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Abstract: We propose a static search model with two types of workers, output sharing (Nash bargaining), and free entry of firms. The matching function is specified so as the unskilled do not create congestion effects for the skilled. An increase in the share of skilled workers has two effects on the welfare of the unskilled: a negative crowding-out effect, and a positive labour demand effect. The former (latter) effect dominates whenever the skill differential is small (large).

Keywords: Matching frictions; Heterogeneity; Congestion effects

J.E.L. classification: J21; J23

1. Introduction

This paper is interested in the following market situation. Good workers are preferred to bad workers in the recruitment process, and the total number of available jobs depends on the profitability of employment relationships. Do good workers hurt bad workers in this environment? I answer this question by means of a very stylized static search model with two types of workers, free entry of firms, and Nash bargaining. The matching technology is specified so as it has constant returns to scale and the unskilled do not create congestion effects for the skilled (the so-called urn-ball technology is a particular case). I show that an increase in the skilled proportion has two effects on the welfare of the unskilled. On the one hand, the negative *crowding-out effect*, according to which the unskilled probability of getting a job decreases at given number of jobs per job-seeker. On the other hand, the positive *labour demand* effect, according to which the total number of advertised jobs raises with the skilled proportion. Indeed, the availability of jobs responds to average profitability through free entry, while rent-sharing implies ex-post profitability

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is increasing in worker's skill. I show that the crowding-out effect is stronger than the labour demand effect if and only if the productivity differential between skill-group is lower than a threshold value.

The crowding-out issue has already been studied in various contributions that differ according to the way the search technology is specified.

In random search models, the search technology is non-discriminant so that all individuals have the same probability of getting a job regardless their characteristics. These papers feature the labour demand effect as a result, but do not predict any crowding-out effect. Gautier (2003) and Dolado et al (2003) for instance consider a model in which skilled and unskilled workers compete for simple jobs, while skilled workers search on the job. There is no crowding-out effect, but the skilled may have an ambiguous impact on simple job profitability. On the one hand, they may be more productive than the unskilled, which tends to attract more vacancies through the labour demand effect. On the other hand, they may quit the job rapidly, which tends to deter firms to post additional vacancies through an inverse labour demand effect. Gautier shows that the former effect dominates the latter whenever the skill differential is sufficiently large.

The crowding-out effect highlighted in this paper differs from Acemoglu (1999) and Rosen and Wasmer (2005), in which holding a vacancy features an option value. In these papers, an increase in the skilled proportion may have a negative impact on the unskilled because firms start rejecting their applications to make a better match in the future (Acemoglu), or because unskilled wage goes down through wage bargaining (Rosen and Wasmer). These papers make predictions that deeply differ from mine. An increase in the skill differential magnifies the labour demand effect in my paper, thereby improving the unskilled economic situation, while it raises the value of a vacancy in Acemoglu and Rosen and Wasmer, thereby deteriorating the unskilled welfare.

The closest paper is Shi (2002), who considers a directed search model. There are two types of jobs and two types of workers. Firms announce wages, workers observe wage offers and send a single application. Finally, firms can select among the pool of applicants, which lead them to favour skilled workers. Shi shows that there are two types of equilibria: a pooling equilibrium in which high-tech and low-tech firms coexist, and a separating equilibrium in which there are only high-tech firms. The separating equilibrium is very close to the equilibrium I study in this paper (the main difference relies on wage determination). However, Shi only provides with comparative statics for the pooling equilibrium, but does not examine the properties of the separating equilibrium. I conjecture that one could reach a similar result to mine in that case[1].

[1] There are a number of directed search models with heterogeneous firms/workers (see for instance Shi, 2001, Lang and Manove, 2003, and Shimer, 2005). These models raise important issues, but do not focus on the impact of changes in the composition of worker types on the extent of mismatch and the crowding-out of lower-skilled workers.

The remaining of the paper is organized in two parts. Section 2 introduces the model, while section 3 shows the results.

2. The model

In this section, we depict a static search model with two types of workers, a single type of job, Nash bargaining over match surplus, and endogenous supply of jobs. The key assumption relates to the matching technology: there is a unique search market, yet the low-skilled do not create congestion effects for the high-skilled.

There are two types of agents seeking a job: n_1 skilled and n_2 unskilled workers. Agents differ with respect to the amount of output they produce if employed. Type-i agents produce $2y_i$. Let $y_2 = (1-\rho)\,y_1$, where $\rho/(1-\rho) \geq 0$ is the skill premium.

There is a large number of firms, each endowed with a single job slot which can be either active or inactive. Each active job costs $cy_1 > 0$, $c \in (0,1)$, and needs to be filled before production starts. Inactive jobs cost nothing.

Active jobs and job-seekers meet each other on the search market. Once a given worker is hired, she starts producing and output is split between the worker and the firm – it corresponds to symmetric Nash bargaining when outside options are nil. This implies that firms prefer to hire skilled workers, as they get more profits with them. The aggregate number of *hires* M is determined by a matching technology which inputs are the total number of job-seekers $n_1 + n_2$ on the one hand, and the number of active jobs v on the other hand:

$$M \equiv \min\left\{m\left(n_1+n_2, v\right), n_1+n_2, v\right\} \tag{2.1}$$

The technology m is strictly increasing and strictly concave in each argument, and has constant returns to scale. In the remaining, we only focus on equilibrium where $M = m\left(n_1+n_2, v\right)$.

Let M_i be the number of hires that involve type-i workers. By construction, $M_1+M_2 = M$. Due to firms' preferences for the skilled, the unskilled do not create congestion for the skilled. Therefore,

$$M_1 = m\left(n_1, v\right) \tag{2.2}$$

The unskilled get the residual number of hires:

$$M_2 = m\left(n_1+n_2, v\right) - m\left(n_1, v\right) \tag{2.3}$$

The strict concavity of m then guarantees M_2 is strictly decreasing in n_1 *at given number of jobs*. Therefore, the skilled crowd-out the unskilled. Note that the directed search model based on the urn-ball statistical model is a particular case of this technology.

Let μ_i denote type-i workers' probability of getting a job, and η_i be firms' probability of recruiting a type-i individual. The number of hires is equiprobably distributed within each side of the market. Hence,

$$\mu_1 \equiv \frac{m(n_1, v)}{n_1} \equiv m(1, \theta/x) \tag{2.4}$$

$$\mu_2 \equiv \frac{m(n_1+n_2, v) - m(n_1, v)}{n_2} \equiv \frac{m(1,\theta) - xm(1,\theta/x)}{1-x} \tag{2.5}$$

$$\eta_1 \equiv \frac{m(n_1, v)}{v} \equiv \frac{m(1,\theta/x)}{\theta/x} \tag{2.6}$$

$$\eta_2 \equiv \frac{m(n_1+n_2, v) - m(n_1, v)}{v} \equiv \frac{m(1,\theta) - xm(1,\theta/x)}{\theta} \tag{2.7}$$

where $x \equiv n_1/(n_1+n_2)$ is the share of skilled agents and $\theta \equiv v/(n_1+n_2)$ is the so-called market tightness. Contact rates depend on either the market tightness θ, or the number of jobs *per skilled worker* θ/x, or both. For instance, the probability of getting a job for a skilled worker is decreasing in the share of skilled workers at given number of jobs. Of course, this reveals the fact that skilled workers create congestion effects for each other, while their employment prospects do not respond to changes in the number of unskilled workers. Conversely, the unskilled are harmed by the presence of skilled workers. The rate of contact μ_2 is therefore decreasing in x *at given tightness.*

Let w_i denote the expected utility of type-i workers, and π be the expected profits of firms.

$$w_i = \mu_i y_i \tag{2.8}$$

$$\pi = \eta_1 y_1 + \eta_2 y_2 - c y_1 \tag{2.9}$$

Finally, the number of active jobs obeys the free entry condition $\pi = 0$.

3. Results

In this section, we solve the model and analyze the impacts of demographic changes on the welfare of each skill group. The main result is the non-monotonous relationship between the share of skilled workers and the welfare of unskilled individuals.

Consider the following function $\psi(z) \equiv m(1,z)/z$, that is the average recruitment rate – it is strictly decreasing in z. From equations (2.6), (2.7) and (2.9), and constant returns to scale in the matching technology, solving reduces to find a market tightness θ such that

$$\psi(\theta/x)\rho + \psi(\theta)(1-\rho) = c \tag{3.1}$$

The properties of the function ψ imply uniqueness whenever there exists an equilibrium – that we assume[2]. Equilibrium tightness displays the following features: it is decreasing in the skill premium $\rho/(1-\rho)$, while it is increasing the share of skilled workers x.

[2]This involves additional restrictions on the matching technology m and the job creation cost c.

What are the effects of x on the welfare of skilled and unskilled workers? To answer this question, we need to know how x alters skill-specific rates of contact. Consider first the skilled workers. From (2.4), we have to find how changes in x alter the ratio $z \equiv \theta/x$. Using (3.1), we get

$$\psi(z)\rho + \psi(xz)(1-\rho) = c \tag{3.2}$$

It is clear that z is strictly decreasing in x. Therefore, the welfare of skilled workers decreases with their share in the workforce. This result is typical of directed search models with mismatch (see Shi, 2002, for instance). This differs from random matching models where the skilled actually benefit from an increase in their proportion.

We now turn to the unskilled. From (2.5),

$$x\frac{d\mu_2/dx}{\mu_2} = \underbrace{x\frac{\partial\mu_2/\partial x}{\mu_2}}_{\substack{\text{crowding-out effect} \\ <0}} + \underbrace{\left(\theta\frac{\partial\mu_2/\partial\theta}{\mu_2}\right)\left(x\frac{d\theta/dx}{\theta}\right)}_{\substack{\text{labour demand effect} \\ >0}} \tag{3.3}$$

with

$$x\frac{\partial\mu_2/\partial x}{\mu_2} = \frac{x}{1-x}\left[1 - \frac{m(1/\theta/x)}{\mu_2}(1-\alpha(\theta/x))\right] < 0 \tag{3.4}$$

$$\theta\frac{\partial\mu_2/\partial\theta}{\mu_2} = \frac{\alpha(\theta)}{1-x}\frac{m(1,\theta)}{\mu_2}\left[1 - x\frac{\alpha(\theta/x)}{\alpha(\theta)}\frac{m(1,\theta/x)}{m(1,\theta)}\right] > 0 \tag{3.5}$$

$$x\frac{d\theta/dx}{\theta} = \frac{\rho}{\rho + (1-\rho)x\frac{(1-\alpha(\theta))m(1,\theta)}{(1-\alpha(\theta/x))m(1,\theta,x)}} > 0 \tag{3.6}$$

where $\alpha(\theta) = \theta m_2(1,\theta)/m(1,\theta) \in (0,1)$ for all $\theta \geq 0$.

Any change in x exerts two conflicting effects on job opportunities for the unskilled. On the one hand, there is a negative *crowding-out effect*. It is a simple partial equilibrium effect due to the fact that the skilled are favoured in the matching process. On the other hand, there is a positive *labour demand* effect. It is a general equilibrium effect according to which the total number of advertised jobs raises with the share of skilled workers. Indeed, job creation is driven by profitability, and rent-sharing implies the profitability of a job is increasing in skill. It follows tightness raises with the share of skilled.

Which of these two effects is the largest? The response depends on the skill premium ρ. Indeed, the labour demand effect is proportional to $\varepsilon_{\theta,x} \equiv x(d\theta/dx)/\theta$, the elasticity of equilibrium tightness with respect to the skill proportion. In turn,

$$\frac{d\varepsilon_{\theta,x}}{d\rho} = x\left(\frac{\varepsilon_{\theta,x}}{\rho}\right)^2 \frac{(1-\alpha(\theta))m(1,\theta)}{(1-\alpha(\theta/x))m(1,\theta,x)} > 0 \tag{3.7}$$

One must check that matching probabilities are well defined, i.e. lower than one. This is so whenever $m(1,\theta/x) < 1$, and $m(1,\theta)/\theta < 1$. These conditions are always satisfied with the urn-ball technology. In this case, there exists an equilibrium iff $0 < c < 1$.

The higher the skill differential, the higher the labour demand effect. Suppose first $\rho = 0$. Then, all workers are equally productive. It follows that $\varepsilon_{\theta,x} = 0$ and equilibrium tightness does not respond to changes in the composition of the workforce. The labour demand effect vanishes as a result. Then, the crowding-out effect implies that the welfare of unskilled workers strictly decreases with the skilled proportion. As ρ increases, the elasticity of equilibrium tightness with the skilled proportion increases too and the magnitude of the labour demand effect raises. In the limit case where ρ tends to 1, we have

$$\begin{aligned} x\frac{d\mu_2/dx}{\mu_2}\bigg|_{\rho=1} &= \frac{x}{1-x}\left[1-\frac{\left(1-\alpha\left(\theta/x\right)\right)m\left(1,\theta/x\right)}{\mu_2}\right] \\ &\quad +\frac{\alpha\left(\theta\right)}{1-x}\frac{m\left(1,\theta\right)}{\mu_2}\left[1-x\frac{\alpha\left(\theta/x\right)}{\alpha\left(\theta\right)}\frac{m\left(1,\theta/x\right)}{m\left(1,\theta\right)}\right] \end{aligned}$$

which has the sign of

$$\phi\left(x\right) = m\left(1,\theta\right)\left[x+\alpha\left(\theta\right)-\alpha\left(\theta\right)x\right]-xm\left(1,\theta/x\right) \tag{3.8}$$

But $\phi(0) > 0$, $\phi(1) = 0$ and $\phi'(x) = (1-\alpha(\theta))\,m(1,\theta) - (1-\alpha(\theta/x))\,m(1,\theta/x) < 0$ because m is strictly concave. Therefore, we have $x\frac{d\mu_2/dx}{\mu_2}\Big|_{\rho=1} > 0$. It follows that there exists a unique skill differential $\widehat{\rho} \in (0,1)$ such that $x\frac{d\mu_2/dx}{\mu_2} > 0$ if and only if $\rho > \widehat{\rho}$. In words, the labour demand effect dominates the crowding-out effect whenever the skill differential is sufficiently strong. This result holds for a very large class of matching technologies, including the Cobb-Douglas function as well as the urn-ball technology.

4. References

Acemoglu, D.T, 1999. Changes in unemployment and wage inequality: an alternative theory and some evidence. American Economic Review 89, 1259-1278

Dolado, J., Jansen, M., Jimeno, J., 2003. On-the-job search in a matching model with heterogenous jobs and workers. IZA working paper

Gautier, P., 2002. Unemployment and search externalities in a model of heterogeneous jobs and workers. Economica 273, 21-40

Rosen, A., Wasmer, E., 2005. Higher education levels, firms' outside options and the wage structure. Labour 19, 621-654

Lang, K., Manove, M., 2003. Wage announcements with a continuum of types. Annales d'Economie et de Statistique 71-72, 223-244

Shi, S., 2001. Frictional assignment. I. Efficiency. Journal of Economic Theory 98, 232-260

Shi, S., 2002. A directed search model of inequality with heterogeneous skills and skill-biased technology. Review of Economic Studies 69, 467-491

Shimer, R., 2005. The assignment of workers to jobs in an economy with coordination frictions. Journal of Political Economy 113, 996-1025

NOT TO BE PUBLISHED

This Appendix is not designed to be published. I discuss with greater details the existence of a non-trivial equilibrium. I mainly provide with constraints on the matching technology m and the cost of creating a job c.

An equilibrium is a tightness/mass number of firms $\theta^* > 0$ which satisfies

$$\psi(\theta/x)\rho + \psi(\theta)(1-\rho) = c \tag{i}$$

$$m(1, \theta/x) < 1 \tag{ii}$$

$$m(1, \theta)/\theta < 1 \tag{iii}$$

Condition (i) is the free-entry condition. There is a non-trivial number of firms that must be compatible with the zero-profit condition.

Conditions (ii) and (iii) check that there is an interior equilibrium. (ii) says that type-1 worker's matching probability is lower than one. This also guarantees that type-2 worker's matching probability is lower than one. (iii) says that firm's matching probability is lower than one. Given that $m(n, v)$ is strictly concave, $m(1, a)/a$ is strictly decreasing in a. This implies uniqueness.

I now deal with the existence of equilibrium. In this goal, I make two (weak) assumptions on the technology m:

A1 There is a unique θ_- such that $m(1, \theta_-)/\theta_- = 1$

A2 There is a unique θ_+ such that $m(1, \theta_+/x) = 1$, or $\lim_{\theta \to \infty} m(1, \theta/x) = 1$. In the latter case, $\theta_+ = \infty$.

I can state the following existence result.

There exists a unique equilibrium iff

$$\theta_- < \theta_+ \tag{a}$$

$$\frac{m(1, \theta_-/x)}{\theta_-/x}\rho + 1 - \rho > c \tag{b}$$

$$\frac{m(1, \theta_+/x)}{\theta_+/x}\rho + \frac{1-\rho}{\theta_+} < c \tag{c}$$

The proof is simple. Let $\phi(\theta) = \psi(\theta/x)\rho + \psi(\theta)(1-\rho) - c$. An equilibrium solves $\phi(\theta^*) = 0$. Condition (a) offers an interval of values for θ that is compatible with conditions (ii) and (iii) exposed above. Condition (b) and (c) together state that $\phi(\theta_+) < 0 < \phi(\theta_-)$. The continuity of ϕ then implies that there is a $\theta^* \in (\theta_-, \theta_+)$ such that $\phi(\theta^*) = 0$. Given that ϕ is strictly decreasing, θ^* is unique.

Let me consider two examples: a Cobb-Douglas matching technology, and the urn-ball matching technology.

In the Cobb-Douglas case, we have $m(1,\theta) = A\theta^{\alpha}$, $A < 0$ and $\alpha \in (0,1)$. One can check that

$$\theta_{-} = A^{\frac{1}{1-\alpha}} \tag{4.1}$$

$$\theta_{+} = xA^{-1/\alpha} \tag{4.2}$$

The set of conditions above is the following

$$A^{\frac{1}{\alpha(1-\alpha)}} < x \tag{a}$$

$$x^{1-\alpha}\rho + 1 - \rho > c \tag{b}$$

$$A^{1/\alpha}\rho + x^{-1}A^{1/\alpha}(1-\rho) < c \tag{c}$$

It is easy to construct an equilibrium. Choose x, α and ρ and pick c such that $x^{1-\alpha}\rho + 1 - \rho > c$. Then, set a sufficiently low A to satisfy conditions (a) and (c).

In the urn-ball technology case, we have $m(1,\theta) = \theta\left(1 - e^{-1/\theta}\right)$. It follows that $\theta_{-} = 0$ and $\theta_{+} = \infty$, which imply that (a) holds. Then, it is easy to check that (b) is equivalent to $c < 1$, and (c) is equivalent to $c > 0$. It follows that there exists a unique equilibrium iff $0 < c < 1$.