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Abstract

This study examines the validity of two conventional approaches to growth accounting under endogenous technical change. We find that the traditional approach to growth accounting, pioneered by Solow (1957), is consistent with the lab-equipment specification for technological progress, whereas the more recent approach, proposed by Hayashi and Prescott (2002) and Kehoe and Prescott (2002), is consistent with the knowledge-driven specification. We develop a unified approach to growth accounting, which in essence takes a weighted average of the Solow approach and the Hayashi-Kehoe-Prescott approach. We show that our unified approach is consistent with a more general specification for technological progress under which the degree of capital intensity in the innovation process determines the relative weight placed on the two approaches. Finally, we apply our unified approach to data of the Chinese economy to explore the quantitative importance of capital accumulation on economic growth in China.

JEL classification: O30, O40
Keywords: growth accounting, endogenous technical change, capital accumulation

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1 Introduction

The traditional approach to growth accounting, pioneered by Solow (1957), decomposes economic growth into the growth rates of factor inputs and technological progress, measured by the Solow residual; see Barro (1999) for a review of this approach. Interpreting these accounting relationships as causal relationships however requires an underlying assumption that the growth rates of factor inputs, e.g., physical capital, are independent from technological progress. An important result from the seminal Solow growth model is that in the long run, growth in output and capital is driven by technological progress. Therefore, interpreting the accounting relationships from the Solow approach as causal relationships may overstate the contribution of capital accumulation to economic growth and understate the contribution of technological progress.[1] A more recent approach to growth accounting, proposed by Hayashi and Prescott (2002) and Kehoe and Prescott (2002), addresses this issue by scaling up the importance of technological progress and measuring the contribution of capital accumulation by the growth rate of the capital-output ratio, rather than the growth rate of the capital stock.

This study examines the validity of these two approaches to growth accounting in the presence of endogenous technical change.[2] In particular, we consider the following two common specifications in the literature for endogenous technological progress: the knowledge-driven specification, and the lab-equipment specification. We find that the Hayashi-Kehoe-Prescott approach to growth accounting is consistent with the knowledge-driven specification that features labor input in the innovation process. Under this knowledge-driven specification, technological progress does not require physical capital, so the Hayashi-Kehoe-Prescott approach that scales down the contribution of capital accumulation and scales up the contribution of technological progress is valid. However, in the case of the lab-equipment specification that features final goods as input in the innovation process, the Solow approach is valid because it captures the contribution of capital accumulation to technological progress via the aggregate production function.

The abovementioned specifications for technological progress involve restrictive assumptions. Specifically, the knowledge-driven specification assumes that innovation does not require capital, whereas the lab-equipment specification assumes that the degree of capital intensity in the innovation process is the same as that of the general production process. To avoid these assumptions, we propose a more general specification for technological progress under which the degree of capital intensity in innovation can be different from production. Furthermore, we develop a unified approach to growth accounting that in essence takes a weighted average of the Solow approach and the Hayashi-Kehoe-Prescott approach. We show that this weight in our unified approach is determined by the degree of capital intensity in the innovation process.

Finally, we apply our unified approach to data of the Chinese economy to explore the importance of capital accumulation on economic growth in China. A study by Zhu (2012) finds that economic growth in China since the late 1970's has been mainly driven by technological progress and that capital accumulation has made almost zero contribution to growth of the

[1] See Aghion and Howitt (2007) for this critique.

[2] See also Barro (1999) and Aghion and Howitt (2007) who discuss the implications of endogenous technological progress on growth accounting.

Chinese economy. We find that the Hayashi-Kehoe-Prescott approach used by Zhu (2012) is valid only when capital accumulation does not contribute to technological progress. We also show that the contribution of capital accumulation to economic growth in China is increasing in the degree of capital intensity in technological progress and that if technological progress is as capital intensive as production, then capital accumulation would be responsible for as much as half the growth in China.

The rest of this study is organized as follows. Section 2 briefly reviews the two approaches to growth accounting. Section 3 explores their validity under endogenous technical change. Section 4 proposes a unified approach to growth accounting and then applies this approach to data in China. Section 5 considers an extension of the model. Section 6 concludes.

2 Review of growth accounting

In this section, we briefly review the two conventional approaches to growth accounting. Let's start with the following aggregate production function:

$$Y = K^{\alpha}(AL)^{1-\alpha}, \tag{1}$$

where Y denotes output, A denotes technology, K denotes physical capital, and L denotes effective labor, which includes human capital and raw labor. The parameter $\alpha \in (0,1)$ determines capital intensity in the production process. In the following subsections, we present the Solow and Hayashi-Kehoe-Prescott approaches to growth accounting and show their different implications on the contribution of capital accumulation to economic growth.

2.1 The Solow approach to growth accounting

We take the log of (1) and differentiate it with respect to time to obtain

$$\frac{\dot{Y}}{Y} = (1-\alpha)\frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1-\alpha)\frac{\dot{L}}{L}, \tag{2}$$

where $\dot{x}/x$ denotes the growth rate of variable $x \in \{Y, A, K, L\}$. In other words, (2) decomposes the growth rate of output into three components: the growth rate of technology, the growth rate of physical capital, and the growth rate of effective labor. Given that our focus is on the relative importance of technological progress and capital accumulation, we start with a constant effective labor L for simplicity.[3]

Under the Solow approach to growth accounting, the share of growth that capital accumulation is responsible for is measured by $\alpha(\dot{K}/K)/(\dot{Y}/Y)$. On the balanced growth path, the capital-output ratio is constant, which in turn implies that capital accumulation is responsible for the share α of growth in output in the long run whereas the rest is due to technological progress. However, this Solow approach to growth accounting may underestimate the importance of technological progress and overestimate the importance of capital

[3] In Section 5, we extend the analysis by allowing for growth in effective labor L.

accumulation. The reason is that the accumulation of physical capital is partly driven by technological progress. For example, a well-known result from the seminal Solow growth model is that in the long run, economic growth and capital accumulation are driven by technological progress. In the next subsection, we consider an alternative approach to growth accounting that addresses this issue.

2.2 The Hayashi-Kehoe-Prescott approach to growth accounting

Hayashi and Prescott (2002) and Kehoe and Prescott (2002) consider an alternative approach to account for the sources of economic growth. In essence, they divide both sides of (1) by Y^{α} to obtain

$$Y^{1-\alpha} = A^{1-\alpha}(K/Y)^{\alpha}L^{1-\alpha}. \tag{3}$$

Then, taking the log of (3) and differentiating it with respect to time yield

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\alpha}{1-\alpha}\frac{(\dot{K/Y})}{(K/Y)}, \tag{4}$$

where we have applied the assumption $\dot{L}/L = 0$. An interpretation of (4) is that capital accumulation is driven by technological progress. Therefore, we should scale up the importance of A by a factor of $1/(1-\alpha)$. If capital accumulation has made an additional contribution to the growth rate of output, then K should have grown at a faster rate than Y in the short run. In the long run, the capital-output ratio is constant, so that capital accumulation does not contribute to growth on the balanced growth path. In the next section, we examine the merit of each of these two approaches when technological progress is endogenous.

3 Growth accounting under endogenous technical change

In the previous section, we have shown that the two conventional approaches to growth accounting give rise to drastically different implications on the contribution of capital accumulation to economic growth. The difference arises for the following reasons. The Solow approach to growth accounting does not take into consideration the underlying determinant that drives capital accumulation, whereas the Hayashi-Kehoe-Prescott approach assumes that capital accumulation is driven by technological progress but not vice versa. In reality, technological progress is an endogenous process. In this section, we consider two common specifications in the literature for technological progress and explore the validity of the two approaches to growth accounting under each specification.

3.1 Knowledge-driven technological progress

We now modify the aggregate production function as follows:

$$Y = K^{\alpha}(AL_Y)^{1-\alpha}, \tag{5}$$

where $L_Y = (1 - s_A)L$ denotes production labor and $s_A \in (0, 1)$ is the share of labor devoted to improving technology A. The law of motion for technology is given by

$$\dot{A} = \overline{\theta} A L_R, \tag{6}$$

where $L_R = s_A L$ denotes R&D labor. The term $\overline{\theta} \equiv \theta/L$ denotes R&D productivity, where $\theta > 0$ is a productivity parameter and $1/L$ captures a dilution effect[4] that removes a counterfactual scale effect from the model. The term A on the right hand side of (6) captures an intertemporal externality of knowledge spillovers from existing technologies A to new technology $\dot{A}$ as in the knowledge-driven R&D specification in Romer (1990).[5] Let's denote the steady-state growth rate of technology as $g_A \equiv \dot{A}/A = \theta s_A$.[6]

The law of motion for capital accumulation is given by

$$\dot{K} = I - \delta K, \tag{7}$$

where I denotes capital investment and the parameter $\delta \in (0, 1)$ denotes the capital depreciation rate. Manipulating (7) yields

$$\frac{\dot{K}}{K} = \frac{I}{K} - \delta. \tag{8}$$

In the long run, the steady-state capital growth rate g_K is constant, which in turn implies a constant steady-state investment-capital ratio I/K. Together with a constant investment-output ratio I/Y in the long run, we have established that the steady-state capital-output ratio K/Y must be constant, which in turn implies that output and capital share the same steady-state growth rate (i.e., $g_Y = g_K$). Taking the log of (5) and differentiating the resulting expression with respect to time yield

$$\frac{\dot{Y}}{Y} = (1 - \alpha)\frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} + (1 - \alpha)\frac{\dot{L}_Y}{L_Y}. \tag{9}$$

We assume that s_A is constant,[7] which in turn implies $\dot{L}_Y/L_Y = 0$. Finally, we substitute the long-run condition $g_Y = g_K$ into (9) to obtain

$$g_Y = g_K = g_A = \theta s_A. \tag{10}$$

Therefore, although technological progress is endogenous in this model, it is independent of capital accumulation. In contrast, capital accumulation is driven by technological progress.

[4] In the appendix, we sketch out a so-called second-generation R&D-based growth model that provides a microfoundation for this dilution effect; see Dinopoulos and Thompson (1998), Peretto (1998), Young (1998) and Howitt (1999) for early studies on the second-generation model and also Laincz and Peretto (2006) and Ha and Howitt (2007) for empirical evidence that supports this model.

[5] Romer (1990) develops the seminal variety-expanding R&D-based growth model, whereas Aghion and Howitt (1992), Grossman and Helpman (1991) and Segerstrom *et al.* (1990) develop the seminal quality-ladder growth model. Technological progress in our model can be viewed either as quality improvement or variety expansion.

[6] Without the dilution effect $1/L$, g_A would be increasing in L, which is inconsistent with empirical evidence; see for example Jones (1995).

[7] Here we assume a constant share s_A, which needs not be exogenous. In a market equilibrium, s_A is determined by household preference, market structure and government policies, etc.

We now examine the validity of the Solow and Hayashi-Kehoe-Prescott approaches to growth accounting within the context of this model. Under the Solow approach to growth accounting, we have the following condition in the long run:

$$\frac{\dot{Y}}{Y} = (1-\alpha)\frac{\dot{A}}{A} + \alpha\frac{\dot{K}}{K} \Rightarrow g_Y = (1-\alpha)g_A + \alpha g_K. \tag{11}$$

As we can see, the Solow approach to growth accounting assigns the share α of growth to capital accumulation g_K, which should in fact be assigned to technological progress g_A as (10) shows.

Under the Hayashi-Kehoe-Prescott approach to growth accounting, we have the following condition in the long run:

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\alpha}{1-\alpha}\frac{(\dot{K/Y})}{(K/Y)} \Rightarrow g_Y = g_A. \tag{12}$$

In this case, the Hayashi-Kehoe-Prescott approach to growth accounting correctly assigns the entire long-run growth in output to technological progress g_A. As for the short run, any growth in K/Y would capture the contribution of capital accumulation.

3.2 Lab-equipment technological progress

We now consider an alternative model of endogenous technological progress. The aggregate production function is given by

$$Y = K^{\alpha}(AL)^{1-\alpha}. \tag{13}$$

The law of motion for technology is modified to capture the lab-equipment R&D specification in Rivera-Batiz and Romer (1991) as follows:

$$\dot{A} = \overline{\theta}R = \frac{\theta R}{L}, \tag{14}$$

where $R = s_A Y$ and $s_A \in (0,1)$ is now the share of output devoted to improving technology. Substituting $R = s_A Y$ and (13) into (14) yields

$$\frac{\dot{A}}{A} = \theta s_A \left(\frac{K}{AL}\right)^{\alpha}, \tag{15}$$

which in turn implies that in the case of a constant steady-state growth rate of technology, the capital-technology ratio K/A must be constant in the long run.

The law of motion for capital is the same as in (7). For simplicity, we define $s_K \in (0,1)$ as the constant share of output devoted to capital accumulation (i.e., capital investment net of depreciation). Formally,

$$s_K Y \equiv \dot{K} = I - \delta K, \tag{16}$$

which in turn implies that

$$\frac{\dot{K}}{K} = s_K \frac{Y}{K}. \tag{17}$$

Therefore, we can now combine (15) and (17) to obtain

$$\frac{\dot{A}}{A} = \frac{\dot{K}}{K} \Leftrightarrow \theta s_A \left(\frac{K}{AL}\right)^{\alpha} = s_K \left(\frac{AL}{K}\right)^{1-\alpha}. \tag{18}$$

Then, we derive the steady-state capital-technology ratio given by[8]

$$\frac{K}{A} = \frac{s_K}{\theta s_A} L. \tag{19}$$

Substituting (19) into (15) yields the steady-state growth rate of technology given by

$$g_A = (\theta s_A)^{1-\alpha} (s_K)^{\alpha}, \tag{20}$$

which in turn determines the steady-state growth rate of output and capital as $g_Y = g_K = g_A$. If we take a log-linear approximation of (20), we have[9]

$$\ln g_A = (1-\alpha)\ln(\theta s_A) + \alpha \ln(s_K) \Rightarrow g_A \approx (1-\alpha)(\theta s_A) + \alpha s_K. \tag{21}$$

In this model, technological progress and capital accumulation follow a two-way process: technological progress drives capital accumulation (i.e., $g_K = g_A$) but capital accumulation also drives technological progress (i.e., g_A depends on s_K).

We now evaluate the validity of the two approaches to growth accounting within the context of this model. Under the Hayashi-Kehoe-Prescott approach to growth accounting, we have the following condition in the long run:

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + \frac{\alpha}{1-\alpha} \frac{(\dot{K/Y})}{(K/Y)} \Rightarrow g_Y = g_A, \tag{22}$$

where $g_A \approx (1-\alpha)(\theta s_A) + \alpha s_K$ depends on the capital-investment rate s_K. However, all the growth in output is wrongly attributed to technological progress g_A under the Hayashi-Kehoe-Prescott approach to growth accounting.

Under the Solow approach to growth accounting, we have the following long-run condition:

$$\frac{\dot{Y}}{Y} = (1-\alpha)\frac{\dot{A}}{A} + \alpha \frac{\dot{K}}{K} \Rightarrow g_Y = (1-\alpha) g_A + \alpha g_K, \tag{23}$$

where $(1-\alpha)g_A + \alpha g_K = g_A \approx (1-\alpha)(\theta s_A) + \alpha s_K$. As we can see, the Solow approach to growth accounting correctly assigns the share α of growth to capital accumulation g_K, which captures the effect of s_K in (21), and the remaining share $1-\alpha$ of growth to technological progress g_A, which captures the effect of θs_A in (21). Under the lab-equipment specification that features goods as input in the innovation process, the Solow approach is valid because it captures the contribution of capital accumulation to technological progress via the aggregate production function of goods.

[8] It can be shown that the capital-technology ratio K/A must converge to this steady state.

[9] Here we use $\ln(1+x) \approx x$.

4 A unified approach to growth accounting

In the previous section, we consider two technological specifications that involve restrictive assumptions. Specifically, the knowledge-driven specification assumes that innovation does not require physical capital, whereas the lab-equipment specification assumes that the degree of capital intensity in innovation is the same as in the production of goods. To avoid these assumptions, this section first considers a more general specification for technological progress. Then, we show that a unified approach to growth accounting is consistent with this general specification for technological progress.

4.1 A general specification for technological progress

In this subsection, we consider a more general specification for technological progress. The aggregate production function is given by

$$Y = K_Y^{\alpha}(AL_Y)^{1-\alpha}, \tag{24}$$

where $L_Y = (1 - s_A)L$, $K_Y = (1 - s_A)K$ and $s_A \in (0, 1)$ is now the share of both labor and capital devoted to improving technology. The law of motion for capital is as before given by

$$\dot{K} = s_K Y = s_K(1 - s_A)K^{\alpha}(AL)^{1-\alpha}. \tag{25}$$

The law of motion for technology is generalized to allow for a different degree of capital intensity $\beta \in (0, 1)$ from the production of goods such that

$$\dot{A} = \overline{\theta} K_R^{\beta}(AL_R)^{1-\beta} = \frac{\theta K_R^{\beta}(AL_R)^{1-\beta}}{L}, \tag{26}$$

where $L_R = s_A L$ and $K_R = s_A K$. Manipulating (26) yields

$$\frac{\dot{A}}{A} = \theta s_A \left(\frac{K}{AL}\right)^{\beta}, \tag{27}$$

which in turn implies a constant steady-state capital-technology ratio K/A. Then, combining (25) and (27) yields the following long-run condition:

$$\frac{\dot{A}}{A} = \frac{\dot{K}}{K} \Leftrightarrow \theta s_A \left(\frac{K}{AL}\right)^{\beta} = s_K(1 - s_A)\left(\frac{AL}{K}\right)^{1-\alpha}. \tag{28}$$

Therefore, the steady-state capital-technology ratio is given by

$$\frac{K}{A} = \left[\frac{s_K(1 - s_A)}{\theta s_A}\right]^{1/(1-\alpha+\beta)} L. \tag{29}$$

Substituting (29) into (27) yields the steady-state growth rate of technology given by

$$g_A = (\theta s_A)^{(1-\alpha)/(1-\alpha+\beta)} \left[s_K(1 - s_A)\right]^{\beta/(1-\alpha+\beta)}. \tag{30}$$

Once again, taking a log-linear approximation, we obtain

$$\ln g_A = \frac{(1-\alpha)\ln(\theta s_A) + \beta \ln s_K + \beta \ln(1-s_A)}{1-\alpha+\beta} \Rightarrow g_A \approx \frac{(1-\alpha)(\theta s_A) + \beta \widetilde{s}_K}{1-\alpha+\beta}, \tag{31}$$

where $\widetilde{s}_K \equiv s_K - s_A$ is the log-linear approximation of $s_K(1-s_A)$ in (25) which is basically the share of capital and labor devoted to capital accumulation. Therefore, $\beta\widetilde{s}_K/(1-\alpha+\beta)$ in (31) captures the importance of capital accumulation on technological progress.

4.2 A unified approach to growth accounting

Previously, we have shown that the two conventional approaches to growth accounting give rise to different implications on the importance of capital accumulation. Here we propose a unified approach to nest the two previous approaches as special cases. To begin, we divide both sides of (1) by $Y^{\alpha-\beta}$ to obtain

$$Y^{1-\alpha+\beta} = A^{1-\alpha} K^{\beta} \left(\frac{K}{Y}\right)^{\alpha-\beta} L^{1-\alpha}, \tag{32}$$

where $\beta \in (0,1)$ also represents a weight parameter here. Taking the log of (32) and differentiating the resulting expression with respect to time yield

$$\frac{\dot{Y}}{Y} = \frac{1-\alpha}{1-\alpha+\beta}\frac{\dot{A}}{A} + \frac{\beta}{1-\alpha+\beta}\frac{\dot{K}}{K} + \frac{\alpha-\beta}{1-\alpha+\beta}\frac{(\dot{K/Y})}{(K/Y)}. \tag{33}$$

If we set $\beta = \alpha$, then we have the Solow approach, under which the importance of capital accumulation is measured by the growth rate of capital K. If we set $\beta = 0$, then we have the Hayashi-Kehoe-Prescott approach, under which the importance of capital accumulation is measured by the growth rate of the capital-output ratio K/Y. More generally, the value of β is given by the degree of capital intensity in the innovation process. To see this, we consider the long-run version of (33) given by

$$g_Y = \frac{(1-\alpha)g_A + \beta g_K}{1-\alpha+\beta}. \tag{34}$$

Equation (34) shows that our unified approach to growth accounting correctly assigns the share $\beta/(1-\alpha+\beta)$ of growth to capital accumulation g_K, which captures the effect of $\widetilde{s}_K$ in (31), and the remaining share $(1-\alpha)/(1-\alpha+\beta)$ of growth to technological progress g_A, which captures the effect of θs_A in (31).

4.3 Importance of capital accumulation in China

We now consider data of the Chinese economy to explore the quantitative importance of capital accumulation on economic growth in China. From Brandt, Hsieh and Zhu (2008), the average value of α in China is about 0.5. From Zhu (2012), the average growth rates of output and physical capital have been roughly the same since 1978.[10] Therefore, we consider the following stylized facts for China: $\alpha = 1/2$, and a constant K/Y since the late 1970's.

Under the Solow approach to growth accounting, one would conclude that capital accumulation $\dot{K}/K$ has been responsible for about half of the growth in China. To see this,

$$\text{Solow approach: } \frac{\alpha \dot{K}/K}{\dot{Y}/Y} \approx \alpha \approx \frac{1}{2}.$$

In contrast, using the Hayashi-Kehoe-Prescott approach, Zhu (2012) concludes that the growth rate of output is mainly driven by growth in technology A because K/Y has been roughly constant since 1978. More formally,

$$\text{Hayashi-Kehoe-Prescott approach: } \frac{\alpha}{1-\alpha} \frac{\dot{(K/Y)}}{(K/Y)} \frac{1}{\dot{Y}/Y} \approx 0.$$

To sum up, according to the Solow approach to growth accounting, capital accumulation has contributed to about half the growth in China, whereas according to the Hayashi-Kehoe-Prescott approach, capital accumulation has made almost zero contribution to growth in China.

We now consider our unified approach to growth accounting for different values of $\beta \in [0, \alpha]$. Under our approach, the importance of capital accumulation can be expressed as

$$\text{Unified approach: } \frac{\beta}{1-\alpha+\beta} \frac{\dot{K}/K}{\dot{Y}/Y} + \frac{\alpha-\beta}{1-\alpha+\beta} \frac{\dot{(K/Y)}}{(K/Y)} \frac{1}{\dot{Y}/Y} \approx \frac{\beta}{1-\alpha+\beta}.$$

Table 1 reports the percent of growth in China for which capital accumulation is responsible for. As β increases, the importance of capital accumulation increases. In the case of Zhu (2012), an implicit assumption is that technological progress does not depend on capital accumulation (i.e., $\beta = 0$) in which case capital accumulation has made almost zero contribution to growth. On the other hand, if technological progress turns out to be as capital intensive as production, then capital accumulation would have been responsible for about half the growth in China. Even in the conservative case in which technological progress is half as capital intensive as production (i.e., $\beta = 0.25$),[11] capital accumulation would have contributed to one third of economic growth in China over the past decades.

Table 1: Importance of capital accumulation in China

β	0	0.1	0.2	0.3	0.4	0.5
percent	0.0%	16.7%	28.6%	37.5%	44.4%	50.0%

[10] To be more precise, the average annual growth rate of the capital-output ratio K/Y in China from 1978 to 2007 was 0.04%.

[11] In the case of an emerging economy like China, technological progress should be viewed more as the adoption of foreign technology than original domestic innovation. However, even technology adoption requires the use of capital (e.g., setting up manufacturing plants to adopt foreign production methods and technologies).

5 Growth in effective labor

In this section, we extend the analysis to allow for growth in effective labor L. Here effective labor L is the product of human capital h and raw labor l, such that $L = hl$. Therefore, the growth rate of effective labor is

$$\frac{\dot{L}}{L} = \frac{\dot{h}}{h} + \frac{\dot{l}}{l} = n, \tag{35}$$

where $n > 0$ is defined as the composite growth rate of human capital and raw labor. The model is the same as in Section 4.1 except that we now allow for growth in L. In this case, we modify the definition of $s_K \in (0,1)$ to be the share of output devoted to the accumulation of capital net of both depreciation and effective labor growth; in other words,

$$s_K Y \equiv \dot{K} - nK = I - (\delta + n)K, \tag{36}$$

which in turn implies that

$$\frac{\dot{K}}{K} = s_K \frac{Y}{K} + n. \tag{37}$$

From (27), a constant steady-state growth rate of technology A implies a constant steady-state ratio of $K/(AL)$, which in turn implies

$$\frac{\dot{A}}{A} + n = \frac{\dot{K}}{K} \Leftrightarrow \theta s_A \left(\frac{K}{AL}\right)^{\beta} + n = s_K(1 - s_A)\left(\frac{AL}{K}\right)^{1-\alpha} + n. \tag{38}$$

where the second equality follows from (27) and (37). Manipulating (38) yields (29), which in turn can be substituted into (27) to obtain (30). Therefore, we have the same result as before that the steady-state growth rate of technology is given by

$$g_A = (\theta s_A)^{(1-\alpha)/(1-\alpha+\beta)} \left[s_K(1 - s_A)\right]^{\beta/(1-\alpha+\beta)} \approx \frac{(1-\alpha)(\theta s_A) + \beta \widetilde{s}_K}{1 - \alpha + \beta}, \tag{39}$$

where $\widetilde{s}_K \equiv s_K - s_A$.

As for growth accounting, we first divide both sides of (1) by L to obtain

$$y = A^{1-\alpha} k^{\alpha}, \tag{40}$$

where $y \equiv Y/L$ and $k \equiv K/L$ are output and capital per unit of effective labor. Then, we follow the same procedure as in Section 4.2 to divide both sides of (40) by $y^{\alpha-\beta}$ to obtain

$$y^{1-\alpha+\beta} = A^{1-\alpha} k^{\beta} \left(\frac{k}{y}\right)^{\alpha-\beta}, \tag{41}$$

which in turn implies that the growth rate of y can be decomposed into

$$\frac{\dot{y}}{y} = \frac{1-\alpha}{1-\alpha+\beta}\frac{\dot{A}}{A} + \frac{\beta}{1-\alpha+\beta}\frac{\dot{k}}{k} + \frac{\alpha-\beta}{1-\alpha+\beta}\frac{(\dot{k/y})}{(k/y)}, \tag{42}$$

where by definition the growth rate of k/y is the same as the growth rate of K/Y. Therefore, in the long run, we have

$$g_y = \frac{(1-\alpha)g_A + \beta g_k}{1-\alpha+\beta}. \tag{43}$$

In other words, the unified approach to growth accounting correctly assigns the share $\beta/(1-\alpha+\beta)$ of growth in output *per effective labor* to capital accumulation g_k, which captures the effect of $\widetilde{s}_K$ in (39), and the remaining share $(1-\alpha)/(1-\alpha+\beta)$ of growth in output *per effective labor* to technological progress g_A, which captures the effect of θs_A in (39). Finally, growth in output Y can be decomposed into

$$\frac{\dot{Y}}{Y} = \frac{\dot{y}}{y} + n \Rightarrow g_Y = \frac{(1-\alpha)g_A + \beta g_k}{1-\alpha+\beta} + n = \frac{(1-\alpha)g_A + \beta g_K + (1-\alpha)n}{1-\alpha+\beta}, \tag{44}$$

where $g_k = g_K - n$. Therefore, the share of output growth g_Y that capital accumulation g_K is responsible for continues to be given by $\beta/(1-\alpha+\beta)$.

6 Conclusion

In this study, we have revisited two conventional approaches to growth accounting and explored their validity under endogenous technical change. We find that the Solow approach is consistent with the lab-equipment specification for technological progress, whereas the Hayashi-Kehoe-Prescott approach is consistent with the knowledge-driven specification. We also develop a unified approach to growth accounting and show that this approach is consistent with a general specification for technological progress under which the degree of capital intensity in the innovation process is the key parameter for growth accounting. Finally, we apply our unified approach to data of the Chinese economy and find that capital accumulation has made a quantitatively important contribution to economic growth in China.

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Appendix: Dilution effect and the second-generation R&D-based growth model

In this appendix, we provide a microfoundation for the dilution effect on R&D productivity using a variant of the second-generation R&D-based growth model. The aggregate production function of final goods is given by

$$Y = \int_0^N K_Y^\alpha(i)[A(i)L_Y(i)]^{1-\alpha} di, \tag{A1}$$

where $\{A(i), K_Y(i), L_Y(i)\}$ are the technology level, capital and labor inputs of intermediate goods $i \in [0, N]$. The variable N denotes the number of varieties of these intermediate goods. The law of motion for technology of intermediate goods $i \in [0, N]$ is given by

$$\dot{A}(i) = \widetilde{\theta} K_R^\beta(i)[A(i)L_R(i)]^{1-\beta}, \tag{A2}$$

where $\{K_R(i), L_R(i)\}$ are the capital and labor inputs devoted to improving the technology of intermediate goods $i \in [0, N]$ and $\widetilde{\theta} > 0$ is a productivity parameter.

We consider a symmetric equilibrium in which $L_R(i) = s_A L/N$, $L_Y(i) = (1 - s_A)L/N$, $K_R(i) = s_A K/N$, $K_Y(i) = (1 - s_A)K/N$ and $A(i) = A$ for all $i \in [0, N]$.[12] Substituting these conditions into (A1) and (A2) yields

$$Y = N \left[\frac{(1 - s_A)K}{N}\right]^\alpha \left[\frac{A(1 - s_A)L}{N}\right]^{1-\alpha} = (1 - s_A)K^\alpha (AL)^{1-\alpha}, \tag{A3}$$

$$\dot{A} = \widetilde{\theta} \left(\frac{s_A K}{N}\right)^\beta \left(\frac{A s_A L}{N}\right)^{1-\beta} = \frac{\widetilde{\theta}}{N} s_A K^\beta (AL)^{1-\beta}. \tag{A4}$$

Equation (A4) shows that R&D productivity $\widetilde{\theta}/N$ is diluted by the number of varieties of intermediate goods. The law of motion for N is given by

$$\dot{N} = \phi L - \delta_N N, \tag{A5}$$

where $\phi > 0$ measures the efficiency of the society in creating new varieties and $\delta_N > 0$ is the obsolescence rate of varieties. In the steady state, we have $N = \phi L/\delta_N$.[13] Substituting this condition into (A4), we have

$$\dot{A} = \frac{\theta}{L} s_A K^\beta (AL)^{1-\beta}, \tag{A6}$$

where we have defined $\theta \equiv \delta_N \widetilde{\theta}/\phi$. Manipulating (A6) yields (27). Taking the log of (A3) and differentiating the resulting expression with respect to time yield

$$\frac{\dot{Y}}{Y} = (1 - \alpha)\frac{\dot{A}}{A} + \alpha \frac{\dot{K}}{K}. \tag{A7}$$

The law of motion for capital is given by (25), which in turn implies a constant capital-output ratio K/Y in the long run. Therefore, the steady-state growth rate of output and capital is given by $g_Y = g_K = g_A$ as before.

[12] A common assumption in the literature is that newly invented intermediate goods have access to the technology of existing intermediate goods.

[13] If labor L increases at the rate n, then the balanced-growth value of N becomes $N = \phi L/(n + \delta_N)$.