Loss of Load Probability of a Power System

Vijayamohanan Pillai N

Centre for Development Studies, Prasanth Nagar, Ulloor, Trivandrum, Kerala, India

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Abstract

By virtue of the vital nature of electric power, both to our economic and personal well being, a power system is expected to supply electrical energy as economically as possible, and with a high degree of quality and reliability. The developed countries in general place higher reliability standards on the performance of electricity supply. However, there has been no significant study in the context of the Indian power sector to analyze reliability in terms of loss of load probability; the technical appraisal of the State power systems in general is confined to examining the plant load factor (PLF) as a measure of capacity utilization only. The present study is a modest attempt to evaluate the reliability of the Kerala power system in the framework of a theory-informed methodology – the first of its kind.

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1. Introduction

Electric power is vital to both our economic and personal well-being and hence a power system is expected to supply electrical energy as economically as possible, and with a high degree of quality and reliability. In fact, few products have a greater need for quality and reliability. Reliability in its broad sense refers to the probability that a component or system comprising components is able to perform its intended function satisfactorily during a specified period of time under normal operating conditions. Thus the reliability assessment of a power system is mainly concerned with its capability, which is related to the existence and availability of sufficient facilities to satisfy customer load. The basic facilities of a system are in the three sectors of its function, viz., generation, transmission and distribution, which are usually vertically integrated. Electric power produced at the generation end is carried to the consumers via transmission and distribution facilities. In this paper our focus is only on the generation sector.

A modern power system is very large and complex, composed of *n* power generating stations, where power is generated from fuels (fossil or nuclear) or by hydroelectric stations. Each generating station or plant consists of *M* plant units or generators, each with a rated capacity. Each of the *N* stations has an installed capacity K_i megawatts (mw), which is the sum of the rated capacities of its *M* units, and the system installed capacity to supply power is the sum of the installed capacities of all the stations. In the case of a hydropower system, each power station has usually associated with it a big reservoir behind a dam that supplies hydraulic power to drive each of the *M* generators.

A power system is unique in that its product is one that must be generated the instant its service is demanded. Another significant characteristic is that the demand for electricity varies greatly at random according to the time of the day and the season of the year.

Therefore a power system is designed to supply instantaneously the power demanded by consumers. However, failures in the system do occur when demand exceeds supply as in the case of any other goods and services.

Demand can exceed supply for two main reasons. One is the random deviations of the demand from its expected level such that a very high peak demand exceeds the installed capacity of the system. Capacity of a power system is in general determined after taking due considerations of such unforeseen fluctuations in demand. This is effected by means of reserve or standby capacity over and above the expected peak period demand that is to be met.

Shortage may still occur, even if the load is not far from its expectation; a high demand that does not exceed the installed capacity of the system can exceed the *available capacity* at that moment. This is due to generator deratings, scheduled preventive maintenance and forced outages of generators. Generator deratings result from equipment problems and changes in operating conditions, and are a function of the age of the equipment. Outage refers to a certain state of a unit when it becomes unavailable to perform its intended function due to some event directly associated with it. An outage may be either a scheduled one or a forced one. Scheduled outage (or maintenance outage) is a planned event, whereby a component/unit is deliberately taken out of service at a chosen time for preventive maintenance or overhaul or repair; this is to keep the generating units in proper running condition. Forced outage, on the other hand, results when a unit falls out of service due solely to random events such as breakdown, malfunction of equipment, etc.

In the case of a hydropower system, besides these two scenarios, shortage can still occur if the hydraulic power in any storage is not sufficient to turn the concerned generator. The plant unit is then shut down, and the system capacity falls accordingly.

A modest attempt is made in this paper to evaluate the reliability of the Kerala power system. Following a detailed discussion of the methodology used in this study, the maximum likelihood estimates of availability and forced outage rates as well as loss of load probability measures are calculated for the 10 hydropower plants of Kerala.

2. Loss of Load Probability: Theory

Availability and Outage Measures

In a Markov process, the life history of a repairable electric power system component during its useful life period is represented by a two-state model, the two possible states being labeled ‘up’ or ‘functioning’ and ‘down’ or ‘unavailable’, denoted by 1 and 0 respectively. Thus when the component fails, it is said to undergo a transition from the up to the down state, and conversely, when repairs are over, it is said to return from the down to the up state. This idea then facilitates to interpret the concept of reliability in terms of the fraction of total time the component remains in the up state. The length of functioning period is also referred to as the time-to-failure, and that of the period under repair as the downtime.

The probabilistic approach to power system reliability analysis views the system as a stochastic process evolving over time. At any moment the system may change from one state to another because of events such as component outages or planned maintenance. Corresponding to a pair of states, say (i, j), there is a conditional probability of transition from the state i to the state j.

Suppose the performance of a power plant is continuously monitored to record the sequence of failures and repairs during sustained operation in order to assess its performance. During each failure-repair cycle, the time to failure (when the plant is in up state) and the time to repair (when the plant is in down state) are recorded. The number of failures per unit of time is known as the failure (or hazard) rate, and the number of repairs per unit of time, the repair rate. The reliability of a power plant is often measured in terms of two availability indices, viz., instantaneous availability, $A(t)$, and steady-state (long-run) availability, $A(\infty)$. The former refers to the probability that the power plant is available for operation at any time t and the latter to its avaiability for large values of t, that is, in long run. Thus,

$$A(t) = \text{Prob(available at time } t\text{), and}$$

$$A(\infty) = \lim_{t \to \infty} A(t).$$

The first step in an availability study is to specify certain probability models for the two variables, time-to-failure, denoted by *X* and time-to-repair, denoted by *Y*. The second step is the derivation of the availability indices, which in general are the functions of the parameters of the statistical models specified for *X* and *Y*.

Usually the failure and repair rates are assumed to be constant; this leads to the assumption that the time-to-failure and the time-to-repair variables follow exponential distribution. The exponential distribution is one of the two (the other being the geometric distribution) unique distributions with the memoryless or no-ageing property. That is, future lifetime of a component remains the same irrespective of its previous use, if its lifetime distribution is exponential.

Thus we assume that the time-to-failure, *X*, is an exponential variable with parameter λ, so that its density function, *viz*., failure (hazard) density function, f(x), is given by f(x) = $\frac{1}{\lambda}\exp\left(\frac{-x}{\lambda}\right)$, for $x > 0$. The parameter $1/\lambda$ is the constant failure (hazard) rate. For an exponential distribution of the above form, the mean is given by λ. Hence the mean-time-to-failure (MTTF) of the power plant is equal to λ; this is also known as the expected survival time. The probability of a plant surviving at time t in a constant failure rate environment, i.e., its survival function, denoted by $R(t)$, is then obtained by integrating the failure density function, f(x), and is given by $R(t) = \exp(-x/\lambda)$. The complement of this survival probability is the probability of failure in time t, given by $1-\exp(-x/\lambda)$.

Similarly we assume an exponential model with parameter μ for the time-to-repair variable *Y*, so that the density function of *Y*, *viz*., the repair density function, g(y), is g(y) $= \frac{1}{\mu}\exp\left(\frac{-y}{\mu}\right)$, for $y > 0$. In this model, $1/\mu$ is the constant repair rate and its reciprocal, μ, is the mean down (repair) time (MDT) or the expected outage time. The sum of MTTF and MDT is termed the mean-time-between-failures (MTBF) or cycle time.

Shooman (1968, Chapter 6) and Gnedenko, Belyayev and Solovyev (1969, Chapter 2) have shown that for the above exponential models, the instantaneous availability of a power plant is

$$A(t) = \frac{\lambda}{(\lambda + \mu)} + \frac{\mu}{(\lambda + \mu)} \exp\left\{-(\frac{1}{\lambda} + \frac{1}{\mu})t\right\}.$$

The steady-state availability is obtained by taking the limit of $A(t)$ as t approaches infinity. This gives

$$A(\infty) = \frac{\lambda}{(\lambda + \mu)} = \frac{MTTF}{MTBF}.$$

Corresponding to these availability measures, we can also define two down-state probabilities, instantaneous forced outage, denoted by $R(t)$ and steady-state forced outage, denoted by $R(\infty)$ (Pillai,1992, Chapter 4). Thus the instantaneous forced outage rate of a plant is

$$R(t) = \frac{\mu}{(\lambda + \mu)} + \frac{\lambda}{(\lambda + \mu)} \exp\left\{-(\frac{1}{\lambda} + \frac{1}{\mu})t\right\},$$

and the long-run (steady-state) forced outage is

$$R(\infty) = \frac{\mu}{\lambda + \mu} = \frac{MDT}{MTBF}.$$

Now if we let $P_{ij}(t)$, $(i, j = 0,1)$ be the probability of the transition of state from i to j in a small interval of time t, where 1 denotes 'up' and 0, 'down' state in a Markov chain, it can be shown (*ibid.*) that the instantaneous availability and instantaneous forced outage rate, as obtained above, are nothing but the same state transition probabilities, $P_{11}(t)$ and $P_{00}(t)$ respectively. That is, $P_{11}(t) = A(t)$ and $P_{00}(t) = R(t)$. This gives us the remaining two transition probabilities (from 'up' to 'down' state and from 'down' to 'up' state) of the Markov chain:

$$P_{01}(t) = 1 - P_{00}(t) = \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu} \exp\left\{-\left(\frac{1}{\lambda}+\frac{1}{\mu}\right)t\right\}$$

$$P_{10}(t) = 1 - P_{11}(t) = \frac{\mu}{\lambda+\mu} - \frac{\mu}{\lambda+\mu} \exp\left\{-\left(\frac{1}{\lambda}+\frac{1}{\mu}\right)t\right\}$$

It is significant to note that the initial state probabilities obtained for $t = 1$ are nothing but the state transition probabilities, P_{ij}. That is $P_{01}(t = 1) = P_{01}$; $P_{00}(t = 1) = P_{00}$; $P_{10}(t = 1) = P_{10}$; and $P_{11}(t = 1) = P_{11}$.

When $t \to \infty$, these probabilities are known as limiting state probabilities that give the steady-state (or stationary or long-run) probabilities of the Markov chain:

$$\lim_{t\to\infty} P_{00}(t) = P_{00}(\infty) = \lim_{t\to\infty} P_{10}(t) = P_{10}(\infty) = R = \mu/(\lambda+\mu),$$

gives the forced outage rate (FOR) as defined earlier, and

$$\lim_{t\to\infty} P_{11}(t) = P_{11}(\infty) = \lim_{t\to\infty} P_{01}(t) = P_{01}(\infty) = A = \lambda/(\lambda+\mu).$$

is the availability rate.

Now it can be shown that $R = \dfrac{\mu}{\lambda + \mu} = \dfrac{1/\lambda}{1/\lambda + 1/\mu} = \dfrac{P_{10}}{P_{10} + P_{01}}$, and

$$A = \frac{\lambda}{\lambda + \mu} = \frac{1/\mu}{1/\lambda + 1/\mu} = \frac{P_{01}}{P_{10} + P_{01}},$$

where $P_{ij} = P_{ij}(1)$, $(i, j = 0,1)$, as specified above.

From this it follows that $P_{10} = 1/\lambda$, and $P_{01} = 1/\mu$, with $P_{00} = 1 - P_{01}$ and $P_{11} = 1 - P_{10}$, where $1/\lambda$ is the failure rate and $1/\mu$ is the constant repair rate. That is, we are now able to estimate all the four state transition probabilities simply by using the MTTF and MDT. This is an important result.

Maximum Likelihood Estimators

In practice, however, the parameters λ and μ of the exponential models assumed for the time-to-failure (X) and time-to-repair (Y) are usually unknown. Thus the availability and outage indices are also unknown for most practical problems. Hence we need to estimate these measures from a sample of values on X and Y. Note that both $A(t)$ and $A(\infty)$, as well as $F(t)$ and $F(\infty)$, are functions of λ and μ, the parameters of the exponential models assumed for X and Y. We can, therefore, obtain the maximum likelihood estimates of these measures by substituting the maximum likelihood estimators (MLE) of λ and μ in the above results (Zehna 1966).

To calculate the maximum likelihood estimators (MLE) of λ and μ, we observe the power plant unit through n failure-repair cycles, and collect the data on T time-to-failure (x_1, x_2, ….., x_T) and T time-to-repair (y_1, y_2, ….., y_T). Actually the data sets are two independent exponential samples.

The maximum likelihood procedure as developed in Kendall and Stuart (1967, Chapter 18) gives the following estimators:

the MLE of MTTF (λ): $\hat{\lambda} = \sum x_i/T = \bar{x}$, and

the MLE of MDT (μ): $\hat{\mu} = \sum y_i/T = \bar{y}$.

Then the maximum likelihood estimators of availability and outage are obtained by substituting λ and μ into the above results.

Thus the MLE of availability, $\hat{A} = \dfrac{\sum x_i}{\sum x_i + \sum y_i}$.

Thus the MLE of outage, $\hat{R} = \dfrac{\sum y_i}{\sum x_i + \sum y_i}$.

The steady-state (long-run) forced outage is generally known as forced outage rate (FOR), computed as a ratio of the unit's average down-time to the total available time, say, 720 hours a month; that is,

FOR = average forced outage hours/available hours.
The availability measure is then obtained as A = 1 – FOR.

Mobility

A measure of (what we call) 'propensity to down' (mobility) is given by

$D = \sum_{i=1}^{k} \sum_{j=1}^{k} P_i P_{ij} \,|\, i-j \,|$, where k is the number of states of nature, P_i is the long run probability and P_{ij} the transition probability, In the case of $k = 2$, with $i, j = 0, 1$, we have

$$P_1 = \frac{P_{01}}{P_{10}+P_{01}}, \quad P_0 = \frac{P_{10}}{P_{10}+P_{01}}, \text{ and hence } \quad D = \frac{2P_{01}P_{10}}{P_{10}+P_{01}}.$$

D varies between 0 for immobility (least propensity to down) and 1 for extreme propensity to down.

Capacity Outage Distribution

The next step in the generation reliability model is to combine the capacity and availability of the individual units to estimate expected available generation capacity in the system. Thus we obtain a capacity model, in which each generating unit is represented by its *nominal* capacity k_j and its FOR, R_j, j =1…N. Note that for each of the N units of the generating station, the *expected available* capacity k_j^A, j =1…N, is a random variable that can take the value 0 with probability R_j and the value k_j with probability $A_j = 1 - R_j$ as shown below:

$$k_i^A(k_i, R_i) \begin{cases} (k_j, A_j = 1 - R_j), \text{ if unit is available;} \\ \\ (0, R_j), \text{ if unit is in outage.} \end{cases}$$

Then the expected available capacity of a plant unit j is $k_j^A = k_j A_j$ and the expected total generating capacity available at the plant level is: $K^A = \sum_j^N k_j^A$.

Note that the available capacity at both the unit k_j^A and the plant level K^A is a random variable; and the units fail and get repaired independently of such events of other units. These conditions help us obtain the probability distribution of K^A by combining the independent individual probabilities of k_j^A. This in turn gives us a discrete (available) capacity distribution $K^A = (K_i, R_i\}$, $i = 1,\ldots,2^N$. The available capacity states takes on 2^N values, equal to the number of combinations of up and down units (due to forced outages) in an N-unit system. Each capacity state represents an outage event with one or more units unavailable. This capacity probability distribution is tabulated and referred to as the *capacity outage probability table*.

The capacity of the *i*th state, K_i, with M available units and $N - M$ failed units is the sum of the capacities of the M available units, that is,

$$K_i = K_1 + K_2 + \ldots\ldots + K_M$$

Given the outage or availability probabilities, the probability corresponding to each available capacity state can be calculated. Remember that the probability of the simultaneous occurrences of two or more independent events is the product of the respective event probabilities. Thus the probability of the *i*th state is equal to the product of the availabilities A_i of the M available units and the FORs R_i of the $N - M$ out-of-service units, that is:

$$P_i = A_1 A_2 \ldots A_M R_1 R_2 \ldots R_{N-M}.$$

For illustration, below we give the capacity outage probability tables for a 2-unit and 3-unit plants and their generalization:

1. Case of a 2-unit Plant

Capacity state			Plant availability
	Unit 1	Unit 2	
All up	Up	Up	A_1A_2
1 up, 1 down	Up	Down	A_1R_2 +
	Down	Up	A_2R_1 =
			$A_1R_2 + A_2R_1$
All Down	Down	Down	R_1R_2

Note: $R_j = 1 - A_j$ is the FOR and A_j is the steady state availability of unit *j*.

2. Case of a 3-unit Plant

Capacity state	Unit 1	Unit 2	Unit 3	Plant availability
All up	Up	Up	Up	$A_1A_2A_3$
2 up, 1 down	Up	Up	Down	$A_1A_2R_3$ +
	Up	Down	Up	$A_1R_2A_3$ +
	Down	Up	Up	$R_1A_2A_3$ =
				$A_1A_2R_3+A_1R_2A_3+R_1A_2A_3$
1 Up, 2 Down	Up	Down	Down	$A_1R_2R_3$ +
	Down	Up	Down	$R_1A_2R_3$ +
	Down	Down	Up	$R_1R_2A_3$ =
				$A_1R_2R_3+R_1A_2R_3+R_1R_2A_3$
All Down	Down	Down	Down	$R_1R_2R_3$

Note: $R_j = 1 - A_j$ is the FOR and A_j is the steady state availability of unit j.

In general,

Plant availability (capacity state probability, P_i)

when all plant units are up = ΠA_j for all j.

when all plant units are down = $\Pi (1 - A_j)$ for all j.

for a 2-unit plant, when 1 unit is up and 1 unit down = $\sum_{i \neq j} A_i (1 - A_j)$; $i, j = 1, 2$.

for a 3-unit plant, when 2 units are up and 1 unit down =

$$\sum_{i \neq j \neq k} A_i A_j (1 - A_k); \; i, j, k = 1, 2, 3.$$

for a 3-unit plant, when 2 units are down and 1 unit up =

$$\sum_{i \neq j \neq k} A_i (1 - A_j)(1 - A_k); \; i, j, k = 1, 2, 3.$$

__Loss of Load Probability (LOLP)__

The unreliability of a system in this context is viewed as its inability to meet the daily peak load. A loss of load occurs whenever the system load exceeds the available generating capacity. The overall probability that there will be a shortage of power (loss of load) is called the Loss-of-Load Probability or LOLP. It is usually expressed in terms of days per year, hours per day or as a percentage of time. When expressed as the expected accumulated amount of time during which a shortage of power is experienced, the measure is more correctly referred to as the loss of load expectation (LOLE). The LOLP measure was first introduced by Calabrese (1947).

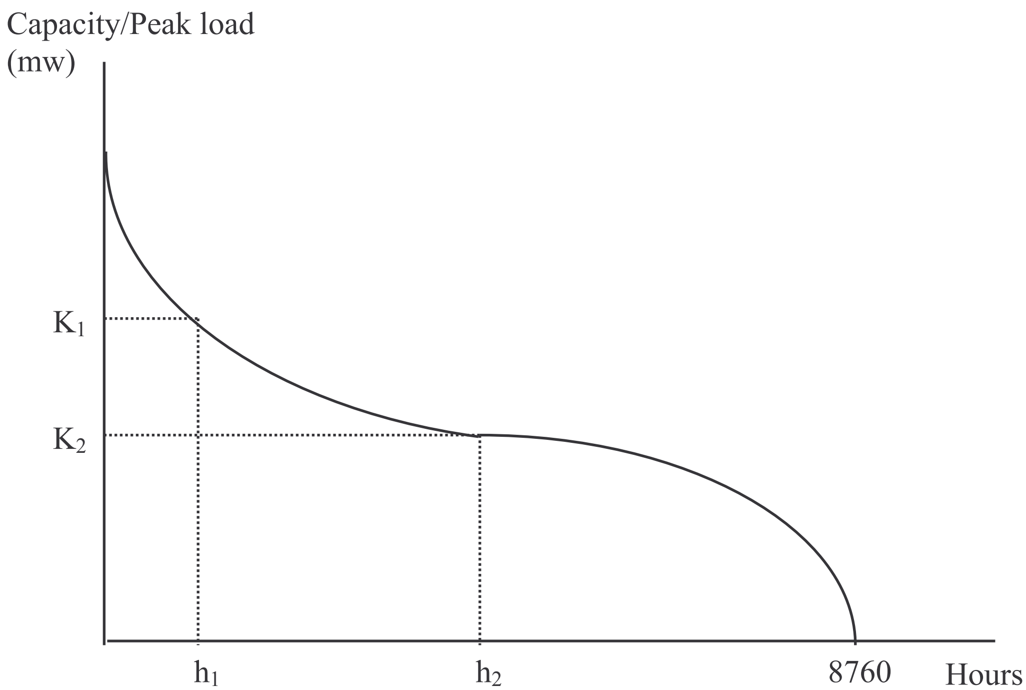
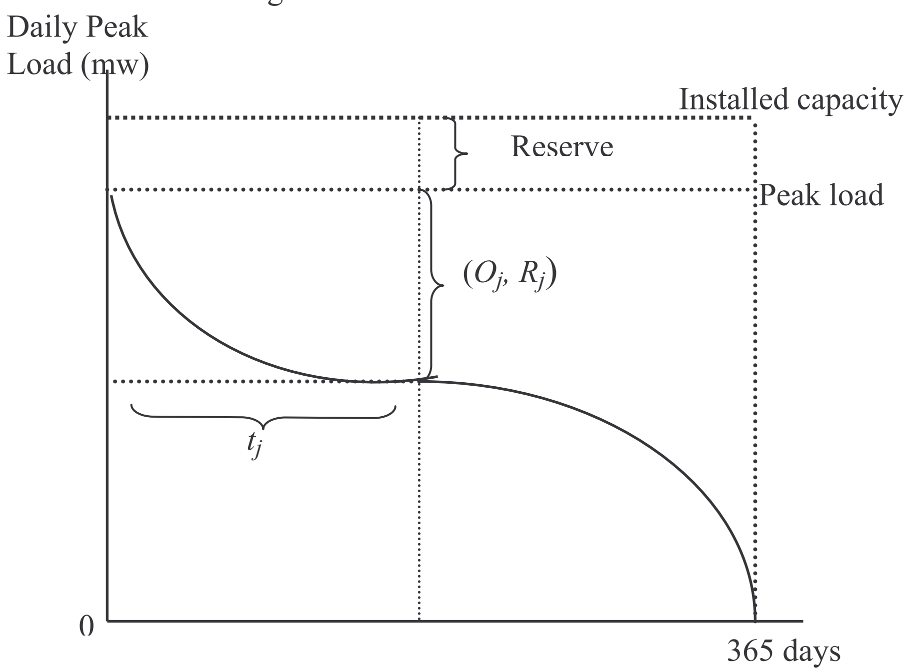


Fig. 1: Load duration curve

By combining the availability of each capacity state with the system load duration curve (LDC), we obtain the expected risk of loss of load. A load duration curve is defined as a function whose abscissa specifies the width of the time interval, usually the number of hours in a year, during which customer's (peak) demand for power (D) equals or exceeds the associated level of available capacity (K^A) given on the ordinate (Fig. 1). Thus it shows the time duration for which a capacity outage would cause a loss of load (D ≥ K). By normalizing the time variable as a proportion of the total, the value at any point on the abscissa can be taken as the (cumulative) probability that the corresponding load will be equaled or exceeded (D ≥ K). When the daily peak load curve is used, the value of LOLE is in days for the period of study, usually days per year. Because of its monotonicity and continuity, the function can be inverted to obtain the proportion of this time interval. This inverse function can in turn be interpreted as the complementary cumulative density function (i.e., the distribution function) of the customer's demand.

Fig. 2: Estimation of LOLP



The LOLP can be used to measure loss-of-load risk as illustrated in Fig. 2 with a daily peak load curve. O_j is the magnitude of the jth outage in the system, R_j is the probability of a capacity outage of magnitude O_j, and t_j is the number of days that an outage of magnitude O_j would cause a loss of load in the system. Note that capacity outages less than the reserve will not lead to a loss of load; a particular capacity outage greater than the reserve contributes to the overall risk by the amount P_j t_j. Then the system LOLP for the period is:

$$\text{LOLP} = \sum_j P_j t_j .$$

Now how to estimate the outage duration, t_j ?

Suppose the customer's daily maximum demand on a power system over one year can be represented by a normal distribution with mean η and standard deviation σ. Then the proportion of time during which a capacity outage would cause a loss of load (i.e., $D \geq K^A$) is given by

$$\text{Prob}(D \geq K^A) = 1 - \text{Prob}(D < K^A) = 1 - \phi(z),$$

where $z = (K^A - \eta)/\sigma$, and $\phi(z)$ is the area under the standard normal curve given by

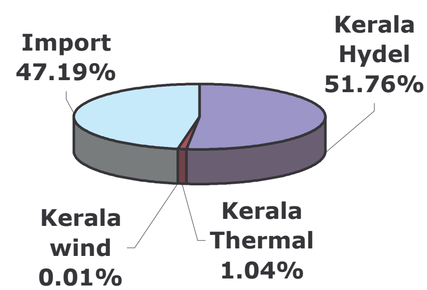
$$\phi(z) = (\sqrt{2\pi})^{-1} \int_{-\infty}^{z} \exp(-x^2/2)dx,$$

which can be read off a standard statistical table. K^A here denotes the available capacity in a certain capacity state; thus we obtain the 'outage duration' that is, the proportion of time during which a forced outage results in a loss of load ($D \geq K^A$) in each of the possible capacity states. The relative contribution of this outage to the overall system loss of load is then obtained by multiplying the availability of a certain capacity state by the corresponding proportion of time that available capacity level is equaled or exceeded. The total LOLP is the sum of all contributions due to the different capacity outages. Multiplying the LOLP by 365 then gives the expected cumulative number of days in a year when loss of load is experienced due to forced outage.

3. LOLP of the Kerala Hydro-Power System

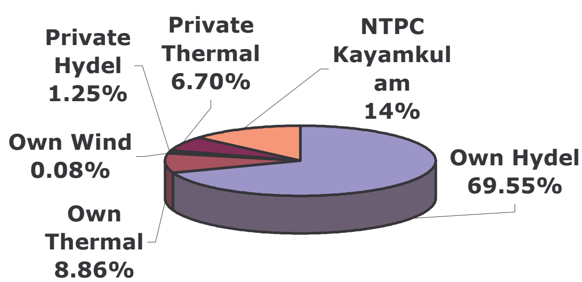
Till the mid-1980s, Kerala had a predominantly hydroelectric power system; with the increased dependency on energy import, the hydro-thermal mix has come down, and now hovers around 52:48 (Fig. 3). However, if we consider installed capacity, the system is still predominantly hydel (Fig. 4). There are 30 power generating stations, including 24 hydel, five thermal and one wind, with an installed capacity of 2649.25 MW; out of

Fig. 3: Energy Availability: 2005-06



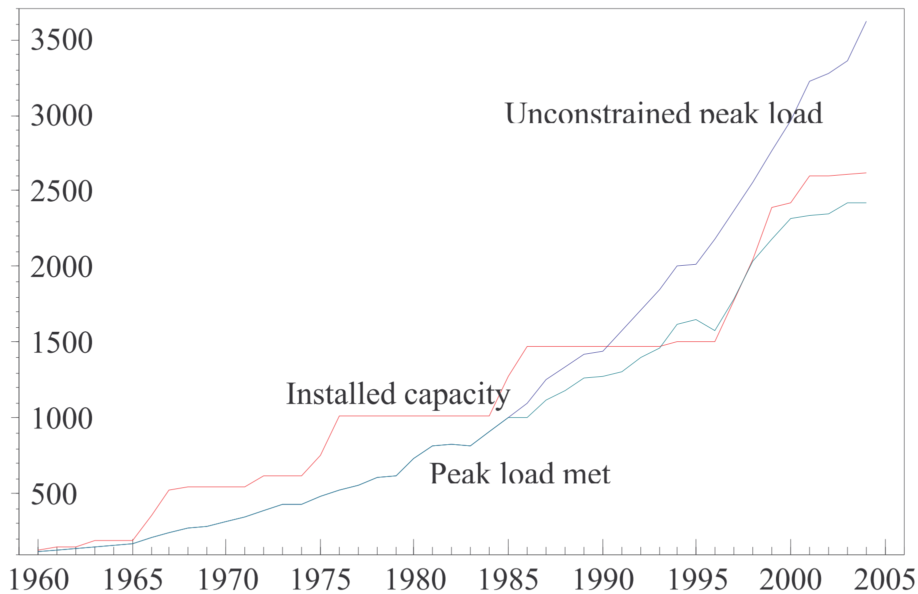
Source: Government of Kerala, *Economic Review 2006*

Fig. 4: Installed capacity: 2006



Source: Government of Kerala, *Economic Review 2006* which the Kerala State Electricity Board (KSEB) owns one wind, two thermal and 22 hydel stations, accounting for 78.5 percent of the total installed capacity. The present study considers only the 10 old hydropower stations of the State, in view of the

Fig. 5: Installed capacity vs. Peak load (mw) of Kerala Power system



Source: Kerala State Electricity Board, *Power System Statistics*;
Central Electricity Authority, *Electric Power Survey of India*,
various issues

availability of sufficiently large time series data. These power stations in the descending order of age (the last plant, Idamalayar, was commissioned in 1987) with their important characteristics are given in Table 1. These ten power stations, with an installed capacity (IC) of 1476.5 megawatt (mw), accounts for about 80 percent of the total own hydel IC (1842.6 mw) and 71 percent of the total own IC (2079.23 mw) of the Kerala power system.

Table 1: Characteristics of the 10 Hydropower Stations

Power plants	Unit Capacity (No x mw)	Installed Capacity (mw)	Average Designed Generation Potential (ADGP)		Storage capacity	
			mu	% to IC	mu	% to ADGP
Pallivasal	3x5+3x7.5	37.5	284.7	86.67	79.54	27.94
Sengulam	4 x 12	48	182.2	43.33	49.61	27.23
Neraiamangalam	3 x 15	45	237.4	60.22	67.58	28.47
Panniar	2 x 15	30	157.7	60.01	45.47	28.83
Poringalakuthu	4 x 8	32	171.7	61.25	63.43	36.94
Sholayar	3 x 18	54	233	49.26	99.47	42.69
Sabarigiri	6 x 50	300	1337.7	50.90	770.32	57.59
Kuttiyadi	3 x 25	75	268.1	40.81	41.46	15.46
Idukki	6 x 130	780	2397.6	35.09	2147.88	89.58
Idamalayar	2 x 37.5	75	380.2	57.87	254.45	66.93
Total		1476.5	5650.3	43.69	3619.21	64.05

Since the early 1980s, Kerala has been suffering from severe capacity shortage in the power sector. Even by 1984-85, the State had an installed capacity of only 1011.5 mw against an estimated demand of 1122 mw. During the two decades from 1976, Kerala's installed capacity in the power sector was growing at an exponential rate of only 3 per cent per annum against 6 per cent of the maximum demand, which in effect was restricted in many ways by power shortage (see Fig. 5). Only since the late 1990s has there been some perceptible addition to the IC.

That a hydropower system is at the mercy of the vagaries of the monsoon is a foregone conclusion, especially with an insufficient storage capacity. Once the monsoon goes dry, close on the heels follow severe power shortages as was the case in the 1980s and thereafter in Kerala; power cut/load shedding has become the rule of the day since 1982-

83, even with very large import of thermal power, often more than 60 percent of the total available power.

Availability and Forced Outage Rates.

The relevant data for a period of 26 years from 1978-79 (to 2003-04) were collected from the Kerala State Electricity Board (KSEB); the data on plant unit capacity, hours of operation and forced outage are from the KSEB's annual publication, viz., '*System Operations*' and the data on daily maximum demand on each of the 10 power stations for three years from 2001-02 were collected from the (unpublished) records from the KSEB head office (*Vydhyuthi Bhavan*) in Trvandrum. In the case of Idukki II stage (3 units) and Idamalayar, commissioned during 1985-87 period, the data are from this period onwards.

It should be noted here that a study of this dimension (theory-informed methodology) is the first of its kind in the case of most of the State Electricity Boards in India, especially the KSEB; the technical appraisal of these power systems in general is confined to examining the plant load factor (PLF) as a measure of capacity utilization only. It goes without saying that PLF is by no means directly comparable with LOLP, as the two methodologies totally differ from each other; this precludes us from attempting at any comparison with the official measure.

The estimated mean-down-time (MDT; forced outage time), measured in hours per month, of each of the units of the 10 hydro power generating stations are given in Table 2; with a cycle time (MTBF) of 720 hours in general, we derive the corresponding forced outage and availability rates.

The long run availability is the highest, coming very close to unity, for (all the units of) Kuttiadi power plant. Neriamangalam, Poringalkuthu, Idukki, Sabarigiri and Idamalayar plants also have higher availability for all their units (above 90 percent). All but two (units III and IV) of the 6 units of Pallivasal also have higher availability; Sengulam also follows suit with one unit (IV) having the highest FOR of 0.29. The remaining two plants, Panniar and Sholayar, bear the whole brunt of higher FOR. Sholayar unit I has the second highest FOR of 0.24; Panniar unit I follows with only 79 percent of availability.

It is worth finding that an average outage of more than one month in one year occurred in the case of 8 out of the total 39 plant units; Sengulam unit IV has the fate of having the maximum mean outage of a cumulative period of more than 3 months a year and Sholayar unit I, nearly 3 months; the credit of having the minimum mean outage of less than one day in one year goes to the 3 units of Kuttiadi, with 7-18 hours a year only.

Table 2: Long run Availability and Forced Outage Rates

Plant	Units	Monthly MDT (hours)	FOR	Availability
Pallivasal	1	39.97	0.0555	0.9445
	2	23.23	0.0323	0.9677
	3	72.53	0.1007	0.8993
	4	74.76	0.1038	0.8962
	5	40.43	0.0562	0.9438
	6	29.09	0.0404	0.9596
Sengulam	1	49.59	0.0689	0.9311
	2	8.07	0.0112	0.9888
	3	40.05	0.0556	0.9444
	4	207.72	0.2885	0.7115
Neraiamangalam	1	11.24	0.0156	0.9844
	2	15.48	0.0215	0.9785
	3	26.39	0.0367	0.9633
Panniar	1	153.43	0.2131	0.7869
	2	95.87	0.1332	0.8668

Poringalakuthu	1	55.69	0.0773	0.9227
	2	20.83	0.0289	0.9711
	3	14.18	0.0197	0.9803
	4	14.10	0.0196	0.9804
Sholayar	1	173.58	0.2411	0.7589
	2	127.19	0.1767	0.8233
	3	79.39	0.1103	0.8897
Sabarigiri	1	7.58	0.0105	0.9895
	2	37.59	0.0522	0.9478
	3	33.96	0.0472	0.9528
	4	41.12	0.0571	0.9429
	5	48.27	0.0670	0.9330
	6	56.67	0.0787	0.9213
Kuttiyadi	1	1.52	0.0021	0.9979
	2	1.09	0.0015	0.9985
	3	0.58	0.0008	0.9992
Idukki	1	12.44	0.0173	0.9827
	2	45.02	0.0625	0.9375
	3	25.29	0.0351	0.9649
	4	9.82	0.0136	0.9864
	5	24.82	0.0345	0.9655
	6	14.83	0.0206	0.9794
Idamalayar	1	31.17	0.0433	0.9567
	2	38.96	0.0541	0.9459
Total		46.24	0.0642	0.9358

In the case of Kuttiadi, instantaneous availability readily collapses on the steady-state one, owing to the least MDT (or FOR). For all other plant units, the long run evolves through time limit. For Sabarigiri unit I and Sengulam unit II, it takes a cumulative period of nearly one month to reach the steady state; other units take much more time. In the case of Sengulam unit IV and Sholayar unit I, with very high FOR, the long-run is evolved across a cumulative period of more than 3 months.

Note that minimum mean outage time does not necessarily mean higher mean operating period, as the case of Kuttiadi clearly shows. Even though Kuttiadi has the highest availability (and the least FOR) as per definition, its service period, when accumulated, amounts on an average to only about 7 months a year; that is, all the three units of this plant remain shut down for a cumulative mean period of about 5 months for scheduled maintenance and/or for want of sufficient water in the reservoir. Panniar and Sholayar with lower levels of service time (MTTF) are also shut down for about 3 – 4 months. The shutdown period of other plants in general accumulate up to 1 – 3 months a year.

Averaging all the data, we find that the whole system has an annual MDT of 46.24 hours per month per unit, with a FOR of 6.4 percent and availability of about 93.6 percent. We can also find that a cumulative time of about 1 month is required for translating the 'instant' into the long-run for the system. However, a 6 percent FOR in the face of capacity shortage imposes a heavy tax on the system. Assuming an annual average generation of 6100 million units (mu; average generation of the last 6 years from 2001-02), with sufficient hydraulic power capacity, this level of FOR implies a potential energy shortage to the tune of $6100 \times 0.0642/0.9358 \approx 420$ mu a year. At an average revenue of Rs. 3.25 per unit as at present, this represents a financial loss to the system of Rs. 1365 million a year. Moreover, the potential energy (lost) of 420 mu is equivalent to about 80 mw installed capacity at 60 per cent load factor. A zero FOR in this case then implies that it could dispense with the investment requirement of adding about 80 mw capacity to the system, saving immensely in capital costs and working expenses. Note that this saving is in addition to the gain in sales revenue.

Table 3 reports the state transition probabilities along with the measures of propensity to down. The initial instantaneous availability (P_{11}, when $t = 1$) for all the plant units is much higher, close to unity, with very low measure of D, the propensity to down; so is the initial instantaneous forced outage rate, but less than P_{11}: it takes some time for repair. The system average follows suit.

Table 3: Transition Probabilities and Propensity to Down

Plant	Units	Transition Probabilities				Propensity to down
		P_{00}	P_{01}	P_{11}	P_{10}	
Pallivasal	1	0.975	0.025	0.9985	0.0015	0.00274
	2	0.958	0.042	0.9986	0.0014	0.00272
	3	0.986	0.014	0.9985	0.0015	0.00276
	4	0.987	0.013	0.9985	0.0015	0.00276
	5	0.976	0.024	0.9985	0.0015	0.00274
	6	0.966	0.034	0.9986	0.0014	0.00273
Sengulam	1	0.980	0.020	0.9985	0.0015	0.00275
	2	0.884	0.116	0.9987	0.0013	0.00261
	3	0.975	0.025	0.9985	0.0015	0.00274
	4	0.995	0.005	0.9981	0.0019	0.00277
Neraiamangalam	1	0.915	0.085	0.9987	0.0013	0.00266
	2	0.937	0.063	0.9986	0.0014	0.00269
	3	0.963	0.037	0.9986	0.0014	0.00272
Panniar	1	0.994	0.006	0.9982	0.0018	0.00277
	2	0.990	0.010	0.9984	0.0016	0.00276
Poringalakuthu	1	0.982	0.018	0.9985	0.0015	0.00275
	2	0.953	0.047	0.9986	0.0014	0.00271
	3	0.932	0.068	0.9986	0.0014	0.00268
	4	0.932	0.068	0.9986	0.0014	0.00268
Sholayar	1	0.994	0.006	0.9982	0.0018	0.00277
	2	0.992	0.008	0.9983	0.0017	0.00276
	3	0.987	0.013	0.9984	0.0016	0.00276
Sabarigiri	1	0.876	0.124	0.9987	0.0013	0.00260

	2	0.974	0.026	0.9986	0.0014	0.00274
	3	0.971	0.029	0.9986	0.0014	0.00274
	4	0.976	0.024	0.9985	0.0015	0.00274
	5	0.980	0.020	0.9985	0.0015	0.00275
	6	0.983	0.017	0.9985	0.0015	0.00275
Kuttiyadi	1	0.518	0.482	0.99898	0.0010	0.00203
	2	0.401	0.599	0.99909	0.0009	0.00182
	3	0.178	0.822	0.99934	0.0007	0.00132
Idukki	1	0.923	0.077	0.9986	0.0014	0.00267
	2	0.978	0.022	0.9985	0.0015	0.00275
	3	0.961	0.039	0.9986	0.0014	0.00272
	4	0.903	0.097	0.9987	0.0013	0.00264
	5	0.961	0.039	0.9986	0.0014	0.00272
	6	0.935	0.065	0.9986	0.0014	0.00268
Idamalayar	1	0.968	0.032	0.9986	0.0014	0.00273
	2	0.975	0.025	0.9986	0.0014	0.00274
Total		0.979	0.021	0.9985	0.0015	0.00275

Capacity-Outage Probability and LOLP

The first step in the estimation of LOLP is to find out the available capacity state probabilities. Table 4 reports the distributed levels of available capacity with the corresponding probability of occurrence in accordance with the different combinations of up and down (due to forced outages) units of each of the 10 power stations, estimated as per the section on ‘Capacity outage distribution’. Note that the available capacity probabilities of all the states add up to unity. Given the availability and the nominal capacity of each unit, we can find the available capacity (k_j^A) corresponding to each state.

Pallivasal, as shown in Table 4, has different levels of available capacity and availability in each of the possible capacity states due to different unit capacities – it has 3 units of 5 mw each and another 3 of 7.5 mw each. Thus in the '1-unit-down' capacity state, we have two levels of available capacity, depending on the capacity of the unit that goes down; if a 5 mw unit fails, the available capacity will be 32.5 mw and in the other case, 30 mw. Also note that in two capacity states ('3 units up', and '2 units up'), the same level of available capacity (15 mw) is obtained; in the '3-units-up' state, it may so happen that all the 3 units of 5 mw each may be in operation (with the other 3 units of 7.5 mw each in outage) and in the next '2-units-up' state, 2 of the 3 units of 7.5 mw each may be in service. All other plants have same-capacity units and hence each capacity state has a unique level of available capacity and probability.

As is already evident, Kuttiadi has the highest availability (almost nearing unity) of maximum capacity (when all units are up). Only 3 plants have an all-units-up availability of more than 90 per cent (Neriamangalam, Kuttiadi and Idamalayar), and 5 plants, of more than 80 per cent (including Poringalkuth and Idukki). Sholayar is the only plant with an all-up availability of less than 60 per cent.

Table 4: Estimation of LOLP by Capacity States

Plant	Capacity state	Nominal Capacity mw	Availability	Available Capacity mw k_j^A	Standard Normal Variate zi	Outage duration t_j	LOLP of Capacity State
(1)	(2)	(3)	(4)	(5) = (3) x (4)	(6)	(7)	(8) = (4) x (7)
Pallivasal	All 6 units up	37.5	0.6671	25.02	2.32	0.010	0.007
	5 up, 1 down	32.5	0.1362	4.426	1.46	0.072	0.010
		30	0.1451	4.352	1.03	0.152	0.022
	4 up, 2 down	27.5	0.00819	0.225	0.60	0.274	0.002
		25	0.02962	0.7404	0.17	0.433	0.013
		22.5	0.00953	0.214	-0.26	0.603	0.006
	3 up, 3 down	22.5	0.000146	0.00329	-0.26	0.603	0.000
		20	0.00178	0.03562	-0.69	0.755	0.001
		17.5	0.00194	0.03403	-1.12	0.869	0.002

		15	0.000194	0.002905	-1.55	0.939	0.000
	2 up, 4 down	15	0.0000318	0.000478	-1.55	0.939	0.000
		12.5	0.000117	0.00146	-1.98	0.976	0.000
		10	3.953E-05	0.000395	-2.41	0.992	0.000
	1 up, 4 down	7.5	2.090E-06	0.0000157	-2.84	0.998	0.000
		5	2.377E-06	0.0000119	-3.27	0.999	0.000
	All 6 units down	0	4.250E-08	0	-4.12	1.000	0.000
Sengulam	All 4 units up	48	0.6186	29.69	2.62	0.004	0.003
	3 up, 1 down	36	0.3401	12.24	0.57	0.284	0.097
	2 up, 2 down	24	0.0398	0.955	-1.49	0.932	0.037
	1 up, 3 down	12	0.00150	0.0180	-3.54	1.000	0.002
	All 4 units down	0	1.239E-05	0	-5.59	1.000	0.000
Neraiamangalam	All 3 units up	45	0.9279	41.76	0.90	0.184	0.171
	2 up, 1 down	30	0.0704	2.11	-1.47	0.929	0.065
	1 up, 2 down	15	0.00166	0.0249	-3.84	1.000	0.002
	All 3 units down	0	1.230E-05	0	-6.21	1.000	0.000
Panniar	All 2 units up	30	0.6821	20.46	0.96	0.169	0.115
	1 up, 1 down	15	0.2895	4.34	-1.01	0.844	0.244
	All 2 units down	0	0.0284	0	-2.97	0.999	0.028
Poringalakuthu	All 4 units up	32	0.8611	27.56	0.87	0.192	0.165
	3 up, 1 down	24	0.1323	3.18	-0.66	0.745	0.099
	2 up, 2 down	16	0.00642	0.103	-2.19	0.986	0.006
	1 up, 3 down	8	0.00013	0.001003	-3.72	1.000	0.000
	All 4 units down	0	8.630E-07	0	-5.25	1.000	0.000
Sholayar	All 3 units up	54	0.5560	30.02	3.15	0.001	0.000
	2 up, 1 down	36	0.3648	13.13	-0.14	0.556	0.203
	1 up, 2 down	18	0.0746	1.34	-3.43	1.000	0.075
	All 3 units down	0	0.00470	0	-6.71	1.000	0.005
Sabarigiri	All 6 units up	300	0.7242	217.26	1.77	0.038	0.028
	5 up, 1 down	250	0.2412	60.30	-0.01	0.504	0.122
	4 up, 2 down	200	0.0323	6.46	-1.79	0.963	0.031
	3 up, 3 down	150	0.00220	0.330	-3.57	1.000	0.002
	2 up, 4 down	100	7.916E-05	0.00792	-5.35	1.000	0.000
	1 up, 5 down	50	1.363E-06	0.0000681	-7.14	1.000	0.000
	All 6 units down	0	7.811E-09	0	-8.92	1.000	0.000

Kuttiyadi	All 3 units up	75	0.9956	74.67	0.69	0.245	0.244
	2 up, 1 down	50	0.00442	0.221	-0.30	0.618	0.003
	1 up, 2 down	25	6.109E-06	0.000153	-1.30	0.903	0.000
	All 3 units down	0	2.573E-09	0	-2.29	0.989	0.000
Idukki	All 6 units up	780	0.8291	646.72	2.54	0.006	0.005
	5 up, 1 down	650	0.1586	103.07	0.42	0.337	0.053
	4 up, 2 down	520	0.0118	6.162	-1.70	0.955	0.011
	3 up, 3 down	390	0.000446	0.174	-3.81	1.000	0.000
	2 up, 4 down	260	8.934E-06	0.002323	-5.93	1.000	0.000
	1 up, 5 down	130	9.084E-08	0.000012	-8.05	1.000	0.000
	All 6 units down	0	3.674E-10	0	-10.17	1.000	0.000
Idamalayar	All 2 units up	75	0.9049	67.87	1.71	0.044	0.039
	1 up, 1 down	37.5	0.0927	3.48	-1.55	0.939	0.087
	All 2 units down	0	0.00234	0	-4.82	1.000	0.002

The second step in the estimation of LOLP is to bring in the load duration curve (LDC) and derive from it the complementary distribution function of customers' demand. This we accomplish by assuming that the customers' daily maximum demand on the Kerala power system follows a normal distribution. Thus data on the daily maximum demand on each of the 10 power stations for three years from 2001-02 were averaged to avoid variability; and then the respective mean and standard deviation were estimated (Table 5). The maximum demand on Kuttiadi powerhouse is the most variable (coefficient of variation: 43.6%; due to seasonal operation necessitated by insufficient storage) and that on Idukki, the least (coefficient of variation: 9.8%).

Now using these parameters (the mean daily maximum demand and standard deviation), the expected available capacity (k_j^A) in each possible state is transformed into its corresponding standard normal variate, z_j, and the associated area under the normal curve, $\phi(z_j)$, is found from a standard statistical table. Then $1 - \phi(z_j)$ represents the (cumulative) proportion of the outage duration, *i.e.*, the proportion of time during which the load equals or exceeds the available capacity, determined by forced outages in a certain capacity state (Table 4). Thus, in the case of Panniar, about 16.9 percent of the time the maximum demand is likely to equal or exceed the available capacity when all the units are in operation; or, in other words, in the ‘all-units-up’ capacity state of Panniar, about 17 percent of the time a forced outage is likely to result in a loss of load. It increases to 84 percent in case any one unit falls down.

The proportion of non-supply duration during which loss of load is caused by different capacity outages in the case of all the 10 power plants are given in the penultimate column of Table 4. This outage duration, when all the units are in operation, is negligible for only 6 plants – Pallivasal, Sengulam, Sholayar, Idukki, Sabarigiri and Idamalayar. About 25 percent of the time a loss of load is experienced in the case of Kuttiadi even when all the units are in operating condition. For Poringalkuthu it is about 19 percent, and for Neriamangalam and Panniar, about 18 and 17 percent respectively. Obviously, the factors determining the extent of the non-supply duration are the capacity-demand gap and the variability (standard deviation) of the demand distribution. The smaller the capacity-demand gap, the larger will be the non-supply duration. A surplus capacity coupled with low demand variability or a deficit capacity with high demand variability results in a short non-supply duration. For Kuttiadi the major influencing factor is obviously the higher demand variability, whereas in the case of Neriamangalam and Poringalkuthu, the smaller capacity-demand gap appears to be the main culprit for larger non-supply duration. Both the factors seem to act on Panniar.

If one unit is thrown out of service, demand is likely to exceed for more than 50 per cent of the time in the case of as many as 7 plants and for more than 80 per cent of the time in the case of 3 plants – Neriamangalam, Panniar and Idamalayar. If the available capacity is only one-half of the installed capacity, then demand tends to exceed it for more than 80 – 90 per cent of the time in general.

LOLP

The expected loss of load in each capacity state is calculated by multiplying the outage duration by the respective availability in that state, given in the last column of Table 4. Summing this over all the capacity states of a plant yields the measure of LOLP; the estimates of LOLP, both as a proportion of time and in terms of number of days a year, for all the 10 plants are given in Table 5. For example, a LOLP of 0.39 for Panniar means that on the whole about 39 percent of the time a loss of load is expected due to forced outages in the case of Panniar. On an annual basis, the expected loss of load is 141.5 days in one year, the expected accumulated amount of time during which demand equals or exceeds the available capacity causing a loss of load due to forced outages; this is the maximum among the 10 plants, followed a little afar by Sholayar (103 days), and Poringalkuthu (99 days). Evidently, the major determinants of this measure are the distribution of availability and non-supply duration in the capacity states. Thus, for example, in the case of Panniar, larger non-supply durations, coupled with the associated, not so small, availability of the respective capacity states contributed to its higher LOLP; that is, the relative contribution to LOLP of larger non-supply duration of lower capacity states is significantly high in this case. On the other hand, for Kuttiadi, the relative contribution to LOLP of larger non-supply durations are negligibly smaller. The minimum LOLP is enjoyed by Pallivasal (23 days a year) and Idukki (25 days a year). A simple average of the LOLPs of all the 10 plants gives the system LOLP of 0.20 or 73.3 days a year with a coefficient of variation of 51.3 percent.

Table 5: Loss of Load Probability and Expected Available Capacity

	Daily maximum demand (mw)		Loss of load		Expected available capacity	
Plant	Mean	SD	Probability	Days/Year	mw	% to IC
Pallivasal	24.01	5.82	0.063	22.95	25.02	66.71
Sengulam	32.69	5.85	0.138	50.38	29.69	61.86
Neraiamangalam	39.32	6.34	0.238	86.83	41.76	92.79
Panniar	25.66	7.62	0.388	141.46	20.46	68.21

Poringalakuthu	27.44	5.22	0.271	98.75	27.56	86.11
Sholayar	36.77	5.48	0.282	103.07	30.02	55.60
Sabarigiri	261.23	31.60	0.183	66.71	217.26	72.42
Kuttiyadi	57.61	25.12	0.247	90.06	74.67	99.56
Idukki	624.12	61.39	0.070	25.49	646.72	82.91
Idamalayar	55.33	11.49	0.129	47.06	67.87	90.49
System	1184.19		0.20	73.28	1181.03	79.99

Table 5 also reports the expected available capacity of the 10 plants (when all units are up; also see column (5) of Table 4); as many as 5 plants have available capacity less than 80 percent of the installed capacity: Pallivasal, Sengulam, Panniar, Sholayar and Sabarigiri. Note that in these cases, the available capacity is barely sufficient to meet the peak load. For the system as a whole, only about 80 percent of the capacity is expected to be available, again not up to the system peak load.

4. Conclusion

The vital nature of electric power, both to our economic and personal well being, has prompted the developed countries to place higher reliability standards on the performance of electricity supply. For example, most of the U.S. electric power utilities are designed on the technical assumption that the total accumulated time of supply interruptions (forced outages) should be no more than 1 day in 10 years (see, for example, Vardi and Avi-Itzhak 1981: 18). This evidently appears to be a very strict design criterion even for developed countries. Some studies (for example, Telson 1975) have in fact shown this 1-day-in-10-years reliability target as economically unjustified, and that it could reasonably be reduced without adversely affecting the economy. Though the reliability performance of an under-developed electricity supply system such as Kerala's is by no means comparable with that of the developed countries, the estimates of LOLP reported here seem on all counts to be stupendously higher. That the expected cumulative outage time of the power generating system in Kerala amounts to 73 days a year is a shocking revelation of the kind of service rendered.

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