How Political Parties Shape Electoral Competition

Motz, Nicolas

Universidad Carlos III de Madrid

February 2016

Online at https://mpra.ub.uni-muenchen.de/69351/
MPRA Paper No. 69351, posted 10 Feb 2016 05:44 UTC
How Political Parties Shape Electoral Competition

Nicolas Motz*

Department of Economics, Universidad Carlos III de Madrid

February 2016

Abstract

This paper provides a model of party formation that can explain the contrast observable in the US between highly competitive presidential elections and state elections that are often dominated by one party. The puzzling aspect of this pattern is that the barriers to entry that seem to exist at the state level do not apply to the federal level. The explanation that the model provides rests on the career concerns of politicians: state politicians would like to advance their career to the federal level, but only have the opportunity of doing so as a member of a federally successful party. If politicians value such career opportunities sufficiently strongly, entry of additional parties at the state level does not occur. There then exists an equilibrium with two parties, one centre-left and one centre-right, where each party dominates some states. When career concerns are weak, on the other hand, the number of parties in equilibrium will be larger with a tendency towards parties with a narrower ideological profile. In addition to explaining the patterns observable in election results, the model also makes empirical predictions regarding the sorting of politicians into parties across different regions.

Keywords: Political parties, electoral competition.

JEL Classification: D72.

*This paper formed a chapter of my doctoral thesis submitted at University College London. I am grateful to my supervisors Ian Preston and Guy Laroque for their support. Omer Ali was immensely helpful in providing feedback as well as some of the data used in this paper. I would also like to thank Marco Bassetto, Antonio Cabrales, Boris Ginzburg, Anders Jensen, Gilat Levy, Aureo de Paula, Michael Ting, and Lukas Wenner for helpful comments and discussions.
1 Introduction

In the United States, there exists a clear and persistent contrast between elections at the federal level and elections at the state level: federal-level elections are typically highly competitive, while elections at the state level are often strongly dominated by one party. The competitiveness of federal elections is most pronounced for presidential elections where the vote shares of Democratic and Republican candidates are generally fairly close to each other, but it is also true that seat shares in Congress rarely exceed 60 percent. In contrast, state governors are frequently elected with wide margins of victory and it is not uncommon that one party holds 70 to 80 percent of all seats in a state legislature. To demonstrate the strength and persistence of this contrast between the federal and the state level, figure 1 displays a measure of competitiveness for presidential and gubernatorial elections across three 20-year periods, namely the average difference in the vote share of Democratic and Republican candidates.\(^1\) The figure clearly shows that the outcomes of presidential elections are generally fairly close and do not consistently favour one party. This is also illustrated by the fact that the margins of victory achieved by Lyndon Johnson and Richard Nixon of slightly above 22 percent still stand as historical exceptions. In contrast, there is no lack of states where the candidates of one party achieve margins of victory of above 20 percent on average.\(^2\)

The contrast between state and federal elections raises two closely linked questions. How is it possible that a party practically monopolizes elections in a state? And if something allows parties to dominate a region, why is this force not at play at the national level as well? While there are many potential barriers to entry that could serve as an answer to the first question, most of them apply equally well to any election and thus fail to answer the second question.\(^2\) In contrast, the model that I present in this paper is able to fully reproduce the pattern in the data.

My model is based on a particular concept of what exactly the role of political parties is. In line with a growing body of empirical research to be discussed below, the policy choices of a politician are assumed to be determined by this politician’s preferences. Voters therefore care about the preferences of politicians, but are initially poorly informed about these. Following Snyder & Ting\(^1\) Considering these elections has the advantage that they are not influenced by gerrymandering.\(^2\) I discuss some alternative explanations in section 2.
Figure 1: Average Differences in Vote Shares of Democratic and Republican Candidates

Notes: Each circle represents gubernatorial elections in a given state, while crosses stand for presidential elections. In the latter case, the numbers are based on popular vote shares.
Sources: Presidential elections - www.ropercenter.uconn.edu/elections/common/popvote.html; Gubernatorial elections up to 1990 - ICPSR (1995); Gubernatorial elections after 1990 - library.cqpress.com/elections/

(2002), parties provide some of this information by not allowing politicians of all political shades to join. Seeing that a politician is a member of a particular party thus tells voters that this politician must fall into a specific region of the political spectrum. In contrast, there would be no role for parties in the model if voters were fully informed.

In order to compete, parties thus need to attract the right kind of politicians. Importantly, politicians also care about their chances of getting elected. This concern is a main driver of the choice of party affiliation. A key insight that emerges from the model is that political parties that are successful in national elections can maintain regional monopolies because they offer career prospects at the federal level. As an example, consider a state like Nebraska where the
Republican Party currently controls all major elected offices. Suppose strongly conservative members of the Republican Party in Nebraska could form a separate party and do equally well in state elections. This would have the benefit of eliminating internal competition for nominations from politicians belonging to the more moderate wing of the party. It would, however, also deprive members of the newly formed party of access to many attractive positions available at the federal level. Most obviously, members of a third party will stand little chance of being elected president. In addition, and perhaps more importantly, assignments to congressional committees and the congressional leadership as well as cabinet posts are firmly in control of the established parties. If these career opportunities are valuable enough, conservative politicians in Nebraska prefer to remain a part of the Republican Party, which can then maintain its hold on the state.

While the moderate wing of the Republican Party is a burden to conservatives in a right-leaning state, the conservative wing is a detriment to the electoral chances of the party at the national level. It makes the party more extreme and thus less attractive to voters in the political centre. The national party itself might therefore have an incentive to try to exclude its most conservative members. But this would result in the establishment of a more extreme party and the risk of a split in the conservative vote. This threat of entry is the force that prevents the national party from moderating itself in the model and may explain why the establishment of the Republican Party has been relatively accommodating towards the radical Tea Party movement.

The main result of the paper is that there exists an equilibrium where two parties are formed, one centre-left and one centre-right. Both parties win with equal probability at the federal level while dominating some state elections. This equilibrium is maintained by the forces described above: neither party can shift further towards the centre without inducing entry of a third party, while in equilibrium such entry is precluded as politicians have no incentive to deviate towards joining a new party. State monopolies exist in this equilibrium because parties have strongly differentiated ideological profiles. This enables each party to capture a large share of votes in particular states. For example, in a state with a median voter located far to the left, the centre-left party dominates state elections, while states with a more moderate median voter will be more competitive. The model is thus able to recreate the pattern displayed in figure 1. As I will argue in section 2, other factors may be at play as well, but cannot convincingly explain the data by themselves.
To the best of my knowledge, the contrasting patterns of political competition at the state and at the federal level have previously not been demonstrated as clearly as in figure 1. Besley et al. (2010) discuss the wide variation in the degree of competition observable across US states, but do not refer to the federal level. Their empirical results are nevertheless closely related to the current paper, in that they show that in states where competition is lower policies tend to be less favourable to growth and actual growth is reduced as well. This indicates that the dominance of one party has negative consequences and highlights the need for a better understanding of how such political monopolies emerge.

A related theoretical paper is provided by Callander (2005), who studies competition between two parties in multiple single-member districts with threat of entry at the district level. Parties, which are not explicitly modelled, are free to choose any platform. Callander finds that the threat of entry leads to the divergence of party platforms, similar to this paper. The mechanism through which entry is deterred is different though. In addition, the equilibrium presented by Callander requires specific assumptions on the distribution of voters across districts, while the restrictions imposed on voter distributions in this paper are mild. This is because entry in the model of Callander implies the loss of one district, while entry has much wider consequences in the current model as explained above. Previous contributions to the literature on political competition with entry consider only a single district (Palfrey 1984, Osborne 1993, 2000).

Political parties clearly form a central element of the political system of democratic countries, yet they have received surprisingly little attention, at least in terms of formal modelling. Few papers have attempted to fully endogenize the number parties existing in equilibrium as I do here (Jackson & Moselle 2002, Levy 2004, Morelli 2004, Osborne & Tourky 2008, Eguia 2011). As mentioned above, the concept of political parties that I employ is taken from Snyder & Ting (2002). These authors, as well as other contributions building on their approach (Ashworth & Bueno de Mesquita 2008, Bernhardt et al. 2009), consider the behaviour of a given number of parties. I show how the concept of parties as “informative labels” can yield an equilibrium with two parties that looks very similar to what we observe in a number of countries. Furthermore, I demonstrate that career concerns of politicians can be a driving force behind the number and shapes of parties that form in equilibrium. Previously, attention has mainly focused on variations in the electoral system as a determinant of the number of parties (see Morelli 2004). Overall, I feel that the success of the model presented
here in reproducing and explaining empirical regularities indicates that thinking of parties as informative labels is a fruitful approach.

The rest of the paper is organized as follows: section 2 addresses some alternative explanations of the pattern visible in figure 1 while section 3 discusses a number of empirical results that lend support to some of the assumptions made in the model. The model itself is presented in section 4. Section 5 gives the theoretical results. Robustness of the results to relaxing some of the assumptions made in the basic version of the model is discussed in section 6. Section 7 concludes.

2 Alternative Explanations

A number of reasons can explain why parties are able to dominate elections in a particular state. For example, a party could simply offer exactly the candidates and policies that voters want, voters could develop preferences for particularly parties over time, or there could be economies of organisational scale\(^3\). Alternatively, campaign funds that established parties have at their disposal could be used to prevent successful entry of additional parties. The problem is that these explanations apply equally to federal elections and therefore do not speak to the contrast between state and federal elections.

A factor that could potentially explain this contrast is the role of incumbents running for re-election. It is well known that incumbents tend to enjoy an electoral advantage. Some states in the US also have less strict term limits for governors. This raises the question to what extent the difference between federal and state elections is a consequence of the presence of incumbents at either level. To investigate this issue I run regressions for the three periods shown in figure 1 of presidential and gubernatorial vote margins on a set of state dummies as well as indicators for the presence of Democratic and Republican incumbents. The coefficients on the incumbent dummies vary widely across parties and periods, both in terms of magnitude and significance. The effect on the remaining coefficients, however, is moderate, as can be seen in figure 2. For each period, the column to the left reproduces the average vote margins as shown before, while the right column plots predicted vote margins with the incumbent dummies set to zero. The presence of incumbents does not appear to be a main driver of contrast between gubernatorial and presidential elections.

\(^3\)Osborne & Tourky (2008) show how economies of scale can limit the number of competing parties in a model featuring a single election.
Figure 2: Controlling for the Role of Incumbents

Notes: Each circle represents gubernatorial elections in a given state, while crosses stand for presidential elections. For each period, the figures plotted on the right-hand side are calculated controlling for the presence of incumbents as described in the text.

A final concern I want to address is that differences in the rate of turnout between state and federal elections might be of importance. Lower turnout could potentially make election results more volatile and thus result in higher average vote margins. This would not explain the persistently high vote margins in favour of one party though, as they are observable across a range of states. It is harder to rule out that differential rates of turnout among different groups of voters could result in larger vote margins. However, the results of Levine & Palfrey (2007) point in the opposite direction: in an experiment on voter turnout, these authors find that those favouring a disadvantaged candidate are more likely to vote. In addition, turnout is lower in the experiment when the expected closeness of the election is reduced. This suggests that low turnout in state elections might be a consequence rather than a cause of high vote margins.
3 Related Empirical Evidence

This section will discuss empirical evidence supportive of some of the assumptions featured in the model or the general ideas behind it. First of all, a growing literature investigates the determinants of the policy choices of elected officials. Chattopadhyay & Duflo (2004) and Bhalotra & Clots-Figueras (2014) find that policy preferences of politicians matter. Both papers establish that an exogenous increase in female representation in India leads to a greater provision of public services typically utilised by women. The results of Lee et al. (2004) go further: according to their estimates the voting behaviour of individual members of the US House of Representatives is independent of their electoral odds. This seems to indicates that the preferences of politicians not only influence but largely determine policy choices. Otherwise one would expect legislators in close election to alter their voting behaviour in an attempt to cater to voter tastes. Similar results are obtained by Levitt (1996) for the US Senate, who additionally controls for a potential role of party discipline imposed on legislators.

A second strand of evidence relates to the nature of political parties. Casual observation suggests that in many countries the vast majority of votes is cast for two parties, one located left and one located right of the political centre, which tend to alternate in power. In view of the studies cited above, the interpretation of the statement that a party is centre-left would have to be that the politicians belonging to such a party prefer centre-left policies, at least on average. This is also formally confirmed by research estimating the ideological positions of politicians. Poole & Rosenthal (1997, 2001) do so using voting records from the US Congress. According to their results the membership of both the Republican and the Democratic Party spans a wide range of positions, but with little overlap between them. Barberá (2015) finds very similar results using data from the social network Twitter. He also applies his method to five European countries, and again a similar picture emerges.\(^4\)

All of this is consistent with the view of parties as collections of similar-minded politicians. In this case voters can learn something about a politician’s views from observing which party she is a member of, even if they cannot observe preferences of politicians directly. This is the idea formalised by Snyder & Ting (2002) and applied in this paper. Snyder & Ting also demonstrate empirically

\(^4\text{As the author remarks, the reason that he finds somewhat more overlap between parties in Europe may be due to the possibility that variation in political preferences is less well captured by a single dimension there.}\)
that voters’ knowledge of an individual politician’s position is almost entirely captured by her party affiliation. They use estimates of these positions to predict how voters place candidates on an ideological scale. As it turns out, a simple dummy for party affiliation does just as well in explaining the variation in voter knowledge.

Finally, career concerns of politicians are a driving force behind the results. In the context of the model, it seems natural to assume that politicians progress from the regional to the national level. After all, winning a regional election reveals information about a politician, which is a strong advantage when facing a competitor who is unknown to voters. At least for the US, it is well documented that politicians indeed use elections as stepping stones towards higher offices. For example, Diermeier et al. (2005) collect a sample of members of Congress in the period 1947 to 1994. They find that 78 percent of these politicians held a different local, state, or federal elected office before joining Congress, where it seems safe to assume that local and state offices should account for the vast majority of these cases. About ten percent of representatives in their sample run for a Senate seat. Of those who leave Congress, 35 percent stay in politics.

4 The Model

A federal state consisting of \( S \geq 4 \) states selects federal and state governments through plurality rule elections. Candidates for these elections are nominated by political parties. I divide the game into two main stages: a party formation stage and an election stage. Each stage will be described in more detail below once some of the basic elements of the game have been introduced. Note that the model features no “states of nature” and the word state therefore always refers to a geographical unit.

4.1 Voters, Politicians, and Parties

Each state \( s \) has a set of citizens that is large, finite, and odd. Each citizen votes in two elections: the election for the government of state \( s \) and the election for the federal government. Let \( p_s \) and \( p_f \) denote the policies that are implemented in state \( s \) and at the federal level, respectively. The objective of voters in election \( l \in \{ s, f \} \) is to maximize

\[
E[u(\vert p_l - i \vert)]
\]
where \( u : \mathbb{R} \rightarrow \mathbb{R} \) is a decreasing function while \( i \in \mathbb{R} \) is the ideal policy of the voter.\(^5\)

Each state also has a finite set of politicians. Every politician is endowed with a fixed platform and once elected to any office a politician is committed to implementing this platform.\(^6\) There are three possible platforms, namely -1, 0, and 1, and each state has three politicians, none of which share the same platform.\(^7\) Put differently, there is one politician located at each of the possible policies -1, 0, and 1. I will apply the labels centrist, extremist, rightist, and leftist to politicians in the obvious manner.

For an election at level \( l \in \{s, f\} \) the winning candidate receives a payoff of \( y_l > 0 \). These payoffs subsume the material and immaterial benefits of holding office. For example, purely local concerns that might motivate a politician could form part of \( y_s \). I discuss this possibility further at the end of section 5.2. A politician who has won a state election but does not win the nomination of her party for the federal election nevertheless receives a payoff of \( y_P > 0 \) if her party wins the federal election. This payoff has a number of interpretations. It may represent opportunities to move upwards in the party hierarchy that arise when a party wins the federal election or the chance of becoming a member of the federal government. For example, the party that holds a majority of seats in the House of Representatives appoints committees chairs and the House leadership. A second interpretation of this payoff is that career opportunities in the private sector become more valuable if a politician is well connected within the party in power. It is also possible to think of \( y_P \) as representing “pork”: the leader of a state government will be likely to receive more federal money if her party is in control of the federal government.

\(^5\)As will become clear later, the outcomes of state elections may affect events at the federal level, but it is assumed that voters do not take this interdependence into account when voting at the state level. This is done simply to make the proofs presented in subsequent sections more compact, but is otherwise not necessary. If voters were forward-looking, this would only change their behaviour where voting for their less preferred candidate at the state level would somehow yield a sufficiently large benefit at the federal level. In all of the specific situations dealt with below it can be shown that this is impossible.

\(^6\)The platform of a politician is supposed to represent the ideal policy of a politician. As discussed in section 3, preferences over policies appear to be the main driver of the choices that politicians make in office. Of course it would be preferable that this behaviour emerges as part of an equilibrium, rather than being imposed from the outset. I will allow politicians to be more flexible in their policy choices in section 6.

\(^7\)The number of cases to consider increases rapidly in the number of possible platforms and is already large with three platforms. A possible interpretation of this assumption is that voters have a coarse perception of the policies chosen by politicians. Evidence from psychology indicates that people tend to think in simplifying categories (For a discussion of some of this research see Fryer & Jackson 2008).
In order to clearly define the utility of a politician, let $\pi_s$ be the probability that a politician is nominated for and wins the state election in her state. Conditional on doing so, let $\pi_n$ give the probability that a politician is nominated for the federal election. $\pi_f$ is then the likelihood of winning conditional on receiving the nomination, while $\pi_P$ is the probability that the party wins conditional on some other candidate having received the nomination. All of these probabilities will later be determined in equilibrium. The expected utility of a politician who has joined a party is given by

$$\pi_s(y_s + \pi_n \pi_f y_f + (1 - \pi_n) \pi_P y_P),$$

while a politician who is not part of a party receives a payoff of zero. It is assumed that $y_f > 2y_P$.

A political party is basically a subset of the policy space and only politicians whose platforms fall within this subset can join. This idea is based on Snyder & Ting (2002), where the leadership of a party chooses a platform and politicians pay a cost for joining the party that depends on the distance between this platform and their own ideal policy. Two interpretations of this cost are given: first, politicians could find it costly to be members of an organisation that pursues goals that differ from their personal views. Second, parties could be actively screening their members and only promote those who agree with the party line. As a result, only politicians with an ideal policy belonging to an interval centred around the party platform join. The size of the interval depends on the membership cost in the first interpretation, or the effectiveness of screening in the second interpretation. I simplify things by giving parties full control over the size of the interval that represents the party. Given that the space of platforms consists of integers, parties will be given by “integer intervals”: for $a, b \in \{-1, 0, 1\}$ define

$$[a..b] \equiv \{ p \in \{-1, 0, 1\} : a \leq p \leq b \}.$$

If $a$ equals $b$ I simply write $[a]$. The set of all possible shapes a particular party can have is

$$\mathcal{I} = \{[-1], [0], [1], [-1..0], [0..1]\}. \quad \text{8}$$

\footnote{Note that parties that allow all types of politicians to join are not allowed for. Including them would give rise to two additional equilibria, which have some implausible features and...}
Parties are organized nationally, meaning that the interval that represents the party is the same in all states. The set of politicians that joins a party does not have to be the same across states, however, as politicians in different states might face different incentives. Individual parties will be denoted by capital letters. For any such party \( P \) the shape of the party is given by \( I_P \in \mathcal{I} \). Multiple parties are allowed to have the same shape.

### 4.2 The Election Stage

In order to describe the election stage, let there be \( N \) existing parties, collected in the set \( \mathcal{P} \). Let \( \mathcal{P}(p) \) denote the possibly empty set of existing parties that include the policy \( p \). The strategy set of a politician with platform \( p \) in this subgame is then given by \( \mathcal{P}(p) \). Note that this means that a politician can join at most one party and that a politician who has the ability to join at least one party must do so.\(^9\)

The election stage starts with politicians making their affiliation decisions, followed by simultaneous state elections, which in turn are followed by the federal election. Immediately prior to each election every party nominates a candidate, who is drawn uniformly at random from the candidate pool of the party for the election in question.\(^10\) For a particular state the candidate pool of a party consists of all politicians of that state who have joined this party. Each winner of a state election then becomes a member of the candidate pool of their party for the federal election.

The policy that is implemented in a state is equal to the platform of the politician elected in the state election, just like the policy at the federal level is equal to the platform of the politician elected in the federal election. The winner of each election is the candidate that achieves the highest number of votes with ties resolved randomly.\(^11\)

---

\(^9\)The latter assumption is made for convenience and could easily be replaced with a small payoff that a politician receives once she joins a party.

\(^10\)Of course, candidate selection is generally an important strategic decision. As it turns out, however, parties have an incentive to commit to a candidate selection mechanism that gives extremists a sufficiently high chance of being nominated. This can been seen as a rational for the introduction of primaries. I explain this in more detail in section 6.

\(^11\)Even in a system of first-past-the-post elections the implementation of policies requires a majority in parliament. With more than two parties competing the choice of policy may therefore require a process of coalition formation. I abstract from such issues here. At least the two-party equilibria presented below, however, do not depend on what is assumed about the process of policy formation when no party achieves a majority. This is because voters will
4.3 The Party-Formation Stage

Parties are formed by “founders” of which there is an infinite number. Founders can choose to propose a party or remain passive. Consequently, their action space is given by \( I \cup \emptyset \), where \( \emptyset \) stands for the decision to not propose a party. Once a founder has proposed a party, I will also refer to this founder as a party leader.

Each founder pays a cost \( c > 0 \) for proposing a party, while she receives a payoff of \( x_w > 0 \) for every state election that her party wins, as well as a payoff of \( x_f > 0 \) if her party wins the federal election. Denoting by \( \rho_r \) with \( r \in \{1, \ldots, S, f\} \) the equilibrium probability that the party of a founder wins election \( r \), the expected utility of a founder who proposes a party is given by

\[
\sum_{s=1}^{S} \rho_s x_w + \rho_f x_f - c
\]

Passive founders always achieve a utility of zero. I assume \( c < x_w \), so that a founder whose party wins at least one state election does not want to deviate to remaining passive.

The party formation stage simply consists of all founders simultaneously deciding to propose a party or stay passive. Denote by \( P_F \) the set of parties proposed by founders. The set of all existing parties \( P \) will then be given by the union of \( P_F \) and a set of three “latent” parties with respective shapes of \([-1, 0, 1]\). As explained in the introduction, the results rely on the threat of entry of additional parties and this threat is most easily modelled through the inclusion of these latent parties. It is possible to think of them as parties that are already active at the local level. Once a founder deviates by changing the shape of her party this may give a party previously of only local relevance a chance to become active at the state level. Founders will take this possibility into account when considering a deviation.

4.4 Information

A crucial feature of the concept of political parties employed here is that voters have limited information about politicians. At the beginning of the game, the electorate cannot distinguish between different politicians, but knows how be allowed to vote strategically. This always allows for the possibility of one party winning with a strict majority, even off the equilibrium path when a third party has entered.
their platforms are distributed. In contrast, politicians and founders observe platforms directly. Everything else is common knowledge. In particular, voters know which parties have been proposed and how many politicians have joined each one of them in each state. Knowing that a candidate belongs to a certain party therefore allows voters to update their beliefs about this politician’s platform prior to casting their vote for the state-level election. The winner of the election then implements her platform at the state level, thus revealing it to voters. Voters accordingly have full information about candidates at the federal level. All agents are also fully informed about the distribution of voters in all states and at the federal level.

4.5 Equilibrium

The timing of the game is summarised in figure 3. Any set of parties \( P \) that gets proposed in the first stage of the model leads to a proper subgame comprised of the steps previously referred to as the election stage. I will refer to this as the election subgame under the set of parties \( P \).

Given that the game features incomplete information, the appropriate equilibrium concept is perfect Bayesian equilibrium. By itself, this would entail the possibility of a huge number of equilibria that exist when voters are allowed to vote strategically. Other papers in the literature on party formation assume sincere voting to avoid this problem. I generally allow for strategic voting, but impose three plausible restrictions: first of all, I consider only equilibria in weakly undominated strategies. The exclusion of weakly dominated strategies
is a common way of refining voting equilibria and eliminates the possibility that voters vote for their least preferred candidate. The second restriction reads as follows: if a candidate is the unique most preferred option of a strict majority of voters, then this candidate wins the election. In general, there may exist voting equilibria where a different candidate gets elected in this situation, but it nevertheless seems likely that voters will be able to solve the coordination problem in this case. The third restriction is a tie breaking rule: I assume that voters who are indifferent between all candidates vote for all candidates with equal probability, but only if the election takes place along the equilibrium path. Imposing this restriction along the equilibrium path only can be interpreted as “party loyalty”: if an additional party enters, indifferent voters may continue to vote for one of the previously existing parties out of habit.

The following definition gives the equilibrium concept:

**Definition 1.** A party-formation equilibrium is a perfect Bayesian equilibrium of the party-formation game that satisfies the following conditions:

i) No player uses a weakly dominated strategy.

ii) If a candidate in some election is the unique most preferred option of a strict majority of voters, then this candidate wins the election.

iii) Along the equilibrium path all voters who are indifferent between all candidates vote for all candidates with equal probability.

iv) Along the equilibrium path no latent parties win any elections.

Given the non-strategic nature of latent parties, it seems natural to focus on equilibria where they play no active role. But while this restriction does rule out some equilibria, all of these are actually redundant in the following sense: suppose, for example, that there was an equilibrium where founders propose the set of parties \( P_F^* \) and the latent party of shape \([1]\) wins at least one state election. Then there exists an equilibrium that is identical to the previous equilibrium with the exception that a previously passive founder proposes an additional party of shape \([1]\) and all politicians who previously affiliated themselves to the latent party of the same shape now join this additional party.\(^{12}\) When stating the number of parties that exist in an equilibrium I always refer to the number of parties proposed by founders.

\(^{12}\)This claim would not be true if and only if the founder of the party that replaces a latent party has an incentive to change the shape of their party. As will become clear below, the presence of latent parties renders such deviations futile.
Equilibrium objects are indicated by stars. In particular, \( P^*_F \) will denote the set of parties formed be founders in equilibrium, while \( N^* \equiv |P^*_F| \). In addition, \( w^*_p \) will denote the equilibrium number of state elections won by party \( P \).

4.6 Voter Distributions

A crucial input of the model is the set of voters. I will only make relatively weak assumptions in this regard. More specifically, the results in subsequent sections require a minimum amount of heterogeneity in voter tastes. Figure 1 seems to indicate that actual heterogeneity across US states is substantial.

Before stating my assumptions, I need to take a step towards analysing behaviour in the model. This will also illustrate how parties reveal information about their members. Suppose there are two parties, \( A \) and \( B \), contesting a state election. Party \( A \) has the politician with platform -1 as the unique member, while the remaining two politicians have joined party \( B \). In any perfect Bayesian equilibrium voters hold correct beliefs about the strategies of politicians and therefore know which politician has joined which party, even though they are unable to tell exactly which member of party \( B \) has which platform. Each voter then knows that the candidate nominated by party \( A \) has platform -1. The candidate of party \( B \), on the other hand, is equally likely to have either platform 0 or 1 due to the assumption of random candidate selection. Let \( p^- \) be the unique real number such that a voter with ideal policy equal to \( p^- \) is indifferent between the candidate of either party, that is \( p^- \) solves

\[
u(1 - p^-) = \frac{1}{2} \left[ u(| - p^-|) + u(|1 - p^-|) \right].
\]

As the utility of voters is symmetric around their ideal policy and \( u \) was assumed to be decreasing, it must be the case that \( p^- \in (-0.5, 0) \). Next, consider the situation that would result if the politician with platform 0 were to switch from party \( B \) to party \( A \). In this case a voter with ideal policy \( |p^-| \) would be indifferent between voting for either party. Denote this policy by \( p^+ \).

I assume that the set of voters in any state \( s \) can be described by a measure \( V_s \) that assigns to any subset of \( \mathbb{R} \) the number of voters whose ideal policy lies in this subset. Let \( m_s \) denote the ideal policy of the median voter of state \( s \).

\( ^{13} \)These assumptions imply that a voter with ideal policy equal to -0.5 is indifferent between the policies -1 and 0, but strictly prefers -1 over 1. She must therefore strictly prefer the candidate of party \( A \) over the candidate of party \( B \). A voter with ideal policy zero, on the other hand, is indifferent between the policies -1 and 1 and therefore strictly prefers party \( B \) over party \( A \) due to the possibility that party \( B \) nominates a centrist candidate.
Similarly, let $V_f$ be the measure of voters at the federal election with median $m_f$ where

$$V_f(D) = \sum_{s=1}^{S} m_s(D)$$

for any $D \subset \mathbb{R}$. It is assumed that $m_f$ is equal to zero. It will often be important to know what share of voters in some region $r \in 1,..,S, f$ is located in the interval $[-0.5,0.5]$. I will therefore define

$$\Lambda_r([-0.5,0.5]) \equiv \frac{V_r([-0.5,0.5])}{V_r[\mathbb{R}]}.$$

The first more substantial assumption regarding voter preferences specifies that there is some minimum amount of heterogeneity in voter distributions across states: let there be at least one state $s$ such that $m_s < -0.5$, at least one state $s'$ such that $m_{s'} \in (p^-,p^+)$ and $\Lambda_{s'}([-0.5,0.5]) > 0.5$, and at least one state $s''$ such that $m_{s''} > 0.5$. Note that -0.5 (0.5) is the ideal policy at which a voter is indifferent between the platforms -1 and 0 (0 and 1). I will refer to states with median voter below -0.5 or above 0.5 as extremist states, while states with median voter between $p^-$ and $p^+$ are called centrist states. Purely for convenience, I will also assume that there are no voters located exactly at $p^-, 0, \text{ or } p^+.$

The second assumption on voter distributions says that voters at the federal level are not too concentrated in the centre of the policy space: $\Lambda_f([−0.5,0.5]) \leq 0.5$. To illustrate, this requirement would be satisfied, for example, if all voters were spread out equally between -1 and 1.

## 5 Results

The model described in the previous section has many equilibria. This should come as no surprise: after all, it features two coordination problems—one between politicians and one between voters—as well as out-of-equilibrium beliefs that can be chosen to support a specific equilibrium. In particular, voters may believe that a politician who deviates has a platform that the median voter of the state dislikes, which makes it unlikely that the deviation is successful.\(^{14}\)

\(^{14}\)Such beliefs are not entirely unrestricted though. Consider, for example, the case where all politicians in some state have joined a party that allows any politician to join. If one politician deviates and joins a party with shape $[-1,0]$, voters find themselves at an information set that has two nodes; one for the case in which politician -1 has deviated and one for the case in
Given the multiplicity of equilibria, I will proceed as follows: given their empirical relevance, my main interest is in equilibria with two parties. It turns out that this class can be fully characterized, as I discuss in the next section. This section contains the main results of the paper. In general, however, the number of equilibria is large. Section 5.2 will discuss this in more detail and suggest a refinement.

5.1 Equilibria with Two Parties

Following the discussion in section 3, a natural starting point is to consider a situation with a centre-left and a centre-right party. The most obvious formalisation of this would be an equilibrium where the set of proposed parties is equal to \( \{L, R\} \) with \( I_L = [-1,.0] \) and \( I_R = [0,.1] \).

Suppose for now that parties \( L \) and \( R \) were the only existing parties. How would politicians behave? Those with platform -1 and 1 will become members of the unique party available to them by assumption. In a state with median voter below \( p^- \) it then does not matter which party the politician with platform 0 joins: a very left-leaning median voter always prefers party \( L \) as it has the leftist politician of the state as a member.\(^{15}\) As this is her only chance of getting elected, politician 0 will therefore always join party \( L \) in such states. Analogously, a politician with platform 0 will join party \( R \) in a state where the ideal policy of the median voter is greater than \( p^+ \).

In centrist states, in contrast, politician 0 can make either party the winner of the state election by joining. The probability with which politician 0 is nominated for and wins the state election in such states is thus the same independent of which party politician 0 becomes a member of. Conditional on receiving the nomination at the federal level, the probability of winning is also independent of the choice of party. This is because each party has a moderate and an extremist member in states where it wins and also nominates these with equal probability for the state election. Accordingly, both parties have an equal number of politicians of either type in their federal candidate pool in expectation. This means that the “expected opponent” at the federal election is equally strong no matter which party a politician joins. The only factor affecting the utility of a politician with platform 0 that may differ between parties is the proba-

\(^{15}\)Note that voters always know who has joined which party in this setting as there are no information sets that contain more than one node.
bility of being nominated at the federal level. This probability is decreasing in the number of states elections won by the party. It follows that politicians with platform 0 in centrist states will join the party that wins fewer elections, a force that works towards equalizing the number of states won by each party. Equilibrium is reached if centrist politicians in centrist states have either joined the party that wins fewer elections or are distributed across parties such that the number of state elections won differs by at most one between them. For example, suppose there are $2n$ states, in $n$ of which the median voter’s most preferred policy is -1 and party $L$ accordingly wins the state election, while in all other states the median voter is located above $p^-$. Then party $R$ must win all of the latter states, no matter what exactly the ideal policies of the median voters in these states are.

That politicians choose their affiliations as described in the previous paragraph is a necessary condition for the existence of an equilibrium where parties $L$ and $R$ are proposed. Otherwise some politicians would have an incentive to switch between the two parties. These findings are summarized in the following lemma.

**Lemma 1.** Suppose $\mathcal{P}_F^* = \{L, R\}$ with $I_L = [-1..0]$ and $I_R = [0..1]$. Then it must be true that

1. politician -1 (politician 1) joins party $L$ (party $R$) in every state,
2. politician 0 joins party $L$ (party $R$) in any state $s$ such that $m_s < p^-$ ($m_s > p^+$),
3. in any state $s$ such that $m_s \in (p^-, p^+)$ politician 0 joins party $L$ (party $R$) if $w^*_L < w^*_R$ ($w^*_R < w^*_L$) and may be a member of either party if $w^*_L - 1 \leq w^*_R \leq w^*_L + 1$,
4. and in each state the party that politician 0 joins wins.

The behaviour given in lemma 1 has an interesting feature: parties are more extreme in states where the distribution of voters does not favour them. For example, members of the centre-left party are more left-leaning on average in states where the distribution of voters is strongly skewed towards the right. This is because centrist politicians join party $R$ in this case as this is the only party that gives them a chance of winning elections. As a consequence, only politicians with a left-wing platform remain in party $L$. Translated to the context of US politics, this would imply that members of the Democratic Party are
more moderate on average in a very liberal state like Massachusetts, and more 

extreme in a conservative state like Texas. It should be noted that this prediction 

does not necessarily carry over to nominated candidates, if the assumption 

that candidates are selected randomly is relaxed. There would be an additional 

selection issue when trying to test this using observations on elected politicians. 

Regarding the membership itself, this seems like a robust prediction of the model 

though. While it is difficult to test this prediction directly, there exists some 

empirical support for a comparative-statics result that follows directly from this 

feature of the behaviour of centrist candidates. If the median voter of a state 

shifts from left to right, there must exist a point between $p^-$ and $p^+$ where the 

centrist politician in the state switches their affiliation from party $L$ to party $R$. 

If one interprets the realignment of voters in the US from the Democratic 

to the Republican Party as such a shift in voter preferences, then the model 

would predict the occurrence of state politicians changing their affiliation from 

Democrat to Republican. This is exactly what happened in the nineties and 

early two-thousands, as documented by McKee & Yoshinaka (forthcoming). 

It was explained above that each party nominates a moderate or an extremist 

politician with equal probability at the federal level, which is true independent 

of the number of states won by either party. Given the behaviour imposed 

on completely indifferent voters, it follows that each party wins the federal 

election with equal probability ex-ante.$^{16}$ While the assumption that candidates 

are chosen randomly clearly plays a role in generating this result, it is no less 

important that both parties are able to recruit moderate politicians that are 

attractive to the federal median voter. In contrast, one party would gain a 

substantial advantage if it were able to attract all centrists. As described above, 

electoral concerns of politicians make this impossible. Due to the importance 

of this result in the context of the paper, I restate it as a proposition. 

\textbf{Proposition 1.} Suppose $\mathcal{P}_F = \{L, R\}$ with $I_L = [-1,0]$ and $I_R = [0,1]$. Then 
each party wins the federal election with equal probability. 

A constellation with a centre-left and a centre-right party is thus able to 

reproduce the pattern observable in US election results: while the preceding 

proposition establishes that the federal election is competitive, at least some 

$^{16}$There are three cases to consider. If both parties nominate a centrist, all voters are 

indifferent and either candidate wins with equal probability. If both parties nominate an 

extremist, a tie results due to the assumptions that $m_f = 0$ and that there are no voters 

located at zero. If one party nominates a centrist but the other nominates an extremist, the 

centrist wins with certainty. However, as both parties are equally likely to nominate either 
type of candidate, they are also equally likely to be the party that nominated the centrist.
state elections will be dominated by one party, namely elections in states with median voters located in the extremes of the policy space. One of the central questions raised in the introduction has not been answered yet though: what prevents entry of additional parties in states where one party dominates? The model can only provide an answer if it can be shown that there actually exists an equilibrium of the game as a whole where parties $L$ and $R$ are formed and no other parties enter. This is confirmed in the following proposition, subject to a condition on payoffs being satisfied.

**Proposition 2.** An equilibrium of the party formation game where $P^*_F = \{L, R\}$, with $I_L = [-1..0]$ and $I_R = [0..1]$, exists if

$$\frac{1}{2S}y_f + \left(1 - \frac{1}{S}\right)y_P \geq 2y_s.$$ 

**Proof.** First, it needs to be established that no politician has an incentive to join any additional parties. It is sufficient to show that conditional on the affiliation behaviour given in lemma 1, no politician wants to deviate to joining a party that allows only one type of politician to join. Politicians with platform -1 do not gain by joining a party with shape $[-1]$ if they are in a state with median greater than -0.5 as they would subsequently lose the state election. In a state with median voter below -0.5, on the other hand, a majority of voters would strictly prefer the new party, which would then win the state election by assumption. In equilibrium a politician with platform -1 in such a state achieves

$$\frac{1}{2} \left[ y_s + \frac{1}{4w^*_L}y_f + \left(1 - \frac{1}{w^*_L}\right)\frac{1}{2}y_P \right],$$

as each member of party $L$ gets nominated with equal probability and there are two members in the state; each of the $w^*_L$ state-winners of party $L$ are nominated for the federal election with equal probability; extremists can only win the federal election if party $R$ also nominates an extremist (which happens with probability one-half) and a tie results; and both parties win the federal election ex-ante with equal probability. If the same politician joins the latent party of shape $[-1]$ her payoff is $y_s$ as she loses the federal election with certainty. This is because there will then be three parties competing at the federal level and a politician with platform -1 can never be strictly preferred over the other two candidates by a strict majority. Accordingly, there always exists a voting equilibrium where some other party wins. As it was assumed that $y_f > 2y_P$, the equilibrium utility decreases as $w^*_L$ increases. As no party can win more state elections than there are states, a sufficient condition for deviations of this type
not being profitable is

\[
\frac{1}{2} \left[ y_s + \frac{1}{4S} y_f + \left( 1 - \frac{1}{S} \right) \frac{1}{2} y_P \right] \geq y_s ,
\]

which can be rewritten to give the condition in the statement of the proposition.

For politicians with platform 0 the equilibrium payoff is given by

\[
\frac{1}{2} \left[ y_s + \frac{3}{4w_j^*} y_f + \left( 1 - \frac{1}{w_j^*} \right) \frac{1}{2} y_P \right] ,
\]

with \( j \in \{ L, R \} \). In case such a politician deviates to a party with shape \([0]\) she can at best hope to win the state election. This is because it was assumed that \( \Lambda_f([-0.5, 0.5]) \leq 0.5 \) and there accordingly exists a voting equilibrium of the federal election where one of the other two parties wins. This is true even if all parties nominate a candidate with platform 0, as the restriction that all candidates receive an equal share of all votes when all voters are indifferent was only imposed on the equilibrium path. The payoff from the deviation is then \( y_s \), which is the same as the highest possible deviation payoff for extremist members of the same party. As the equilibrium payoff of centrist politicians is greater than the equilibrium payoff of extremist politicians, it follows that centrists do not deviate as long as extremists do not deviate.

It remains to be checked whether any active founders want to deviate. Any such deviation can be punished by an equilibrium of the election subgame where only latent parties attract members, which exists due to the proof of proposition 5 in appendix A.

I will refer to the equilibrium in the preceding proposition as the \( L-R \) equilibrium. Under this constellation of parties no third party can successfully enter for reasons easily illustrated in an example: suppose a party were to enter that admits only politicians with platform -1 as members. Given that such a party attracts members, it may do well at the state level, but would not be able to win the federal election due to the presence of the two already established parties. Politicians with platform -1 in states with very left-leaning median voters may nevertheless be tempted to join the entering party as this eliminates competition from centrist politicians for the party nomination at the state level. The incumbent party prevents them from defecting by offering career prospects at the federal level. These include the possibility of becoming the party’s nominee for the federal election as well as the payoff \( y_P \). This payoff, it was argued above, represents access to positions such as committee assignments, which are controlled by federally powerful parties.
While the set of parties \( \{L, R\} \) is robust to entry in many cases, the threat of entry is nevertheless required to support the equilibrium. Either party leader would otherwise have an incentive to exclude extremist politicians from the party, winning at least as many state elections as before and increasing the probability of winning the federal election. With entry this move is not profitable: suppose, for example, that party \( L \) were to exclude politicians with platform -1. The politicians now excluded from the party would join a latent party, which in turn would win at least some state elections. Consequently, this party will also be able to nominate a candidate for the federal election. With three candidates competing at the federal level there exists a voting equilibrium where the candidate of party \( L \) loses. My interpretation of this is that entry of even an extremist party poses a serious threat because of the danger that the established party loses its core support.\(^{17}\)

Real-life examples of this abound: prior to the general election of 2015, the UK Conservative Party feared the rise of the UK Independence Party (UKIP), not so much because UKIP itself was expected to win many seats in parliament, but because the votes lost to UKIP could have played into the hands of the Labour Party.\(^{18}\) In Germany, the market-oriented reforms unexpectedly pushed through by chancellor Gerhard Schroeder between 2002 and 2003 alienated the socialist wing of his party and fostered the formation of left-wing party The Left. The Social Democrats have been trailing the Christian Democrats in federal elections ever since.

Defeat at the general election due to a split in the left-wing vote implies that party \( L \) becomes less attractive and all eligible politicians prefer to join party \( R \). Even entry of an extremist party would consequently deter the leader of party \( L \) from deviating. There thus exists a mutual dependence: extremist politicians prefer being members of a more moderate party due to the opportunities that come with being a member of federally powerful party. The party, on the other hand, is happy to offer these opportunities as it benefits from having extremist politicians as members rather than as competitors.

The set of all equilibria with two parties is characterized in appendix A and illustrated in figure 4. The constellation in panel I) is the \( L-R \) equilibrium discussed previously. If career opportunities at the federal level are not sufficiently

\(^{17}\)For simplicity, the proof uses the fact that a harsher punishment can be assigned after this deviation by the leader of party \( L \). However, even entry of a party of shape \([-1]\) as it is discussed here is sufficient to make this deviation not profitable.

\(^{18}\)In the event, a late swing in voter sentiment handed a surprisingly strong victory to the Conservative Party. UKIP, on the other hand, seems to have hurt Labour candidates in at least some constituencies; a possibility previously overlooked by commentators.
valuable to prevent extremists from joining smaller parties, this equilibrium fails to exist. There then nevertheless exists a two-party equilibrium, which is exemplified in panel II) of figure 4. Here an extremist left-wing party (right-wing party) faces a centre-right (centre-left) party. I will refer to this equilibrium as the $M-E$ equilibrium. The moderate party wins the federal election with higher probability than parties in the $L-R$ equilibrium do, increasing the payoff of its members. This can prevent them from joining a third party, even when the $L-R$ equilibrium does not exist.

If even the $M-E$ equilibrium is not robust to members of the larger party defecting to a smaller party, no equilibrium with two parties exists. This is illustrated in figure 5 for $y_s$ equal to one. In the upper panel the number of states is equal to four, while the lower panel corresponds to the case where the number of states is infinite. Under combinations of values for the payoffs $y_f$ and $y_P$ that fall into region I the $L-M$ and the $M-E$ equilibrium both exist, while in region II the $M-E$ equilibrium is the unique equilibrium with two parties. While the $M-E$ equilibrium exists more broadly, the $L-R$ equilibrium seems more empirically relevant, as it generates competitive federal elections and also allows for party switching of moderate politicians, as is observable in the US for example. Comparing both panels illustrates how internal competition increases with the number of states. When the number of states is low, each state winner has a high chance of being nominated for the federal election. This prospect alone can be enough to keep politicians from deviating to joining smaller parties, who would offer less competition at the state level. As the number of states increases, the probability with which each individual politician wins the federal nomination of her party goes to zero. A sufficiently high value for the payoff $y_P$ is then required in order to prevent defections. In region III of the figure, no equilibrium with two parties exists. The space above the diagonal line, on the
Figure 5: Existence of Two-Party Equilibria

Notes: Both panels show equilibrium existence in $y_f$-$y_P$ space for $y_s = 1$ and a fixed number of states. In the upper panel the number of states is equal to four. The lower panel presents the case where the number of states is infinite.

other hand, is not part of the parameter space due to the assumption $y_f > 2y_P$.

5.2 Equilibria with Any Number of Parties

This section considers the possibility of equilibria where the number of parties formed is not equal to two. A result that can easily be established is that equilibria where only one founder proposes a party do not exist.

**Proposition 3.** There is no equilibrium such that $N^* = 1$.

**Proof.** Suppose only one party exists and call this party $A$. As party $A$ cannot cover all platforms, there must be some policy $p$ such that politicians with this platform are not allowed to join party $A$. These politicians would therefore join the latent party of shape $[p]$, which would then win at least one state election: by assumption, there exists a state with a median voter who strictly prefers platform $p$ over any other platform. In this state the latent party of shape $[p]$ would accordingly win the state election, as there are only two parties nominating candidates. □
If there was only one party, this party would not be able to allow all politicians to join by assumption. But if some politicians are unable to join any party, a latent party that allows just these politicians to join could easily recruit them. Due to the assumption on heterogeneity in voter preferences across states, this party would then also win at least one state election.

It should be stressed that the result that no single party can fend off entry would continue to hold even if parties were allowed to cover all policies. The reason is that such a party would create intense competition for nominations within the party, making at least some politicians willing to join a second party.

No such clear-cut results can be established once equilibria with three or more parties are considered, as this set is large. What is more, the equilibrium number of parties can also be large as demonstrated by the following example.

**Lemma 2.** Suppose there are $S$ states, with two and only two states $s$ such that $m_s \in (p^-, p^+)$. Also, let it be true for any state $s'$ such that $m_{s'} \notin (p^-, p^+)$ that $\Lambda(s', [-0.5, 0.5]) \leq 0.5$. Then there exists an equilibrium such that $N^* = S$.

**Proof.** Consider the following strategy profile: $S$ founders propose parties. Two of them propose a party of shape [0]. Call these centrist parties $C_1$ and $C_2$. All other parties either have the shape $[-1]$ or $[1]$. Call these parties leftist and rightist. The number of parties with shape $[-1]$ is equal to the number of states $s$ such that $m_s < p^-$. Collect these states in the set $S_l$. The number of parties with shape $[1]$ equals the number of states $s$ such that $m_s > p^+$. Collect these states in the set $S_r$. Each of the centrist parties wins the election in one of the two states that do not belong to the set $S_l \cup S_r$ and accordingly has a member in that state. Each leftist (rightist) party has a member in one and only one of the states belonging to $S_l$ ($S_r$) and wins the state election in that state. At the federal election the candidates of the centrist parties tie and no other parties receive any votes.

Given that each election features at least three candidates and the assumptions on voter distributions, there always exists a voting equilibrium such that the specified candidate wins. Centrist politicians do not gain from changing their party affiliation as they either continue to lose the state election or simply substitute external competition for internal competition at the federal level. Similarly, extremist politicians do not benefit from switching parties either. Any additional parties would not win at the federal level due to strategic voting and all politicians who win state elections are already the sole member of their party in their state, implying that they would not increase their utility by joining a newly formed party. No founder wants to deviate to remaining passive, as each founder wins one state election. Finally, no founder wants to change the shape of their party, as any such deviation can be punished by an
equilibrium of the election subgame where only latent parties attract members, which exists due to the proof of proposition 5 in appendix A.

The proof of the preceding proposition illustrates the possibility that a large number of parties forms, only a few of which play a role at the federal level. What may be the reason that such an equilibrium is not typically observed in reality? One possibility is that constellations with many parties do not persist because there are incentives for parties to merge. In fact, mergers are commonly observed in reality. To name just a few examples: the Liberal Democrats of the UK were formed as a fusion of the Liberal Party and the Social Democratic Party. The Conservative Party of Canada came into existence as a merger of the Progressive Conservative Party and the Canadian Alliance. In Australia, the coalition between the Liberal Party and the National Party has existed for so long that it is often treated as a single party.

One gain from a merger may be that a group of parties can win more elections by entering into an alliance than the total number won by them individually. A second potential reason for a merger of parties lies in synergies such as reduced administrative costs, more effective fund-raising and advertising campaigns, and greater recognition by voters in states where the party does not have a strong presence. In the model, such an incentive for a merger comes from the presence of the fixed cost $c$ that founders pay when forming a party. Avoiding this cost can make a merger profitable even if the newly formed party does not enjoy greater success.

Note that any merger that happens for reasons of increased electoral success also leads to a reduction in fixed costs, while the converse is not true. In trying to integrate mergers into the model I will therefore focus on fixed costs as the driving force, which is also simpler to implement than the case where electoral motivations play a role. Doing so requires that leaders are somehow able to share the benefits of their joint ventures. I allow for this by giving founders the ability of committing to transfers to other founders in the beginning of the game that are conditional on whether or not the founder receiving the transfer proposes a party. In reality these transfers will often take the form of powerful positions in the party hierarchy or in government, which are pledged to party leaders who agree to enter into an alliance. The ability to credibly commit to these transfers, on the other hand, can be justified by reputational concerns. Formally, such a promise takes the shape of a map $\{0, 1\} \to \mathcal{R}_+$, which assigns an amount of utility to be transferred to a founder to an indicator for whether
this founder has proposed a party or not. Accordingly, the strategy space of each founder is extended to allow for a choice of a vector of infinite length consisting of such maps. Let \( \tilde{t} \) be the sum of all transfers a founder receives net of any transfers the founder carries out. The utility of this founder is then given by

\[
\sum_{s=1}^{S} \pi_s x_w + \pi_f x_f - c + \tilde{t},
\]

using the same notation as in section 4. Call this version of the game the extended party-formation game.

Beyond transfers, mergers also require the possibility of joint deviations by groups of founders. One way to allow for this would be to look for equilibria that are coalition-proof among founders as defined by Bernheim et al. (1987). However, a much simpler approach, which turns out to be equivalent in this particular case, is to look for equilibria that are Pareto efficient among founders in the extended party-formation game. I say that such an equilibrium is robust to party mergers.

As is shown in appendix B, it is necessary and sufficient for an equilibrium to be robust to party mergers that there exists no other equilibrium with a lower number of parties (Lemma 3 in appendix B). Only an equilibrium with a lower number of parties generates a greater sum of utilities among founders, which can then be redistributed through transfers such that a Pareto improvement results. The reason that the sum of utilities is greater when there are fewer parties is due to the fixed cost \( c \) that any party leader has to pay.

This result makes it possible to fully characterize the number of parties formed in party-formation equilibria that are robust to party mergers.

**Proposition 4.** The number of parties in any party-formation equilibrium that is robust to party mergers is

i) no lower than two and no greater than three,

ii) equal to two whenever a party-formation equilibrium exists in which two parties are formed.

**Proof.** See appendix B. \( \square \)

A second look at figure 5 illustrates the set of equilibria that are robust to party mergers. In regions I and II, two-party equilibria exist and all other types of equilibria are therefore ruled out. In region III the only equilibria that are
robust to party mergers feature three parties. The equilibrium used to establish this result is one where all existing parties allow only one type of politician to join, there is one such party for each possible platform, and the candidates of the two extremist parties tie at the federal election. Other three-party equilibria exist as well. An interesting possibility are equilibria which are “almost identical” to one of the two-party equilibria. As an example, consider the $M$-$E$ equilibrium with $I_M = [-1..0]$ and $I_E = [1]$. A comparison of the upper and the lower panel of figure 5 shows that this equilibrium sometimes exists only if the number of states is sufficiently low. Otherwise too much internal competition makes extremist members of party $M$ willing to defect to a newly formed party of shape $[-1]$. This suggests the existence of an equilibrium where this smaller party forms in addition to the parties of the $M$-$E$ equilibrium and voters at the federal level behave as if the smaller party did not exist (this is possible under strategic voting). While not successful at the federal level, the additional party wins some state elections and thereby reduces internal competition among members of party $M$ sufficiently to prevent further deviations.

The model thus allows for the formation of parties that win state elections and play no role federally, but only if the expected value of career opportunities at the federal level is sufficiently low. Thinking more broadly, the model suggests an additional reason for the existence of parties whose success is confined to specific regions. Take the Scottish National Party (SNP) as an example. A politician who firmly believes that Scotland should be an independent country cannot expect to have a career at the national level of the UK. Consequently, such politicians will be much more willing to establish a party of their own, explaining the existence of the SNP. The same reasoning applies to other regions with separatist movements, such as Catalonia in Spain, Quebec in Canada, or South Tyrol in Italy. While perhaps more adequately modelled through a more general policy space, this phenomenon could simply be captured through an increased magnitude of the payoff $y_s$ for politicians residing in such states.

6 Robustness

The basic model of party formation presented here requires a number of simplifying assumptions for tractability. This section will discuss some of these in more detail.
6.1 Comprehensive Parties

Parties that allow all politicians to join were ruled out in the basic version of the model. This section will discuss what additional equilibria exist if parties of shape \([-1..1]\) were included in the action space of founders. As was already argued above, even a party that allows all politicians to join would not be able to deter entry of additional parties due to intense internal competition. However, focusing on the class of two-party equilibria, two additional stable constellations of parties emerge. One features either the equilibrium set of parties \{\([-1..1], [-1..0]\)\} or \{\([-1..1], [0..1]\)\}. This constellation can actually be essentially equivalent to the M-E equilibrium. This is the case when all eligible politicians join the smaller party in all states. Out-of-equilibrium beliefs of voters then prevent any deviations by politicians towards joining the larger party.

The second case is that an equilibrium with two parties of shape \([-1..1]\). This is possible because out-of-equilibrium beliefs can be used to make both parties have only one member in any state, as in the previous paragraph. It seems unlikely though that a party could maintain such widely varying ideological profiles across different states. If both parties had three members in most states, on the other hand, entry of a party of shape \([0]\) would be possible. This is because the larger parties then create too much internal competition. Furthermore, a centrist party could also do well federally, as it is relatively likely that both of the larger parties nominate a candidate with platform -1 or that both of them nominate a candidate with platform 1. In both cases a strict majority of voters would prefer a centrist candidate, who would accordingly win by assumption.

A way of eliminating the equilibria just discussed would be to allow politicians within a state to make joint-deviations. This would make it impossible that a party of shape \([-1..1]\) wins a state election with a single member in that state, as the remaining two politicians would be better off by coordinating on joining the winning party. By the same logic as in the preceding paragraph, entry of a party of shape \([0]\) would then occur. Other equilibria discussed above, on the other hand, are robust to allowing for such joint deviations. On the equilibrium path, the constellations of parties considered actually give no scope for joint deviations, as at most one politician has a choice between different parties. Off the equilibrium path, there is no point in making joint deviations. The reason why politicians may find a deviation to an additional party attractive is that this can reduce the degree of internal competition they face at the state
level. A joint-deviation would eliminate this benefit.

6.2 A Greater Number Of Politicians or Platforms

Allowing for a greater number of politicians or platforms is difficult, as this leads to a greater number of cases to consider. It also increases the scope for coordination failure at the stage at which party affiliations are chosen and thus the number of equilibria. Nevertheless, one effect of a higher number of politicians populating each state is clear: More politicians imply greater internal competition for nominations at the state level. This would make existence of the two-party equilibria discussed above less likely, as the magnitude of the payoffs $y_s$ and $y_P$ required in order to keep politicians from joining smaller parties increases proportionally with the number of party members. This may not be too much of a concern: the politicians in the model should be thought of as those who have already achieved some prominence within state parties and are therefore in a position to be considered for nominations. At any given point in time the number of such individuals will be limited. In addition, an emerging party focused on issues already covered by an existing party would find it hard to achieve credibility if it fails to attract any of the more prominent members of that party. In fact, the formation of new parties is typically the product of a whole faction of an existing party defecting jointly. It is possible to interpret each politician in the model as representing factions who coordinate their actions.

6.3 Policy Choices

The assumption that politicians are committed to implementing their platform is not satisfying. While the empirical literature presented in section 3 seems to suggest that policy preferences of politicians are the main driver of their choices in office, it would be more appealing to see this behaviour emerge as part of an equilibrium rather than imposing it from the outset. In the model, extremist politicians can often increase their chances of winning the federal election by pretending to be a centrist when choosing state policies. To address this concern I will consider a more general utility function for politicians that includes both career concerns and policy preferences. For a politician with ideal policy $i$ let
the utility function now be given by

$$\pi_s(y_s + \pi_n \pi_f y_f + (1 - \pi_n) \pi_p y_p) - \alpha \sum_{i \in \{s, f\}} (p_i - i)^2,$$

where $\alpha$ measure the the relative weight that politicians attach to policy and
the notation is otherwise the same as in section 4. In addition, assume that
politicians can freely choose the policy they implement at any stage. All other
elements of the game remain unchanged. This more general version of the
model is challenging to solve in its entirety. I will present results for the election
subgame reached after parties $L$ and $R$ have been proposed.

Proceeding by backwards induction, it is clear that any politician elected
at the federal level will implement her ideal policy. All other stages are less
straightforward. I will start by asking under what conditions a separating equi-
librium exists where politicians implement their ideal policy at the state level
and otherwise behave as in lemma 1. In this case a politician with ideal policy
-1 achieves a continuation utility of

$$\frac{1}{w^*_L} \left[ \frac{1}{4} y_f + \frac{1}{4} (-\alpha) + \frac{1}{2} (-\alpha) \right] + \left( 1 - \frac{1}{w^*_L} \right) U_n$$

after winning a state election, where the term $U_n$ captures the utility in case
the politician is not nominated for the federal election. Choosing the policy 0
after the state election results in a utility of

$$-\alpha + \frac{1}{w^*_L} \left[ \frac{3}{4} y_f + \frac{1}{4} (-\alpha) \right] + \left( 1 - \frac{1}{w^*_L} \right) U_n.$$

The politician now incurs a cost for a suboptimal policy choice but increases
her chance of winning the federal election, simultaneously reducing the prob-
ability that an extremist from party $R$ gets elected. Note that voters do not
observe that a deviation has taken place as the politician elected in the state
has ideal policy 0 with positive probability ex-ante. It can then be shown that
the separating equilibrium exists as long as the ratio $\alpha/y_f$ is no smaller than
$$\max_{j \in \{L, R\}} 2/(4w^*_j - 5).$$

The greater the number of elections won by a party,
the lower the probability that any given politician will get nominated for the
federal election, which in turn makes extremist politicians less likely to benefit
from pretending to be a centrist.

\[19\] This condition applies as long as each party wins at least two state elections, which implies
that $4w^*_j - 5$ is positive.
Proceeding as above it can be shown that if the ratio \( \alpha/y_f \) is smaller than 
\[ \min_{j \in \{L,R\}} \frac{2}{(4w_j^* - 5)} \] 
then there only exists a pooling equilibrium where all politicians implement the policy 0 at the state level.\(^{20}\) Behaviour nevertheless remains very close to the one given in lemma 1. Dropping the assumptions that voters vote myopically (which was introduced above purely as a simplification), voters in state elections will base their choice on considerations regarding federal policies as all politicians implement the same policy at the state level. Federal policies are determined by the winner of the federal election, who is effectively picked at random from among state winners. State voters thus benefit from adding politicians to this pool that have similar preferences to them.\(^{21}\) For example, a voter with ideal policy -1 in a state where politician 0 has joined party \( L \) would not vote for party \( R \) as a victory by party \( R \) increases the probability that a politician with platform 1 wins the federal election. As long as politicians in both parties pool, the only difference between them is the degree of internal competition for the federal nomination, at least from the perspective of a centrist politician in a centrist state. As before, these politicians will therefore tend to sort into the party that wins fewer state elections.

The empirical evidence is in favour of the separating equilibrium, where all politicians implement their own ideal policies. Nevertheless, the discussion above suggests that the overall results do not necessarily change much even if extremist politicians try to pass off as centrists.

### 6.4 Candidate Selection

Parties are assumed to nominate candidates randomly in the basic version of the model. This hurts the chances of either party of winning the federal election. However, if parties were to nominate a centrist politician for the federal election whenever one is available, this would reduce the expected utility of extremist party members and potentially induce them to join a third party. In order to prevent this, the party leader may have an incentive to commit ex-ante to nominating extremists with a sufficiently high probability. In practice this could be achieved through choosing a particular mechanism for candidate selection, such as primaries or nomination through voting by delegates at the party convention.

\(^{20}\)Cases where politicians in one party separate and politicians in the other party pool are more complicated and will not be considered here.

\(^{21}\)Voters at the federal election are split at zero between both parties as they cannot distinguish between candidates exactly, but know that the candidate of either party may be a centrist or an extremist with equal probability.
As it turns out, the requirement to keep extremist party members satisfied may lead to extremists being nominated with even greater probability than in the basic model. This possibility is demonstrated in appendix C, which analyses an extension of the model where the party leader commits to a probability of nominating either type of politician at the federal election.

A second and closely related possibility of keeping the extremist wing of the party satisfied would be to increase the probability with which extremist politicians get nominated in extremist states. To some extent I feel that the assumption that the nomination process is fairly noisy from the perspective of party-outsiders is more appropriate at the state level than at the federal level. After all, state-level candidates are often little known to the public and different factions within the party will be pulling in different directions, with the party leader (the party establishment) certainly favouring moderate candidates due to their electability at the federal election. Nevertheless, nominating candidates through primary elections may be one way to ensure that candidates fit the preferences of the state median voter well. If this is the case, then it becomes even easier to see how national parties maintain regional monopolies: extremist politicians in extremist states would basically face no internal competition from moderates and the existence conditions for the two-party equilibrium of proposition 2 would be greatly relaxed. It is noteworthy though that even in this case no single party would be able to monopolize all elections. Intense internal competition for the federal nomination would still lead to the successful entry of a second party and proposition 3 continuous to hold.

7 Conclusion

The aim of this paper was to explain why US federal elections are typically strongly competitive, while state elections are often dominated by one party. It was shown that an equilibrium with two parties, one centre-left and one centre-right, can explain the main features of the data. In this equilibrium, states with centrist median voters will display small vote margins while state elections with more extreme median voters are dominated by one party. The federal level, on the other hand, turns out to be competitive as both parties are able to attract symmetric candidate pools centred around the federal median voter.

Two key forces are involved in stabilising this situation. The threat of entry of additional parties makes it necessary that parties “cover their flanks”:
parties need to extend their membership far enough to the extremes such that any politician who appeals to a state median voter is allowed to join. As a consequence, new parties can only compete successfully if they manage to attract members of the existing parties. The major disadvantage of such entrants is that they might do well at the state level, but would find it hard to compete federally. This means that incumbent parties have exclusive control over attractive positions at the federal level, which they can allocate to their members. If politicians find such career opportunities sufficiently valuable, newly formed parties are unable to recruit members and entry fails.

More generally, career concerns of politicians are the main driver of the number of parties that form in equilibrium. Using an additional refinement, it was shown that two parties are the unique outcome when career concerns are sufficiently strong. On the other hand, more than two parties will form if politicians care less strongly about opportunities at the federal level.

While this paper has focused on the US, the pattern that provided the motivation is observable in other countries as well. In the German state of Bremen, the Social Democratic Party has been in control of the state legislature for more than 60 years. Similarly, the Austrian People’s Party has ruled the states of Tyrol and Vorarlberg ever since the end of the Second World War. At the same time, neither country features a dominant party at the federal level. Extending the results presented here to these countries is not straightforward due to the technical difficulties involved in modelling systems of proportional representation. Nevertheless, it seems possible in principle. Both countries feature two main parties that traditionally receive the vast majority of votes. Importantly, this is true federally as well as at the state level. It thus seems that the major parties allow for and attract a membership that is ideologically broad enough to ensure a strong position across states. Preventing entry of any additional parties, in contrast, would require extending the party membership far to the extremes under proportional representation and could be too costly in terms of votes lost at the federal level and in more moderate states. Career concerns would then again be an important factor in that they prevent fringe parties from luring politicians away from the major parties. There would clearly be many additional issues to be taken into consideration, making this a challenging subject for further study.
Appendix

A Additional Results for the Basic Model

This appendix provides a full characterization of equilibria with two parties beyond the L-R equilibrium analysed in the main text. Due to the usefulness of this result in subsequent proofs I will start by establishing the existence of an equilibrium with three parties.

**Proposition 5.** An equilibrium such that \( P^* = \{A, B, C\} \) with \( I_A = [-1], I_B = [0], \) and \( I_C = [1] \) always exists.

**Proof.** Suppose the set of parties \( \{A, B, C\} \) is formed in equilibrium with \( I_A = [-1], I_B = [0], \) and \( I_C = [1] \). Then the affiliation behaviour of politicians is trivial. Assume the candidates of parties A and C tie at the national level, party B wins all centrist states and states \( s \) such that \( \Lambda_s([-0.5, 0.5]) > 0.5 \), and party A (party C) wins all state elections in states \( s \) such that \( m_s < p^- (m_s > p^+) \) and \( \Lambda_s([-0.5, 0.5]) \leq 0.5 \).

No politician has an incentive to join any additional parties. There would be at least three parties competing at the federal level, implying that there exists a voting equilibrium such that the newly formed party loses. Politicians therefore have no incentive to join this new party, as they either already win the election in their state with certainty or would not win even after the deviation.

It remains to check whether any founder has an incentive to reposition their party. Without loss of generality, suppose that the founder of party A changes the shape of their party. Subsequently, let all politicians who were previously members of party A join the latent party of shape \([-1] \), while all other politicians behave as before. By virtue of the proceeding steps of the proof, this constitutes an equilibrium of the election subgame reached. As party A has lost all its members, the deviation is not profitable for the founder of party A.

An additional equilibrium with two parties is given by the following proposition.

**Proposition 6.** An equilibrium of the party-formation game where \( P^* = \{M, E\} \) with either \( I_M = [-1.0] \) and \( I_E = [1] \), or \( I_M = [0.1] \) and \( I_E = [-1] \) exists whenever

\[
\frac{1}{S} y_f + \left(1 - \frac{1}{S}\right) \frac{3}{2} y_p \geq 2 y_s.
\]

**Proof.** As the equilibria in the statement of the proposition are symmetric to each other, the proof will focus on the case \( I_M = [-1.0] \) and \( I_E = [1] \). In this case the affiliation behaviour of politicians is trivial. Party M wins all states \( s \) such that

\[
\frac{1}{S} y_f + \left(1 - \frac{1}{S}\right) \frac{3}{2} y_p \geq 2 y_s.
\]


\( m_s < p^+ \) and wins the federal election with probability three-fourth. It will first be shown that entry of additional parties is impossible.

The equilibrium utility of extremist members of party \( M \) is

\[
\frac{1}{2} \left[ y_s + \frac{1}{2w^*_M} y_f + \left( 1 - \frac{1}{w^*_M} \right) \frac{3}{4} y_P \right].
\]

This expression is decreasing in \( w^*_M \) under the assumption that \( y_f > 2y_P \). If this politician joins an entering party of shape \([-1]\), she may win the state election, but can be made the certain loser of the federal election. As party \( M \) cannot win more state election than there are states, it follows that a sufficient condition for this deviation not being profitable is

\[
\frac{1}{S} y_f + \left( 1 - \frac{1}{S} \right) \frac{3}{2} y_P \geq 2 y_s.
\]

Centrist politicians achieve a higher utility in equilibrium than extremist members of party \( M \) as they win the federal election with higher probability. Due to the assumption that \( \Lambda_f([-0.5,0.5]) \leq 0.5 \) and the presence of three candidates at the federal level, there exists a voting equilibrium such that centrist candidates of a third party do not win at the federal level. Their deviation payoff is accordingly also \( y_s \).

This shows that centrist members of party \( M \) do not deviate to joining a third party whenever politicians with platform \(-1\) refrain from doing so.

Finally, members of party \( E \) do not gain from joining a party of shape \([1]\) as they at best win all elections with the same probability as before.

It remains to be checked whether any active founders want to deviate. Any such deviation can be punished by an equilibrium of the election subgame where only latent parties attract members, which exists due to the proof of 5.

The equilibrium in the preceding proposition will be referred to as the \( M\)-\( E \) equilibrium. Any other constellation of two parties not considered so far can never form an equilibrium by itself.

**Proposition 7.** No constellation of two parties other than \( \{[-1..0],[0..1]\} \), \( \{[-1..0],[1]\} \), and \( \{[-1],[0..1]\} \) is part of an equilibrium where those two parties win all elections.

**Proof.** It is straightforward to verify that under any constellation of two parties not listed in the statement of the proposition there must be some platform \( p \) such that politicians with this platform cannot join any party proposed by founders. Politicians with platform \( p \) will therefore join the latent party of shape \( [p] \). Due to the assumptions on voter distributions there exists at least one state where a strict majority of voters strictly prefers platform \( p \) over the expected platform of a candidate of any other party. It then follows from the restrictions on voting behaviour that the latent party of shape \( [p] \) wins at least one state election. \( \square \)
B Robustness To Party Mergers

This appendix derives properties of equilibria of the extended party formation game that are robust to party mergers, as defined in section 5.2. The following result shows that the requirement of robustness to party mergers selects those party-formation equilibria with the lowest number of parties, but does not discriminate between equilibria within that class.

Lemma 3. A party-formation equilibrium is robust to party mergers if and only if there exists no other equilibrium in which a smaller number of parties is formed.

Proof. Assume there is a party-formation equilibrium $E$ such that $N^* = k$ and there is a potentially empty set $F_T$ of founders who receive transfers from other founders. Let $F_E$ be the set of founders who propose parties in this equilibrium. All founders who make transfers must belong to the set $F_E$. Any founder not belonging to this set could lower the transfers she makes to zero. This would not affect the transfers she receives as these can depend only on whether or not a founder proposes a party. The total utility of all members of the set $F_E \cup F_T$ is therefore equal to $Sx_w + x_f - kc$.

To show necessity, suppose that there exists a second party-formation equilibrium $E'$ in which a number of parties $k' < k$ is formed. Now consider the following equilibrium: in the first step, $k'$ members of the set $F_E$ promise transfers and subsequently the same $k'$ founders propose the set of parties that exists along the equilibrium path of $E'$ and politicians accordingly behave as in the equilibrium $E'$ as well. The transfers promised are such that any founder belonging to $F_E \cup F_T$ achieves a strictly greater utility than in the equilibrium $E$. This is possible because the total utility of all founders is higher in the equilibrium $E'$ than in the equilibrium $E$ by the amount $(k - k')c$. If any deviation occurs at the first stage, the same set of parties as in the equilibrium of proposition 5 in the appendix is formed by three founders not belonging to the set $F_E \cup F_T$. This ensures that no founder can gain from deviating from the transfers promised in equilibrium. Finally, it must be true that no passive founder has an incentive to form an additional party. If there was such an incentive, this would require that some politician strictly gains from joining the new party for any belief that voters could form. This would imply in turn that the same politician also gains from joining a latent party, which contradicts that $E$ or $E'$ is an equilibrium. This shows that the equilibrium $E$ is not robust to party mergers.

For sufficiency, note that any equilibrium $E'$ in which the number of parties is equal to or greater than $k$ generates a total utility that is no greater than the total utility achieved in the equilibrium $E$. It follows immediately that it is impossible that the equilibrium $E'$ Pareto dominates the equilibrium $E$. □

The sufficient and necessary condition in the previous result makes it possible to fully characterize the number of parties formed in party-formation equilibria that are
Proposition 4. The number of parties in any party-formation equilibrium that is robust to party mergers is

i) no lower than two and no greater than three,

ii) equal to two whenever a party-formation equilibrium exists in which two parties are formed.

Proof. The proof of proposition 3 applies to the extended party formation game, implying that it is impossible that only one founder proposes a party and this party wins all elections. Consequently, there must be at least two parties in any party-formation equilibrium that is robust to party mergers. Combined with lemma 3 this establishes claim ii). In order to show that there cannot be more than three parties it needs to be demonstrated that there exists a party-formation equilibrium of the extended game in which three parties are formed whenever no equilibrium with two parties exists. This result has been established in proposition 5 in appendix A. □

C Candidate Selection

Consider a version of the basic model described in section 4, where the founder of a party $P$ makes an additional strategic choice in committing to a probability $q_P$. In any situation where party $P$ has both extremist and moderate politicians in its candidate pool for the federal election, the candidate for this election will be randomly drawn from among centrists with probability $q_P$ and from among extremists with probability $1 - q_P$. The choice of $q_P$ is made simultaneous to the proposal of the party. This appendix will provide a proof for the claim made in section 6.4 that this extended version of the model may have an equilibrium where extremist politicians are nominated with probability greater than one-half. To do so I will derive an equilibrium of this more general model where parties $L$ and $R$ as defined in the main text get proposed, no other parties can successfully enter, and $q_L = q_R = q$ for some probability $q$. Throughout this section it will be assumed that there are four states, one with a median voter located below $p^-$, one with a median voter located above $p^+$, and two with centrist median voters. It will also be assumed that $y_f > \frac{2p}{3} y_P$.

Start by considering the affiliation behaviour of politicians. Let $\pi_f(q|p)$ be the probability that a candidate for the federal election with platform $p$ wins, given that the other party uses the nomination probability $q$. In the case where party $L$ wins the state election in one other state, the utility of a member of party $L$ with platform -1 in a state where $L$ wins is

$$\frac{1}{2} \left( y_s + \frac{1}{2} (1 - q) + \frac{1}{4} \pi_f(q|-1)y_f + \left( \frac{1}{2} q \pi_f(q|0) + \frac{1}{4} \pi_f(q|-1) \right) y_P \right)$$,
with \( \pi_f(q| -1) = \frac{1}{8} + \frac{1}{2} (1 - q) \frac{1}{2} \) and \( \pi_f(q|0) = \frac{3}{8} + \frac{1}{2} - \frac{1}{4} q \). For \( q = 0 \) this expression becomes

\[
\frac{1}{2} \left( y_s + \frac{9}{32} y_f + \frac{3}{32} y_P \right).
\]

In the state with median voter at or below -0.5 such a candidate can achieve a utility of at most \( y_s \) by deviating to joining a new party. The deviation utility is no greater than the equilibrium utility for \( q = 0 \) as

\[
\frac{1}{2} \left( y_s + \frac{9}{32} y_f + \frac{3}{32} y_P \right) > \frac{1}{2} \left( y_s + \frac{29}{32} y_f + \frac{3}{32} y_P \right) = \frac{1}{2} (y_s + y_P),
\]

where the first inequality holds due to the assumption that \( y_f > \frac{29}{9} y_P \). Deviating to joining a new party is consequently worse as long as \( y_P \geq y_s \). It follows from the continuity of payoffs in \( q \) that politicians with platform -1 or 1 either do not want to deviate even if \( q = 1 \) or that there exists some threshold \( q^c \in (0,1) \) such that the deviation is not undertaken for \( q = q^c \), but occurs for any \( q > q^c \). In the former case set \( q^c = 1 \).

Politicians with platform 0 in states with centrist median voters have a choice between both joining the same or separate parties. In the latter case each achieves a utility of

\[
\frac{1}{2} \left\{ y_s + \left( \frac{1}{2} q + \frac{1}{4} \right) \pi_f(q|0)y_f + \left[ \frac{1}{2} (1 - q) \pi_f(q| -1) + \frac{1}{4} \pi_f(q|0) \right] y_P \right\}, \tag{1}
\]

with \( \pi_f(q| -1) \) and \( \pi_f(q|0) \) as given above. For \( q = 0 \) this simplifies to

\[
\frac{1}{2} \left( y_s + \frac{7}{32} y_f + \frac{13}{32} y_P \right).
\]

If both politicians join the same party their utility becomes

\[
\frac{1}{2} \left\{ y_s + \left( \frac{1}{12} + \frac{1}{2} q \right) \pi_f(q|0)y_f + \left( \frac{1}{8} + \frac{3}{16} q + \frac{3}{16} (1 - q) \right) y_P \right\}. \tag{2}
\]

Setting \( q \) to zero yields

\[
\frac{1}{2} \left( y_s + \frac{1}{16} y_f + \frac{5}{16} y_P \right).
\]

Both politicians thus prefer being in separate parties for \( q = 0 \). It follows from the continuity of payoffs in \( q \) that politicians with platform 0 in centrist states either both want to be members of the same party even if \( q = 1 \) or that there exists some threshold \( q^c \in (0,1) \) such that they are indifferent at \( q = q^c \), but would prefer being members of the same party for any \( q > q^c \). In the former case set \( q^c = 1 \).

It can be shown that \( q^c > 0.5 \). To do so evaluate expressions (1) and (2) at \( q = 0.5 \). This yields \( \frac{1}{2} (y_s + \frac{7}{4} y_f + \frac{1}{2} y_P) \) and \( \frac{1}{2} (y_s + \frac{1}{4} y_f + \frac{5}{16} y_P) \), respectively. The first utility is greater than the second utility as long as \( y_f > y_P \). This shows that centrist politicians...
in centrist states prefer to be members of the same party for \( q = 0 \) and \( q = 0.5 \). It is easy to show that expression (1) is concave in \( q \) as long as \( y_f > y_P \), while expression (2) is linear in \( q \). It immediately follows that the former utility must be greater than the latter utility for any \( q \in [0, 0.5] \). This demonstrates that the threshold \( q^c \) must be greater than 0.5.

Now assume \( y_s = y_P = 1 \) and \( y_f = 5 \). In this case it can be calculated that \( q^e \approx 0.38 \). This is below the threshold \( q^c \), which must be greater than 0.5. Is there an equilibrium such that \( \mathcal{P}_F = \{L, R\} \) and the founder of each party sets \( q \) equal to \( q^e \)? As long as the affiliation behaviour of politicians does not change, the utility of a founder is increasing in his choice of \( q \), as centrist politicians win the federal election with higher probability. Accordingly, neither founder would want to deviate to choosing a lower value of \( q \) than the equilibrium one. Increasing the level of \( q \) beyond \( q^e \) would lead to the entry of a new party, as extremist politicians in the party under consideration would then be willing to deviate by the definition of the cut-off \( q^e \). As in the proof of proposition 2, the entry of such a party reduces the utility of the party leader undertaking the deviation to zero. As \( q^e < q^c \), centrist politicians will join different parties. A straightforward calculation based on the expressions derived above shows that their utility is greater than \( y_s \), which is the utility they could achieve by joining a third party.
References


ICPSR (1995), Candidate name and constituency totals, 1788-1990, Dataset ICPSR00002-v5, Inter-University Consortium for Political and Social Research.


