residually from the total government outlays target, because the value of \( \tau, \) is determined residually from the government budget constraint, and because the normalization of the matrix \( \Gamma_{w,0} \) allows us to determine the values of three of the transition probabilities directly.
Consequently, we tried many different initial values in order to find the best parameterization possible. We report the numerical choices for the 29 model economy parameters in Table 5, for 9 in Table 6 and for the remaining 12 in Table 7. In Section 5 below and we discuss the results of our calibration exercise.

<table>
<thead>
<tr>
<th>Table 7: The Stochastic Component of the Endowment Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>$\omega = 1$</td>
</tr>
<tr>
<td>$\omega = 2$</td>
</tr>
<tr>
<td>$\omega = 3$</td>
</tr>
</tbody>
</table>

$\pi^*(\omega)^2$ denotes the invariant distribution of $\omega$.

5 Calibration results

5.1 The stochastic component of the endowment process

The procedure used to calibrate our model economy identifies the stochastic component of the endowment of efficiency labor units process. Since this is an important feature of our model economy we start off this section describing its main properties which we report in Table 7. We find that to replicate the Spanish Lorenz curves of the income and earnings distributions in our model economy, the differences in the realizations of $\omega$ need not be very large. Specifically, the highest realization is only 3.2 times the lowest realization of the process (see the first column of Table 7). In the next three columns of that table, we report the conditional transition probabilities of the process. We find that the process is not persistent at all. Specifically, the expected durations of the shocks are 1.3, 1.5, and 1.0 years respectively. The last column of the table reports the invariant distributions of the shocks. We find that approximately 99 percent of the workers are in states $\omega = 1$ and $\omega = 2$ and that only one percent is in state $\omega = 3$.

5.2 Aggregates and ratios

We report the values of our aggregate targets for Spain and for the benchmark model economy in Table 8. We find that every ratio is very similar in Spain and in the model economy. In our model economy the only source of government revenues that we do not report in that table is the unintentional bequests, $E$, which amount to 3.6 percent of $Y$. In Spain every source of government revenues reported in Table 4 is accounted for.
Table 8: Macroeconomic Aggregates and Ratios in 1997 (%)

<table>
<thead>
<tr>
<th>Spain</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>18.8</td>
<td>2.38</td>
<td>32.2</td>
<td>20.6</td>
<td>10.1</td>
<td>5.4</td>
<td>4.2</td>
<td>11.1</td>
</tr>
<tr>
<td>19.6</td>
<td>2.38</td>
<td>30.3</td>
<td>20.6</td>
<td>10.4</td>
<td>5.4</td>
<td>5.3</td>
<td>11.1</td>
</tr>
</tbody>
</table>

The $K/Y$ ratio is expressed in natural units and not in percentage terms.

The ratio $INT/Y$ is the ratio of the interest payments on the stock of public debt to GDP.

For the Spanish economy, this ratio is the sum of the revenues levied by the Impuesto sobre la Renta de las Personas Físicas, the Impuesto Sobre Sociedades, and three additional points corresponding to social contributions, apart from those collected by the Spanish public pension system, as reported by the INE. For the model economy, it is the sum of the capital and the labor income tax revenues (see Table 4).

For the Spanish economy, this ratio is the sum of all revenues obtained by the Spanish public sector other than the Impuesto sobre la Renta de las Personas Físicas, the Impuesto Sobre Sociedades, and three additional points corresponding to social contributions. For the model economy, it is the consumption tax revenues (see Table 4).

5.3 Incequality

In Table 9 we report the Gini indexes and selected points of the Lorenz curves of earnings, income and wealth in Spain and in our model economy. Our main finding is that our model economy successfully replicates the Spanish earnings and income distributions in very much detail. If we look at the fine print, we find that earnings is somewhat more unequally distributed in Spain.

On the other hand, we find that wealth is significantly more concentrated in Spain than in our model economy. This result was completely expected for three reasons. First, we have argued elsewhere (see Castañeda et al., 2003) that, in general, overlapping generations economies fail to replicate the large concentrations of wealth observed in the data. Second, in our calibration choices we did not target any of the points of the Lorenz curve of wealth. Finally, the Spanish Survey of Family Finances oversamples the rich and therefore gives a very accurate description of the top tail of the distribution.

5.4 Retirement behavior

Perhaps the single most important feature of the Spanish economy that our model economy should replicate if we are to take its results seriously, is the retirement behavior of Spanish households. To describe this behavior, we use some labor market statistics and the conditional probabilities of retirement.

Average retirement age. We find that our model economy does a good job in accounting for the average retirement age of the Spanish households. Specifically, the average retirement
Table 9: The distributions of earnings, income and wealth in Spain and in the model economy in 1997

<table>
<thead>
<tr>
<th></th>
<th>Bottom Tail</th>
<th>Quintiles</th>
<th>Top Tail</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gini</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td><strong>The Earnings Distributions (%)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.57</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Model</td>
<td>0.53</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td><strong>The Income Distributions (%)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.39</td>
<td>0.0</td>
<td>0.6</td>
</tr>
<tr>
<td>Model</td>
<td>0.39</td>
<td>0.1</td>
<td>0.6</td>
</tr>
<tr>
<td><strong>The Wealth Distributions (%)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.57</td>
<td>-0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Model</td>
<td>0.52</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

*The source of data for the Spanish income and earnings distribution is the 1997 European Community Household Panel as reported in Budría and Díaz Giménez (2006a).

*The source of data for the Spanish income and earnings distribution is the 2004 Encuesta Financiera de las Familias Españolas as reported in Budría and Díaz Giménez (2006b).

age is 60.4 years in Spain and 59.9 years in the model economy. Moreover we find that the average retirement age is increasing in the number of years of education. Specifically, the average retirement ages for non-high school, high school, and college workers are 58.9, 61.3, and 62.5 years. We do not have the corresponding data for the Spanish economy but this increasing relationship seems to be intuitively plausible.

The sixty year old retirees. In 1995 in Spain 29.5 percent of the 60 year old workers choose to retire, and in our model economy this number is 37.7. Of these early-retirees, 67.7 percent receive the minimum pension in Spain and in our model economy this number is 59.6 percent. This significant discrepancy between model and data could be due to features of the retirement decision that are absent from our model economy. As far as the educational distribution of the 60 year-old retirees is concerned, we find that in our model economy the vast majority (81.9 percent) have not completed high school. We also find that most of these households (70.0 percent) receive the minimum pension. In contrast, the shares of the 60 year old retirees who have completed high school and college and receive the minimum pension are very much smaller (13.3 percent and 9.8 percent only).

25The Spanish average retirement age has been computed for both male and female workers, it corresponds to the year 1995 and it is reported in Blöndal and Scarpetta (1997). Every number reported in this section for our model economy corresponds to the year 1997.

26The share of the Spanish 60 year old retirees who receive the minimum pension corresponds to the year 1995 and it is reported in Sánchez-Martín (2003).
Table 10: Distribution of the participation rates in the 60-64 age cohort in 1997 (%)

<table>
<thead>
<tr>
<th></th>
<th>Spain</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>28.1</td>
<td>30.5</td>
</tr>
<tr>
<td>Non-High School</td>
<td>25.9</td>
<td>23.9</td>
</tr>
<tr>
<td>High School</td>
<td>38.5</td>
<td>36.8</td>
</tr>
<tr>
<td>College</td>
<td>57.7</td>
<td>60.2</td>
</tr>
</tbody>
</table>

*The Spanish data is the average of the four quarters of the 1997 Encuesta de la Población Activa.*

The labor market behavior of the households in the 60-64 age cohort. In 1997 in Spain the average employment rate of the households in the 60-64 age cohort was 26.0 percent and their average participation rate was 28.1 percent. In our model economy the average employment rate was 30.5 percent. These numbers confirm that old people work more in our model economy than in Spain. This discrepancy could be due to features of the retirement decision that are absent from our model economy and that induce Spanish households to retire early.

In Table 10 we report the distribution of these participation rates by educational types. We find that our model economy matches the Spanish participation rates very closely. However, this means that our model economy overestimates the Spanish employment rates since we abstract from unemployment. This notwithstanding, we find that both in our model economy and in the data the participation rates of the elderly are clearly increasing in education. Two reasons justify this relationship. First, most non-high school workers are entitled to minimum pensions only, they are not affected by the early-retirement penalties and, consequently, they choose to retire as early as possible. And second, even though all the educational types value leisure equally, the foregone labor income— which is the opportunity cost of leisure—is smaller for the households with less education. Consequently, the less educated workers choose to retire earlier than their more educated colleagues.

The retirement behavior of disabled households. As far as the retirement behavior of disabled household is concerned, it turns out that in our model economy, all disabled households choose to retire at age 65 and, consequently, they collect their full pensions. We have not found data on the retirement behavior of Spanish disabled households, but we can safely guess that probably some of them choose to retire early.

---

27^Since in our model economy we abstract from unemployment, the employment rates and the participation rates coincide.
Retirement hazards. Finally, in Figure 6 we compare the conditional probabilities of retirement in Spain and in our model economy.²⁵ We find that our model economy replicates reasonably closely the retirement peak observed in Spanish data at age 60. Specifically, the observed probability of retirement at age 60 in Spain is 29.5 percent and in our model economy it is 37.7 percent. Our model economy also replicates the retirement peak observed in Spain at age 65. But in this case it is by construction. The probability of retiring at age 65 is 85.0 percent in Spain, and in our model economy it is 100 percent, since every household is forced to retire at that age. Our model economy also accounts for the increasing probability of retirement between ages 61 and 64 observed in the data. This is because of the concavity of the efficiency labor units endowment profile, which reduces the rewards to working at older ages.²⁶ However, we find that the probabilities of retiring between ages 61 to 64 are higher in our model economy than in the Spanish data.

6 Transitions and the pension system

In this section we simulate the consequences of the demographic and educational transitions for the sustainability the Spanish public pension system. To do this, we use the following strategy: we simulate three different transitions after our calibration target year, and we compare the pensions, the payroll tax collections, the pension system deficit, the pension fund and the consumption tax collections of each simulation (see Table 11 and Figures 7, 8 and 9).

²⁵The Spanish data corresponds to the year 1995 and it is reported in Sánchez-Martín (2008).
²⁶See Boldrin, Jiménez-Martín and Peracchi (1999) for a discussion of this feature of the Spanish pension system.
Table 11: The transitions and the pension system

<table>
<thead>
<tr>
<th></th>
<th>1997</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pensions (% of Y)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No transitions</td>
<td>10.3</td>
<td>10.4</td>
<td>10.7</td>
<td>11.0</td>
<td>11.2</td>
<td>11.2</td>
<td>11.2</td>
<td>11.2</td>
</tr>
<tr>
<td>Educational only</td>
<td>10.4</td>
<td>10.5</td>
<td>10.7</td>
<td>10.7</td>
<td>10.5</td>
<td>10.2</td>
<td>10.2</td>
<td>10.5</td>
</tr>
<tr>
<td>Educational and Demographic</td>
<td>10.1</td>
<td>10.7</td>
<td>10.9</td>
<td>11.9</td>
<td>13.6</td>
<td>16.1</td>
<td>18.2</td>
<td>19.0</td>
</tr>
<tr>
<td><strong>Payroll Tax Collections (% of Y)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No transitions</td>
<td>11.0</td>
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Figure 7: No Transitions (% Y)

Panel A: Pensions and contributions

Panel B: The pension deficit

Panel C: The pension fund

Panel D: Consumption tax collections
6.1 No transitions

In the first simulation, we assume that there is no demographic transition whatsoever and that the educational shares of working-age households always remain at their 1997 values. These assumptions have two implications. First, since the age and education distribution of native workers must be stationary in 1997 and the duration of the working-life is 45 years, the educational shares of the native newborns must be constant from the year 1953 (\(= 1997 - 45 + 1\)) onwards. Second, the educational transition ends in year 2033. This is because the educational shares of the retirees change for another 36 years after 1997.30 In Figure 7 we report the pensions, the payroll tax collections, the pension system deficit, the pension system fund and the consumption tax collections that we obtain in this simulation. Notice that in this simulation, after 1997 in our model economy the age distribution is time invariant; the asset distribution changes because the retirees that leave the economy are replaced by retirees that are more educated and, consequently, richer; and the distribution of pension claims also changes for the same reason.

We find that, if the educational and the population shares had remained in their 1997 values, the Spanish public pension system would have been perfectly sustainable. More specifically, in our model economy in 1997 there would have been a pension system surplus or 0.8 percent. This surplus would have decreased gradually until the year 2021, and the pension deficit would have grown very slowly from that date onwards. By the year 2060 there would be a small pension system deficit of 0.2 percent of the model economy output. In spite of these deficits, the value of the pension fund would have grown steadily throughout the entire period to reach 33.6 percent of the model economy output by year 2060. This is because the fund’s interest income was more than enough to finance the deficits. Finally we find that the changes in both the consumption tax rates and the consumption tax collections are very small (see Table 11 and Figure 7).

The main reasons that justify all these results are that the old-age dependency ratio is always time invariant at its 1997 value of 26.5 percent and that the retirees become increasingly educated. Specifically in 1997, 11.7 percent of the retirees had completed high school and only 4.5 percent had completed college. In 2060 these numbers had grown to 24.0 and 13.4 percent. Our findings lead us to conclude that the original design of the current Spanish pension system was essentially correct, taking into account the population structure of the nineteen nineties and that it would have been perfectly sustainable had there been no transitions.

\(^{30}\)Recall that the educational shares of the immigrants are always time-invariant.
Figure 8: The Educational Transition Only (% GDP)

Panel A: Pensions and contributions

Panel B: The pension deficit

Panel C: The pension fund

Panel C: Consumption tax collections
6.2 The educational transition

In the second simulation, we still assume that there is no demographic transition after 1997, but we allow for a complete educational transition that starts in 1951. The educational transition proceeds as we describe in Section 4.2 until it ends in the year 2131. The educational transition implies that the shares of high school and college households are higher in 2060 than in 1997 both for working-age households and for retirees. It also implies that these educational shares are higher throughout the entire period when compared with the “No transitions” simulation. In Figure 8 we report the pensions, the payroll tax collections, the pension system deficit, the pension system fund and the consumption tax collections that we obtain in this simulation.

Panel A of Figure 8 shows that the payroll tax collections are higher than pension payments throughout the entire 1997–2060 period. Specifically in 2060 the public pension system has a surplus of 3.0 percent of the model economy output. Moreover, Panel B of that same figure shows that, except in the first five or six years, the pension system surplus is significantly larger when we simulate the educational transition than when we simulate no transitions. As a result of these sustained sequence of surpluses, the pension system fund would have grown steadily reaching 150.0 percent of the model economy output by the year 2060. Notice that the changes in both the consumption tax rates and the consumption tax revenues needed to balance the government budget in this simulation are also very small.

From these results we conclude that, because of the progressivity introduced in the system by the maximum and minimum pensions, the educational transition would have made the Spanish public pension system even more sustainable than what it would have been if there had been no transitions. Notice that these intragenerational transfer has been documented by several studies such as Monasterio and Sáez (1992), Melis and Díaz (1993), and Bandrés and Cuenca (1996).

6.3 The educational and the demographic transitions.

Finally, we simulate both the demographic and the educational transitions that we describe in Section 4. In Figure 9 we plot the pensions, the payroll tax collections, the pension system deficit, the pension system fund and the consumption tax collections that obtain in this simulation.

Panels A and B show that the aging of the population makes the Spanish public pension system completely unsustainable. In spite of the large numbers of immigrants that enter the economy (a total of 17.7 millions between 1997 and 2060), payroll tax collections expressed as a share of output increase by only 0.5 percentage points of output. Since total expenditure in pensions increases by a startling 8.6 percentage points, in the year 2060 the public pension
Figure 9: The Demographic and the Educational Transitions (% Y)

Panel A: Pensions and contributions

Panel B: The pension deficit

Panel C: The pension fund

Panel D: Consumption tax collections
system deficit is 7.4 percent of the model economy output, up from a 0.6 percent surplus in 1997. Panel B shows that the first public pension deficit appears in the year 2016, and Panel C shows that the pension system fund is depleted in the year 2029. Moreover, as a result of this sequence of sustained deficits, the pension system debt follows an explosive path reaching a shocking 277.1 percent of the model economy output by the year 2060.

Consequently, this simulations lead as conclude that the current Spanish public pension system is completely unsustainable, and that it is safe to bet that it will experience large changes in the coming decades.

7 Conclusions

In this paper we study an overlapping generation model with native and immigrant households that differ in their education, receive an uninsurable, idiosyncratic endowment of efficiency labor units, understand the link between the payroll taxes they pay and the public pensions that they receive, and decide when to retire from the labor force optimally. We calibrate this model economy to Spanish data so that it replicates the main Spanish macroeconomic aggregates and ratios, and the Spanish Lorenz curves of income and earnings. We then use the model economy to simulate the consequences of the Spanish demographic and educational transitions for the sustainability of the public pension system. We find that even though the educational transition plays an important role and reduces the public pension system deficit somewhat, the aging of the Spanish population makes the current public pension system completely unviable. In our model economy the Spanish pension system shows a deficit for the first time in the year 2016, by 2020 the deficit is 0.4 of the model economy output, by 2040 it is 4.1 and by 2060 it is 7.1. This leads us to conclude that it is safe to bet that the Spanish public pension system will experience large changes in the coming decades.

References


