Borrowers’ Participation in Group Borrowing

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12 February 2016

Online at https://mpra.ub.uni-muenchen.de/69506/
MPRA Paper No. 69506, posted 13 February 2016 12:36 UTC
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Abstract

Borrowers’ participation in MFI group lending credit market is not insured because of the alternative sources of credit available. The question arises what is the ideal MFI interest rate to ensure borrowers’ participation which at the same time being financially viable for MFI. The paper attempts to answer this question and analyzes the borrowers’ trade-off of borrowing from MFI or from moneylender (ML). Results show that borrowers may find comparative advantage in borrowing individually from ML as compared to borrowing in a group from MFI if the transaction cost burden is high and their credit requirement is low.

Keywords: Microfinance, Group lending, Informal finance, Transaction cost, Effective cost

JEL classification: G21, O16, O17

*The paper is derived from the fourth Chapter of the dissertation submitted to Centre for International Trade and Development (CITD), Jawaharlal Nehru University (JNU), New Delhi, India in partial fulfillment of the requirements for the award of the degree of Master of Philosophy (M.Phil) by the author. I am grateful to Dr. Mandira Sarma, Associate Professor, CITD, JNU for her valuable guidance.
1. Introduction

The concept of microfinance and group lending was popularized by the Grameen Bank of Bangladesh in response to the problems faced by poor and marginal borrowers who are in need for small credit and who are unable to pledge any collateral to secure credit. Under a typical Grameen Bank lending contract, lending happens in groups with joint liability, regular repayment schedule and dynamic incentive clause. This model proved to be very successful as shown by repayment rates of over 95% (Besley and Coate, 1995). The success of microfinance group lending has led to an extensive and growing literature on the subject. The models of Stiglitz (1990), Besley and Coate (1995), Ghatak (1999), Aghion (1999) and Aghion and Gollier (2000) show how Grameen type group lending with joint liability helps to mitigate the effect of information asymmetry between the lender and the borrower by exploiting the local information about the borrowers. This is made possible through borrowers’ participation in group formation, peer monitoring, and imposing social sanctions on the defaulting borrowers, among others.

Notwithstanding the extensive and still growing literature on microfinance and group lending, most theoretical literature has approached the group based lending from the lenders’ perspective. It shows how group lending contracts with joint liability help to solve lenders’ agency problems like adverse selection, ex-ante moral hazard and ex-post moral hazard. On the other hand, the borrowers’ perspective considers the additional burden to the borrowers in the form of transaction costs when borrowing happens in a typical group lending contract with joint liability and regular repayment schedule. Transaction costs include the opportunity cost of attending weekly repayment meetings, cost of travelling to attend meetings, cost of monitoring the group member etc. The problem of borrowers’ transaction costs in group lending has been discussed by Chung (1995), Bhatt and Tang (1998) (who term these costs as ‘hidden beasts’), Pal (2002), Karduck and Seibel (2004), Dehem and Hudon (2013), among others. However there is very limited theoretical literature which has explicitly incorporated this information in the MFI’s decision framework.

Against this background, we attempt to provide a theoretical framework around the borrowers’ trade-off of group borrowing from MFI versus individual borrowing from ML when both MFI and ML co-exist in the credit market and when both are equally competent to meet the borrowers’ funding requirements. We assume that borrowing from MFI comes at lower interest cost but with
the additional transaction cost while borrowing from ML comes at higher interest cost without incurring any transaction cost.

We solve the MFI’s optimization problem (profit maximization) considering borrowers’ participation constraint and repayment feasibility constraint to produce threshold or optimum MFI interest rate. The threshold MFI interest rate is interpreted as the maximum rate which an MFI can charge to ensure borrowers’ participation in the group lending contract while at the same time maximizing its profits. To extend this further, we perform numerical simulations. The parameter estimates to perform simulations are taken from transaction cost estimation studies done in the Indian context, primarily Karduck and Seibel (2004), Shankar (2007) and Dehem and Hudon (2013).

Results show that the increased transaction cost burden negatively impacts poor and marginal borrowers who are believed to have lower credit requirement. It posits the possibility of poor and marginal borrowers being excluded from the MFI credit market even if it does not require them to offer any collateral as in the other sources of financing from the formal sector. This also partly explains the relative stable dependence of borrowers’ funding requirements on ML as observed in many developing countries.

The paper contains six sections. This introductory section talks about the objective and also gives a brief mention of results. Section 2 talks about the MFI group lending contract and ML individual lending contract. It sets up the expressions for expected utility function of representative borrower when she borrows from MFI and when she borrows from ML. Section 3 lists some of the necessary assumptions deployed to derive optimum MFI interest rate. Section 4 solves the MFI’s optimization problem by maximizing its profit function subject to borrowers’ participation constraint and repayment feasibility constraint. The solution to MFI’s optimization problem results in threshold/optimum MFI interest rate. Also, it lists some of the parameter restrictions and the results on comparative statics. The next section presents the simulation results performed on optimum MFI interest rate. Section 6 concludes the paper.

2. MFI and ML contracts
We assume there are two borrowers and are considering of investing in a project. They do not have initial wealth and hence cannot offer any collateral. Therefore, the borrowers are credit rationed from commercial banking sources. They have a choice of borrowing from MFI as a group (group of 2 people) or borrowing from ML individually. The interest rate charged by MFI is lower than that of ML. However, there are additional costs that the members have to bear when they borrow from MFI. This includes costs associated with joint liability, transaction costs like weekly repayment, monitoring the other member, opportunity cost of time to attend weekly meeting, among others. There is no additional burden in the form of transaction cost when borrowing happens from ML.

It is assumed that there is an indivisible project which requires an investment of amount K (K > 0) at the beginning of period 1. The indivisibility of the project implies project will not produce any returns when there is an investment of amount lower than K. When successful, it will realize positive returns of amount Y at the end of period 2. The probability of project being successful (generating sufficiently high returns to repay MFI and ML) is p, where p ∈ (0,1). Returns realized Y depends upon the ability factor α and amount borrowed K (Gine, 2011). The ability factor α (α > 0) presents the ability of the individual borrower to convert capital invested into successful realization of returns. A representative borrower requires a loan of amount K at the beginning of period 1.

Case I: When borrower partners with the other borrower and takes a group loan from MFI

- Borrower forms a group with the other member and takes a joint or group loan of amount 2K from MFI at an interest rate of r in the beginning of period 1. Borrowers divide the loan equally and invest in the project individually.

- The representative borrower needs to repay some amount, s, to MFI with interest r (r > 0) as an installment at the end of period 1, and the remaining (K - s) with interest at the end of period 2 (Jain and Mansuri, 2003). MFI is assumed to charge interest rate on flat rate basis, implies fixed proportion of the amount borrowed.

- Since returns are only realized at the end of period 2, therefore she needs to borrow from ML to repay the MFI installment at the end of period 1. It is assumed that among the informal sources of lending, moneylenders constitute the largest share (Pradhan, 2013). Suppose ML gives loans at an interest rate of m (m > 0), where m > r. The borrower
borrows an amount of \( s(1+r) \) from ML at the end of period 1 and needs to repay an amount of \( s(1+r)(1+m) \) to ML at the end of period 2.

- The amount due to MFI at the end of period 2 for the individual borrower reduces to \( (K-s)(1+r) \).

- When both the borrower and her partner are successful (happens with probability \( p^2 \)), there is no joint liability payment. However, when one partner is unsuccessful while the other borrower is successful (happens with probability \( p^*(1-p) \)), the successful member needs to repay an additional amount of \( (K-s)(1+r) \) to MFI on behalf of her unsuccessful peer. This is because the MFI offers loans involving dynamic incentive clause, implies future loans are made only when current dues are fully paid.

- There are fixed transaction costs \( T_c \) \((T_c > 0)\) involved when borrower borrows from MFI. \( T_c \) is transaction cost burden per member.

The expected utility function of each borrower takes the following form:

\[
\begin{align*}
\text{EU}_{\text{MFI}} &= \left[ Y - s(1+r)(1+m) - (K-s)(1+r) - T_c \right] p^2 \\
&\quad + \left[ Y - s(1+r)(1+m) - 2(K-s)(1+r) - T_c \right] p(1-p)
\end{align*}
\]

Assuming \( Y = \alpha K \) (Gine, 2011), the above expression is re-written as:

\[
\begin{align*}
\text{EU}_{\text{MFI}} &= \left[ \alpha K - s(1+r)(1+m) - (K-s)(1+r) - T_c \right] p^2 \\
&\quad + \left[ \alpha K - s(1+r)(1+m) - 2(K-s)(1+r) - T_c \right] p(1-p)
\end{align*}
\]  

The first expression inside square bracket denotes the case when both members in the group are successful (happens with probability \( p^2 \)), and hence there is no joint liability payment. The second expression represents the case when the representative borrower is successful while her peer is unsuccessful (which happens with probability \( p^*(1-p) \)) and involves an extra joint liability cost of \( (K-s)(1+r) \) at the end of period 2. Joint liability share is assumed to be 100 percent (Aghion, 1999; Aghion and Gollier, 2000) and project returns (when successfully realized) are assumed to be sufficiently high.

Case II: When borrower borrows from ML
• A representative borrower borrows individually an amount of K at an interest rate of m in the beginning of period 1.
• The borrower needs to repay K with interest at the end of period 2.
• As in the MFI case, p is the probability of successful return realization.

The expected utility in this case takes the following form:

\[ EU_{ML} = [\alpha K - K(1 + m)]p \]  

(2)

As in the MFI case, Y is assumed to be equal to \( \alpha K \).

3. Some Assumptions

We deploy some of the assumptions in an attempt to analyze group lending contracts from the borrowers’ perspective while at the same time keeping MFI’s group lending contracts financially viable. Borrowers are assumed to be risk-neutral and identical (Stiglitz, 1990; Ghatak, 1999), however their projects returns are not correlated. Probability of success (p) is assumed to be same for both borrowers by the positive assortative matching argument put forth in Ghatak (1999). It is assumed that borrowers have limited options in terms of number of projects and, hence the opportunity cost of putting effort is assumed to be zero.

MFI is assumed to be profit maximizer (Jain, 1999; Aghion, 1999), while ML is assumed to break even (Gine, 2011). Additionally, MFI is assumed to be profit maximizer period by period which implies repayments happen with interest both at the end of first and second period. It is assumed that MFI and ML are in direct competition with each other and are equally competent to meet borrowers’ capital requirement. Also, borrowers’ outside options in terms of competing MFIs are assumed to be limited (Field and Pande, 2008). We have assumed perfectly elastic supply of loanable funds to refrain from the possibility of any equilibrium credit rationing (Ghatak, 1999). The cost of funds is assumed to be negligible for both MFI and ML (Aghion, 1999).

The assumptions of positive assortative matching (Ghatak, 1999) and MFI’s observability over the borrowers’ capital requirement (Gine, 2011) ensure there is no adverse selection problem. The assumptions on ML being the principal source of lending among informal sources (Pradhan, 2013),
borrowers’ inability to offer any collateral (Bose, 1998), and no compulsory savings deposits in case of MFI borrowing (Pal, 2002) cumulatively imply borrowers have to take recourse to ML to repay MFI installments. Since ML is assumed to enjoy better monitoring capabilities and lend only when borrowers invest in safe projects (Jain and Mansuri, 2003), this ensures there is no ex-ante moral hazard problem for the MFI. The assumption of negligible enforcement costs ensures borrowers cannot engage in strategic default and implies there is no ex-post moral hazard problem (Ghatak, 1999). These assumptions imply MFI is aware of borrowers’ characteristics like probability of project being successful, borrowers’ transaction cost and their ability to produce returns through ML involvement.

4. Solving for threshold/optimum MFI interest rate

The MFI profit function takes the following form:

\[
\pi = 2 \left[ s(1+r) + \left(1-(1-p)^2\right)(K-s)(1+r) - K \right]
\]  

(3)

The first term, \(s(1+r)\), represents the amount of first installment at the end of period 1 which is received with full certainty, because of borrowing from ML. The second term, \((K-s)(1+r)\), is the remaining amount to be received from borrower at the end of second period provided returns are realized successfully for at least one member (happens with probability of \(1-(1-p)^2\)). The third term, \(K\), is the amount of funds lent. Since there are two borrowers in the group, hence the three terms are multiplied by 2.

To solve for the threshold MFI interest rate or the optimum MFI interest rate \(r\) at which the representative borrower becomes indifferent between borrowing from MFI versus from ML, the expected utility functions, \(EU_{MFI}\) and \(EU_{ML}\) are equated. Also, while determining the optimum interest rate, MFI needs to ensure that repayment is feasible. Therefore, the MFI’s optimization problem is solved subject to the following two constraints:

- Borrower’s participation/indifference constraint which ensures that borrower is indifferent between borrowing from MFI and ML i.e.

\[
EU_{MFI} = EU_{ML} \text{ or,}
\]
\[
\alpha K - s(1+r)(1+m) - (K-s)(1+r) - T_c \right] p^2 + \left[ \alpha K - s(1+r)(1+m) - 2(K-s)(1+r) - T_c \right] p(l-p)
\]
\[
= [\alpha K - K(1+m)] p
\]
\[
K[(1+r)(p-2)+(1+m)] - s(1+r)(p+m-1) - T_c = 0
\]

- Repayment feasibility constraint which implies that returns (when successfully realized) are high enough to repay MFI and ML i.e.
\[
Y \geq s(1+r)(1+m) + 2(K-s)(1+r)
\]

The first term on the right side of the inequality \( s(1+r)(1+m) \) represents the payment due to ML and the second term \( (K-s)(1+r) \) is the payment due to MFI at the end of period 2. The second term is multiplied by 2 because of the assumption of full joint liability.

Putting \( Y = \alpha K \), the repayment feasibility inequality is re-written as:
\[
\alpha K \geq s(1+r)(1+m) + 2K(1+r) - 2s(1+r), \text{or}
\]
\[
K[\alpha - 2(1+r)] + s(1+r)(1-m) \geq 0
\]

MFI will maximize its profit function \( \pi \) and determines optimum interest rate \( r^* \) and optimum installment amount \( s^* \) subject to the borrower’s indifference constraint and the repayment feasibility constraint. The MFI’s optimization problem is written as:

Max \( r,s \)
\[
\pi = 2 \left[ s(1+r) + \left\{1-(1-p)^2\right\}(K-s)(1+r) - K \right] \quad \text{subject to:}
\]
\[
K[(1+r)(p-2)+(1+m)] - s(1+r)(p+m-1) - T_c = 0
\]
\[
K[\alpha - 2(1+r)] + s(1+r)(1-m) \geq 0
\]

The optimum \( r^* \) and \( s^* \) are as follows:
\[
r = r^* = \frac{(p+m-1)K\left\{\alpha - (1+m)\right\} - (1-m)T_c}{pK(1+m)}
\]
\[
s = s^* = K\left[\frac{K\alpha(p-2) + 2K(1+m) - 2T_c}{K\alpha(p+m-1) + (1-m)K(1+m) - (1-m)T_c}\right]
\]
The parameter restriction of \( p(1+m) > 1 \) ensures \( p \) to be sufficiently high (> 1/2) when ML interest rate is less than 100%. This is in consonance with some of the empirical studies like Ahmed (1989) and Pradhan (2013). Ahmed (1989) gave empirical evidence of ML interest rate of nearly 40% in
Bangladesh while Pradhan (2013) showed empirical evidence of ML interest rate of around 25-30% on an average in the Indian context. The parameter restriction 2 ensures borrowers’ participation constraint to be binding in the MFI’s optimization problem. Conditions 1 and 2 are derived from the assumption of MFI being profit maximizer and determine its optimum interest rate considering both borrowers’ participation constraint and repayment feasibility constraint. The inequalities in 3 and 4 are expected to hold true when K is sufficiently high relative to transaction cost, Tc and α is sufficiently high relative to per unit cost of borrowing from ML (1 + m). Similarly, Condition 5 is derived to satisfy the requirement of MFI interest rate to be lower than ML interest rate in conjunction with the requirement of maximum installment size of K. Restrictions on ability factor (conditions 6, 7), α ensures expected utility of borrowing from ML and transaction costs are positive. The last two conditions, 8 and 9, ensure meaningful values of all the parameters.

The parameter restrictions listed above help to determine how the threshold MFI interest rate r* changes when there is a marginal change in one of the parameter, keeping other parameters constant. We derive the following lemmas:

**Lemma 1:** With the increase in amount borrowed K, threshold interest rate r* also increases when p is sufficiently high (p > 1/2).

**Lemma 2:** r* falls with the increase in transaction costs Tc when p > 1/2.

**Lemma 3:** With the increase in ability factor α, threshold interest rate r* increases.

**Lemma 4:** There is a positive relation between p and r*.

**Lemma 5:** The relation between ML interest rate m and threshold interest rate r* is ambiguous.

Proof: See Appendix 3

The above lemmas establish that threshold MFI interest rate r* increases with the amount borrowed K, probability of successful return realization p, and the ability factor α, keeping other parameters constant. With the increase in transaction cost, threshold MFI interest rate goes down to keep the borrower indifferent between borrowing from MFI and from ML. The comparative statics results are largely dependent on the threshold level of probability of 1/2. This indicates the possibility of having one member’s project as successful out of the two member group. With the assumption of full joint liability and project returns (when successful) being sufficiently high, the repayment to
MFI happens with full certainty if at least one member’s project is successful. An interesting case is of lemma 5 wherein the relation between \( r^* \) and \( m \) could be positive or negative. At lower values of \( m \) the relationship is expected to be positive, however after a particular threshold of \( m \), the relationship becomes negative. This is explained better with the help of numerical simulations in next section.

5. Simulation results on threshold MFI interest rate

We perform simulations on the threshold MFI interest rate, changing one of the parameters among \( p, \alpha, K, m \) and \( T_c \) while keeping others at some constant value. The range of parameter estimates are taken from Karduck and Seibel (2004), Banerjee and Duflo (2010), Dehem and Hudon (2013), Pradhan (2013) and Ahlin (2013). In particular, we consider \( K \) to vary between 3800 and 6800, \( T_c \) to vary between 150 and 400, and \( m \) to vary between 0.2 and 0.6. The estimates on the ability factor and probability of successful return realization are limited. The estimates around marginal productivity of capital (MPK) are taken as a proxy for the ability factor (\( \alpha \)). Banerjee and Duflo (2010) peg the MPK estimates at 75%-90% of net and gross returns respectively while Ahlin (2013) in his simulation study use a wider range of 60%-100%. Since we have made the assumption of ML borrowing to repay MFI installments and ML is expected to have better monitoring capabilities, hence borrowers are expected to invest in projects having higher probability of success. In consonance with Banerjee and Duflo (2010) and Ahlin (2013), we use \( \alpha \) range of (1.6, 2) and \( p \) range of (0.5, 0.99). We review the simulation results below. The graphs below put threshold MFI interest rate on Y-axis and the parameter considered on X-axis.

We consider five cases in total described in Table 1 and 2. These five cases are shown in figures 1 till 5. We fix four parameters at a time and change any one of the parameters among \( m, K, T_c, p \) and \( \alpha \). This is shown under header Fixed (parameters fixed at a particular value) and Variable (parameter changing value in a continuous range) in the tables below. We consider four to six parameter combination values for each case.

| Table 1: Parameter combinations considered in simulation results |
Figure 1: Threshold MFI interest rate as a function of ML interest rate $m$

<table>
<thead>
<tr>
<th></th>
<th>Figure 1</th>
<th>Figure 2</th>
<th>Figure 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed</td>
<td>$p$, $\alpha$, $K$, $T_c$</td>
<td>$m$, $\alpha$, $K$, $T_c$</td>
<td>$m$, $p$, $K$, $T_c$</td>
</tr>
<tr>
<td>Variable</td>
<td>$m$ (0.2, 0.6)</td>
<td>$p$ (0.5, 0.99)</td>
<td>$\alpha$ (1.6, 2)</td>
</tr>
<tr>
<td>a)</td>
<td>$K = 3900$, $T_c = 350$, $p = 0.85$, $\alpha = 1.7$</td>
<td>$K = 3900$, $T_c = 350$, $m = 0.3$, $\alpha = 1.7$</td>
<td>$K = 3900$, $T_c = 350$, $m = 0.3$, $p = 0.85$</td>
</tr>
<tr>
<td>b)</td>
<td>$K = 3900$, $T_c = 290$, $p = 0.85$, $\alpha = 1.7$</td>
<td>$K = 3900$, $T_c = 160$, $m = 0.3$, $\alpha = 1.7$</td>
<td>$K = 3900$, $T_c = 290$, $m = 0.3$, $p = 0.85$</td>
</tr>
<tr>
<td>c)</td>
<td>$K = 6700$, $T_c = 350$, $p = 0.85$, $\alpha = 1.7$</td>
<td>$K = 6700$, $T_c = 350$, $m = 0.3$, $\alpha = 1.7$</td>
<td>$K = 6700$, $T_c = 350$, $m = 0.3$, $p = 0.85$</td>
</tr>
<tr>
<td>d)</td>
<td>$K = 6700$, $T_c = 290$, $p = 0.85$, $\alpha = 1.7$</td>
<td>$K = 6700$, $T_c = 160$, $m = 0.3$, $\alpha = 1.7$</td>
<td>$K = 6700$, $T_c = 290$, $m = 0.3$, $p = 0.85$</td>
</tr>
<tr>
<td>e)</td>
<td>$K = 6700$, $T_c = 160$, $p = 0.85$, $\alpha = 1.7$</td>
<td></td>
<td>$K = 6700$, $T_c = 160$, $m = 0.3$, $p = 0.85$</td>
</tr>
</tbody>
</table>

Source: The author

Figure 1 represents threshold MFI interest rate as a function of ML interest rate $m$. We observe a concave relationship between $r^*$ and $m$ as supported by the comparative statics results derived. Initially, with the increase in ML interest rate $m$, there is an increase in threshold MFI interest rate. However, beyond a certain level of ML interest rate, the threshold MFI interest rate goes down to keep the borrower indifferent in borrowing from two sources and to ensure repayments of the borrowed debt.
For the given values of $K$, $p$ and $\alpha$, a higher transaction cost amount shifts the $r^*$ curve downwards. This is shown by the graph for case a lying below the graph for case b at lower values of $K$ (3900) and graph for case c lying below the graph for case a and b. Higher values of $K$ result in higher level of threshold interest rate as compared to the case when $K$ is lower as shown in the graphs for cases c, d and e lying above the graphs for cases a and b.

Figure 2 shows threshold MFI interest rate as a function of probability $p$. The threshold interest rate is positive for sufficiently high $p$ values. A lower transaction cost ($T_c = 160$) combined with higher amount borrowed ($K = 6700$) leads to positive $r^*$ when probability of success is at least of the level of 74% (case d), while for higher transaction costs ($T_c = 350$), the corresponding probability level got increased up to 79% (case c).

Figure 2: Threshold MFI interest rate as a function of probability $p$
For lower values of $K$ (3900), the probability level beyond which $r^*$ is positive increases to 86% in case a and 77% in case b. These results are in consonance with Jain and Mansuri (2003) wherein the ML is expected to lend to borrowers to repay MFI installments only when they invest in safe projects having higher probability.

Figures 3 shows the threshold MFI interest rate curve as a function of ability factor $\alpha$. With the increase in transaction cost (keeping $K$, $m$ and $p$ fixed), ability factor $\alpha$ level also increases beyond which threshold $r^*$ is positive.

**Figure 3: Threshold MFI interest rate as a function of ability factor $\alpha$**

Source: The author

Threshold $r^*$ is positive when $\alpha$ crosses the threshold of 1.72 in case a and 1.65 in case b. If the amount borrowed is high enough ($K = 6700$), threshold $r^*$ is positive in the entire range of $\alpha$ considered (1.6, 1.99) in all the three cases (c, d and e).

The graphs in figure 4 show the relation between the threshold MFI interest rate and amount borrowed $K$. We observe that with the lower transaction cost ($T_c = 290$), the rate of change in $r^*$ with respect to $K$ is smaller (the curve is flatter) as compared to the case when transaction cost is high ($T_c = 350$) keeping rest of the parameters at some constant level. This is shown for graphs in cases d, e and f have relatively lower slope than the graphs in cases a, b and c for the given level of $m$, $p$ and $\alpha$.

**Table 2: Parameter combinations considered in simulation results (contd.)**
An increase in ML interest rate or ability factor shifts the $r^*$ curve upwards keeping rest of the parameters fixed at particular level. This is shown by the graphs for cases b and c lying above the graph for case a when $T_c$ is fixed at 350.

Figure 5 shows the threshold interest rate $r^*$ curve with respect to transaction cost. With the given level of $K$ and $p$, a higher ability factor or higher ML interest rate shifts the threshold interest rate curve upwards. However, the extent of increase in threshold $r^*$ is higher in the case of higher ability factor as compared to the case of higher ML interest rate. This is shown in the graphs for
case b and c lying above the graph of case a (assume $K = 6700$), however graph for case b is still above the graph for case c.

**Figure 5: Threshold MFI interest rate as a function of transaction cost $T_c$**

Source: The author

Threshold $r^*$ is positive for each value of $T_c$ when $K$ is higher at 6700 (cases a, b and c). At lower $K$ (3900), $r^*$ is positive in the entire range of $T_c$ considered (150, 400) when either $\alpha$ is high (1.8) as in case e or $m$ is high (0.35) as in case f. In case d, $r^*$ is positive if $T_c$ lies in the range of (150, 330).

6. Conclusion

To conclude, results show that with increased transaction costs, and relative lower credit requirement combined with low average productivity (ability factor), threshold MFI interest rate and the corresponding MFI profit becomes negative and hence becomes unviable to offer group lending contract. This is due to the assumption of MFI being profit maximizer. However, if MFI wants to offer such a contract, it needs to raise its interest rate above the threshold interest rate. When the actual interest rate charged is higher than the threshold MFI interest rate, the effective cost of borrowing from ML becomes relatively lower and the objective of reducing dependence on ML gets diluted.

Therefore, to begin with both MFI and ML are equally competent to meet borrowers’ credit requirement, but with the inclusion of borrowers’ participation constraint, MFI lending becomes feasible only under certain conditions. Results show there is higher probability of MFI lending to be feasible when there is relative higher credit requirement or higher ability of borrowers to
produce returns or lower transaction costs of borrowers. The additional transaction cost burden involved in MFI borrowing works against the poor borrowers in particular, who are believed to have lower ability to produce returns and also have small credit requirement. Therefore, from the policy standpoint, these results reiterate the importance of reducing transaction costs to enhance borrower welfare as Bhatt and Tang (2001), Field and Pande (2008), Laureti (2012) and several other authors have pointed out.

Appendix 1

Deriving optimum MFI interest rate r* (equation 6) and optimum installment amount s* (equation 7)

Lagrange function L is written as:

\[
L = 2\left[ s(1+r) + \left\{ 1 - (1 - p)^2 \right\} (K - s)(1+r) - K \right] \\
- \lambda_1 \left[ K \left\{ (1+r)(p-2) + (1+m) \right\} - s(1+r)(p+m-1) - T \right] \\
+ \lambda_2 \left[ K \alpha - 2(1+r) \right] + s(1+r)(1-m)
\]

\[
\frac{\partial L}{\partial r} = 2s + 2(2p-p^2)(K-s) - \lambda_1 [K(p-2)-s(p+m-1)] + \lambda_2 [-2K+s(1-m)]
\]

\[
\frac{\partial L}{\partial s} = 2(1+r) + 2(2p-p^2)(1+r)(-1) - \lambda_1 [-(1+r)(p+m-1)] + \lambda_2 [(1+r)(1-m)]
\]

Putting \( \frac{\partial L}{\partial r} = 0 \) and \( \frac{\partial L}{\partial s} = 0 \)

Getting \( \lambda_1 \) from both equations,

\[
\lambda_1 [K(p-2)-s(p+m-1)] = 2s + 2(2p-p^2)(K-s) + \lambda_2 [-2K+s(1-m)]
\]

\[
\lambda_2 (1+r)(p+m-1) = -2(1+r) + 2(2p-p^2)(1+r) - \lambda_2 [(1+r)(1-m)] \text{ or },
\]

\[
\lambda_2 (p+m-1) = -2 + 2(2p-p^2) - \lambda_2 (1-m)
\]
Equating the two equations for $\lambda_1$ and solving for $\lambda_2$, 

$$\lambda_2 = \frac{2(p - 2)(1 - p(1 + m))}{(p + mp)}$$

The Kuhn- Tucker conditions for constrained maximization ensure $\lambda_2 > 0$ for repayment feasibility constraint to be binding at the optimum solution. This implies $p(1 + m) > 1$.

Solving for $\lambda_1$ using $\lambda_2$ equation yields the following:

$$\lambda_1(p + m - l) = -2 + 2(2p - p^2) - \lambda_2(l - m)$$

$$\lambda_1(p + m - l) = -2 + 2(2p - p^2) - (l - m).2(2 - \frac{1 - p(1 + m)}{p + mp})$$

$$\lambda_1 = \frac{2[p(1 + m)(2 - p) - 2]}{p(1 + m)}$$

$\lambda_1$ is Lagrange multiplier for equality constraint, and hence it can take any real number. However for constraint to be binding, $\lambda_1$ should be non-zero which implies $p(1 + m)(2 - p) \neq 2$

Solving for $r$ and $s$ using the following two constraint equations;

$$K[(1 + r)(p - 2) + (1 + m)] - s(1 + r)(p + m - 1) - T_c = 0 \text{ and}$$

$$K[\alpha - 2(1 + r)] + s(1 + r)(1 - m) = 0$$

From 1$^{st}$ equation getting the value of $s$ and putting in 2$^{nd}$ equation,

$$s(1 + r) = \frac{1}{(p + m - 1)}\left[K\{1 + r)(p - 2) + (1 + m)\} - T_c \right]$$

From 2$^{nd}$ equation,

$$K[\alpha - 2(1 + r)] + \frac{(1 - m)}{(p + m - 1)}\left[K\{1 + r)(p - 2) + (1 + m)\} - T_c \right] = 0$$

Since $(p + m - 1) \neq 0$, therefore,

$$K(p + m - 1)[\alpha - 2(1 + r)] + (1 - m)\left[K\{1 + r)(p - 2) + (1 + m)\} - T_c \right] = 0$$
Solving for $r$,

$$(1+r) = \frac{K\alpha(p+m-1) + (1-m)K(1+m) - (1-m)T_c}{pK(1+m)}$$

$$r = r^* = \frac{(p+m-1)K\{(\alpha-(1+m)} - (1-m)T_c}{pK(1+m)}$$

Putting the value of $(1+r)$ in $s$ equation and solving for $s$ yields,

$$s(1+r) = \frac{1}{(p+m-1)}\left[ K\{(1+r)(p-2) + (1+m)\} - T_c \right]$$

$$s(p+m-1)\left[ \frac{K\alpha(p+m-1) + (1-m)K(1+m) - (1-m)T_c}{pK(1+m)} \right] = K(p-2)\left[ \frac{K\alpha(p+m-1) + (1-m)K(1+m) - (1-m)T_c}{pK(1+m)} \right] + K(1+m) - T_c$$

$$s = s^* = K\left[ \frac{K\alpha(p-2) + 2K(1+m) - 2T_c}{K\alpha(p+m-1) + (1-m)K(1+m) - (1-m)T_c} \right]$$

Since there are two equations to solve (borrower’s participation constraint and repayment feasibility constraint) with two unknowns ($r, s$), hence the first order conditions with constraints on Lagrange multipliers as shown above are sufficient conditions for constrained maximization (Kim, n.d.)

**Appendix 2**

Restrictions on parameters $K, \alpha, p, m, T_c$ (Section 4):

Following are the restrictions imposed on parameters values to establish $r^* \geq 0, s^* \geq 0, r^* \leq m, s^* \leq K, \lambda_2 > 0$ and $\lambda_1 \neq 0$

From $\lambda_2$ equation,

$$\lambda_2 = \frac{2(p-2)\{1-p(1+m)\}}{(p+mp)}$$
Since (p-2) is negative, therefore \(\{1-p(1+m)\}\) has to be negative as well which implies,

\[ p(1+m) > 1 \]  \hspace{1cm} (1)

From the above inequality, we have \(m \geq \frac{(1-p)}{p}\). For \(m < 1\), it should satisfy \((1-p)/p < 1\) which implies \(p > 1/2\).

From \(\lambda_1\) equation,

\[
\lambda_1 = \frac{2[p(1+m)(2-p)-2]}{p(1+m)}
\]

For \(\lambda_1\) to be non-zero, it should satisfy,

\[ p(1+m)(2-p) \neq 2 \]  \hspace{1cm} (2)

From \(r^* \geq 0\)

\[
r^* = \frac{(p+m-1)K\{\alpha-(1+m)\}-(1-m)T_c}{pK(1+m)} \geq 0
\]

\[(p+m-1)K\{\alpha-(1+m)\} \geq (1-m)T_c \]  \hspace{1cm} (3)

From \(r^* \leq m\)

\[
r^* = \frac{(p+m-1)K\{\alpha-(1+m)\}-(1-m)T_c}{pK(1+m)} \leq m
\]

\[(p+m-1)K\{\alpha-(1+m)\} \leq mpK(1+m)+(1-m)T_c \]  \hspace{1cm} (4)

From \(s^* \geq 0\)

\[
s^* = K \left[ \frac{K\alpha(p-2)+2K(1+m)-2T_c}{K\alpha(p+m-1)+(1-m)K(1+m)-(1-m)T_c} \right] \geq 0
\]

\[ K[\alpha(p-2)+2(1+m)] \geq 2T_c \]  \hspace{1cm} (5)

The above inequality is established since, the denominator of \(s^*\) is positive by inequality (3) above.
From $s^* \leq K$

$$s^* = K \left[ \frac{K\alpha(p-2) + 2K(1+m) - 2T_c}{K\alpha(p+m-1) + (1-m)K(1+m) - (1-m)T_c} \right] \leq K$$

$$K\alpha(p-2) + 2K(1+m) - 2T_c \leq K\alpha(p+m-1) + (1-m)K(1+m) - (1-m)T_c$$

$$(1+m)[-K\alpha + K(1+m) - T_c] \leq 0$$

$$K[(1+m) - \alpha] \leq T_c \quad (5')$$

For $EU_{ML}$ to be positive, it should satisfy the following

$$\alpha > (1+m) \quad (6)$$

From inequality (6) and assumption of $T_c > 0$, inequality (5’) is always satisfied and hence superfluous.

From inequality (5) in this appendix, we have,

$$T_c \leq \frac{K[\alpha(p-2) + 2(1+m)]}{2}$$

Since $T_c$ is assumed to be strictly positive, therefore it must satisfy

$$\frac{K[\alpha(p-2) + 2(1+m)]}{2} > 0$$

which implies,

$$\alpha < \frac{2(1+m)}{(2-p)} \quad (7)$$

The above parameter restrictions are valid for $K, \alpha, m, T_c$ all strictly positive and $p \in (0,1)$

**Appendix 3**

Comparative statics results of $r*$ (Section 4)

1. $\frac{\partial r}{\partial K}$
\[
\frac{\partial r}{\partial K} = \frac{1}{p(1+m)} \left[ \frac{K(p+m-1)(\alpha-(1+m)) - K(p+m-1)(\alpha-1+m) + (1-m)T_c}{K^2} \right]
\]

\[
\frac{\partial r}{\partial K} = \frac{(1-m)T_c}{p(1+m)K^2}
\]

Therefore,

\[
\frac{\partial r}{\partial K} > 0 \text{ if } m < 1 \text{ and, }
\]

\[
\frac{\partial r}{\partial K} < 0 \text{ if } m > 1
\]

\[
m < 1 \text{ implies } p > 1/2 \text{ and vice-versa from inequality (1) in Appendix 2}
\]

2. \[
\frac{\partial r}{\partial T_c}
\]

\[
\frac{\partial r}{\partial T_c} = \frac{-(1-m)}{p(1+m)K}
\]

\[
\frac{\partial r}{\partial T_c} < 0 \text{ if } m < 1 \text{ and, }
\]

\[
\frac{\partial r}{\partial T_c} > 0 \text{ if } m > 1
\]

\[
m < 1 \text{ implies } p > 1/2 \text{ and vice-versa from inequality (1) in Appendix 2}
\]

3. \[
\frac{\partial r}{\partial \alpha}
\]

\[
\frac{\partial r}{\partial \alpha} = \frac{(p+m-1)}{p(1+m)} > 0
\]

This is unambiguously positive because of \(p(1+m) > 1\), inequality (1) in Appendix 2
4. \( \frac{\partial r}{\partial p} \)

\[
\frac{\partial r}{\partial p} = \frac{1}{K(1+m)} \left[ \frac{pK \{ \alpha - (1+m) \} - (p + m - 1)K \{ \alpha - (1+m) \} + (1-m)T_c}{p^2} \right]
\]

\[
\frac{\partial r}{\partial p} = \frac{(1-m) \left[ K \{ \alpha - (1+m) \} + T_c \right]}{p^2 K(1+m)}
\]

The expression \( \left[ K \{ \alpha - (1+m) \} + T_c \right] \) is positive from \( s\leq K \) condition. Hence the sign of partial derivative is dependent upon if \((1-m)\) is positive or negative. Therefore,

\[
\frac{\partial r}{\partial p} > 0 \text{ if } m < 1 \text{ and }
\]

\[
\frac{\partial r}{\partial p} < 0 \text{ if } m > 1
\]

\( m < 1 \) implies \( p > 1/2 \) and vice-versa from inequality (1) in Appendix 2

5. \( \frac{\partial r}{\partial m} \)

\[
\frac{\partial r}{\partial m} = \frac{1}{pK} \left[ \frac{(1+m) \left\{ -K(p + m - 1) + K(\alpha - (1+m)) + T_c \right\}}{-(p + m - 1)K \{ \alpha - (1+m) \} + (1-m)T_c} \right]
\]\n
\[
\frac{\partial r}{\partial m} = \frac{1}{pK(1+m)^2} \left[ -K\alpha(p + m - 1) + K\alpha(1+m) - K(1+m)^2 + 2T_c \right]
\]

\[
\frac{\partial r}{\partial m} = \frac{1}{pK(1+m)^2} \left[ 2T_c - K(1+m)^2 - K\alpha(p - 2) \right]
\]
\[
\frac{\partial r}{\partial m} > 0 \text{ if } 2T \cdot K(2 - p) > K(l + m)^2 , \text{ and }
\]
\[
\frac{\partial r}{\partial m} < 0 \text{ if } 2T \cdot K(2 - p) < K(l + m)^2
\]

References


