Use of maximum entropy in estimating production risks in crop farms

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USE OF MAXIMUM ENTROPY IN ESTIMATING PRODUCTION RISKS IN CROP FARMS

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Abstract: *The entropic value of the production risk is closely linked to the farmer's aversion to this type of risk. Since risk aversion is difficult to quantify, it is preferable to use the MaxEnt model as a quantitative benchmark in assessing and covering the production risk through adequate financial resources. The classification of the Selyaninov index value as measure of the production risk based on the MaxEnt model utilization makes it possible to evaluate the production risk and the transfer decision to an adequate market implicitly. The authors' previous research investigated the risk coverage through derivative financial instruments that diminish the farmer's exposure to the production risk; the present paper adds to previous research by investigating an equally important issue: sizing the risk that is the object of coverage. Through the utilization of the stochastic methods in estimating the risk measure, a less rigid method is obtained that can be adapted and applied to the risk management processes in agriculture.*

Key words: *production risk, crop farms, Markov models, MaxEnt*

INTRODUCTION

By HMM (Hidden Markov Model) utilization in the continuous evaluation of the weather risk and by the application of advanced computational mechanisms one can evaluate with higher accuracy the weather phenomena that induce agricultural production risks. The random aspects that characterize the dynamics of weather factors determine implicitly the size, quality and price of farm production and make the weather risk be found in the agricultural production risk at farm level. Wheat production variability in Romania in the period 2005-2009 ranged from 50% to 130% (Rutten, 2012); the same source mentions Swiss Re[3], which estimates an 80% extreme weather phenomena risk in Ukraine in the year 2010, compared to only 20% political risk. The performance of insurance schemes mainly in the case of production risk leaves room for significant improvements mainly in the emergent economy area.

Starting from a risk assessment and identifying a special risk (weather risk, for instance), a strategy for this risk management is chosen, practically its transfer in the conditions in which an agricultural insurance product cannot be identified on the agricultural insurance market. The insurance index is a simplified insurance form in which the compensation payment is based on the values of an index that acts as a proxy for losses and not on the losses estimated according to the individual insurance policy. The insured amount is based on the production cost on the basis of convened value (established in advance) and the payments are made on the basis of a pre-established grid in the insurance policy.

In a World Bank document on the agricultural insurances (World Bank, 2011), WII (Weather Index Insurance) is presented, which originates in the weather derivatives market, in a scenario in which the speculative funds take over the weather risk. The interest for WII in agriculture increased on the background of certain non-performant traditional insurance products, mainly in the countries with emergent economies, and where the limited commercialization and the small farm size are major obstacles to the sustainable development of certain performant insurance products for the farm production. In the papers by Hurduzeu (2014) and Kevorchian et al. (2013), the Selyaninov index for wheat is used as weather index insurance:

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[3] The Swiss Re Group is a leading wholesale provider of reinsurance, insurance and other insurance-based forms of risk transfer

$$\mathrm{SHR}_{\text{wheat}} = \frac{\sum_{\text{April15 - June 30}} \text{Daily rainfall.}}{0{,}1 * \sum_{\text{April15 - Jume30}} \text{Daily average temperature}}$$

where the impact (e.g. on production) is determined according to the formula:

$$I(SHR) = \begin{cases} \max\{M, (1 - SHR) * \theta\} & SHR \in [0,1) \\ 0 & SHR \in [1,1.4] \\ \max\{M, (SHR - 1.4) * \theta\} & SHR \in (1.4,2) \end{cases}$$

where M is the contract value, and θ being a table variable as index value (Kevorchian et al., 2013). The daily value modelling in the vegetation period can be described on the basis of a Markov model.

Starting from the Markov model described by Kannan et al. (2015), we considered that the stochastic modelling of the Selyaninov index series is important for the management of crop development conditions in the vegetative period. We consider that the stochastic models can be successfully used to describe the development of crops in the vegetation cycle. When the observed available values of the Selyaninov index are inadequate, it is possible to work with the series of index values in a synthetic variant that has statistical properties comparable to those of the existing values.

MATERIAL AND METHOD

A stochastic process with the Markov property is designated as Markov chain.

A Markov chain is a particular case of a finite automaton whose associated transition graph is weighted and for which the entrance sequence is indicating the passage state of the automaton. From the formal point of view we have:

*	$S = s_1, s_2, \ldots, s_n$	family of states
*	$A = \begin{pmatrix} a_{01} & \cdots & a_{0m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix}$	matrix of transition probabilities where a_{ij} represents the probability of transition from state *i* to state *j* and $\sum_{j=1}^{n} a_{ij} = 1$ for any $i=1..n$ and s_0 and s_F are the initial and final states respectively of the Markov chain.

In a first-order Markov chain, the probability of a particular state depends only on the previous state in the sequence of states:

$$P(s_i | s_1, s_2, \ldots, s_{i-1}) = P(s_i | s_{i-1})$$

For any a_{ij} a probability $p(s_j|s_i)$ is associated with $\sum_{j=1}^{n} a_{ij} = 1$ for any $i \in \{1,2..n\}$. A Markov chain provides for the probability associated to an observable sequence of events. In the current practice, the investigated events are not always observable. For instance, the production losses on a farm are not observable at a variation of weather factors associated to a Selyaninov index. Practically the evaluation of production losses is hidden (non-transparent) as it is not directly observable. The objective is to use these observations in estimating the losses due to weather factors.

A HMM (Hidden Markov Model) is revealed by the following elements:

*	$S = s_1, s_2, \ldots, s_n$	represents the family of states
*	$A = \begin{pmatrix} a_{01} & \cdots & a_{0m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix}$	matrix of transition probabilities with a_{ij} represents the probability of transition from state *i* to state *j* and $\sum_{j=1}^{n} a_{ij} = 1$ for any $i=1..n$ and s_0 and s_F are the initial and final states respectively of the Markov chain.

*	$O = o_1, o_2, \dots, o_l$	A sequence of t probable observations, each of them obtained from V* associated to the vocabulary V={v_1, v_2,…, v_w,}
*	B=$b_i(o_t)$	Sequence of probable observations, each representing the probability of an o_t observation, being generated at the state *i* level.
*	s_0, s_f	Initial state, final state respectively, not associated to observations, together with the transition probabilities a_{01}, $a_{02,\dots,}$ a_{0n} , besides the initial state and a_{1f}, $a_{2f,\dots,}$ a_{nf} , in the final state.

Here is an alternative representation for HMM that is not based on the initial and final states, using an explicit distribution on the set of initial and explicitly accepted states:

$\pi = \pi_1, \pi_2, \dots, \pi_n$	an initial probability distribution over the states, π_i is the probability that the Markov chain will start from the state i. Certain states *j* can have $\pi_j = 0$ which means that it is not an initial state, as well as $\sum_{j=1}^{n} \pi_j = 1$
$SA = \{s_x, s_y, \dots, s_z, \dots\} \subset S$	is the set of accepted states.

A first-order HMM implies complying with the following conditions:

1.	$P(s_i \| s_1, s_2, \dots s_{i-1}) = P(s_i \| s_{i-1})$	named ***Markov hypothesis***
2.	$P\left(o_i \middle\| s_1, \dots s_i, \dots s_p, o_1, \dots o_i, \dots o_p\right) = P(o_i \| s_{i-1})$	also named the output independence hypothesis in which the output o_i is dependent only on the state s_i in which it is obtained.

We can consider two “hidden” states of the Markov chain (P, associated to losses for a certain crop and C, associated to gains in the case of the same crop), and the family of observations **O** corresponds to the Selyaninov index associated to the vegetative period for the respective crop. We should mention that the probabilities of transition between the two states will be non-null, which will lead to a completely connected or ergodic transition graph.

Be it a HMM formally defined by the triplet λ={A,B,π} with the description of relation between the sequence of observations and the sequence of states, so that:

i. The alphabet of states, the alphabet of observations is:

$$S = \{s_1, s_2, \dots, s_n\}$$
$$V = \{v_1, v_2, \dots, v_m\}$$

ii. The sequence of form observations constructed over the alphabet *V*

$$Y = y_1 y_2 \ \dots \ y_t \ where \ \ y_t \in V$$

iii. A is the transition vector whose components are the transition probabilities from state *i* to state *j*:

$$A = [a_{ij}] \ \ where \ a_{ij} = P(q_t = s_i | q_{t-1} = s_j)$$

Where $q_i \in Q = \{q_i \ q_2, \dots, q_n\}$ is the family of hidden states.

iv. B is the vector of observations whose components are the probabilities that the observation k is realized in the state j independently of t:

$$B = [b_i(k)] \ \ unde\ b_i(k) = P(x_t = v_i | q_t = s_i)$$

v. π is the vector of initial probabilities:

$$\pi = [\pi_{ij}] \ \ where\ \pi_i = P(q_1 = s_i)$$

We must emphasize the Markov working hypotheses in which the current state depends only on the previous state (model memory) and that of the independence from the previous states and outputs, the output at the time t being dependent only on the current state.

The probability distribution of states at time *t*:

$$\gamma_t(i) = P(x_t = s_i)$$

can be calculated by a complexity algorithm $\boldsymbol{O}(n^2)$. For each object an application is defined:

$$C: S \to \mathbb{R}$$

for the evaluation of the cost determined by the weather-induced losses, while the risk at the moment t is given by the following formula:

$$R = \sum_{i=1}^{n} \gamma_t\ (i) \cdot C(i)$$

In this context, the following problems must be solved up:

- Be it the sequence of observations $O = o_1, o_2, \ldots, o_T$ and a HMM λ={A,B,π} for which an efficient calculation algorithm must be provided for the calculation of the probability of supplying by the model of the sequence of objects $P(O|\lambda)$
- Be it the observations $O = o_1, o_2, \ldots, o_T$ and and a HMM λ; a correspondence must be identified between the sequence of states $S = s_1, s_2, \ldots, s_T$ that provides the ”best characterization” of the given observations.
- Adjusting the parameters of model λ={A,B,π} to maximize the probability $P(O|\lambda)$.

RESULTS AND DISCUSSIONS

Be it the random variables $X_1, X_2, \ldots, X_n$ with identical distribution as follows:

$$X_i = \begin{cases} 1 & if\ in\ the\ i-th\ vegetation\ week\ SHR \in [1,1.4] \\ 0 & if\ in\ the\ i-th\ vegetation\ week\ SHR \in (0,2)\backslash[1,1.4] \end{cases}$$

where $i \in \{1..n\}$.

The working hypothesis is:

$$P(X_n = x_n | X_{n-1} = x_{n-1}, \ldots, X_1 = x_1) = P(X_n = x_n | X_{n-1} = x_{n-1})$$

where $x_1, x_2, \ldots, x_n \in \{0,1\}$.

where the probability to obtain a certain level of the Selyaninov index in the week k of the vegetation period depends only on the level of the Selyaninov index of the previous week. Furthermore, the independence of the event from the history makes the stochastic process be of Markov chain type.

The associated transition matrix is:

$$\begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix}$$

where $P_{ij} = P(X_{k+1} = j | X_k = i)$ with $i, j \in \{1,0\}$, $k \in \{1 \dots n\}$ și $P_{00} + P_{01} = 1, P_{11} + P_{10} = 1$. P_{11} provides for the probability that in a certain week of the vegetation period, the favourable weather conditions for the wheat crop come after a week that is favourable for the vegetation growth phase of the crop. A small probability of this situation does not reveal a favourability meant to ensure the production gain that is foreseen in phyto-technical terms. An index can be generated that can provide for the production loss tendency due to the weather factors that are not totally favourable (Alam et al., 2013):

$$LPI = P_{11} \cdot P_{01}$$

with $LPI \in [0,1]$. For the defined Markov chain, the probability associated to the sequence of observations 1.1_1.5_1.6 is calculated only on the basis of associated states and by the multiplication of the probabilities associated to the connection graph. For a HMM, one can determine the probability of obtaining a certain production in the conditions of an observable sequence of the type 1.1_1.5_1.6. For a sequence of states (of unfavourable-favourable-favourable type) we can calculate the probability of the output 1.1_1.5_1.6. Each "hidden" state supplies an observation; hence the sequence of "hidden" states and the sequence of observations are characterized by equal lengths. For a "hidden" sequence of the states $S = s_1, s_2, \dots, s_T$ and a sequence of observations $O = o_1, o_2, \dots, o_T$, we have:

$$P(O|Q) = \prod_{i=1}^{T} P(o_i |\, s_i)$$

For the calculation of probability associated to the sequence 1.1_1.5_1.6 with the sequence of "hidden" unfavourable-favourable-favourable states we have:

$$P(1.1 - 1.5 - 1.6 | unfavourable - favourable - unfavourable) = \\ = P(1.1|unfavourable) \cdot P(1.5|favourable) \cdot P(1.6|favourable)$$

but it is necessary to calculate the probability of the event 1.1_1.5_1.6 weighted with the associated probability:

$$P(O,S) = P(O|S) \cdot P(S) = \prod_{i=1}^{n} P(o_i |\, s_i) \cdot \prod_{i=1}^{n} P(s_i |\, s_{i-1})$$

hence we have

$$P(1.1 - 1.5 - 1.6, unfavourable - unfavourable - favourable) \\ = P(unfavourable|start) \cdot P(unfavourable|unfavourable) \\ \cdot P(favourable|unfavourable) \cdot P(1.1|unfavourable)) \cdot P(1.5|favourable) \\ \cdot\ P(1.6|favourable)$$

Making a synthesis of the results, the probability associated to observations is:

$$P(O) = \sum_S P(O,S) = \sum_S P(O|S)P(S)$$

For our case we have:

$$P(1.1 - 1.5 - 1.6) \\ = P(1.1 - 1.5 - 1.6, unfavourable, unfavourable, unfavourable) \\ + P(1.1 - 1.5 - 1.6, unfavourable, unfavourable, favourable) \\ + P(1.1 - 1.5 - 1.6, favourable, favourable, unfavourable) + \cdots$$

Starting from the fact that an equiprobable distribution displays maximum entropy:

$$H(X) = -\sum_x P(x) log_2 P(x)$$

it intuitively leads us to the idea to construct a distribution by the continuous adding of properties (complying with the “Occam’s Razor” rule). Each property is a function that reveals a subset of instructing observations, for instance the “production losses”, the “production gain” in the context of the previously constructed HMM. In other words, in order to select a model from a set C of probability distributions, we choose the model $p^* \in C$ characterized by maximum entropy *H(p)*:

$$p^* = argmax\ H(p)\ unde\ p \in C$$

Practically, the MaxEnt model is a classifier of sequences that assign a class (e.g. “production losses”) by calculating a probability associated to an exponential distribution for a weighted set of properties corresponding to the observation. MaxEnt can be trained by convex optimization methods. A MEMM (Maximum Entropy Markov Model) is an extension of MaxEnt that uses Viterbi decoding algorithms.

CONCLUSIONS

The application of stochastic methods in the evaluation of production risk in agriculture in the technological context opened by cloud computing and machine learning adds further accuracy in the process of obtaining the production levels planned by the farmers, and increases the possibility of an improved risk management. Either derivative instruments or financial products adapted to the agricultural production are used, we need further accuracy in evaluating the production and market risks at farm level.

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