Consumer Search and Retail Market Structure

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Abstract

This paper proposes a framework for studying how consumer search frictions affect retail market structure. In our model single-product firms which supply different products can merge to form a multiproduct firm. Consumers wish to buy multiple products and value the one-stop shopping convenience associated with a multiproduct firm. We find that when the search friction is relatively large all firms are multiproduct in equilibrium. However when the search friction is smaller the equilibrium market structure is asymmetric, with single-product and multiproduct firms coexisting. This asymmetric market structure often leads to the weakest price competition, and is the worst for consumers among all possible market structures. Due to the endogeneity of market structure, a reduction in the search friction can increase market prices and decrease consumer welfare.

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JEL classification: D11, D43, D83, L13

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1 Introduction

Many consumers place a high value on one-stop shopping convenience. They are often time-constrained, and so value the opportunity to buy a large basket of products in one place.[1] Consequently product assortment is an important dimension along which retailers compete. Over time there has been a steady increase in the size of retail assortments. The Food Marketing Institute estimates that between 1975 and 2013, the number of products in an average US supermarket increased from around 9,000 to almost 44,000. At least part of this increase can be attributed to retailers stocking new product categories.[2] For example Wal-Mart has expanded into pharmacies and clinical services, whilst drugstores like Walgreens and CVS have started selling fresh food and grocery items. Nevertheless one striking feature of most retail markets is their persistent heterogeneity – large retailers like Wal-Mart or Amazon often coexist alongside many specialist retailers with much narrower product selections.[3]

There is little formal research on how demand-side economies of scope, such as one-stop shopping convenience, might shape the retail market structure. This is partly because multiproduct competition is complicated to analyze in environments where consumers demand multiple products and value one-stop shopping convenience. This paper provides a tractable consumer search framework to investigate this issue. We find that the magnitude of consumer search frictions determines whether the equilibrium market structure is symmetric with all multiproduct firms, or is asymmetric with a mix of single-product and multiproduct firms. We also examine the welfare properties of different market structures, and show that a move towards larger retail assortments is not necessarily beneficial for consumers. In the same vein we show that once endogeneity of market structure is accounted for, a reduction in search frictions (due, for example, to a shift from traditional to online retailing) does not necessarily increase consumer welfare.

Our model starts with a situation where there are two products (or product categories)

[1] Nowadays many consumers buy groceries from big box stores such as Wal-Mart and Target, instead of more traditional grocery stores. For example, a survey by King Retail Solutions shows that 77% of consumers bought groceries from a non-traditional grocery store in 2013 (see http://www.kingrs.com/news/filter/white-paper/study-traditional-retail-categories-are-blurring). Seo (2015) estimates that the value of one-stop shopping convenience from grocery stores being able to sell liquor is about \$2.52 per trip, or 8% of an average household's expenditure on liquor.

[2] Messinger and Narasimhan (1997) provide empirical evidence that time-saving convenience is the most important driver of this growth in supermarket store size. (Another important reason is the adoption of modern distribution technology in the 1980s and 1990s.)

[3] Indeed anecdotally online markets appear even less symmetric than offline ones e.g. in 2012 Amazon sold more than its top 12 online competitors combined.

and each of them is sold by two single-product firms. Each pair of single-product firms which supply different products then choose whether to merge and form a multiproduct firm. This generates one of three possible market structures: either four single-product firms, or two multiproduct firms, or an asymmetric market with one multiproduct firm and two single-product firms. Consumers differ with respect to their search technology. Some consumers ("shoppers") are able to visit all firms without incurring any cost and so buy each product at the lowest price available. Other consumers ("non-shoppers") are time-constrained and are only able to visit one (single-product or multiproduct) firm. It is these non-shoppers who value the one-stop shopping convenience provided by a multiproduct firm. The fraction of non-shoppers is interpreted as a measure of the search friction in the market.

We show that a merger has two distinct effects. Firstly, when two single-product firms which supply different products merge, they provide one-stop shopping convenience and so are searched by more non-shoppers (a "search effect"). Secondly though, the merger also changes market structure and influences price competition (a "price competition effect"). We show that when the first pair of single-product firms merge, this leads to an asymmetric market structure and softens price competition. This is because the resulting multiproduct firm focuses more on exploiting its one-stop shopping convenience through higher prices, which further relaxes competition with the remaining two single-product firms. (In fact, all firms in our model benefit from this first merger.) Consequently the price competition effect works in the same direction as the search effect, and so there is no equilibrium with four single-product firms. More interestingly, the size of the search friction determines whether or not a second merger occurs. When the second pair of single-product firms merge, they win back some non-shoppers, but the resulting market structure with two multiproduct firms also intensifies price competition relative to the asymmetric case. In other words, the price competition effect now works against the search effect. The price competition effect dominates – and so the equilibrium market structure is asymmetric – if and only if the search friction is relatively low. Thus our model is able to generate both symmetric and asymmetric market structures, depending upon the size of consumer search frictions in the market.

By comparing the three possible market structures, we find that the asymmetric market structure is the worst for consumers and often the best for industry profit. This finding has two implications. First, it indicates that a merger between two firms which supply different products can harm consumers, even if it does not reduce the number of competitors in each product market. In antitrust parlance this is called a "conglomerate merger".[4] Our model suggests that if there are search frictions on the demand side, a conglomerate merger can be anti-competitive. (We discuss this point further in the related literature section, below.) Second, our result also suggests that reducing search frictions does not necessarily harm firms and benefit consumers. This is because when the search friction becomes smaller, the market structure can switch from a symmetric one with all big firms to an asymmetric one with both big and small firms. This indirect effect on market structure can work against and even dominate the direct effect of reducing search frictions on firms and consumers. Therefore our study suggests that a welfare assessment of a change in search frictions (e.g. due to a move towards online retailing) should take into account its impact on market structure.

These main insights continue to hold in two extensions which are (i) allowing non-shoppers to be able to visit more than one firm by paying a search cost, and (ii) considering more than two pairs of firms. We also consider two alternative models: one with a non-merger framework where firms can choose their product ranges directly, and the other where firms sell differentiated products and consumers engage in sequential search. The main result that an asymmetric market structure arises in equilibrium when the search friction is relatively small remains true in all these variants of the model.

Related literature: Our benchmark search model with homogeneous products builds on Varian (1980) and Burdett and Judd (1983) which introduce differentially informed consumers, whilst our alternative search model with differentiated products builds on Wolinsky (1986) and Anderson and Renault (1999). (These are the two most common approaches to avoid the Diamond, 1971 paradox.) These papers only study single-product search. We extend them to the multiproduct case where consumers need and firms (may) supply multiple products.

There is a growing literature on multiproduct consumer search. Lal and Matutes (1994) show that multiproduct search can lead to loss-leader pricing when some products are advertised. McAfee (1995) and Shelegia (2012) examine when and how multiproduct firms correlate their prices across products when consumers are heterogeneously informed.[5] Zhou (2014) investigates how multiproduct search generates a joint search ef-

[4]There are two types of conglomerate merger. One involves firms producing totally unrelated products e.g. steel and tissues. The other involves firms producing complementary products, or products which belong to a range of products that are generally purchased by the same set of consumers. (See for example the EU guidelines on non-horizontal mergers.) The merger discussed in our paper is of the second type.

[5]See also Baughman and Burdett (2015) and Kaplan et al. (2015) for more recent work in this direction. The former shows that assuming no consumer recall can greatly simplify the analysis of multiproduct search with price dispersion. The latter offers a search model with high and low valuation consumers which can explain relative price dispersion across retailers.

fect, which creates complementarity between physically independent products, and leads to lower prices compared to the case with single-product search. Rhodes (2015) studies the relationship between the size of a retailer's product range, its pricing, and its advertising decision. He shows that a multiproduct retailer's low advertised prices can signal low prices on its unadvertised products. However all these papers assume an exogenously given market structure where each firm sells the same range of products. We depart from this literature by endogenizing market structure, and show that an asymmetric market structure can emerge as an equilibrium outcome.

There is also research on multiproduct firms and endogenous market structure when consumers have perfect information about firm offerings. Typically these papers consider a duopoly model where each firm can choose which varieties of a product to supply. The varieties are either horizontally differentiated (e.g. Shaked and Sutton, 1990), or vertically differentiated (e.g. Champsaur and Rochet, 1989), or both (e.g. Gilbert and Matutes, 1993). However in these papers there is no notion of one-stop shopping convenience, and moreover an asymmetric market with both large and small firms does not usually arise in equilibrium. (See Manez and Waterson, 2001 for a survey of this literature.) There are also papers on multiproduct competition which introduce shopping frictions whilst maintaining the assumption of perfectly informed consumers. However they assume either an exogenous symmetric market where two firms supply the same range of products (e.g. Lal and Matutes, 1989, Klemperer, 1992, and Armstrong and Vickers, 2010), or an exogenous asymmetric market where one big firm coexists with a competitive fringe of small firms with a narrower product range (see Chen and Rey, 2012).[6]

Our paper is also related to the literature on bundling and market structure. Another potential advantage of forming a multiproduct firm is the ability to use more advanced pricing strategies such as bundling. However if all single-product firms merge and form multiproduct firms, the resulting bundle-against-bundle competition is often fierce and harms all firms. As a result an asymmetric market structure can arise in equilibrium. Nalebuff (2000) and Thanassoulis (2011) make this point in different settings with product differentiation. We argue that even if multiproduct firms do not use bundling (e.g. in many retail markets such as the grocery industry we do not observe store-wide bundling), the existence of search frictions can still favor a multiproduct firm and generate an asymmetric market structure. Our model also predicts that a symmetric market with all big firms can arise in equilibrium, which is not the case in the above two papers.

[6]See also Johnson (2014) for a multiproduct competition model where the market friction is that consumers are boundedly rational and make unplanned purchases. Section 3 of his paper considers an asymmetric market where one firm is exogenously able to carry more products than another.

Also related is the literature on agglomeration. Baumol and Ide (1956) argue that larger retailers may attract more demand, because consumers are more willing to incur the time and transportation costs necessary to visit them. Stahl (1982) shows that due to a similar demand expansion effect, single-product firms have an incentive to co-locate (e.g. in a shopping mall) provided their products are not too substitutable. In a search environment firms may locate near each other either to offer consumers a higher chance of a good product match (Wolinsky, 1983), or as a way of guaranteeing consumers that they will face low prices (Dudey, 1990 and Non, 2010). Moraga-González and Petrikaitė (2013) show that when a subset of firms with differentiated versions of a product merge and sell all their products in a single shop, they become prominent and are searched first by consumers. However in all these papers consumers buy only one product, and so any one-stop shopping convenience does not arise from consumers' need to buy multiple products. Nevertheless this is an important feature of many retail markets.

Finally, our paper is also related to the literature on conglomerate mergers. Since conglomerate mergers do not eliminate competitors and may generate cost synergies, economists and policymakers (especially in the US) often hold a benign view (see Church, 2008 for a survey). However our model shows that conglomerate mergers (which involve firms producing products needed by the same set of consumers) have a potential anti-competitive effect. In independent and concurrent work, Chen and Rey (2015) examine conglomerate merger using a different framework. They find that conglomerate merger can also soften price competition, but that it benefits consumers (at least when bundling is infeasible). In addition, due to their modelling assumptions a second conglomerate merger is never profitable because it leads to Bertrand competition.

The rest of the paper proceeds as follows. Section 2 outlines a benchmark model, characterizes price distributions in various market structures, and solves for the equilibrium market structure. Section 3 considers various extensions and shows the robustness of the main results from the benchmark model, and Section 4 concludes. All omitted proofs are available in the appendix.

2 A Benchmark Model

A unit mass of consumers is interested in buying two products 1 and 2. Each consumer has unit demand, and is willing to pay up to v for each product.[7] Initially there are four single-product firms in the market: two of them, denoted by 1_A and 1_B, sell a homogenous

[7] The analysis can be extended to allow for elastic demand without changing the main result. The details are available upon request.

product 1, and the other two, denoted by 2_A and 2_B, sell a homogenous product 2. The marginal cost of supplying each product is normalized to zero.

As we describe in more detail below, it is too costly for some consumers to visit multiple firms, and so they would benefit from the emergence of multiproduct firms which supply both products. We consider a two-stage game. In the first stage, each pair of firms $(1_k, 2_k)$, $k = A, B$, which supply different products, has the opportunity to merge and form a multiproduct firm.[8] Their merger decisions can be simultaneous (in which case we focus on pure strategy equilibria) or sequential. We assume that merger is costless and does not affect the marginal cost of supplying each product.[9] In the second stage, after observing the market structure firms simultaneously choose their prices and consumers search and make their purchases. We assume that multiproduct firms do not use bundling and charge separate prices for each product.[10]

Consumers differ with respect to their search technology. A fraction $\alpha \in (0, 1)$ of consumers are shoppers, who can search and multi-stop shop freely and so will buy each product at the lowest price available. A shopper randomizes if indifferent about where to buy a particular product. The remaining fraction $1 - \alpha$ of consumers are non-shoppers, who can visit only one firm (but can do so costlessly). Non-shoppers observe each firm's product range, but do not observe prices when deciding which firm to visit.[11] Instead they form (rational) expectations about each firm's pricing strategy, and visit the firm which they believe will give them the highest expected payoff. We assume that a non-shopper randomizes when indifferent between visiting two or more firms. Once they visit a firm they observe all its prices and make their purchase decisions. Each firm sets its price(s) to maximize expected profits, given consumer search strategies and other firms' pricing strategies.

Some remarks on our modeling approach. We have assumed that non-shoppers cannot search beyond the first visited store. This implies that when there are no multiproduct

[8] Or equivalently one firm has the opportunity to acquire the other. We assume that horizontal merger between two firms selling the same product is *not* permitted (or is too costly), for instance due to antitrust policy.

[9] In practice mergers may be costly to propose, but could also generate economies of scope and therefore long-term cost savings. We assume this away to highlight the effect of one-stop shopping convenience. However introducing this into the model would not change the main qualitative insights.

[10] Given multiproduct firms charge separate prices for each product, our model is actually isomorphic to a game of store location choice, where each pair of single-product firms which supply different products can choose whether to locate together (e.g. in a shopping mall) or stay separately.

[11] The assumption that product range is observable but price is not is plausible in many cases, because prices tend to change frequently whereas product ranges are more stable.

firms in the market, non-shoppers can only buy one product even though they want both. This is an extreme way to introduce one-stop shopping convenience from having multiproduct firms. A less extreme approach would be to allow non-shoppers to search more firms if they pay a search cost. One way to do that is to have non-shoppers search sequentially as in for example Stahl (1989). However this is complicated to analyze in a multiproduct context, because typically there are multiple mixed-strategy pricing equilibria which are not outcome equivalent, and moreover their characterizations are complex (see McAfee, 1995). In Section 3.1 we discuss an alternative way to allow non-shoppers to buy both products, and show that the main insights from the benchmark model remain unchanged.

We are using a merger framework to study endogenous retail market structure. There are many examples where retailers expand their product ranges by acquisitions or mergers. For example, in the UK Amazon acquired LoveFilm to create a one-stop service for video streaming, DVD rental, and books. Very recently Sainsbury's offered to acquire Argos to create a combined food and non-food retailer, with the hope of gradually relocating Argos stores into Sainsbury's supermarkets. Of course an alternative modelling approach to endogenize market structure would be to allow each firm in the market to directly choose which products to stock. We explore such a model in Section 3.3 and show that the main insights from our merger model continue to hold. However the merger framework captures those insights in a much more parsimonious way.

2.1 Pricing under different market structures

We first solve for equilibrium at the second stage of the game. There are three market structures we need to consider: (i) if no merger has occurred, a market with four independent single-product firms, (ii) if only one pair of firms has merged, an asymmetric market with one multiproduct firm and two single-product firms, and (iii) if both pairs of firms have merged, a symmetric market with two multiproduct firms.

As a preliminary step, we first consider a simpler game where two single-product firms sell an identical product, some consumers are 'captive' (able to buy from only one exogenously given firm) and others are 'non-captive' (able to buy from either firm). The following lemma reports equilibrium pricing in this game.[12]

Lemma 1 *Consider a simultaneous pricing game between two firms A and B which supply a homogenous product at zero cost. Let N_k be the mass of consumers who can only*

[12] The results in Lemma 1 are not new (but are stated here for completeness), and can be found in Varian (1980), Narasimhan (1988), and Baye et al. (1992).

buy from firm $k = A, B$. Suppose $N_A \geq N_B \geq 0$ with at least one strict inequality. Let $S > 0$ be the mass of consumers who can buy from either firm.
(i) There is no pure-strategy Nash equilibrium.
(ii) If $N_A = N_B = N > 0$, the unique equilibrium is that each firm charges a random price drawn from the atomless price distribution

$$F(p) = 1 - \frac{N}{S}\left(\frac{v}{p} - 1\right) \tag{1}$$

which has support $[\underline{p}, v]$ with

$$\underline{p} = \frac{N}{N+S}v \ . \tag{2}$$

Each firm earns Nv.
(iii) If $N_A > N_B \geq 0$, the unique equilibrium is that firm A charges a random price drawn from a price distribution $F_A(p)$, where

$$F_A(p) = 1 + \frac{N_B}{S} - \left(\frac{N_B}{S} + \lambda\right)\frac{v}{p} \tag{3}$$

for $p \in [\underline{p}, v)$ with

$$\underline{p} = \frac{N_A}{N_A + S}v \tag{4}$$

and $F_A(p)$ has a mass point at v of size

$$\lambda = \frac{N_A - N_B}{N_A + S} \ , \tag{5}$$

while firm B charges a random price drawn from the atomless price distribution

$$F_B(p) = 1 - \frac{N_A}{S}\left(\frac{v}{p} - 1\right) \tag{6}$$

which also has support $[\underline{p}, v)$. Firm A earns $N_A v$ and firm B earns $(N_B + S\lambda)v$.

As usual the two firms randomize over their price, because they face a trade-off between pricing low to attract non-captives, or pricing high to exploit captives. Lemma 1 implies that firms' price distributions can be ranked in a simple way. Firstly when the two firms have the same number of captives (i.e. $N_A = N_B$) they use the same price distribution. Secondly when one firm has more captives than the other, for example $N_A > N_B$, equations (3), (5) and (6) imply that the two density functions satisfy

$$f_A(p) = (1 - \lambda) f_B(p) \tag{7}$$

for $p \in [\underline{p}, v)$. This means that firm A charges higher prices than firm B in the sense of first-order stochastic dominance (FOSD). Intuitively this is because firm A has relatively more incentive to extract surplus from its captive consumers by pricing high, than compete for non-captive consumers by pricing low.

We now return to our set-up, and use Lemma 1 to study equilibrium pricing in each of the three possible market structures outlined above, starting with the simple case of four independent single-product firms.

Lemma 2 *Suppose there are four independent single-product firms. Non-shoppers randomly visit one firm, and each firm uses the mixed pricing strategy in Lemma 1(ii) with* $N = \frac{1}{4}(1-\alpha)$ *and* $S = \alpha$*. Earn firm earns* $\frac{1}{4}(1-\alpha)v$*.*

Proof. Firstly in equilibrium firms 1_A and 1_B must have the same number of non-shoppers. Suppose, in contrast, that 1_A for example has strictly more non-shoppers than 1_B. Using Lemma 1(iii) 1_A charges strictly more in the sense of FOSD than 1_B, which is inconsistent with non-shoppers' search behavior. Secondly for the same reason, 2_A and 2_B must have an equal number of non-shoppers. Thirdly all four firms must have the same number of non-shoppers. Suppose, in contrast, that 1_A and 1_B for example have strictly more non-shoppers than 2_A and 2_B. Using Lemma 1(ii) 1_A and 1_B charge strictly more in the sense of FOSD than 2_A and 2_B, which again yields a contradiction. Lastly then, each firm has $\frac{1}{4}(1-\alpha)$ non-shoppers and so the equilibrium outcome is given by Lemma 1(ii) with $N_A = N_B = \frac{1}{4}(1-\alpha)$ and $S = \alpha$. ∎

Another simple case is when the market has two multiproduct firms. It is without loss of generality to focus on an equilibrium where each firm randomizes independently over the prices of its two products.[13]

Lemma 3 *Suppose there are two multiproduct firms. Non-shoppers randomly visit one firm, and each firm chooses the prices of its two products independently using the mixed pricing strategy in Lemma 1(ii) with* $N = \frac{1}{2}(1-\alpha)$ *and* $S = \alpha$*. Each firm earns* $\frac{1}{2}(1-\alpha)v$ *from each product.*

Proof. The argument that in equilibrium non-shoppers must randomly visit one firm is similar to Lemma 2. Hence the equilibrium outcome is given by Lemma 1(ii) with $N_A = N_B = \frac{1}{2}(1-\alpha)$ and $S = \alpha$. ∎

[13] A firm's payoff depends only on its rival's marginal price distributions. Therefore for any equilibrium in which firm i (for $i = A, B$) uses a joint price distribution $F_i(p_1, p_2)$, we can construct an alternative payoff-equivalent equilibrium in which firm i chooses its two prices independently using the marginal distributions of $F_i(p_1, p_2)$.

In both symmetric market structures non-shoppers visit one firm at random, and therefore (by Lemma 1) all firms draw their price from the same distribution. However the price distribution is lower (in the sense of FOSD) in a market with four single-product firms compared to a market with two multiproduct firms. This is because in the former case each single-product firm gets only one quarter of the non-shoppers, whereas in the latter case each multiproduct firm gets half the non-shoppers and therefore has less incentive to price aggressively to attract shoppers. This is due to the assumption (which we relax in Section 3.1 below) that a non-shopper can visit only one firm, even if all firms supply a single product.

Next consider the asymmetric market structure. Suppose that 1_A and 2_A have merged to form a multiproduct firm A, but 1_B and 2_B remain as single-product firms. A non-shopper chooses between visiting the multiproduct firm and buying both products, for a payoff of

$$2\int_{\underline{p}}^{v}(v-p)f_A(p)dp\ , \tag{8}$$

or visiting a single-product firm i_B (for $i=1,2$) for a payoff of

$$\int_{\underline{p}}^{v}(v-p)f_{i_B}(p)dp\ . \tag{9}$$

Clearly, other things equal, it is more attractive to visit the multiproduct firm and get both products. However, on the other hand, we know from Lemma 1 that if more non-shoppers visit the multiproduct firm than a single-product firm, the multiproduct firm will on average charge a higher price. Using equation (7) to compare the two payoffs, we can state the following result:

Lemma 4 *Suppose there is a multiproduct firm A and two single-product firms 1_B and 2_B.*
(i) A non-shopper visits firm i with probability X_i, where

$$X_A=\begin{cases}1 & \text{if } \alpha\geq\frac{1}{2}\\ \frac{1}{2(1-\alpha)} & \text{if } \alpha<\frac{1}{2}\end{cases}\quad \text{and}\quad X_{1_B}=X_{2_B}=\begin{cases}0 & \text{if } \alpha\geq\frac{1}{2}\\ \frac{1-2\alpha}{4(1-\alpha)} & \text{if } \alpha<\frac{1}{2}\end{cases}.$$

(ii) Firm A uses the mixed pricing strategy $F_A(p)$, and firms 1_B and 2_B use the mixed pricing strategy $F_B(p)$, both given in Lemma 1(iii), with $N_i=(1-\alpha)X_i$ and $S=\alpha$.
(iii) Firm i earns expected profit π_i on each of its products, where

$$\pi_A=\begin{cases}(1-\alpha)v & \text{if } \alpha\geq\frac{1}{2}\\ \frac{1}{2}v & \text{if } \alpha<\frac{1}{2}\end{cases}\quad \text{and}\quad \pi_{1_B}=\pi_{2_B}=\begin{cases}\alpha(1-\alpha)v & \text{if } \alpha\geq\frac{1}{2}\\ \frac{1}{4}v & \text{if } \alpha<\frac{1}{2}\end{cases}.$$

Lemma 4 shows that if there are relatively few non-shoppers (i.e. if $\alpha \geq \frac{1}{2}$) they all buy from the multiproduct firm. However if there are relatively many non-shoppers (i.e. if $\alpha < \frac{1}{2}$) some of them buy from a single-product firm instead. This prevents the multiproduct firm from charging too high prices, and thus rationalizes non-shoppers' search behavior by ensuring that the payoffs (8) and (9) are equal. Nevertheless the multiproduct firm still attracts a disproportionate share of non-shoppers, because it offers them one-stop shopping convenience. One implication of this is that for all $\alpha \in (0,1)$ the multiproduct firm charges higher prices (in the sense of FOSD) than its single-product rivals. This prediction may not fit the casual observation that large retailers are often cheaper than small ones. Remember, however, that to highlight the effect of one-stop shopping convenience our model has assumed away any possible cost synergy from the merger. In reality larger retailers may enjoy economies of scale, and also be able to extract better deals from upstream suppliers. This may lead them to charge lower prices on average.

Finally, for convenience, Table 1 summarizes per-product profit in each of the three market structures. Here we also report total welfare as well as its components industry profit and aggregate consumer surplus. One useful observation is that the asymmetric market structure tends to lead to the weakest price competition, in the sense that it is the worst for consumers, and it is the best for industry profit whenever $\alpha > \frac{1}{4}$. The reason is that in the asymmetric market the multiproduct firm gets a disproportionate share of the non-shoppers, and so charges high prices; by strategic complementarity, this induces the two single-product firms to set relatively high prices as well.[14]

	4 sp firms	2 mp firms	asymmetric ($\alpha \geq 1/2$)	asymmetric ($\alpha < 1/2$)
Per product profit	$\frac{1}{4}(1-\alpha)v$	$\frac{1}{2}(1-\alpha)v$	$A:\ (1-\alpha)v$ $B:\ \alpha(1-\alpha)v$	$A:\ \frac{1}{2}v$ $B:\ \frac{1}{4}v$
Industry profit	$(1-\alpha)v$	$2(1-\alpha)v$	$2(1-\alpha^2)v$	$\frac{3}{2}v$
Consumer surplus	$2\alpha v$	$2\alpha v$	$2\alpha^2 v$	αv
Total welfare	$(1+\alpha)v$	$2v$	$2v$	$(\frac{3}{2}+\alpha)v$

Table 1: Profit and welfare comparison across market structures

[14] Wilson (2011) finds a similar market segmentation effect in a different context, where a single-product firm strategically makes it harder for consumers to search it.

2.2 Equilibrium market structure

We can now examine the equilibrium market structure when both pairs of firms (1_A,2_A) and ($1_B, 2_B$) have the opportunity to merge before engaging in price competition. Using our earlier results, we can state that:

Proposition 1 *(i) When* $\alpha \geq \frac{1}{2}$ *the unique (pure-strategy) equilibrium outcome is that the market has one multiproduct firm and two single-product firms.*
(ii) When $\alpha < \frac{1}{2}$ *the unique equilibrium outcome is that the market has two multiproduct firms.*

Intuitively a merger between a pair of firms leads to two different effects. Firstly there is a "search effect": the merging firms offer one-stop shopping convenience and so become more attractive to non-shoppers. Consequently the merged entity is searched by more non-shoppers. Secondly though, there is a "price competition effect": the merger changes the market structure and hence the intensity of competition. As discussed earlier, at an industry level the asymmetric market structure typically leads to the softest price competition.

Proposition 1 can then be explained as follows. There is no equilibrium with four single-product firms, because if one pair deviates and merges, both effects work in their favor i.e. they secure higher demand and soften overall competition.[15] (In fact Table 1 shows that the remaining single-product firms also benefit from the first merger.) However if the second pair contemplates merging they face a trade-off, since a merger restores symmetry and so intensifies competition. When there are relatively many shoppers ($\alpha \geq \frac{1}{2}$) the second pair of firms do not merge, because it is more important to avoid strong competition for the shoppers. Hence the equilibrium market structure is asymmetric, even though firms start off symmetric.[16] However when there are relatively many non-shoppers ($\alpha < \frac{1}{2}$) the second pair of firms do merge, because it is more important to capture a high share of the non-shoppers. Hence the equilibrium market structure is symmetric.

Corollary 1 *Compared to the initial situation with four single-product firms, the equilibrium market structure in Proposition 1 results in strictly higher welfare and firm profits, but (weakly) lower consumer surplus.*

[15] Of course if we assume that merger involves a sufficiently high fixed cost, then the initial situation can remain as an equilibrium outcome.

[16] Notice that if the two pairs of firms make their merger decisions simultaneously, there are two asymmetric pure-strategy equilibria and one mixed-strategy equilibrium.

The market outcome with merger is better for total welfare due to a positive "market coverage effect": non-shoppers generate more surplus when they visit a multiproduct firm and buy two products instead of one.[17] The market outcome also increases each firm's profit: price competition is weaker, either because of the resulting asymmetric market structure (when $\alpha \geq \frac{1}{2}$), or because more non-shoppers visit each multiproduct firm (when $\alpha < \frac{1}{2}$). However consumers are made worse off because they pay higher prices on average, and this (weakly) dominates the fact that non-shoppers can now buy both products.

Finally, notice that the fraction of non-shoppers (i.e. $1-\alpha$) is a measure of search frictions in this baseline model. Interestingly once we endogenize market structure, a higher search friction does not necessarily harm consumers. This is shown graphically in Figure 1 below, which plots for $v = 1$ total welfare (the top horizontal line), industry profit (the thick solid lines), and aggregate consumer surplus (the dashed lines) against $1-\alpha$. Intuitively when $1-\alpha \leq \frac{1}{2}$ the market structure is asymmetric, and a larger search friction relaxes competition to the detriment of consumers but the benefit of firms. However around the point $\alpha = \frac{1}{2}$ the market structure changes and becomes symmetric with two big multiproduct firms, such that competition intensifies and consumer surplus jumps up and industry profit jumps down in the search friction. However total welfare is constant because demand is inelastic.[18]

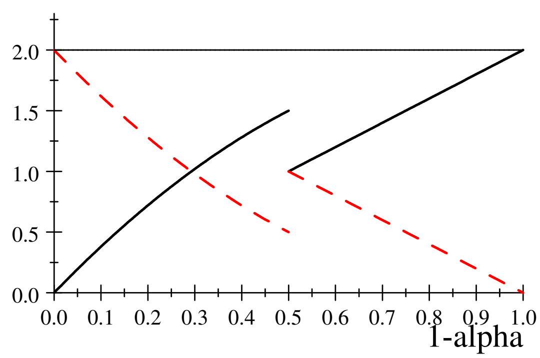

Figure 1: Welfare and the search friction

[17] Notice that in the current setting with inelastic demand, the price competition effect of merger does not affect total welfare.

[18] When demand is elastic, total welfare changes with $1-\alpha$ in a similar way as do the dashed lines in Figure 1.

3 Extensions and Robustness Discussion

This section shows that the main insights from the benchmark model are robust to various extensions.

3.1 Allowing non-shoppers to multi-stop shop

We now relax our earlier assumption that non-shoppers can only visit one firm (and so can only buy one good when all firms are single-product). To make this extension simple, we assume that at the beginning of the pricing game (in each possible market structure) non-shoppers can choose to either (i) visit one (single-product or multiproduct) firm at zero cost, or (ii) visit two single-product firms (if available) at a cost $s > 0$. The model is otherwise the same as before. (Our modeling approach here is therefore ruling out the possibility that non-shoppers can visit two multiproduct firms, or one multiproduct firm and one or two single-product firms. This greatly simplifies the analysis and captures the idea that time-constrained consumers want to buy both products but do not find it worthwhile to search for lower prices.) Notice that we may loosely interpret s as a measure of one-stop shopping convenience generated by having a multiproduct firm.

First consider equilibrium pricing in the two symmetric market structures. When there are four single-product firms, non-shoppers can either visit one firm only or visit two firms with different products by paying the search cost s. We focus on the case where s is small enough such that non-shoppers will visit two firms and buy both products.[19] For reasons analogous to those in the benchmark model, each firm must then receive the same mass of non-shoppers i.e. $N = \frac{1}{2}(1 - \alpha)$. When instead there are two multiproduct firms, non-shoppers choose one firm to visit and the model is identical to the one that we solved earlier, and hence each firm again receives a mass $N = \frac{1}{2}(1 - \alpha)$ of non-shoppers. Consequently equilibrium price distributions and per-product profits are now *identical* in the two symmetric market structures, and given by Lemma 3 from earlier. However consumer surplus is strictly higher when there are two multiproduct firms, because non-shoppers can buy both products without having to pay s.

Second consider the asymmetric market structure. Now non-shoppers have three options: visit the multiproduct firm only, visit one single-product firm only, or visit both single-product firms by paying the search cost s. Let $X(s, \alpha)$ and $Y(s, \alpha)$ denote the fractions of non-shoppers who visit a multiproduct firm and both single-product firms respectively. (Then the remaining $1 - X(s, \alpha) - Y(s, \alpha)$ non-shoppers visit one single-

[19] See the proof of Lemma 5 in the appendix for further details. Our benchmark model corresponds to the case when s is sufficiently high such that non-shoppers visit one store only.

product firm only.) We can then show the following:

Lemma 5 *Suppose there is one multiproduct firm and two single-product firms. There exists a unique equilibrium, whose exact form depends on the thresholds* $\tilde{s}(\alpha) < \dot{s}(\alpha) < \bar{s}(\alpha)$ *which we define in the appendix.*
(i) When $\alpha \geq \frac{1}{2}$ *non-shoppers search either the multiproduct firm, or both single-product firms.* $X(s,\alpha)$ *is strictly increasing in* $s \in (0, \bar{s}(\alpha))$ *and satisfies*

$$s = 2v \frac{2\tilde{X} - \frac{1-\alpha}{\alpha}}{\tilde{X}+1} \left[1 + \tilde{X} \ln \left(\frac{\tilde{X}}{\tilde{X}+1} \right) \right] \tag{10}$$

where $\tilde{X} \equiv \frac{1-\alpha}{\alpha} X(s,\alpha)$, *whilst* $X(s,\alpha) = 1$ *for all* $s \geq \bar{s}(\alpha)$.
(ii) When $\alpha < \frac{1}{2}$ *and* $s \in (0, \tilde{s}(\alpha)]$ *non-shoppers search either the multiproduct firm, or both single-product firms.* $X(s,\alpha)$ *is strictly increasing in* s *and satisfies (10).*
(iii) When $\alpha < \frac{1}{2}$ *and* $s \in (\tilde{s}(\alpha), \dot{s}(\alpha))$ *non-shoppers randomize between searching the multiproduct firm, both single-product firms, and one of the single-product firms.*

$$Y(s,\alpha) = 2X(s,\alpha) - \frac{1}{1-\alpha} \ , \tag{11}$$

whilst $X(s,\alpha)$ *is strictly decreasing in* s *and uniquely solves*

$$s = v \left[1 + \tilde{X} \ln \left(\frac{\tilde{X}}{\tilde{X}+1} \right) \right] \ . \tag{12}$$

(iv) When $\alpha < \frac{1}{2}$ *and* $s \geq \dot{s}(\alpha)$ *non-shoppers search the multiproduct firm with probability* $X(s,\alpha) = \frac{1}{2(1-\alpha)}$ *and otherwise search one randomly chosen single-product firm.*

The pricing equilibrium in the asymmetric market is more complicated than in the benchmark model, but the interpretation of Lemma 5 is straightforward. For instance consider the case where $\alpha < 1/2$. When s is relatively low (below $\tilde{s}(\alpha)$), non-shoppers randomize over where to shop, with some searching the multiproduct firm, and others searching both of the single-product firms. As s increases it becomes more attractive to search the multiproduct firm and avoid paying s. Therefore to ensure that non-shoppers are willing to randomize, the multiproduct firm's relative prices should increase, which is achieved by having more non-shoppers search it. This explains why $X(s,\alpha)$ increases in s. However when s is sufficiently large (above $\tilde{s}(\alpha)$), the multiproduct firm becomes so expensive that non-shoppers also find attractive the option of searching only one single-product firm. Therefore at this point some non-shoppers also search just one single-product firm. As s further increases, fewer and fewer non-shoppers opt to search both single-product firms. Eventually s becomes so large (above $\dot{s}(\alpha)$) that no non-shopper searches both single-product firms. At this point the equilibrium is exactly the same as in our earlier benchmark model.

Now consider the equilibrium retail market structure:

Proposition 2 *There exists an $\alpha^*(s) \in \left(\frac{1}{3}, \frac{1}{2}\right]$ (weakly) increasing in s such that:*
(i) When $\alpha \geq \alpha^*(s)$ the equilibrium market structure is asymmetric.
(ii) When $\alpha < \alpha^*(s)$ the equilibrium market structure is two multiproduct firms.

Qualitatively the market structure is the same as in the benchmark model. A first merger is always profitable, because the merging firms soften competition and attract higher demand. Intuitively, the merged entity attracts a disproportionate share of non-shoppers, because it enables them to buy both products without needing to incur the additional cost $s > 0$. A second merger is then profitable if and only if α is sufficiently low. As in the benchmark model, a second merger has both a price and search effect, with the latter dominating when there are relatively few shoppers in the market. The result that the threshold $\alpha^*(s)$ (weakly) increases in s implies that when one-stop shopping convenience becomes more important, it is more likely that the market has two multiproduct firms. Finally as in the benchmark model, we are also able to show that the asymmetric market structure is the worst for consumers, but the best for industry profit provided that α is not too small.

3.2 More firms and heterogeneous consumers

This section extends the benchmark model in two ways. First we consider $n \geq 2$ pairs of firms, which we denote by $(1_j, 2_j)$ for $j = 1, \cdots, n$. Second we allow for the coexistence of both single-product and multiproduct consumers. In particular a consumer's valuation for a product is now v with probability $\gamma > 0$, and 0 with probability $1-\gamma$. Valuations are drawn independently across products and consumers, and do not depend on whether or not a consumer is a shopper. Therefore γ^2 consumers want to buy both products, $\gamma(1-\gamma)$ want to buy only product i ($i = 1, 2$), and the rest of the consumers want nothing. The model and timing are otherwise the same as in the benchmark model from Section 2 (which therefore corresponds to the special case of $n = 2$ and $\gamma = 1$).

This extended model is less straightforward to analyze than the benchmark model, because price competition in an asymmetric market with $n > 2$ is more complicated. We show in the Online Appendix that in the asymmetric market structure, non-shoppers' search behavior depends on whether $\gamma \gtreqless \frac{k}{n}$ where k denotes the number of multiproduct firms. When $\gamma < \frac{k}{n}$, the ratio of consumers demanding two products to firms supplying two products is relatively low. We show that in this case non-shoppers requiring both products search a multiproduct firm, whilst non-shoppers who want only one product randomly choose between all firms in the marketplace. This mixing is done in such a way that all firms use the same price distribution and earn the same profit. We also show that the case $\gamma > \frac{k}{n}$ is more complicated and depends upon the exact number of multiproduct firms. Nevertheless as is intuitive, multiproduct firms charge more in the sense of FOSD, and non-shoppers wanting only one product buy it from a relevant single-product firm.

The following proposition reports the equilibrium market structure. (Its proof is relegated to the Online Appendix.)

Proposition 3 *(i) When $n = 2$ the equilibrium market structure is asymmetric if $\alpha \geq \frac{\gamma}{1+\gamma}$, and otherwise has two multiproduct firms.*
(ii) Suppose $n \geq 3$ and that a pair of single-product firms choose not to merge when they are indifferent. Then (a) if $\gamma \leq 1 - \frac{1}{n}$, the market has $\lceil n\gamma \rceil$ multiproduct firms. (b) If $\gamma > 1 - \frac{1}{n}$, the market has either $n-1$ or n multiproduct firms. If α is sufficiently large, there are $n-1$ multiproduct firms, and if α is sufficiently small, there are n multiproduct firms.

The $n = 2$ case is thus qualitatively the same as in the benchmark model, except that the critical threshold for α is now a function of consumer needs. When $n \geq 3$, an asymmetric market structure always arises when γ is small i.e. when relatively few consumers are interested in both products, such that 'demand' for multiproduct firms is weak. In a similar spirit when γ is relatively large, at most one pair of single-product firms will remain in the market. Whether all firms merge or not depends on the search friction, in a way that is qualitatively the same as in the baseline model.[20]

3.3 Allowing firms to choose product ranges

We now consider an alternative way to endogenize market structure. Instead of allowing single-product firms to merge, we now let each firm in the market directly choose its product range. We suppose there are three firms A, B, C in the market. (Three is the minimum number required to generate the asymmetric market structure with at least one multiproduct firm and one single-product firm for each product.) We normalize the fixed cost of stocking one product to zero, and then let $\Delta > 0$ denote the incremental fixed

[20]Unfortunately it is difficult to derive a cut-off result on α as we do in the case with $n = 2$, because a non-shopper's search problem is much less tractable.

cost of stocking a second product.[21] The firms play a two-stage game where they first simultaneously choose product ranges, and then observe their rivals' choices and select a price for each of their products. In all other respects the set-up is the same as in the benchmark model.

We focus on deriving conditions under which there exists an asymmetric market structure with one multiproduct firm and two single-product firms supplying different products. Without loss of generality, consider a hypothetical equilibrium where firm A supplies both products, and firms B and C supply product 1 and product 2 respectively. Using Lemma 4 from earlier, firms' expected profits in this market structure are

$$\pi_A = \begin{cases} 2(1-\alpha)v - \Delta & \text{if } \alpha \geq \frac{1}{2} \\ v - \Delta & \text{if } \alpha < \frac{1}{2} \end{cases} \quad \text{and} \quad \pi_B = \pi_C = \begin{cases} \alpha(1-\alpha)v & \text{if } \alpha \geq \frac{1}{2} \\ \frac{1}{4}v & \text{if } \alpha < \frac{1}{2} \end{cases}.$$

There are three possible deviations that we need to check. (i) Suppose a single-product firm, say firm B, deviates by stocking both products. Then the market has two multiproduct firms A and B and a single-product firm C supplying product 2 only. As we show in the proof of the proposition below, in this scenario non-shoppers randomly visit one of the two multiproduct firms, and firm B's deviation profit is $(1-\alpha)v - \Delta$. (ii) Alternatively suppose the multiproduct firm A deviates by dropping one product, say product 2. Then the market has two single-product firms A and B supplying product 1 and one single-product firm C supplying product 2. Clearly firm C will charge the monopoly price v because it is the only supplier of product 2. Hence applying Lemma 1 from earlier, non-shoppers randomize between visiting firm A or firm B, such that firm A's deviation profit is $\frac{1}{2}(1-\alpha)v$. (iii) Finally suppose a single-product firm, say firm B, deviates by dropping its current product and stocking the other instead. Then the market has one multiproduct firm A and two single-product firms B and C both supplying product 2 only. Again since firm A is the only supplier of product 1 it charges the monopoly price v for product 1. Non-shoppers must then randomize between all three firms, such that by the usual logic firm B's deviation profit is $\frac{1}{3}(1-\alpha)v$. Collecting these results together, we can then state the following:

Proposition 4 *If and only if $\alpha \geq \frac{1}{4}$ there exist $\underline{\Delta}(\alpha) < \overline{\Delta}(\alpha)$ such that for $\Delta \in [\underline{\Delta}(\alpha), \overline{\Delta}(\alpha)]$ it is an equilibrium that one firm supplies both products and the other two firms each supply a different product.*

[21] We can show that without this fixed cost, all firms choose to supply both products. This did not happen in the merger framework because there was an opportunity cost of merging, namely the profit that could be made by remaining independent.

Proposition 4 shows that in order to have an asymmetric market structure we require that $\alpha \geq \frac{1}{4}$, and also that the fixed cost Δ is neither too high (otherwise the multiproduct firm will drop one product) nor too low (otherwise a single-product firm will add another product). The requirement that $\alpha \geq \frac{1}{4}$ is consistent with our earlier merger framework, in which an asymmetric outcome arose if and only if the fraction of shoppers was sufficiently high.

Finally, we can also derive conditions under which any of the other possible market structures is an equilibrium. The details are lengthy and so we omit them, but there are two observations. Firstly, for a fixed α the number of multiproduct firms tends to decrease as Δ increases. Secondly, for a fixed (and sufficiently small) Δ the number of multiproduct firms tends to increase as α decreases (i.e. as the search friction increases). This is again consistent with our earlier merger framework.

3.4 Product differentiation and sequential search

This section explores an alternative framework with product differentiation and sequential search. We show that the search friction affects equilibrium market structure in a similar way to what we found in the homogenous goods case. However we also highlight some important differences with our earlier results. For example in an asymmetric market the multiproduct firm charges lower prices than its smaller rivals even if it has no cost advantage.

We return to the merger framework in Section 2. There are two products 1 and 2, and consumers wish to buy one unit of each. Initially there are four single-product firms, with firms i_A and i_B supplying horizontally differentiated versions of product i (for $i = 1, 2$). Following Wolinsky (1986) and Anderson and Renault (1999) we use the random utility framework to model product differentiation. In particular the match utility of each product i is a random draw from a common distribution $G(u)$ with support $[\underline{u}, \overline{u}]$ and density $g(u)$. The realization of the match utility is i.i.d. across consumers, products, and firms, as consistent for example with consumers having idiosyncratic tastes. If a consumer buys a product with match utility u and pays a price p, she obtains a surplus $u - p$. We follow Anderson and Renault (1999) and assume that in equilibrium all consumers buy both products. This is the case if consumers have a sufficiently high basic valuation for each product i.e. $\underline{u}$ is sufficiently high.

The timing is as before: at the first stage each pair of firms which supply different products simultaneously decides whether or not to merge; at the second stage their merger decisions are observed by all parties, and prices are chosen. However unlike in the benchmark model, consumers all have the same search technology. In particular consumers are initially uninformed about firms' prices and match values, although they know the match utility distribution $G(u)$ and also hold rational expectations about each firm's pricing strategy. A consumer can learn a firm's prices and match utilities by incurring a search cost $s > 0$; search is sequential and with costless recall. To capture the idea of one-stop shopping convenience, we assume that the search cost is the same whether a consumer visits a single-product or a multiproduct firm. To have active search in each possible market structure, we assume that the search cost is not too high i.e.

$$s < \int_{\underline{u}}^{\overline{u}} (u - \underline{u}) dG(u) \ . \tag{13}$$

As before we first derive the pricing equilibrium in each possible market structure, and then examine the equilibrium market structure.

A market with four single-product firms. With four single-product firms, a consumer's search process is separable across the two product markets. In each market we have a duopoly version of the sequential search model in Anderson and Renault (1999). Consider the market for product i. We look for a symmetric equilibrium where both firms charge the same price p_0 and consumers search in a random order (i.e. half of the consumers visit firm i_A first and the other half visit firm i_B first). In symmetric equilibrium the optimal stopping rule is characterized by a reservation utility level a which solves

$$\int_{a}^{\overline{u}} (u - a) dG(u) = s \ . \tag{14}$$

(The left-hand side is the expected benefit from sampling the second firm when the first firm offers match utility a.) This equation has a unique solution $a \in (0, \overline{u})$ given the search cost condition (13). In equilibrium a consumer buys immediately at the first visited firm if and only if its match utility is no less than a.

As explained in the appendix, the first-order condition for the equilibrium price is[22]

$$\frac{1}{p_0} = g(a)[1 - G(a)] + 2 \int_{\underline{u}}^{a} g(u)^2 du \ . \tag{15}$$

In equilibrium firms share the market equally, and so each firm earns profit $\pi_0 = \frac{1}{2} p_0$. For example when valuations are uniformly distributed with $G(u) = u$, we have $a = 1 - \sqrt{2s}$ and condition (13) requires $s < \frac{1}{2}$. The first-order condition then implies that

$$p_0 = \frac{1}{2 - \sqrt{2s}} \ . \tag{16}$$

[22] We can show that if $p[1 - G(p)]$ is concave, then the first-order condition is also sufficient for defining the equilibrium price. See Appendix B in Anderson and Renault (1999) for other conditions which ensure the existence of a symmetric pure-strategy pricing equilibrium.

It is depicted as the dashed curve in Figure 2a below.

A market with two multiproduct firms. With two multiproduct firms we have a multiproduct search model as analyzed in Zhou (2014). Let p_m denote the equilibrium price for each product. We first report the optimal stopping rule in an equilibrium where both firms charge the same prices. Consider a consumer who visits firm A first. After visiting firm A she faces the following options: stop searching and buy both products, or buy one product and keep searching for the other, or keep searching for both products. Given that the search cost occurs at the firm level and consumers have free recall, the second option is always dominated by the third. If the consumer continues to visit firm B, she can thereafter freely mix and match among the two firms. Therefore the consumer will stop searching and buy both products at firm A if the match utilities (u_{1A}, u_{2A}) satisfy

$$\int_{u_{1A}}^{\overline{u}} (u_{1B} - u_{1A}) dG(u_{1B}) + \int_{u_{2A}}^{\overline{u}} (u_{2B} - u_{2A}) dG(u_{2B}) \leq s \ .$$

(The left-hand side is the expected benefit from sampling firm B.) This condition defines a reservation frontier $u_{2A} = \phi(u_{1A})$, where $\phi(\cdot)$ is a decreasing and convex function. If the match utilities (u_{1A}, u_{2A}) at firm A are such that $u_{2A} \geq \phi(u_{1A})$ the consumer buys immediately, otherwise she searches firm B.

We refer the reader to Zhou (2014) for details of how to derive the equilibrium price. For a general distribution, the first-order condition for the equilibrium price is[23]

$$\frac{1}{p_m} = \int_a^{\overline{u}} [1-G(\phi(u))] g(\phi(u)) dG(u) + \int_a^{\overline{u}} [1-G(u)] g(\phi(u)) dG(u) + 2 \int_{u_2 \leq \phi(u_1)} g(u_1)^2 g(u_2) d\mathbf{u} \ ,$$

where a is defined in (14). In equilibrium firms share the market equally, so each firm's per product profit is $\pi_m = \frac{1}{2} p_m$. In the uniform distribution example, the first-order condition implies that

$$p_m = \frac{1}{2 - (\frac{1}{2}\pi - 1)s} \ , \tag{17}$$

where $\pi \approx 3.14$ is the mathematical constant. It is depicted as the lowest solid curve in Figure 2a below.

Zhou (2014) proves that $p_m < p_0$ i.e. products become cheaper when single-product firms merge into two multiproduct firms. This differs from the result in the homogenous goods model in Section 2, and arises due to the following joint-search effect. Intuitively when a firm reduces one product's price, more consumers who visit it first will stop

[23] As explained in Zhou (2014), in general it is hard to derive a simple sufficient condition for the existence of a symmetric pure-strategy equilibrium. But for many common distributions (including the uniform distribution) the first-order condition is sufficient for defining the equilibrium price.

searching and buy both products. That is, reducing one product's price can increase the demand for the other product as well. Hence the two products behave like complements, inducing each firm to price more aggressively. This joint-search effect did not arise in the benchmark model, because no consumers had a sequential search decision to make.

An asymmetric market. Consider the asymmetric case with a multiproduct firm A and two single-product firms 1_B and 2_B. Let p_A be the multiproduct firm's price and p_B be each single-product firm's price. We look for an equilibrium where all consumers visit the multiproduct firm first. Notice that the cost of visiting each single-product firm is separable, and so a consumer's search decision when she is at the multiproduct firm is also separable between the two products. This means, for example, that she searches the single-product firm i_B if and only if the multiproduct firm's product i has a surplus less than $a - p_B$, where a is defined in (14). Therefore unlike the case with two multiproduct firms, there is no joint-search effect here. The multiproduct firm competes with its smaller rivals in two separate single-product markets where consumers search non-randomly. (As such the pricing problem is similar to the one studied by Armstrong, Vickers, and Zhou, 2009, where one firm is prominent and always visited first by consumers.)

As explained in the appendix, the first-order conditions for the equilibrium prices (p_A, p_B) are[24]

$$p_A = \frac{Q(\Delta)}{Q'(\Delta)}, \quad p_B = \frac{1 - Q(\Delta)}{Q'(\Delta) - [1 - G(a)]g(a - \Delta)} \ , \tag{18}$$

where $\Delta \equiv p_B - p_A$, and $Q(\Delta) \equiv 1 - \int_{\underline{u}}^{a-\Delta} [1 - G(u + \Delta)] dG(u)$ is the equilibrium demand for firm A's product i. Firm A's per product profit is $\pi_A = p_A Q(\Delta)$ and each single-product firm's profit is $\pi_B = p_B(1 - Q(\Delta))$. This analysis implicitly assumes that all consumers visit firm A first and that $a - \Delta > \underline{u}$. The following result provides a condition for the system of equations in (18) to have a solution $\Delta \in (0, a - \underline{u})$. With $\Delta > 0$ i.e. $p_A < p_B$, the consumer search order is indeed optimal, because the multiproduct firm both offers lower prices and provides one-stop shopping convenience.[25]

Lemma 6 *Suppose* $1 - G$ *is strictly log-concave and condition (13) holds. Then the system of equations in (18) has a solution* $\Delta \in (0, a - \underline{u})$.

Therefore under the regularity condition there is an equilibrium in this asymmetric market where the multiproduct firm is cheaper than its single-product rivals and all

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[24] As in the case with four single-product firms, the first-order conditions are also sufficient for defining the equilibrium prices if $p[1 - G(p)]$ is concave.

[25] Armstrong, Vickers, and Zhou (2009) show a similar result without assuming full market coverage, but they focus on the uniform distribution case.

consumers visit the multiproduct firm first. The prediction that $p_A < p_B$ is different to what we observed in our earlier model with homogenous products. Here, a consumer visits a single-product firm only if she is unsatisfied with the multiproduct firm's product. Therefore when a consumer searches a single-product firm, she reveals something about her preferences. This gives the single-product firm extra market power and induces it to charge a higher price.[26] Nevertheless as we will see below, both p_A and p_B tend to be higher than p_0 and p_m. This is similar to the benchmark model, where price competition was typically softest when the market structure was asymmetric.

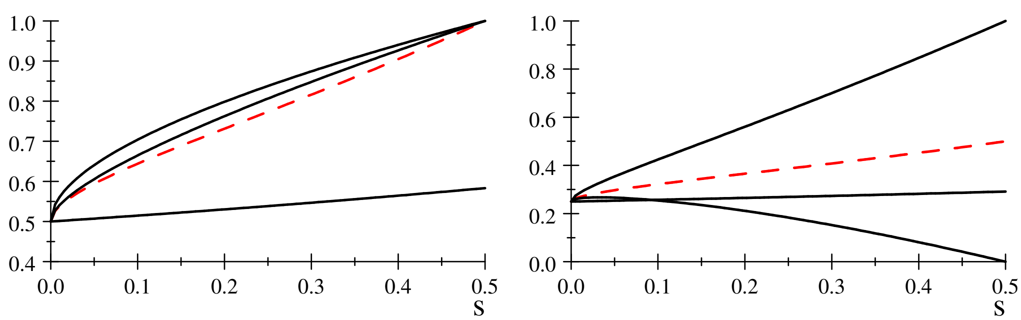
In the uniform distribution example, (18) simplifies to

$$p_A = \frac{1}{1-\Delta}[1 - a + \Delta + \frac{1}{2}(a^2 - \Delta^2)], \quad p_B = 1 - \frac{1}{2}(a - \Delta) \ ,$$

where $a = 1 - \sqrt{2s}$. It has a unique solution:

$$p_A = \frac{1}{16}(3K - 5a - 5), \quad p_B = \frac{1}{16}(K - 7a + 9) \ ,$$

where $K \equiv \sqrt{17a^2 - 30a + 49}$. The prices are depicted as the second highest and the highest solid curves respectively in Figure 2a below.



a: Prices

b: Profits

Figure 2: Price and profit comparison with differentiated products

Equilibrium market structure. For a general match utility distribution, it is hard to compare profits and study the equilibrium market structure. Therefore to make progress

[26]Following this argument, one may conjecture another possible equilibrium in which consumers visit the two single-product firms first and they charge lower prices than the multiproduct firm. In order for this to be an equilibrium, the price difference has to be large enough to compensate consumers for the extra search cost incurred by visiting single-product firms first. It is analytically difficult to exclude this possibility in general, but in the uniform distribution example it can be ruled out.

we focus on the uniform distribution case with $G(u) = u$. It can be verified that $p_A > p_0$ for any $s \in (0, \frac{1}{2})$ and so the four prices can be ranked as $p_m < p_0 < p_A < p_B$. (They are depicted in Figure 2a below.) As in the benchmark model with homogenous products, the asymmetric market structure generates the highest market prices (and so also the highest industry profit, given the assumption of full market coverage). Different from the benchmark model, here the market structure with two multiproduct firms yields the lowest market prices and industry profit.

Figure 2b above compares per product profit across market structures. The dashed curve is π_0 (each firm's profit in the case with four single-product firms), the middle (almost horizontal) solid curve is π_m (each firm's per product profit in the case with two multiproduct firms), and the other high and low curves are respectively π_A and π_B (the multiproduct firm's per product profit and each single-product firm's profit in the asymmetric case). A few observations follow: (i) $\pi_A > \pi_0$, so starting from the initial situation with four single-product firms, each pair of firms have a unilateral incentive to merge. (ii) $\pi_B > \pi_m$ if and only if s is less than about 0.092. We can then deduce that if s is less than 0.092, the (pure-strategy) equilibrium outcome is an asymmetric market with a multiproduct firm and two single-product firms. On the other hand, if s is greater than 0.092, each pair of firms chooses to merge and the market has two multiproduct firms. This leads to the lowest industry profit, and so firms end up in a prisoner's dilemma.

Therefore at least for the case of a uniform distribution, the search friction affects market structure in a similar way as it did in the model with homogeneous products. Again there is a trade-off between the search effect and the price competition effect. In particular when there is already a multiproduct retailer in the market, a merger between the remaining single-product firms (i) makes them more prominent in consumers' search order, boosting their demand, but (ii) also intensifies price competition. The latter effect dominates when s is small, whilst the former effect dominates otherwise.

As far as consumer surplus is concerned, the market structure with two multiproduct firms is the best since it leads to the lowest market prices and also saves search costs for consumers. Numerical simulations show that the asymmetric market structure is the worst for consumers when s is less than about 0.055, and otherwise the initial situation with four single-product firms is the worst. Then as in the benchmark model, due to the endogeneity of the market structure, reducing the search cost does not always improve consumers surplus.

4 Conclusion

This paper offers a simple and tractable framework to study equilibrium retail market structure when consumers buy multiple products and value one-stop shopping convenience. We have shown that the size of the search friction plays an important role in determining the equilibrium market structure. When search frictions are relatively high the market has all large firms. When search frictions are relatively low the market is asymmetric, such that large and small firms coexist. This is because some firms choose to remain unmerged in order to weaken the amount of price competition in the market. Among all possible market structures, the asymmetric market structure delivers the weakest price competition, and as such minimizes consumer surplus and often maximizes industry profit. Consequently our model suggests a potential anti-competitive effect of mergers among firms which supply different products. Our model also suggests that once the endogeneity of market structure is taken into account, reducing search frictions does not necessarily lower market prices and improve consumer welfare.

Appendix: Omitted Proofs and Details

Proof of Lemma 1. These results can be found in the existing literature. We provide proofs here for completeness.

(i) The proof is standard and so omitted.

(ii) We first verify that this is an equilibrium. Since the other firm is using the atomless price distribution F, a firm's profit at $p < v$ is $p[N + S(1 - F(p))]$, whilst its profit at $p = v$ is Nv. The expression for F in (1) equalizes these two profits, such that each firm is indifferent among all prices in $[\underline{p}, v]$, where the lower bound $\underline{p}$ in (2) is derived from $F(\underline{p}) = 0$. It is also clear that neither firm has an incentive to charge a price below $\underline{p}$. Varian (1980) proves there are no other symmetric equilibria, whilst Baye et al. (1992) prove there are no asymmetric equilibria either.

(iii) Again we begin by verifying that this is an equilibrium. Consider firm A first. Given that firm B is using the equilibrium strategy F_B, A's profit at $p < v$ is $p[N_A + S(1 - F_B(p))]$, whilst its profit at $p = v$ is $N_A v$. The expression for F_B in (6) equalizes these two profits. The lower bound of the support $\underline{p}$ in (4) is derived from $F_B(\underline{p}) = 0$. Firm A is then indifferent among all prices between $\underline{p}$ and v, and also has no incentive to charge a price below $\underline{p}$.

Now consider firm B. Given that firm A is using the equilibrium strategy F_A, B's profit at $p < v$ is $p[N_B + S(1 - F_A(p))]$. When p converges to v from below, B's profit converges to $v[N_B + S\lambda]$ since F_A has a mass point of size λ at $p = v$. The expression for F_A in (3) equalizes these two profits. Given the mass point of F_A, firm B never wants to charge a price exactly at $p = v$ because it is dominated by a price slightly below v. Hence the support of F_B is open at v. λ in (5) is derived from $F_A(\underline{p}) = 0$. Then firm B has no incentive to charge a price below $\underline{p}$ either. Narasimhan (1988) establishes uniqueness of this equilibrium. ∎

Proof of Lemma 4. Notice that in equilibrium each single-product firm must have the same number of non-shoppers, such that $N_{1_B} = N_{2_B} = N_B = \frac{1}{2}(1 - \alpha - N_A)$. Notice also that in equilibrium the multiproduct firm must receive some non-shoppers, otherwise it would charge lower prices in the sense of FOSD than the single-product firms, invalidating non-shoppers' search behavior. Notice also that (8) exceeds (9) if and only if

$$2(1 - \lambda) \geq 1 \Leftrightarrow \frac{N_B + S}{N_A + S} \geq \frac{1}{2} \ . \tag{19}$$

(i) An equilibrium with all non-shoppers visiting firm A exists if (19) holds with $N_A = 1 - \alpha$, $N_B = 0$ and $S = \alpha$. This yields the condition $\alpha \geq \frac{1}{2}$. Firm A's profit from each product is $N_A v = (1-\alpha)v$, and each single-product firm's profit is $(N_B + S\lambda)\, v = \alpha(1-\alpha)v$.

(ii) The only other possible equilibrium is that a fraction $X_A \in (0,1)$ of the non-shoppers visit the multiproduct firm A such that $N_A = (1-\alpha)X_A$ and $N_B = \frac{1}{2}(1-\alpha)(1-X_A)$. According to (19), non-shoppers will be indifferent between visiting a multiproduct firm and a single-product firm only if $\lambda = \frac{1}{2}$, or equivalently

$$\frac{N_B + S}{N_A + S} = \frac{1}{2} \Leftrightarrow X_A = \frac{1}{2(1-\alpha)} \ .$$

The requirement $X_A \in (0,1)$ yields the condition $\alpha < \frac{1}{2}$. Then $N_A = \frac{1}{2}$ and $N_B = \frac{1}{4}(1-2\alpha)$. Firm A's profit from each product is $N_A v = \frac{1}{2}v$ and each single-product firm's profit is $(N_B + S\lambda)\, v = \frac{1}{4}v$.

In each case $N_A > N_B$ so equilibrium price distributions follow from Lemma 1(iii). ■

Further details for Table 1. (i) Consider four single-product firms. Shoppers buy two products and non-shoppers one product, hence total welfare is $(1+\alpha)v$. Using Lemma 2 industry profit is $(1-\alpha)\, v$. Aggregate consumer surplus is therefore $2\alpha v$. (ii) Consider two multiproduct firms. All consumers buy two products, so total welfare is $2v$. Using Lemma 3 industry profit is $2\,(1-\alpha)\, v$. Aggregate consumer surplus is therefore $2\alpha v$. (iii) Consider the asymmetric market structure and recall Lemma 4. If $\alpha \geq \frac{1}{2}$ industry profit is $2(1-\alpha^2)v$, total welfare is $2v$, and so aggregate consumer surplus is $2\alpha^2 v$. If $\alpha < \frac{1}{2}$ industry profit is $\frac{3}{2}v$, total welfare is $(\frac{3}{2}+\alpha)v$, and so aggregate consumer surplus is αv.

Proof of Lemma 5. Notice that Lemma 1 still applies in this extension once non-shoppers' search behavior is given. As in the benchmark model, the multiproduct firm must attract a positive mass of non-shoppers, whilst each single-product firm must attract the same number of non-shoppers. Here it is more convenient to use the price density function. Let $f_A(p)$ denote the density of the multiproduct firm's price distribution and $\lambda \geq 0$ its mass point. Let $f_B(p)$ denote the density of a single-product firm's price distribution. A non-shopper's expected payoff from searching the multiproduct firm is

$$2\int_{\underline{p}}^{v} (v-p)\, f_A(p)\, dp = 2\,(1-\lambda)\int_{\underline{p}}^{v} (v-p)\, f_B(p)\, dp \ , \tag{20}$$

where the equality follows from equation (7). A non-shopper's expected payoff from visiting both single-product firms is

$$2\int_{\underline{p}}^{v} (v-p)\, f_B(p)\, dp - s \ , \tag{21}$$

whilst the expected payoff from visiting just one single-product firm is

$$\int_{\underline{p}}^{v} (v-p) f_B(p) \, dp \; . \tag{22}$$

Step 1. Look for an equilibrium where all non-shoppers search the multiproduct firm i.e. $N_A = 1-\alpha$ and $N_B = 0$. Lemma 1(iii) then implies that $\lambda = 1-\alpha$, $f_B(p) = \frac{1-\alpha}{\alpha}\frac{v}{p^2}$ and $\underline{p} = (1-\alpha)v$. Firstly this equilibrium requires that (20) exceeds (22), which holds if and only if $\alpha \geq \frac{1}{2}$. Secondly this equilibrium also requires that (20) exceeds (21), which holds if and only if $s \geq \bar{s}(\alpha)$ where[27]

$$\bar{s}(\alpha) = 2\lambda \int_{\underline{p}}^{v} (v-p) f_B(p) \, dp = 2(1-\alpha)v\left[1 + \left(\frac{1-\alpha}{\alpha}\right)\ln(1-\alpha)\right] \; .$$

Step 2. Look for an equilibrium where $X \in (0,1)$ non-shoppers search the multiproduct firm, and the other $1-X$ search both single-product firms i.e. $N_A = (1-\alpha)X$ and $N_B = (1-\alpha)(1-X)$. Lemma 1(iii) then implies that $\lambda = \frac{(1-\alpha)(2X-1)}{(1-\alpha)X+\alpha}$, $f_B(p) = \frac{(1-\alpha)X}{\alpha}\frac{v}{p^2}$ and $\underline{p} = \frac{(1-\alpha)X}{(1-\alpha)X+\alpha}v$. Firstly this equilibrium requires that (20) equals (21), which holds if and only if X satisfies equation (10) from earlier. It is straightforward to show that the right-hand side of (10) equals 0 when evaluated at $X = \frac{1}{2}$, is strictly increasing in $X \in (0,1)$, and equals $\bar{s}(\alpha)$ when evaluated at $X = 1$. Therefore a necessary condition for the equilibrium is that $s < \bar{s}(\alpha)$. (If we let $X(s,\alpha)$ denote the solution to (10), we also have that $X(s,\alpha) > \frac{1}{2}$ and $\frac{d}{ds}X(s,\alpha) > 0$ for all $s \in (0, \bar{s}(\alpha))$). Secondly this equilibrium also requires that (20) exceeds (22), which holds if and only if

$$X(s,\alpha) \leq \frac{2-\alpha}{3(1-\alpha)} \; . \tag{23}$$

Thirdly then, if $\alpha \geq \frac{1}{2}$ an equilibrium exists if and only if $s < \bar{s}(\alpha)$ (because (23) is automatically satisfied). Fourthly consider $\alpha < \frac{1}{2}$, let $\tilde{s}(\alpha)$ denote the right-hand side of (10) when evaluated at $X = \frac{2-\alpha}{3(1-\alpha)} \in \left(\frac{1}{2}, 1\right)$, and note that $\tilde{s}(\alpha) < \bar{s}(\alpha)$. Hence for $\alpha < \frac{1}{2}$ an equilibrium exists if and only if $s < \tilde{s}(\alpha)$.

Step 3. Look for an equilibrium where $X > 0$ non-shoppers search the multiproduct firm, $Y > 0$ search both single-product firms, and $1-X-Y > 0$ search one single-product firm i.e. $N_A = (1-\alpha)X$ and $N_B = \frac{(1-\alpha)(1-X+Y)}{2}$. Lemma 1(iii) then implies that $\lambda = \frac{(1-\alpha)(3X-Y-1)}{2[(1-\alpha)X+\alpha]}$, $f_B(p) = \frac{(1-\alpha)X}{\alpha}\frac{v}{p^2}$ and $\underline{p} = \frac{(1-\alpha)X}{(1-\alpha)X+\alpha}v$. Firstly this equilibrium

[27] As mentioned in the text, in the case with four single-product firms we assume s is small enough that all consumers buy both products. This requires that $s \leq \int_{\underline{p}}^{v}(v-p)\,dF(p) = v\left[1 + \left(\frac{1-\alpha}{2\alpha}\right)\ln\left(\frac{1-\alpha}{1+\alpha}\right)\right]$, where $F(p)$ is the price distribution given in Lemma 1(ii) with $N = \frac{1-\alpha}{2}$. After some algebra this threshold can be shown to exceed $\bar{s}(\alpha)$.

requires that (20) equals (22), which holds if and only if equation (11) from earlier holds. Secondly since $Y > 0$, equation (11) implies that $X > \frac{1}{2(1-\alpha)}$, however this is only possible if $\alpha < \frac{1}{2}$. Hence $\alpha < \frac{1}{2}$ is a necessary condition for this equilibrium. Thirdly since $Y < 1 - X$, equation (11) implies that $X < \frac{2-\alpha}{3(1-\alpha)}$. Fourthly, the equilibrium also requires that (21) equals (22), and this holds if and only if equation (12) from earlier holds. It is straightforward to show that the right-hand side of (12) is strictly decreasing in $X \in (0,1)$, and (for $\alpha < \frac{1}{2}$) equals $\tilde{s}(\alpha)$ when evaluated at $X = \frac{2-\alpha}{3(1-\alpha)}$. Let $\dot{s}(\alpha)$ denote the right-hand side of (12) when evaluated at $X = \frac{1}{2(1-\alpha)}$. Still assuming that $\alpha < \frac{1}{2}$, we note that $\frac{2-\alpha}{3(1-\alpha)} > \frac{1}{2(1-\alpha)}$ which in turn implies that $\tilde{s}(\alpha) < \dot{s}(\alpha)$, and after some algebra we can also show that $\dot{s}(\alpha) < \bar{s}(\alpha)$. Given the previous steps, it is then immediate that the equilibrium exists if and only if $\alpha < \frac{1}{2}$ and $s \in (\tilde{s}(\alpha), \dot{s}(\alpha))$. (Clearly in addition we have that $X(s,\alpha)$ is strictly decreasing in $s \in (\tilde{s}(\alpha), \dot{s}(\alpha))$.)

Step 4. Look for an equilibrium where $X > 0$ non-shoppers search the multiproduct firm, and the other $1-X > 0$ search one single-product firm. From the benchmark model we know this is only possible if $\alpha < \frac{1}{2}$, in which case $X = \frac{1}{2(1-\alpha)}$. Since $N_A = \frac{1}{2}$ and $N_B = \frac{1}{4}$ Lemma 1(iii) implies that $\lambda = \frac{1}{2(1+2\alpha)}$, $f_B(p) = \frac{1}{2\alpha}\frac{v}{p^2}$ and $\underline{p} = \frac{1}{1+2\alpha}v$. Using this we can show that (20) strictly dominates (22), whilst (21) strictly dominates (22) if and only if $s > \dot{s}(\alpha)$. ■

Proof of Proposition 2. Firstly we derive profits in the asymmetric market structure. As is standard the multiproduct firm A earns a per-product profit of $\pi_A = N_A v$ whilst a single-product firm B earns $\pi_B = (N_B + \alpha\lambda)v$. Recall that $X(s,\alpha)$ and $Y(s,\alpha)$ are the fractions of non-shoppers that visit respectively the multiproduct firm and two single-product firms. Hence we have that $N_A = (1-\alpha)X(s,\alpha)$ and $N_B = (1-\alpha)\frac{1-X(s,\alpha)+Y(s,\alpha)}{2}$. Moreover from the proof of Lemma 5 we know that $X(s,\alpha) > \frac{1}{2}$, such that $N_A > N_B$. This in turn implies that by Lemma 1(iii) we have

$$\lambda = (1-\alpha)\frac{3X(s,\alpha) - Y(s,\alpha) - 1}{2[(1-\alpha)X(s,\alpha) + \alpha]} .$$

Hence we can write

$$\pi_A = (1-\alpha)X(s,\alpha)v > (1-\alpha)\frac{v}{2} , \quad (24)$$

$$\pi_B = (1-\alpha)\frac{v}{2}\left[1 - X(s,\alpha) + Y(s,\alpha) + \alpha\frac{3X(s,\alpha) - Y(s,\alpha) - 1}{(1-\alpha)X(s,\alpha) + \alpha}\right] . \quad (25)$$

Secondly there is no equilibrium with four single-product firms. As argued in the main text, with four single-product firms each product earns profit $(1-\alpha)\frac{v}{2}$. Therefore if one pair deviate and merge, by equation (24) their per-product profit strictly increases.

Hence at least one pair merge. Thirdly consider the incentive of the second pair to merge. As argued in the main text, after a second merger each product earns profit $(1-\alpha)\frac{v}{2}$. Hence the second pair does not merge (such that the market structure is asymmetric) if and only if (25) exceeds $(1-\alpha)\frac{v}{2}$. We now consider several different cases.

Case 1: $\alpha \geq \frac{1}{2}$. From Lemma 5 we have that $Y(s,\alpha) = 1 - X(s,\alpha)$. Substituting this into (25), we find that $\pi_B \geq (1-\alpha)\frac{v}{2}$.

Case 2: $\alpha < \frac{1}{2}$ and $s > \dot{s}(\alpha)$. From Lemma 5 we have that $X(s,\alpha) = \frac{1}{2(1-\alpha)}$ and $Y(s,\alpha) = 0$. Substituting this into (25), we find that $\pi_B < (1-\alpha)\frac{v}{2}$.

Case 3: $\alpha < \frac{1}{2}$ and $s \in (\tilde{s}(\alpha), \dot{s}(\alpha))$. From Lemma 5 we have that $Y(s,\alpha) = 2X(s,\alpha) - \frac{1}{1-\alpha}$. Substituting this into (25), we find that $\pi_B < (1-\alpha)\frac{v}{2}$ provided that $X(s,\alpha) < 1$, which is clearly true because from the proof of Lemma 5 we have $X(s,\alpha) < \frac{2-\alpha}{3(1-\alpha)} < 1$.

Case 4: $\alpha < \frac{1}{2}$ and $s < \tilde{s}(\alpha)$. From Lemma 5 we have that $Y(s,\alpha) = 1 - X(s,\alpha)$. Substituting this into (25), we find that $\pi_B \geq (1-\alpha)\frac{v}{2}$ if and only if

$$X(s,\alpha) \leq \frac{\alpha}{1-\alpha}\ . \tag{26}$$

Clearly (26) fails for $\alpha \leq \frac{1}{3}$ because we know from Lemma 5 that $X(s,\alpha) > \frac{1}{2}$. We also claim that for $\alpha \in (\frac{1}{3}, \frac{1}{2})$, there exists a threshold $s^*(\alpha) > 0$ such that (26) holds if and only if $s \leq s^*(\alpha)$. To see this, recall from Lemma 5 that $X(0,\alpha) < \frac{\alpha}{1-\alpha}$, $\frac{d}{ds}X(s,\alpha) > 0$ and $X(\tilde{s}(\alpha), \alpha) = \frac{2-\alpha}{3(1-\alpha)} > \frac{\alpha}{1-\alpha}$. The threshold $s^*(\alpha)$ is calculated by substituting $X = \frac{\alpha}{1-\alpha}$ into equation (10), whereupon we have that $s^*(\alpha) = \frac{v(3\alpha-1)}{\alpha}(1-\ln 2)$. Notice that $\frac{d}{d\alpha}s^*(\alpha) > 0$ with $\lim_{\alpha\to\frac{1}{2}} s^*(\alpha) = v(1-\ln 2)$.

The above results then imply the following. (i) Consider $s < v(1-\ln 2)$. Cases 2-4 imply that there exists a threshold $\alpha^*(s) = \frac{1-\ln 2}{3(1-\ln 2)-\frac{s}{v}} \in (\frac{1}{3}, \frac{1}{2})$ such that $\pi_B < (1-\alpha)\frac{v}{2}$ if $\alpha < \alpha^*(s)$, and $\pi_B > (1-\alpha)\frac{v}{2}$ if $\alpha^*(s) < \alpha < \frac{1}{2}$. Case 1 further implies that $\pi_B > (1-\alpha)\frac{v}{2}$ when $\alpha \geq \frac{1}{2}$. (ii) Now consider $s \geq v(1-\ln 2)$. When $\alpha < \frac{1}{2}$ we have $s > s^*(\alpha)$, hence whichever of Cases 2-4 s falls into, they all imply that $\pi_B < (1-\alpha)\frac{v}{2}$. When $\alpha \geq \frac{1}{2}$ Case 1 implies that $\pi_B \geq (1-\alpha)\frac{v}{2}$. Hence, the critical threshold is

$$\alpha^*(s) = \begin{cases} \frac{1-\ln 2}{3(1-\ln 2)-\frac{s}{v}} & \text{if } s \in (0, v(1-\ln 2)) \\ \frac{1}{2} & \text{if } s \geq v(1-\ln 2) \end{cases}\ .$$

■

Proof of Proposition 4.

The proof consists of two parts. (i) We start by deriving the pricing equilibrium after the first deviation i.e. when both A and B are multiproduct firms but C sells only product 2. Let F_1 be the price distribution that A and B use for product 1. Let F_2 be the price distribution used by A and B for product 2, and $\hat{F}_2$ be the price distribution used by C for product 2. We first look for an equilibrium where all non-shoppers randomly visit one of the two multiproduct firms. Since each multiproduct firm then has half of the non-shoppers, F_1 satisfies

$$p\left[\frac{1-\alpha}{2}+\alpha(1-F_1(p))\right]=\frac{1-\alpha}{2}v\ . \tag{27}$$

Turning to product 2, since the single-product firm C does not have any non-shoppers, its multiproduct competitors' price distribution has a mass point at the monopoly price v. Following the standard logic, F_2 and $\hat{F}_2$ are determined by the following system of equations:

$$\begin{aligned} p\left[\frac{1-\alpha}{2}+\alpha(1-F_2(p))(1-\hat{F}_2(p))\right] &= \frac{1-\alpha}{2}v \\ p\alpha(1-F_2(p))^2 &= \alpha\lambda^2 v\ , \end{aligned} \tag{28}$$

where λ is the size of F_2's mass point. It is straightforward to solve for F_2, $\hat{F}_2$ and λ. One can also check that all three price distributions have a common lower bound. By comparing (27) and (28), it is easy to see that $\hat{F}_2(p) < F_1(p)$. That is, firm C's product 2 is more expensive than the multiproduct firms' product 1. Therefore all non-shoppers prefer to visit a multiproduct firm, consistent with our initial conjecture. (Using a similar logic we can also show that there is no equilibrium where some non-shoppers visit the single-product firm C.)

(ii) We now derive conditions under which none of the three deviations is profitable. (a) The third deviation is not profitable if and only if $\alpha \geq \frac{1}{4}$. (b) Suppose $\alpha \geq \frac{1}{2}$. The first deviation is not profitable if $\alpha(1-\alpha)v \geq (1-\alpha)v - \Delta$, and the second deviation is not profitable if $2(1-\alpha)v - \Delta \geq \frac{1}{2}(1-\alpha)v$. The two conditions simplify to

$$\underline{\Delta}(\alpha) \equiv (1-\alpha)^2 v \leq \Delta \leq \frac{3}{2}(1-\alpha)v \equiv \overline{\Delta}(\alpha)\ .$$

(c) Suppose $\alpha \in \left[\frac{1}{4}, \frac{1}{2}\right)$. The first deviation is not profitable if $\frac{1}{4}v \geq (1-\alpha)v - \Delta$, and the second deviation is not profitable if $v - \Delta \geq \frac{1}{2}(1-\alpha)v$. The two conditions simplify to

$$\underline{\Delta}(\alpha) \equiv \left(\frac{3}{4}-\alpha\right)v \leq \Delta \leq \frac{1+\alpha}{2}v \equiv \overline{\Delta}(\alpha)\ .$$

■

Further details on the model with product differentiation.

The case with four single-product firms. To derive the equilibrium price for product i, suppose firm i_A unilaterally deviates and charges a price p_0'. (i) Half the consumers visit

i_A first. Those for whom $u_A - p_0' \geq a - p_0$ stop searching and buy immediately, which generates demand of $\frac{1}{2}[1 - G(a - p_0 + p_0')]$. Those for whom $u_A - p_0' < a - p_0$ search firm i_B, but then return and buy from firm i_A if $u_A - p_0' > u_B - p_0$. This generates demand

$$\tfrac{1}{2} \Pr[u_B - p_0 < u_A - p_0' < a - p_0] = \tfrac{1}{2} \int_{\underline{u}}^{a - p_0 + p_0'} G(u_A - p_0' + p_0) dG(u_A) \ .$$

(ii) The other half of consumers visit firm i_B first. Since they hold an equilibrium belief about firm i_A's price, they visit firm i_A if $u_B < a$. They then buy from firm i_A if $u_A - p_0' > u_B - p_0$. This generates demand

$$\tfrac{1}{2} \int_{\underline{u}}^{a} [1 - G(u_B - p_0 + p_0')] dG(u_B) \ .$$

Adding these three demand components together, one can check that in symmetric equilibrium each firm sells to half the consumers, and that the slope of demand is $-\frac{1}{2} g(a)[1 - G(a)] - \int_{\underline{u}}^{a} g(u)^2 du$. Therefore the first-order condition for p_0 is given by equation (15).

The asymmetric case. Consider the market for product i. Demand for the multiproduct firm's product, if it charges p_A' while its single-product rival i_B sets the equilibrium price p_B, is

$$[1 - G(a - p_B + p_A')] + \int_{\underline{u}}^{a - p_B + p_A'} G(u - p_A' + p_B) dG(u) \ . \qquad (29)$$

This is explained as follows. All consumers visit firm A first. The first term is consumers who find $u_A - p_A' \geq a - p_B$ and so buy immediately. The second term is consumers who find $u_A - p_A' < a - p_B$ and so search firm i_B, but who subsequently return to buy from firm A because $u_B - p_B < u_A - p_A'$. Demand for firm i_B's product, if it charges price p_B' while firm A sets its equilibrium price p_A, is

$$\int_{\underline{u}}^{a - p_B + p_A} [1 - G(u - p_A + p_B')] dG(u) \ . \qquad (30)$$

This is because all consumers visit firm A first, and hold an equilibrium belief about firm i_B's price. Therefore they search firm i_B if $u_A - p_A < a - p_B$, and then buy from it if $u_B - p_B' > u_A - p_A$.

Now define $\Delta \equiv p_B - p_A$ and

$$Q(\Delta) \equiv 1 - \int_{\underline{u}}^{a - \Delta} [1 - G(u + \Delta)] dG(u) \ .$$

Here $Q(\Delta)$ is the equilibrium demand for firm A (i.e. (29) evaluated at $p_A' = p_A$), and $1 - Q(\Delta)$ is the equilibrium demand for firm i_B (i.e. (30) evaluated at $p_B' = p_B$). Due to the assumption of full market coverage, they only depend on the price difference Δ. It is then straightforward to derive the first-order conditions stated in the main text.

Proof of Lemma 6. Using (18) we get the following equation:

$$\Delta = \frac{1 - Q(\Delta)}{Q'(\Delta) - g(a - \Delta)[1 - G(a)]} - \frac{Q(\Delta)}{Q'(\Delta)} \equiv \Phi(\Delta) \ .$$

Using

$$Q'(\Delta) = [1 - G(a)]g(a - \Delta) + \int_{\underline{u}}^{a-\Delta} g(u + \Delta)dG(u) \ ,$$

we can show that $\Phi(0) > 0$ if and only if

$$\frac{1}{G(a) - \frac{1}{2}G(a)^2} - \frac{g(a)[1 - G(a)]}{\int_{\underline{u}}^{a} g(u)dG(u)} < 2 \ .$$

This inequality holds because

$$\int_{\underline{u}}^{a} g(u)dG(u) = \int_{\underline{u}}^{a} \frac{g(u)}{1 - G(u)}[1 - G(u)]dG(u) < \frac{g(a)}{1 - G(a)}[G(a) - \frac{1}{2}G(a)^2] \ ,$$

where the inequality uses the fact that logconcavity of $1 - G$ implies $\frac{g(u)}{1-G(u)}$ increases in u.

Using L'Hôpital's rule we also have that

$$\Phi(a - \underline{u}) = \frac{1 - G(a)}{g(a)} - \frac{1}{g(\underline{u})[1 - G(a)]} < \frac{1}{g(\underline{u})}[1 - \frac{1}{1 - G(a)}] < 0 < a - \underline{u} \ ,$$

where the first inequality again uses logconcavity of $1 - F$. Therefore by continuity $\Phi(\Delta) = \Delta$ has a solution between 0 and $a - \underline{u}$. ■

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Online Appendix: Proof of Proposition 3

The number of non-shoppers who need both products is $N_b = (1-\alpha)\gamma^2$, and the number of non-shoppers who need only product i, $i = 1, 2$, is $N_i = (1-\alpha)\gamma(1-\gamma)$. The number of shoppers for each product is $S = \alpha\gamma$. Henceforth we call these two types of non-shoppers N_b and N_i respectively. Denote by $\pi_m(k)$, $k \geq 1$, a multiproduct firm's per product profit when there are k multiproduct firms in the market, and by $\pi_s(k)$, $k \leq n-1$, a single-product firm's profit when there are k multiproduct firms in the market.

We first consider the simple case where no firms merge or all firms merge.

Claim 1 *With all single-product firms ($k = 0$) or all multiproduct firms ($k = n$), non-shoppers search randomly and each firm's (per product) profit is respectively*

$$\pi_s(0) = \frac{1-\alpha}{2n}\gamma\,(2-\gamma)\,v \quad \textit{and} \quad \pi_m(n) = \frac{1-\alpha}{n}\gamma v \ .$$

Proof. With all single-product firms, multiproduct non-shoppers visit a firm randomly and single-product non-shoppers visit a relevant firm randomly,[28] so the number of non-shoppers each firm has is

$$\frac{N_b}{2n} + \frac{N_i}{n} = \frac{1-\alpha}{2n}\gamma\,(2-\gamma) \ .$$

This determines each firm's profit $\pi_s(0)$.

With all multi-product firms, all non-shoppers visit a firm randomly, and so the number of non-shoppers each firm has at the product level is

$$\frac{N_b}{n} + \frac{N_i}{n} = \frac{1-\alpha}{n}\gamma \ .$$

This determines each firm's per product profit $\pi_m(n)$. ■

We now turn to an asymmetric market structure with $1 \leq k \leq n-2$ such that at least two pairs of single-product firms remain. (The asymmetric case with $k = n-1$ will be treated separately.) We need the following two results from Baye et al. (1992).

Claim 2 (Asymmetric Varian Model) *Consider a Varian pricing game where n firms supply a homogenous product and consumers have identical valuations v. Suppose there*

[28] In equilibrium non-shoppers must search randomly. If they did not, one firm would have more non-shoppers than another, such that it would charge higher prices and thereby contradict non-shoppers' search behavior. See Section V of Baye et al. (1992) for a formal proof.

are $S > 0$ shoppers in the market. Suppose each firm $j \in \{1, \cdots, l\}$ has $N_A > 0$ non-shoppers and each firm $j \in \{l+1, \cdots, n\}$ has $N_B < N_A$ non-shoppers. There is an equilibrium where the first l firms use a price distribution F_A and the remaining $n-l$ firms use a price distribution F_B, where F_A FOSD F_B, and if $l \leq n-2$ (i.e. if the second group has at least two firms), F_A degenerates at the monopoly price v.

Proof. See Appendix B of Baye et al. (1992).[29] ■

Claim 3 (Asymmetric Equilibrium in Symmetric Varian Model) *In the Varian pricing game described in Claim 2 with $n \geq 3$, if all firms equally share the non-shoppers (i.e. if $N_A = N_B$), then as well as the standard symmetric equilibrium, there exist asymmetric equilibria where a group of firms $j \in \{1, \cdots, l\}$, $l \leq n-2$, adopt a price distribution F_A with support $[\underline{p}, r] \cup \{v\}$ where $r < v$, and the rest of the firms $j \in \{l+1, \cdots, n\}$ adopt an atomless price distribution F_B with support $[\underline{p}, v]$. Moreover, F_A FOSD F_B, and $F_A = F_B$ for $p \in [\underline{p}, r]$.*

Proof. See Theorem 1 in Baye et al. (1992).[30] ■

The following result reports the equilibrium outcome in an asymmetric market with $1 \leq k \leq n-2$.

Claim 4 *Suppose $1 \leq k \leq n-2$. (i) If $\gamma < \frac{k}{n}$, N_b visit multiproduct firms and N_i randomize, and (ii) if $\gamma > \frac{k}{n}$, N_b randomize and N_i visit single-product firms. All firms have the same per product profit:*

$$\pi_m(k) = \pi_s(k) = \begin{cases} \frac{1-\alpha}{n}\gamma v & \text{if } \gamma < \frac{k}{n} \\ \frac{1-\alpha}{2n-k}\gamma(2-\gamma)v & \text{if } \gamma > \frac{k}{n} \end{cases} .$$

Proof. We first exclude the possibility that all non-shoppers (i.e. both N_b and N_i) search in a deterministic way or in a random way. The proof consists of four steps. (i) It is impossible that all non-shoppers visit multiproduct firms. If that were the case, the single-product firms would sell only to shoppers and so set price equal to zero. The multiproduct firms would sell only to non-shoppers and so charge v. This would contradict the optimality of non-shoppers' search behavior.

[29] When $l \leq n-3$ (i.e., when the second group has at least three firms), there are also equilibria where the firms in the second group use different price distributions. In our subsequent analysis, we focus on the equilibrium stated in the claim.

[30] When $l \geq 2$, there also exist asymmetric equilibria where the firms in the first group use different distributions. In our subsequent analysis, we focus on the equilibrium stated in the claim.

(ii) It is also impossible that all non-shoppers visit single-product firms. If that were the case, single-product firms would charge higher prices than multiproduct firms in the sense of FOSD. (If $k \geq 2$, single-product firms would actually charge the monopoly price.) But this again would render non-shoppers' search behavior non-optimal.

(iii) It is also impossible that all N_b visit multiproduct firms and all N_i visit single-product firms supplying product i (except in the edge case $\frac{N_b}{k} = \frac{N_i}{n-k}$ which we ignore). If that were the case, each multiproduct firm would have $\frac{N_b}{k}$ non-shoppers per product, and each single-product firm would have $\frac{N_i}{n-k}$ non-shoppers. If $\frac{N_b}{k} < \frac{N_i}{n-k}$, then according to Claim 2 single-product firms would charge higher prices than multiproduct firms. But then N_i's search behavior could not be justified. If $\frac{N_b}{k} > \frac{N_i}{n-k}$, then according to Claim 2 multiproduct firms would charge the monopoly price v since $k \leq n-2$. But then N_b's search behavior could not be justified.

(iv) It is also impossible that both N_b and N_i randomize their search behavior. N_i would randomize only if multiproduct firms and single-product firms supplying product i provide the same expected consumer surplus from purchasing their product i. But then N_b would favor visiting a multiproduct firm.

As a result, either N_b or N_i randomize in equilibrium. First, consider an equilibrium where N_i randomize and N_b visit multiproduct firms. Let X be the probability that N_i visit a multiproduct firm. Then a single-product firm has $\frac{(1-X)N_i}{n-k}$ non-shoppers, and a multiproduct firm has $\frac{XN_i+N_b}{k}$ non-shoppers per product. They must be equal to each other, otherwise using Claim 2 one type of firm charges higher prices than the other, and then N_i's search behavior could not be justified. Therefore,

$$\frac{(1-X)N_i}{n-k} = \frac{XN_i + N_b}{k} \Leftrightarrow X = \frac{k/n - \gamma}{1-\gamma} \; .$$

which is only positive if $\gamma < \frac{k}{n}$. In this case, one can verify that the number of non-shoppers each firm has at the product level is $\frac{1-\alpha}{n}\gamma$. This implies the profit outcome.

Second, consider an equilibrium where N_b randomize and N_i visit single-product firms. Let X be the probability that N_b visit a multiproduct firm. Then a multiproduct firm has $\frac{XN_b}{k}$ non-shoppers per product, and a single-product firm has $\frac{(1-X)N_b}{2(n-k)} + \frac{N_i}{n-k}$ non-shoppers. (Notice that those N_b who visit single-product firms will randomly choose one among $2(n-k)$ of them.) Again, they must be equal to each other, otherwise using Claim 2 either multiproduct firms would charge the monopoly price (which would contradict N_b's search behavior), or single-product firms would charge higher prices (which would contradict N_i's search behavior). Therefore,

$$\frac{XN_b}{k} = \frac{(1-X)N_b}{2(n-k)} + \frac{N_i}{n-k} \Leftrightarrow X = \frac{2-\gamma}{2n\gamma/k - \gamma} \; .$$

which is only less than 1 if $\gamma > \frac{k}{n}$. In this case, one can verify that the number of non-shoppers each firm has at the product level is $\frac{1-\alpha}{2n-k}\gamma(2-\gamma)$. This implies the profit outcome.[31] ■

Finally, we study the case with $k = n-1$ such that only one pair of single-product firms remain in the market. By a similar logic as in the proof of Claim 4, one can show that in this case it is impossible that all non-shoppers (i.e. both N_b and N_i) visit multiproduct firms or single-product firms, and it is also impossible that all non-shoppers search in a random way. The only difference compared to the case with $k \le n-2$ is that now it is possible that N_b visit multiproduct firms and N_i visit single-product firms supplying product i. This is because given there is only one pair of single-product firms, with this configuration of non-shoppers' search behavior multiproduct firms will no longer charge the monopoly price, and so N_b's search behavior can potentially be justified. Therefore we now have three possible types of equilibrium to consider.

Before we proceed, it is useful to first study an asymmetric Varian model where each of the first $n-1$ firms has N_A non-shoppers and the last firm has $N_B < N_A$ non-shoppers. (This is the case with $l = n-1$ in Claim 2.) Let F_A and F_B be the price distributions used by the two types of firms respectively. F_A has a mass point at v, and let λ be its size. Then the two indifference conditions are:

$$p\left[N_A + S(1 - F_A(p))^{n-2}(1 - F_B(p))\right] = vN_A \ , \tag{31}$$

and

$$p\left[N_B + S(1 - F_A(p))^{n-1}\right] = v(N_B + S\lambda^{n-1}) \ . \tag{32}$$

From the first condition, we can derive the common lower bound of the two price distributions:

$$\underline{p} = \frac{N_A}{S + N_A} v \ . \tag{33}$$

Substituting this into the second condition yields

$$\lambda^{n-1} = \frac{N_A - N_B}{S + N_A} \ . \tag{34}$$

Then the profit outcome is

$$\pi_A = vN_A \quad \text{and} \quad \pi_B = vN_A \frac{S + N_B}{S + N_A} \ . \tag{35}$$

[31] However note that the pricing equilibrium in this case cannot be symmetric. To justify N_b's search behavior, we need an asymmetric pricing equilibrium where multiproduct firms charge higher prices than single-product firms. Given $k \le n-2$, this is possible according to Claim 3. In this equilibrium each firm's per product profit is $\frac{1-\alpha}{2n-k}\gamma(2-\gamma)v$. Details of the equilibrium characterization are available upon request.

The following result reports the market outcome when $k = n - 1$.

Claim 5 *(i) When $k = n-1$ and $\gamma \le 1 - \frac{1}{n}$, N_b visit multiproduct firms and N_i randomize, and each firm has the same per product profit*

$$\pi_m(n-1) = \pi_s(n-1) = \frac{1-\alpha}{n}\gamma v \ .$$

(ii) When $k = n - 1$ and $\gamma > 1 - \frac{1}{n}$, either N_b visit multiproduct firms and N_i visit single-product firms, in which case the profit outcome is

$$\pi_m(n-1) = \frac{1-\alpha}{n-1}\gamma^2 v \quad \text{and} \quad \pi_s(n-1) = \frac{\gamma^2[1-(1-\alpha)\gamma]}{\gamma + (n-1)\frac{\alpha}{1-\alpha}} v \ , \tag{36}$$

or N_b randomize and N_i visit single-product firms, in which case the profit outcome is

$$\pi_m(n-1) = X\frac{1-\alpha}{n-1}\gamma^2 v \quad \text{and} \quad \pi_s(n-1) = \frac{X\gamma^2\left[1-\frac{1+X}{2}(1-\alpha)\gamma\right]}{X\gamma + (n-1)\frac{\alpha}{1-\alpha}} v \ , \tag{37}$$

where $X \in (\frac{n-1}{n+1}\frac{2-\gamma}{\gamma}, 1)$ is the probability that N_b visit a multiproduct firm.

Proof. (i) For N_b to visit multiproduct firms and N_i to randomize, all firms must have the same number of non-shoppers at the product level, otherwise N_i's search behavior could not be justified. This can happen only if $\frac{N_b}{n-1} \le N_i$, or $\gamma \le 1 - \frac{1}{n}$. Let X be the probability that N_i visit a multiproduct firm. Then we need

$$\frac{N_b + XN_i}{n-1} = (1-X)N_i \ .$$

From this one can solve $X \in (0,1)$ and verify that each firm has $\frac{1}{n}(1-\alpha)\gamma$ non-shoppers per product. This implies the profit result.

(ii) First, consider an equilibrium where N_b visit multiproduct firms and N_i visit single-product firms. In this case, $\gamma > 1 - \frac{1}{n}$ implies that a multiproduct firm has more non-shoppers at the product level than a single-product firm (i.e. $\frac{N_b}{n-1} > N_i$). Let F_s be the price distribution used by a single-product firm, and let F_m be the price distribution used by a multiproduct firm. Then the above general analysis of the asymmetric Varian model applies with $N_A = \frac{N_b}{n-1}$, $N_B = N_i$, $F_A = F_m$, and $F_B = F_s$, since $N_A > N_B$. Then the profit outcome (36) is from (35). For this equilibrium to be sustained, we need

$$2\int_{\underline{p}}^{v} (v-p)\, dF_m(p) \ge \int_{\underline{p}}^{v} (v-p)\, dF_s(p) \ . \tag{38}$$

Second, consider an equilibrium where N_b randomize and N_i visit single-product firms. This equilibrium can happen only if multiproduct firms charge higher prices. This requires that each multiproduct firm has more non-shoppers per product.[32] Let X be the probability that a multiproduct non-shopper visits a multiproduct firm. Then the above general analysis of the asymmetric Varian model applies with $N_A = \frac{XN_b}{n-1}$, $N_B = N_i + \frac{1}{2}(1-X)N_b$, $F_A = F_m$, and $F_B = F_s$, if

$$N_A > N_B \Leftrightarrow X > \frac{n-1}{n+1}\frac{2-\gamma}{\gamma} \ . \tag{39}$$

Then the profit outcome (37) is from (35). For N_b to randomize, we need

$$2\int_{\underline{p}}^{v} (v-p)\, dF_m(p) = \int_{\underline{p}}^{v} (v-p)\, dF_s(p) \ . \tag{40}$$

This determines X. ∎

When $n = 2$, some calculations reveal that (38) holds if and only if $\alpha \geq \frac{3\gamma-2}{3\gamma-1}$, whilst for $\alpha < \frac{3\gamma-2}{3\gamma-1}$ (40) is satisfied with $X = \frac{1+(1-\alpha)(1-\gamma)}{2(1-\alpha)\gamma}$. It is then straightforward to check that $\pi_s(n-1) \geq \pi_m(n)$ (i.e. the last pair of single-product firms will not choose to merge) if and only if $\alpha \geq \frac{\gamma}{1+\gamma}$. This proves result (i) in Proposition 3.

Now consider $n \geq 3$ and prove result (ii) in Proposition 3. Firstly consider $\gamma \leq 1 - \frac{1}{n}$. In this case we have characterized profits under every possible market structure, and so using Claims 1, 4, and 5, one can readily verify that the market has $\lceil n\gamma \rceil$ multiproduct firms.

Secondly consider $\gamma > 1 - \frac{1}{n}$. Due to the complexity of the price distributions, in general it is difficult to derive conditions under which (38) or (40) hold (although one of them must hold). However as an initial step, we can prove that there will be at least $n-1$ multiproduct firms in the market. With $\gamma > 1 - \frac{1}{n}$, we must have $\gamma > \frac{k}{n}$ for $1 \leq k \leq n-2$. Then according to Claim 4, when $1 \leq k \leq n-2$, each firm's per product profit is $\pi_m(k) = \pi_s(k) = \frac{1-\alpha}{2n-k}\gamma(2-\gamma)v$. This is increasing in k. Hence it suffices to show that $\pi_s(n-2) < \pi_m(n-1)$. If the profit outcome (36) applies when $k = n-1$, this condition becomes

$$\frac{1-\alpha}{n+2}\gamma(2-\gamma) < \frac{1-\alpha}{n-1}\gamma^2 \Leftrightarrow \gamma > \frac{2(n-1)}{2n+1} \ ,$$

which is satisfied because $\gamma > 1 - \frac{1}{n}$. If the profit outcome (37) applies when $k = n-1$, the condition becomes

$$\frac{1-\alpha}{n+2}\gamma(2-\gamma) < X\frac{1-\alpha}{n-1}\gamma^2 \Leftrightarrow \frac{2-\gamma}{n+2} < X\frac{\gamma}{n-1} \ .$$

[32] Notice that given $k = n-1$ Claim 3 does not apply. So if each multiproduct firm has the same number of non-shoppers as each single-product firm, there is no asymmetric pricing equilibrium where multiproduct firms charge higher prices.

which is satisfied because from equation (39) we know that $X > \frac{n-1}{n+1}\frac{2-\gamma}{\gamma}$.

To determine whether the last pair of single-product firms will merge, we need to compare $\pi_s(n-1)$ with $\pi_m(n)$. If the profit outcome (36) applies, then one can check that $\pi_s(n-1) < \pi_m(n)$ if and only if $\alpha < \frac{\gamma}{1+\gamma}$. If the profit outcome (37) applies, we do not have a clear comparison between $\pi_s(n-1)$ and $\pi_m(n)$, because the X in $\pi_s(n-1)$ cannot be explicitly solved from (40). Therefore to make progress, we consider two extreme cases with $\alpha \approx 1$ or $\alpha \approx 0$.

When $\alpha \to 1$ (i.e. as the fraction of non-shoppers vanishes) the multiproduct firm's price distribution F_m degenerates around zero.[33] Therefore by continuity for α sufficiently close to 1 condition (38) must hold. (Notice that to make this argument, we do not need to say anything about the behavior of F_s.) Since $\alpha > \frac{\gamma}{1+\gamma}$ we know (from the previous paragraph) that $\pi_s(n-1) > \pi_m(n)$ i.e. the last pair of single-product firms will choose not to merge.

When $\alpha \to 0$ (i.e. as the fraction of shoppers vanishes) both firms' price distributions degenerate at the monopoly price,[34] and so by continuity have most of their mass around v when α is sufficiently close to zero. Hence it is not clear whether (38) holds or not. If the profit outcome (36) applies, we already know that $\pi_s(n-1) < \pi_m(n)$ for $\alpha < \frac{\gamma}{1+\gamma}$. If the profit outcome (37) applies, without solving X we can also show $\pi_s(n-1) < \pi_m(n)$. This is because $\lim_{\alpha\to 0}\pi_m(n) = \frac{1}{n}v$ and $\lim_{\alpha\to 0}\pi_s(n-1) = \left(1 - \frac{1+X}{2}\gamma\right)v$, and in addition

$$1 - \frac{1+X}{2}\gamma < \frac{2}{n+1}\left(1 - \frac{\gamma}{2}\right) < \frac{1}{n}$$

where the first inequality uses the fact that $X > \frac{n-1}{n+1}\frac{2-\gamma}{\gamma}$ (from 39), and the second inequality uses $\gamma > 1 - \frac{1}{n}$. Hence by continuity, for α sufficiently small we have $\pi_s(n-1) < \pi_m(n)$. Therefore regardless of whether (38) or (40) applies, the last pair of single-product firms will choose to merge.

[33] As $\alpha \to 1$, both N_A and N_B go to zero in (32) and (34), while S is strictly positive. Then the right-hand side of (32) goes to zero. To sustain (32), we must have $F_A(p) \to 1$ for any $p \in (0, v)$.

[34] As $\alpha \to 0$, S in (33) goes to zero, while for a given γ, N_A is always bounded away from zero. Then (33) implies that $\underline{p}$ converges to the monopoly price v.