Inflation expectations derived from a portfolio model

Covarrubias, Enrique and Hernández-del-Valle, Gerardo

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E. Covarrubias‡ and G. Hernández-del-Valle§

[1]Banco de México

Abstract

This paper proposes a new methodology for extracting inflation expectations from financial markets. For this purpose, a synthetic financial asset is built whose returns are matched with the inflation rate by construction. The methodology estimates the implicit return expected by the market on this asset through a portfolio valuation approach; in other words, implicit inflation expectations are obtained. This approach clarifies the mechanisms behind a negative risk premium: an inflation-linked bond is attractive to an investor when high inflation is expected or when generalized low returns are observed; in both cases, a yield below expected returns is observed.

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‡ Electronic address: `ecovarrubias@banxico.org.mx`; Corresponding author.

§ Electronic address: `gerardo.hernandez@banxico.org.mx`.

1 Introduction

The aim of this paper is to propose a new methodology to extract long-term inflation expectations implicit in financial markets. There are two sources of data that could allow a first approximation of inflation expectations. On the one hand, analyst surveys give a forecast of expected inflation. However, although this is an ongoing area of research, it has been documented even in policy forums that expectations derived from surveys show high stickiness.[1] On the other hand, the difference between the yield of nominal bonds and inflation-indexed bonds, known as *Breakeven Inflation* (BEI), could be considered. While inflation expectations implicit in financial instruments may better reflect immediate movements in inflation expectations of economic agents, Breakeven Inflation includes not only inflation expectations but also other premia including those arising from liquidity or inflation risk concerns (Evans, 1998, 2003; Christensen et al., 2004; Christensen, 2008; Christensen et al., 2010).

To better understand the intuition behind Breaven Inflation, consider an investor who owns an inflation-linked bond. If such investor were to replace this bond by a nominal bond of the same maturity, and be indifferent between them, he would have to be compensated for his expected average inflation for the maturity of the bond. In addition, he would have to receive a premium for inflation risk since expecting long-term inflation to be reached in a stable manner is not the same as expecting it with a path of high volatility. However, it is important to notice that the expected compensation for such volatility is not the same in periods of low or high volatility in financial markets, or in an environment of generalized low or high yields. Altogether, this suggests explaining Breakeven Inflation as an indirect utility function obtained through market instruments.

Theoretical and empirical literature about extracting inflation expectations implicit in market instruments is increasing. On the one hand, in studies that precede even the issuance of inflation-linked bonds (TIPS) in the United States, (Campbell and Shiller, 1996), financial theory was required to estimate the inflation risk premium. In the same vein, Foresi et al. (1997) use a principal components methodology to value bonds, inflation expectations

[1] See Crump et al. (2013) and Yellen (2015).

and the real interest rate. These two factors are called respectively “price of risk in the market" and “inflationary risk", whose nomenclature is replicated in this study. Another branch of literature has studied the term structure of inflation expectations through an affine Nelson-Siegel type model, including Adrian and Wu (2009), Christensen et al. (2011), and Chernov and Muller (2012). These methodologies have been applied to France (Alonso et al., 2001), Canada (Spiro, 2003), Chile (Jervis, 2007), Colombia (Melo Veladia and Moreno Gutiérrez, 2010), United Kingdom (Joyce et al., 2010), Australia (Devlin and Patwardhan, 2012), the Eurozone (Pericoli, 2012, 2014), as well as Mexico (Aguilar et al.,2016).

Through the literature, including this work, it has been recognized that it is difficult to extract expectations about the volatility (instead of the level) of the path that inflation is expected to follow. This contrasts with the relative ease of obtaining implied volatility for other asset classes, such as exchange rates, with a liquid market for options. Therefore, several authors seek to model implied inflation volatility in terms of variables for which an option market exists, including Kitzul and Wright (20013) and Azoulay et al. (2014). There also exist studies that extract expectations through the inflation swap market (Haubrick et al., 2012) or from rate caps (Li and Zhao, 2009)(see also Bauer and Christensen, 2014). However, these markets are not very prevalent in the world and in many countries nonexistent. Additionally, there is a role that inflation-linked bonds play because of their diversification benefits. This idea has been explored by Hunter and Simon (2005) and Bière and Signori (2009).

Finally, it is worth noticing that there is a growing literature, theoretical and empirical, which seeks to explicitly model inflationary risk premia. These results differ in size, term volatility and, in some cases, even in sign. As mentioned by Hördal and Tristani (2007) this variation may simply be due to variations in samples or countries. Indeed, Kandel et. al (1996) found that during high inflation episodes in the United States, the inflationary risk premium was about 34 basis points per month. In contrast, for low inflation periods it was only about 5 basis points per month. In the same line of thinking, Chen et al. (2005), also for the United States, used a Cox-Ingersoll-Ross model with two factors to find an inflationary risk premium between -1 and 132 basis points. For comparison, in this study we find that for Mexico there exist premia as low as -3 basis points in 2016 and as high as 49 basis points in 2013. It should be noted that Evans (1998) using both nominal and inflation-indexed bonds, from the United Kingdom, as well as data from expectation surveys, found a positive inflationary risk premium that is significant and varies positively with Breakeven Inflation. Later, in another study of the same author (2003), but using a regime change model, he found a significant but negative inflationary risk premium.

Our paper provides a methodology that clarifies the mechanisms behind a negative risk premium by noticing that an inflation-linked bond is attractive to an investor when high inflation is expected or when generalized low returns are observed in the market; in both cases, a yield below expected returns (equivalently, a negative risk premium) is observed.

The rest of this document is organized as follows. The next Section presents the methodology for decomposing Breakeven Inflation into two components: inflation expectations and an inflationary risk premium based on a Markowitz type portfolio model. In turn, Subsection 2.2.1 details the estimation of the market price of risk while Subsection 2.2.2 presents the estimation behind the inflation volatility inferred by the market. An error correction model is explained in Subsection 2.2.3. Then, in Section 2.3 we derive the inflation expectations with computations from the previous Section. Applications of this methodology can be found in Section 3. Subsection 3.1 presents all the estimations and results for Mexico. Finally, we conclude in Section 4 with some final remarks.

2 Methodology

2.1 Valuation of the Inflation Asset

With the goal of extracting average inflation expectations for the next T years from financial markets at time t, this document proposes a methodology that decomposes the difference in yield between “nominal" bonds (RN_t) and inflation-linked bonds (RP_t) into

$$\underbrace{RN_{t,T} - RP_{t,T}}_{\text{Breakeven Inflation}} = \underbrace{\mathbb{E}[\Delta P_T | \Delta P_t]}_{\text{Inflation expectations}} + \underbrace{IRP_t}_{\text{Inflation risk premium}} \tag{1}$$

where P_t represents price level, ΔP_t is the annual change in the price level, $\mathbb{E}[\Delta P_T|\Delta P_t]$ is the expectation of the annual change in the price level for the next T years given information at time t, and IRP_t is the inflation risk premium at t. To simplify notation, we also use the expression $\mathbb{E}[\Delta P_T|\Delta P_t] := E[\pi_{t,T}]$ to denote inflation expectations. In particular, through the remainder of the paper we will use the abbreviation

$$\Delta P_t \;=\; \log(P_t) - \log(P_{t-12}) \approx \frac{P_t - P_{t-12}}{P_{t-12}} =: \pi_t.$$

With the goal of studying an investor's decision to invest in nominal or real bonds, we assume that this investor can invest in a portfolio, say Y, that replicates payments of the Inflation Asset, containing a riskless asset A as well as a risky asset B, so that $\text{var}(A) = 0$ and $\text{var}(B) > 0$. Asset B will play the role of the market portfolio. Therefore, the value of the portfolio Y is given by the expression

$$Y = \alpha A + (1-\alpha)B$$

where $\alpha \in [0,1]$ is the share invested in asset A, and $(1-\alpha)$ represents the share invested in asset B. Note that if the perceived risk increases, then our investor will seek to rebalance his portfolio more heavily towards the riskless asset A. This is, in risky conditions, α increases. Notice also that the expected return $\mathbb{E}[Y]$ on this portfolio, as well as its variance $\text{var}[Y]$, are given by

$$\begin{aligned} \mathbb{E}[Y] &= \alpha A + (1-\alpha)\mathbb{E}[B] \\ \text{var}[Y] &= \alpha^2 \text{var}[B]. \end{aligned}$$

Now, the goal of the investor is to maximize his utility subject to a constant level of risk, $\text{var}[Y] = K$, which corresponds to the optimization problem

$$\begin{aligned} \max_{\alpha} \quad & E[Y] \\ \text{subject to} \quad & \text{var}[Y] = K \quad ; \quad K \text{ constant.} \end{aligned} \tag{2}$$

In this sense, the corresponding Lagrangian is given by

$$\mathcal{L}(\alpha, \lambda) = E[Y] - \lambda \text{var}[Y].$$

Since the solution to problem (2) is equal to setting the gradient of the previous Lagrangian to zero, we define the indirect utility function as

$$\begin{aligned} U(\alpha) &= \max_{\alpha}(\mathbb{E}[Y] - \lambda \mathrm{var}[Y]) \\ &= \max_{\alpha}(\alpha A + (1-\alpha)\mathbb{E}[B] - \lambda \alpha^2 \mathrm{var}[B]), \end{aligned} \tag{3}$$

where the solution to (3) is given by

$$\alpha^* = \frac{A - \mathbb{E}[B]}{2\lambda \mathrm{var}[B]} \iff \lambda^* = \frac{A - \mathbb{E}[B]}{2\alpha \mathrm{var}[B]}.$$

Thus, given the share $\alpha \in [0, 1]$, the indirect utility function equals

$$\begin{aligned} U(\lambda^*) &= \mathbb{E}[Y] - \lambda^* \mathrm{var}[Y] \\ &= \mathbb{E}[Y] - \frac{A - \mathbb{E}[B]}{2\alpha \mathrm{var}[B]} \cdot \mathrm{var}[Y] \\ &= \mathbb{E}[Y] + \frac{\mathbb{E}[B] - A}{2\alpha \mathrm{var}[B]} \cdot \mathrm{var}[Y]. \end{aligned} \tag{4}$$

While the liquidity of market instruments is not explicitly modeled in this paper, the parameters α and λ implicitly capture increases in liquidity premia. As will be shown in the empirical Section of this paper, our liquidity estimates are in line with those found by D'Amico et al. (2010), Christensen and Gillan (2011), and Kajuth and Watzka (2011).

Our next goal is to relate expressions (1) and (4). To this end, recall that Breakeven Inflation *BEI* is given by

$$BEI_{t,T} := RN_{t,T} - RP_{t,T}.$$

with $RN_{t,T}$ the nominal bond yield and $RP_{t,T}$ the inflation-linked bond yield. Hence, we can relate the elements of equations (1) and (4) as follows

$$\begin{array}{ccccc} U(\lambda^*) & = & \mathbb{E}[Y] + & \dfrac{\mathbb{E}[B] - A}{2\alpha \mathrm{var}[B]} & \cdot \mathrm{var}[Y] \\ \Updownarrow & & \Updownarrow & \Updownarrow & \\ BEI_{t,T} & = & EI_{t,T} + & IRP_t. & \end{array}$$

Observe in the previous expression that the inflation risk premium IRP_t can be in turn decomposed into:

$$\begin{aligned} IRP_t &= \frac{1}{2\alpha} \cdot \left(\frac{\mathbb{E}[B] - A}{\text{var}[B]} \right) \cdot \text{var}[Y] \\ &:= \frac{1}{2\alpha} MPR_t \cdot \sigma^2_{\Delta P_T | \Delta P_t} \\ &:= \frac{1}{2\alpha} MPR_t \cdot \sigma^2_{\pi_{t,T}} \end{aligned}$$

where the process MPR_t is called "market price of risk" and $\sigma^2_{\pi_{t,T}}$ is the inflation variance *inferred* by the market. Intuitively, MPR_t can be understood as the "price of risk" while $\sigma^2_{\pi_{t,T}}$ can be seen as the "quantity of risk".

Note that when $\mathbb{E}[B] > A$, namely, when the inflation risk premium is positive (or when the market portfolio yields exceed those of the risk-free asset), the demanded compensation by an investor will be positive. However, in an environment of generalized low yields ($\mathbb{E}[B] < A$) the search for higher yields and diversification benefits will cause a negative inflation risk premium.

Additionally, it should be noted that using arguments of conditional expectations, we can get an estimate of the conditional inflation expectation $EI_{t,s}$ for arbitrary times $0 \leq t < s < \infty$. Indeed, given $0 < u < t < s$ and the conditional expectation of a stationary process ω, given by $\mathbb{E}(\omega_s|\omega_u)$ y $\mathbb{E}(\omega_t|\omega_u)$, respectively, we wish to estimate $\mathbb{E}(\omega_s|\omega_t)$. To this end, consider the following linear estimator

$$\mathbb{E}(\omega_s|\omega_t) = \alpha + \beta\omega_t.$$

It can be shown, using a similar derivation of the Kalman filter, that

$$\begin{aligned} \alpha &= \mathbb{E}(\omega_s|\omega_u) - \beta\mathbb{E}(\omega_t|\omega_u) \\ \beta &= \frac{\text{cov}(\omega_s, \omega_t|\omega_u)}{\text{var}(\omega_t|\omega_u)}. \end{aligned}$$

Therefore the best linear estimator of $\mathbb{E}(\omega_s|\omega_t)$ is given by

$$\mathbb{E}(\omega_s|\omega_t) = \mathbb{E}(\omega_s|\omega_u) + \frac{\text{cov}(\omega_s, \omega_t|\omega_u)}{\text{var}(\omega_t|\omega_u)} \left(\omega_t - \mathbb{E}(\omega_t|\omega_u)\right).$$

Finally, since the process π is stationary it holds that

$$\frac{\text{cov}(\omega_s, \omega_t|\omega_u)}{\text{var}(\omega_t|\omega_u)} = \rho_\omega(s - t|u),$$

where ρ_ω is the autocorrelation function of the process ω.

2.2 Estimation of the inflationary risk premium IRP_t

In Section 2.1 we showed that Breakeven Inflation T is given by inflation expectations plus an inflationary risk premium, that is, $BEI_{t,T} = EI_{t,T} + IRP_t$. We showed that the inflation risk premium, IRP_t, is in turn given by

$$IRP_t = \frac{1}{2\alpha} \cdot \left(\frac{\mathbb{E}[B] - A}{\text{var}[B]} \right) \cdot \sigma^2_{\pi_{t,T}} = \frac{1}{2\alpha} MPR_t \cdot \sigma^2_{\pi_{t,T}},$$

where MPR_t is the market price of risk and $\sigma^2_{\pi_{t,T}}$ is the variance of inflation inferred by the market. The aim of this Section is to estimate the inflation risk premium with market information. To do this, in Subsection 2.2.1 we estimate the market price of risk MPR_t while in Subsection 2.2.2 we estimate $\sigma^2_{\pi_{t,T}}$.

2.2.1 Market Price of Risk MPR_t

As mentioned above, in this Section we estimate the market price of risk MPR_t. Rearranging terms and following the methodology of Siegel and Warner (1977), the market price of risk, MPR_t, is given by

$$MPR_t = \frac{(\bar{R}^t_m - \Delta\bar{P}_t) - (\bar{R}^t_f - \Delta\bar{P}_t)}{\text{var}(R^t_m - \Delta P_t)} = \frac{\bar{R}^t_m - \bar{R}^t_f}{\text{var}(R^t_m - \pi_t)},$$

where $\bar{R}^t_m$ is the average return of the market portfolio; $\Delta\bar{P}_t$ is the average monthly inflation rate; and $\bar{R}^t_f$ is the average yield of the risk-free asset.

2.2.2 Estimation of the inflation volatility inferred by the market

Recall that the inflation risk premium (IRP_t) can be expressed as

$$IRP_t = \frac{1}{2\alpha} MPR_t \cdot \sigma^2_{\pi_{t,T}}.$$

While MPR_t has been defined in the previous Subsection, we now focus on the intuition behind the estimation of $\sigma^2_{\pi_{t,T}}$. In general, following Azoulay et al. (2014), we seek long-term relationships (cointegration) between the core price level (in logarithms) P_t and other nominal variables, say $X_1, X_2, \ldots, X_n$.

Once those variables are identified, an error correction model will be estimated in order to explain inflation (independent variable) in terms of the cointegrated variables and the error correction term (EC) through the expression

$$\Delta P_t = \theta_1 \Delta X_1 + \cdots + \theta_n \Delta X_n + \theta_{n+1} \Delta P_{t-1} + \theta_{n+2} EC_{t-1} + \chi_t, \tag{5}$$

where the error correction term will be obtained as the residual when trying to explain inflation only in terms of the cointegrated variables. Once the model is estimated, and rearranging terms in equation (5), the volatility of inflation is computed ($\mathrm{var}(\Delta P_t) = \sigma^2_{\pi_{t,T}}$).

Note that ideally we would have estimated the inferred volatility of inflation through implicit volatility of inflation options by inverting the Black-Scholes formula. However, since these instruments do not exist in many countries, for the estimation of the right-hand side in (5), we will use the criterion that variables ΔX_i are estimated with implicit volatility if there is an options market, or otherwise, with historical data.

2.2.3 Error correction model

For a small open economy, like Mexico or Canada, a natural starting point for the choice of variables X_i would be US inflation (ΔP^*_t) and changes in exchange rates (ΔS_t) with respect to US dollar. Naturally, if reasonable, other variables could be included such as oil prices, but we focus on the most parsimonious model to anchor ideas.

Thus, having established that the variables P_t, P^*_t and S_t are cointegrated, we estimate an error correction model of the form

$$\Delta P_t = \theta_1 \Delta S_t + \theta_2 \Delta P^*_t + \theta_3 \Delta P_{t-1} + \theta_4 \nu_{t-1} + \chi_t, \tag{6}$$

where ν_{t-1} are the residuals lagged one period and χ_t is the corresponding error term for the regression. The next step is to calculate $\mathrm{var}(\Delta P_t)$. To do this, we rearrange the terms in equation (6) to get

$$\Delta P_t - \theta_3 \Delta P_{t-1} = \theta_1 \Delta S_t + \theta_2 \Delta P_t^* + \theta_4 \nu_{t-1} + \chi_t.$$

and calculating the variance on both sides of the previous equation we get

$$\operatorname{var}\left(\Delta P_t - \theta_3 \Delta P_{t-1}\right) = \operatorname{var}\left(\theta_1 \Delta S_t + \theta_2 \Delta P_t^* + \theta_4 \nu_{t-1} + \chi_t\right). \tag{7}$$

For the left hand side of (7) and taking into account that $\operatorname{var}(\Delta P_t) = \operatorname{var}(\Delta P_{t-1})$ we have

$$\begin{aligned}
\operatorname{var}(\Delta P_t - \theta_3 \Delta P_{t-1}) &= \operatorname{var}(\Delta P_t) + \theta_3^2 \operatorname{var}(\Delta P_{t-1}) - 2\theta_3 \operatorname{cov}(\Delta P_t, \Delta P_{t-1}) \\
&= \operatorname{var}(\Delta P_t) + \theta_3^2 \operatorname{var}(\Delta P_{t-1}) - 2\theta_3 \operatorname{cor}(\Delta P_t, \Delta P_{t-1}) \\
&\quad \sqrt{\operatorname{var}(\Delta P_t)}\sqrt{\operatorname{var}(\Delta P_{t-1})} \\
&= \operatorname{var}(\Delta P_t) + \theta_3^2 \operatorname{var}(\Delta P_t) - 2\theta_3 \operatorname{cor}(\Delta P_t, \Delta P_{t-1}) \\
&\quad \sqrt{\operatorname{var}(\Delta P_t)}\sqrt{\operatorname{var}(\Delta P_t)} \\
&= \left[1 + \theta_3^2 - 2\theta_3 \operatorname{cor}(\Delta P_t, \Delta P_{t-1})\right] \operatorname{var}(\Delta P_t).
\end{aligned} \tag{8}$$

For the right hand side, applying a similar reasoning, we get

$$\begin{aligned}
\operatorname{var}(\theta_1 \Delta S_t + \theta_2 \Delta P_t^* + \theta_4 \nu_{t-1} + \chi_t) &= \theta_1^2 \operatorname{var}(\Delta S_t) + \theta_2^2 \operatorname{var}(\Delta P_t^*) \\
&\quad + \theta_4^2 \operatorname{var}(\nu_{t-1}) + \operatorname{var}(\chi_t) \\
&\quad + 2\theta_1 \theta_2 \operatorname{cov}(\Delta S_t, \Delta P_t^*) \\
&\quad + 2\theta_1 \theta_4 \operatorname{cov}(\Delta S_t, \nu_{t-1}) \\
&\quad + 2\theta_2 \theta_4 \operatorname{cov}(\Delta P_t^*, \nu_{t-1}).
\end{aligned} \tag{9}$$

In particular, re-arranging covariances in function of correlations and standard deviations for the last three terms in (9), we can find an expression for the right hand side that is a function only of the volatility of the exchange rate given by

$$\mathrm{var}(\theta_1 \Delta S_t + \theta_2 \Delta P_t^* + \theta_4 \nu_{t-1} + \chi_t) = \eta_1 \mathrm{var}(\Delta S_t) + \eta_2 \sqrt{\mathrm{var}(\Delta S_t)} + \eta_3 \quad (10)$$

where

$$\begin{aligned}
\eta_1 &= \theta_1^2 \\
\eta_2 &= 2\theta_1\theta_2 \mathrm{cor}(\Delta S_t, \Delta P_t^*)\sqrt{\mathrm{var}(\Delta P_t^*)} + 2\theta_1\theta_4 \mathrm{cor}(\Delta S_t, \nu_{t-1})\sqrt{\mathrm{var}(\nu_{t-1})} \\
\eta_3 &= \theta_2^2 \mathrm{var}(\Delta P_t^*) + \theta_4^2 \mathrm{var}(\nu_{t-1}) + \mathrm{var}(\chi_t) \\
&\quad + 2\theta_2\theta_4 \mathrm{cor}(\Delta P_t^*, \nu_{t-1})\sqrt{\mathrm{var}(\Delta P_t^*)\mathrm{var}(\nu_{t-1})}.
\end{aligned}$$

Therefore, matching the right hand side of (8) with the right hand side of (10), and solving for $\mathrm{var}(\Delta P_t)$,we finally get that the implied inflation volatility is given by the following identity

$$\sigma^2_{\Pi_{t,T}} = \frac{\eta_1 \mathrm{var}(\Delta S_t) + \eta_2 \sqrt{\mathrm{var}(\Delta S_t)} + \eta_3}{[1 + \theta_3^2 - 2\theta_3 \mathrm{cor}(\Delta P_t, \Delta P_{t-1})]}. \quad (11)$$

2.3 Inflation risk premium and inflation expectations

Given the calculations of the previous Section, it is now possible to finally derive the point-estimate of inflation expectations EI, at time t for a T-year horizon, which is defined as

$$EI_{t,T} := BEI_{t,T} - IRP_t$$

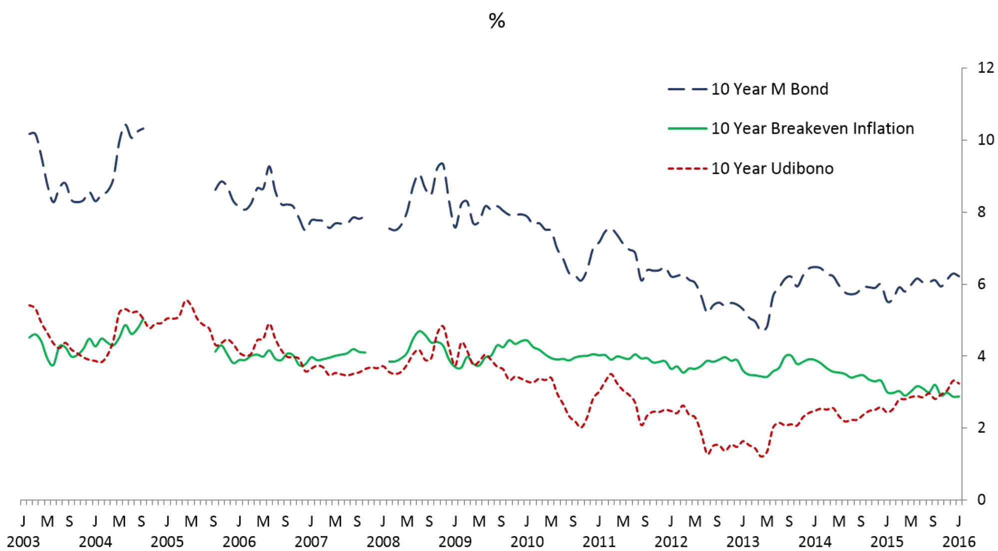
since Breakeven Inflation is observed in the market, and the components of the inflation risk premium $IRP_t = \frac{1}{2\alpha} MPR_t \cdot \sigma^2_{\Delta P_t}$ were previously estimated.

3 Application: Long-term inflation expectations in Mexico

3.1 Introduction to Mexico

Figure 1 shows the evolution of 10-year nominal bonds (M Bond) and 10-year inflation-linked bonds (*Udibonos*) in Mexico. Although yields of nominal bonds and Udibonos have varied considerably in this period, we see that the 10-year Breakeven Inflation (BEI10) has remained relatively stable. To highlight this, Figure 2 compares the Mexican BEI10 with other financial risk variables, in this case VIX and the exchange rate of Mexican peso to US dollars (MXNUSD), showing that BEI10 in Mexico has remained relatively stable even in periods of financial uncertainty or during temporary increases in the annual inflation rate. The latter could be interpreted as evidence of anchored long-term inflation expectations.

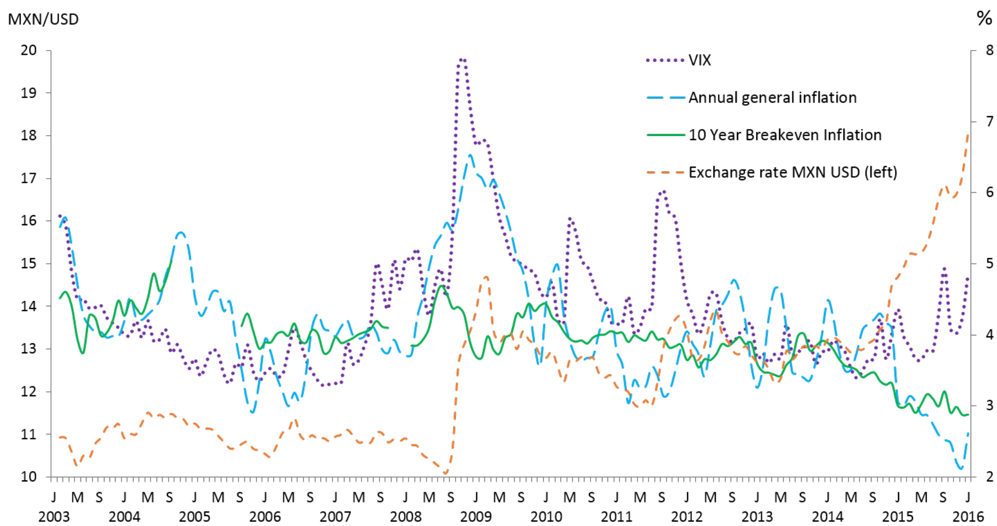
Fig. 1: Nominal bonds, inflation-indexed bonds and BEI10



Source: Own calculations with data from Bloomberg

Notably, BEI10 has increasingly approached the one reported in Banco de México's Survey of Private-Sector Analysts Inflation Expectations *(Encuesta sobre las Expectativas de los Especialistas en Economía del Sector Privado,* *EEEESP)*, as Figure 3 shows. Indeed, the responses in the last two years of private sector specialists show inflation expectations for Mexico at around 3.4 percent for the next 5 to 8 years. Moreover, given that the BEI10 has been oscillating between the values reported in surveys, if these were the true market expectations for inflation it would imply that the inflation risk premium, IRP_t, has been sometimes positive (when BEI10 is above survey expectations) and other times negative (when, on the contrary, BEI10 is below survey expectations).

Fig. 2: BEI10, inflation and risk measures

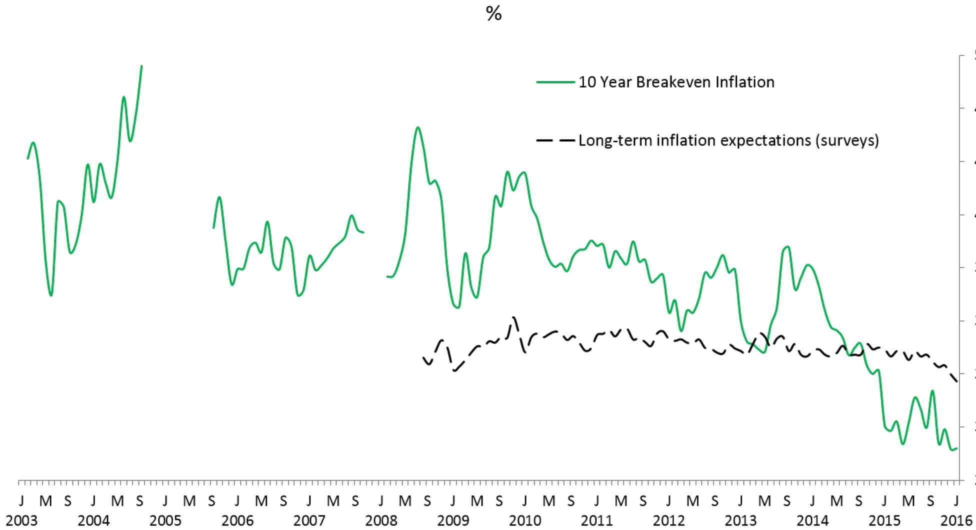


Source: Own calculations with data from Banco de México, INEGI and Bloomberg.

3.2 Data sources

With regard to debt instruments, the corresponding rates of return on bonds and the 28-day Cetes (our proxy for the risk-free rate), monthly data were obtained through a simple average of the daily series reported in Bloomberg and Banco de México respectively. In the case of the nominal and inflation-linked bonds, we consider instruments with a maturity of 10 years and whose rate of return included coupon rate and taxes.

Fig. 3: BEI10 and analyst's answers to surveys



Source: Own calculations with data from Banco de México and Bloomberg.

Regarding the level of peso-dollar exchange rate, the daily exchange rate used to settle liabilities denominated in foreign currency (FIX) reported by Banco de México was used. Implied volatility, the one expected in the future by the market, of the exchange rate in a year, was obtained from the daily series reported by Bloomberg. The monthly series for both, of the level and of the implied volatility of the exchange rate, were obtained through a simple average of daily data.

In the case of inflation in Mexico and the United States, we made use of the Índice Nacional de Precios al Consumidor (INPC) and the Consumer Price Index (CPI), respectively. Making use of these indices, monthly and annual percentage changes were obtained using the log differences of the monthly series. Data for Mexico corresponds to those published by the National Institute of Statistics and Geography (Instituto Nacional de Estadística y Geografía, INEGI), while for the United States correspond to those published by the Federal Reserve Economic Data (FRED).

To obtain the data for the monthly performance of the Mexican Stock Exchange (IPC), monthly averages of the index values at the close of each day reported by Bolsa Mexicana de Valores were obtained. Subsequently, through a log difference of the series, the monthly returns of the IPC were obtained. The latter were given by a 60-months moving average not centered. Similarly, we made use of the Market Volatility Index Options of Chicago (VIX), whose monthly data were obtained through a simple average of the square roots of the daily values reported on Bloomberg.

In this paper, we use the 5-year moving average of the local stock market index (*Índice de Precios y Cotizaciones*, IPC) as proxy for the mean return of the market portfolio ($\bar{R}_m^T$), the 5-year moving average of the monthly yield of 28-day *Cetes* as proxy for the risk-free asset ($\bar{R}_f^T$). In addition, the standardization $\mathrm{var}(R_m^t - \pi_t)$ with data from the entire sample is usted, where π_t is estimated as the difference of the logarithm of the core price index in Mexico.

3.3 Estimation of the inflationary risk premium IRP_t

In this Section we proceed to estimate de inflationary risk premium for the case of Mexico. As explained in previous Sections, we need to first estimate the market price of risk, MPR_t, and the inflation volatility inferred by the market, $\sigma^2_{\Pi_{t,T}}$.

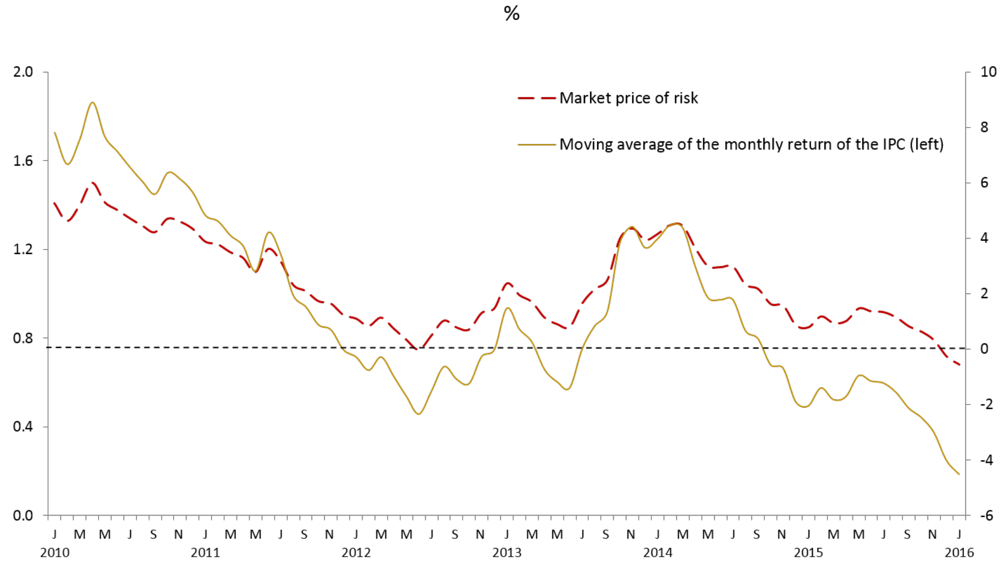
3.3.1 Market Price of Risk MPR_t

Figure 4 shows the estimated market price of risk series, ($\overline{MPR_t}$), as well as the 60-month moving average of the IPC. As can be seen, the price of risk in the market decreases during low returns of the stock market. For the case of Mexico, the price of risk remains positive through most of the sample until the end of 2015.

3.3.2 Estimation of the inflation volatility inferred by the market and Error Correction Model

As before, we now seek to find financial variables that are cointegrated with the core price index in Mexico. To this end, we propose the peso-dollar exchange rate S_t, as well as the price level in the United States P_t^*. As shown

Fig. 4: Market price of risk



Source: Own calculations with data from BMV, Cleveland Federal Reserve Bank, INEGI and Banco de México.

in Table 1, the Dickey-Fuller test states that variables S_t, P_t, P_t^* are $I(1)$, while their first differences, $\Delta S_t, \Delta P_t, \Delta P_t^*$ are $I(0)$. Although not reported, other nominal variables were included, such as oil prices, without significantly changing our results.

To interpret the results in Table 1, remember that the Dickey-Fuller test has as its null hypothesis that the process is $I(1)$. Therefore, the larger the magnitude of the statistic, the more evidence there is to reject the null hypothesis. For example, note that the variable S_t has a statistic value of 1.15, which is to the right of the 5% critical level. This implies that there is not enough statistical evidence to reject the hypothesis that the process is non-stationary. In contrast, note that the statistic for the variable ΔS_t is -7.04 and is located to the left of the critical level 5%. This indicates that there is enough statistical evidence to reject the hypothesis that the process is non stationary, or $I(1)$. This results are similar, for example, to those found by Azoulay et al. (2014) for the case of Israel.

Now, to test cointegration, we estimate the regression

Tab. 1: Unit root tests.

		Critical Values	
Variable	ADF	1%	5%
P_t	6.71	-2.58	-1.95
S_t	1.15	"	"
P_t^*	2.08	"	"
ΔP_t	-2.20	-2.58	-1.95
ΔS_t	-7.04	"	"
ΔP_t^*	-5.68	"	"

$$P_t = \beta_0 + \beta_1 S_t + \beta_2 P_t^* + \nu_t, \tag{12}$$

where ν_t (the residuals of the regression) corresponds to the error correction term (EC) in equation (6). Thus, once the model (12) is obtained, we proceed to perform an augmented Dickey-Fuller test to the error ν_t to verify the cointregration between P_t, S_t, P_t^*. The Dickey-Fuller test result had a statistic with a value of -3.4 and a corresponding critical value of -2.58 at a significance level of 1%. Therefore, we conclude that P_t, S_t, P_t^* are indeed cointegrated. The estimated coefficients of (6), as well as their t-statistics can be found in Table 2. Recall that the coefficients θ_1, θ_2, and θ_3, in (6), capture short-term effects. In turn, coefficient θ_4 refers to the speed at which the system converges to equilibrium. In particular, since $\hat{\theta}_4 = -0.019$ (significant), it follows that approximately 2% of the equilibrium deviation is corrected per month. Finally, we note that, even though the short-term effect on the exchange rate is not very strong, it is still significant at 5%. The magnitude of this effect is even smaller than that found in Capistrán et al. (2012), for the case of Mexico.

Tab. 2: Estimated coefficients of equation (6)

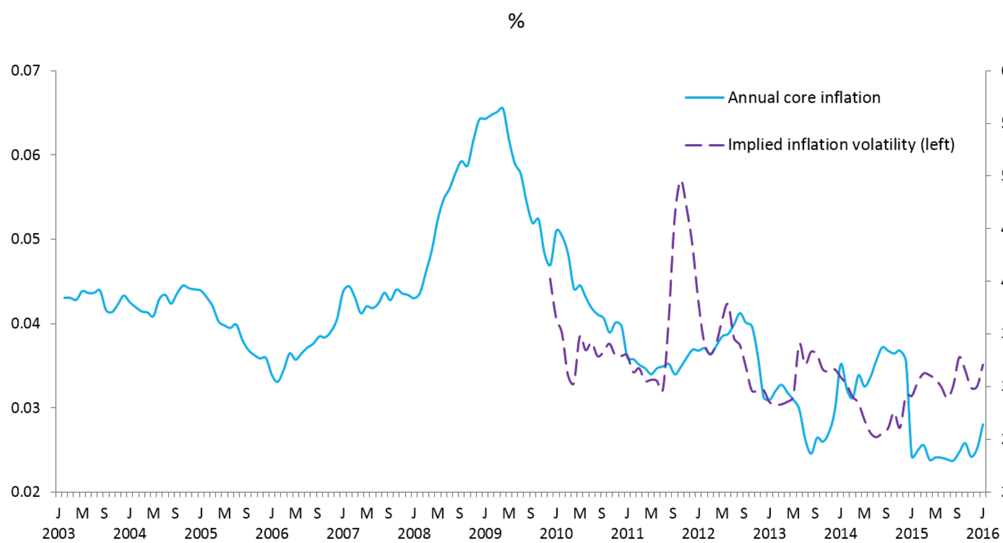
	ΔS_t	ΔP_t^*	ΔP_{t-1}	ν_{t-1}
Coefficient	0.010	0.096	0.813	-0.019
t-statistic	1.968	2.558	17.870	-1.943
p-value	0.051	0.011	0.000	0.054
R^2-adj.	0.777			

As discussed, our criterion is that if there are options on the market for a nominal variable, then we use the implied volatility of these instruments. Otherwise, we estimate the variance from historical data. Therefore, we use implied volatility of peso-dollar exchange rate options, but historical information on US inflation. Notice that to emphasize the dependence of the implied volatility of the exchange rate, we can substitute the coefficients on Table 2 into equation (11) to get

$$\sigma^2_{\Delta P_t} = (2.11 \times 10^{-6}) - (1.72 \times 10^{-6})\sigma_{\Delta S_t} + (9.69 \times 10^{-5})\sigma^2_{\Delta S_t} \qquad (13)$$

That is, equation (13) indicates that the implied inflation volatility of Mexico can be expressed as a function of the implied volatility in the options of the exchange rate, if so desired. Figure 5 shows the implied volatility of the modeled inflation using this methodology. In particular, note that it has remained at around 0.0322 since 2013.

Fig. 5: Implied inflation volatility



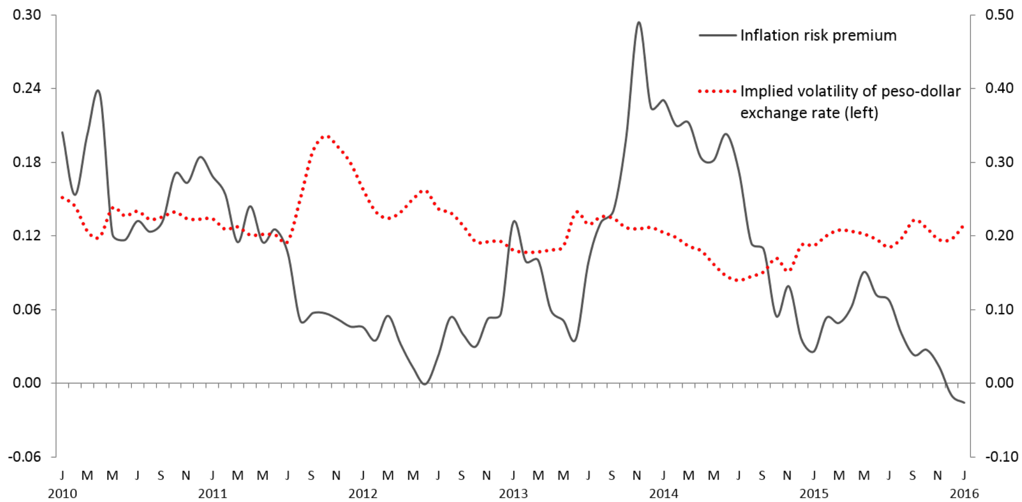
Source: Own calculations with data from Banxico, Cleveland Federal Reserve Bank, INEGI and Bloomberg.

3.4 Inflation risk premium and inflation expectations

As discussed above, the inflation risk premium (IRP_t) is given by

Fig. 6: Inflation risk premium

%



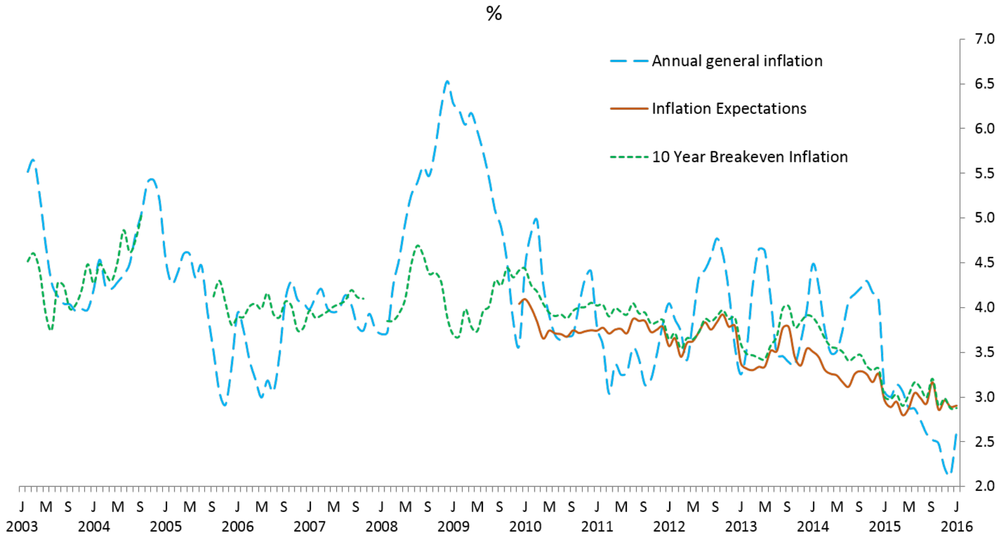
Source: Own calculations with data from Banxico, Cleveland Federal Reserve Bank, INEGI and Bloomberg.

$$IRP_t = \frac{1}{2\alpha} MPR_t \cdot \sigma^2_{\pi_{t,T}}.$$

Also, remember that the "price of risk" MPR_t has already been estimated as well as the "quantity of risk" $\sigma^2_{\pi_{t,T}}$. In this Section we present the product of both, which can be interpreted as the compensation demanded by an investor for inferred volatility. Figure 6 shows the results. As it can be seen, although the inflation risk premium is generally positive, during episodes of generalized low yields, the inflation asset can provide diversification benefits (i.e. risk-adjusted high yield benefits) so an investor is willing to pay for this benefit.

Finally, Figure 7 also shows that the expectations of long-term inflation are anchored, even during periods with transitory inflation shocks. Indeed, expectations have had a downward trend towards the inflation target of Banco de México moving from around 3.5% at the beginning of the decade to practically 3% during as of 2016. In most cases, these expectations have fluctuated within the range ±1% around the 3% target. Table 3 summarizes the results obtained for Mexico.

Fig. 7: Inflation Expectations



Source: Own calculations with data from Banxico, Cleveland Federal Reserve Bank, INEGI and Bloomberg.

4 Final Considerations

This paper proposed a new method for extracting inflation expectations using only information implicit in financial instruments. In particular, this methodology explicitly modeled an inflation risk premium considering the benefits of diversification that inflation-linked bonds have for an investor. The application of this methodology to the case of Mexico seems to suggest that, as of 2016, long-term inflation expectations are anchored at a level close to the permanent inflation target of 3 percent of Banco de México. Future work will seek to apply this methodology to other economies with deep financial markets.

Tab. 3: Results obtained for Mexico

	Breakeven Inflation $BEI_{t,T}$ (5)=(4)+(3)	Inflation Expectations $EI_{t,T}$ (4)	Inflationary Risk Premium IRP_t (3)=(1)*(2)	Market Price of Risk MPR_t (2)	Implied Inflation Volatility $\sigma^2_{\pi_{t,T}}$ (1)
2010	4.048	3.778	0.269	4.879	0.037
2011	3.938	3.773	0.166	2.761	0.041
2012	3.773	3.712	0.061	0.833	0.037
2013	3.683	3.466	0.217	2.295	0.034
2014	3.521	3.273	0.248	2.857	0.029
Jan	3.891	3.507	0.384	4.165	0.034
Feb	3.796	3.446	0.350	4.483	0.033
Mar	3.658	3.305	0.353	4.454	0.031
Apr	3.565	3.260	0.306	3.651	0.031
May	3.545	3.243	0.302	3.003	0.029
Jun	3.504	3.165	0.338	2.967	0.027
Jul	3.405	3.116	0.289	2.981	0.027
Aug	3.448	3.257	0.191	2.332	0.027
Sep	3.470	3.289	0.181	2.176	0.028
Oct	3.348	3.257	0.091	1.632	0.029
Nov	3.299	3.167	0.132	1.569	0.028
Dec	3.318	3.258	0.060	0.868	0.031
2015	3.015	2.943	0.072	0.899	0.033
Jan	3.007	2.963	0.043	0.793	0.031
Feb	2.978	2.889	0.089	1.184	0.033
Mar	3.031	2.949	0.082	0.948	0.034
Apr	2.903	2.800	0.104	1.021	0.034
May	3.024	2.873	0.151	1.474	0.033
Jun	3.166	3.046	0.120	1.371	0.032
Jul	3.101	2.988	0.113	1.338	0.031
Aug	2.997	2.929	0.068	1.147	0.033
Sep	3.205	3.166	0.039	0.825	0.036
Oct	2.907	2.861	0.046	0.633	0.034
Nov	2.988	2.965	0.023	0.322	0.032
Dec	2.876	2.892	-0.016	-0.263	0.033
2016	2.879	2.905	-0.026	-0.552	0.035
Jan	2.879	2.905	-0.026	-0.552	0.035

References

[1] Adrian, T. and H. Wu (2009) "The term structure of inflation expectations", Staff Reports 362, Federal Reserve Bank of New York.

[2] Aguilar, A., R. Elizondo and J. Roldán-Peña (2016). "Descomposición de la Compensación por Inflación y Prima por Riesgo Inflacionario". Mimeo

[3] Alonso, F., R. Blanco and A. del Río (2001) "Estimating inflation expectations using French government inflation-indexed bonds", Banco de España Working Papers, 0111.

[4] Azoulay, E., M. Brenner, and Landskroner and R. Stein (2014) "Inflation risk premium implied by options", *Journal of Economics and Business* 71-C, 90-102.

[5] Bauer, M. D. and J. H. Christensen (2014) "Financial market outlook for inflation", *FRBSF Economic Letter*, Federal Reserve Bank of San Francisco, issue may12.

[6] Bière, M. and O. Signori (2009) "Do inflation-linked bonds still diversify?", *European Financial Management* 15-2, 279-297.

[7] Campbell, J. Y. and R. J. Shiller (1996) "A scorecard for indexed government debt", en S. Bernanke, Ben and Rotemberg, Julio(Eds.). NBER Macroeconomic Annual. MIT Press, 155-197.

[8] Capistrán, C., R. Ibarra-Ramírez and M. Ramos-Francia (2012) "El traspaso de movimientos del tipo de cambio a los precios. Un análisis para la economía mexicana", *El Trimestre Económico* 74, 813-838.

[9] Chen, R., B. Liu and X. Cheng (2010) "Pricing the term structure of inflation risk premia: theory and evidence from TIPS", *Journal of Empirical Finance* 17-4, 702-721.

[10] Chernov, M. and P. Mueller (2012) "The term structure of inflation expectations", *Journal of Financial Economics* 106-2, 367-394.

[11] Christensen, I., F. Dion and C. Reid (2004) "Real return bonds, inflation expectations, and the break-even inflation rate", Bank of Canada Working Paper Series 2004-43.

[12] Christensen J. H. (2008) "Treasury bond yields and long-run inflation expectations", *FRBSF Economic Letter*, Federal Reserve Bank of San Francisco, 2008-25.

[13] Christensen, J. H., F. X. Diebold and G. Rudebusch (2011) "The affine arbitrage-free class of Nelson-Siegel term structure models", *Journal of Econometrics* 164-1, 4-20.

[14] Christensen, J. H. and J. Gillan (2011) "TIPS liquidity, breakeven inflation, and inflation expectations", *FRBSF Economic Letter*, Federal Reserve Bank of San Francisco, 2011-19.

[15] Christensen, J. H., J. A. López and G. Rudebusch (2010) "Inflation expectations and risk premiums in an arbitrage-free model of nominal and real bond yields", *Journal of Money, Credit and Banking* 42, 143-178.

[16] Crump, R., S. Eusepi and E. Moench (2013) "Making a Statement: How Did Professional Forecasters React to the August 2011 FOMC Statement?", Federal Reserve Bank of New York Liberty Street Economics; January 7, 2013.

[17] D'Amico, S., D.H. Kim and M. Wei (2010) "Tips from TIPS: the informational content of Treasury Inflation-Protected Security prices", Finance and Economics Discussion Series 2010-19, Federal Reserve Board.

[18] Devlin, W. and D. Patwardhan (2012) "Measuring market inflation expectations", *Economic Roundup*, The Treasury Australian Government 2012-2, 5-17.

[19] Evans, M. (1998) "Real rates, expected inflation and inflation risk premia", *Journal of Finance* 53, 187-218.

[20] Evans, M. (2003) "Real risk, inflation risk and the term structure", *Economic Journal* 113, 345-389.

[21] Foresi, S., A. Penati and G. Pennacchi (1997) "Estimating the Cost of U.S. Indexed Bonds", Staff Reports 9701, Federal Reserve Bank of Cleveland.

[22] Haubrich, J. G., G. Pennacchi and P. Ritchken (2012) "Inflation expectations, real rates, and risk premia: evidence from inflation swaps", *The Review of Financial Studies* 25-5, 1588-1629.

[23] Hördal, P. and O. Tristani (2012) "Inflation risk premia in the term structure of interest rates", *Journal of the European Economic Association* 10-3, 634-657.

[24] Hunter, D. and D. Simon (2005) "Are TIPS the "real" deal?: a conditional assessment of their role in a nominal Portfolio", *Journal of Banking and Finance* 29-2, 347-368.

[25] Imakubo, K. and J. Nakajima (2015) "Estimating inflation risk premia from nominal and real yield curves using a shadow-rate model", Bank of Japan Working Paper Series 15-E-1.

[26] Jervis, P. (2007) "Inflation compensation and its components in Chile", *Journal Economía Chilena*, Central Bank of Chile 10-2, 27-56.

[27] Joyce, M., P. Lildholdt and S. Sorensen (2010) "Extracting inflation expectations and inflation risk premia from the term structure: A joint model of the UK nominal and real yield curves", *Journal of Banking and Finance*, 34-2, 281-294.

[28] Kajuth, F. and S. Watzka (2011) "Inflation expectations from indexed-linked bonds: Correcting for liquidity and inflation risk premia", *The Quarterly Review of Economics and Finance* 51-3, 225-235.

[29] Kandel, S., A. R. Ofer and O. Sarig (1996) "Real interest rates and inflation: An exante empirical analysis", *Journal of Finance* 51, 205-225.

[30] Kitsul, Y. and J. H. Wright (2013) "The economics of options-implied inflation probability density functions", *Journal of Financial Economics* 110-3, 696-711.

[31] Kwan, S. (2005) "Inflation expectations: how the market speaks", *FRBSF Economic Letter*, Federal Reserve Bank of San Francisco, issue october3.

[32] Li, H. and F. Zhao (2009) "Nonparametric estimation of state-price densities implicit in interest rate cap prices", *The Review of Financial Studies* 22-11, 4335-4376.

[33] Melo Veladia, L. F. and J. F. Moreno Gutiérrez (2010) "Actualización de la descomposición del BEI cuando se dispone de nueva información", Borradores de Economía 007333, Banco de la República.

[34] Pericoli, M. (2012) "Expected inflation and inflation risk premium in the euro area and in the United States", Temi di discussione (Economic working papers) 842, Bank of Italy.

[35] Pericoli, M. (2014) "Real term structure and inflation compensation in the Euro Area", *International Journal of Central Banking* 10-1, 1-42.

[36] Reschreiter, A. (2010) "Indexed bonds and revisions of inflation expectations", *Annals of Finance* 6-4, 537-554.

[37] Siegel, J. J. and J. B. Warner (1977) "Indexation, the risk free asset, and capital market equilibrium", *Journal of Finance* 32-4, 1101-1107.

[38] Spiro, P. (2003) "Evidence on inflation expectations from Canadian real return bonds", Macroeconomics 0312004, EconWPA.

[39] Viceira, L. and Campbell, J. (2001) "Who should buy long-term bonds?", *The American Economic Review* 91-1, 99-127.

[40] Yellen, J.L. (2015) "Inflation Dynamics and Monetary Policy", Philip Gamble Memorial Lecture, University of Massachusetts, Amherst, Amherst, Massachusetts; September 24, 2015.