When will privatization maximize the government’s net revenues?

Leon Taylor
KIMEP University

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Leon Taylor  
Department of Economics  
KIMEP University  
Almaty, Kazakhstan

Abstract

Governments often sell assets for revenues or economic efficiency. When the capital is durable, potential buyers may wait for the government to cut its price, since they know that as a monopoly it will initially price above marginal cost. Rather than sell, the government could continue to lease the capital to the public – that is, to sell the services that the capital generates, in exchange for a tax payment. Comparative statics indicate that a government maximizing its net revenues may prefer leasing to selling for a large inventory of capital-intensive products that buyers view as vital. For example, a socialist government contemplating a transition to markets must consider the impact on its own revenues. If its major assets are capital-intensive, the impact may be negative.

JEL categories: P26, P35, H20

Keywords: Durable-goods monopoly, privatization, leasing versus selling, government revenues, transition economy

1. Introduction

Governments dominate some forms of durable capital, especially those supporting such natural monopolies as the generation of electricity, the treatment of water, and the support of telecommunications. Socialist governments also monopolize such “commanding heights” industries as steel. A practical problem for governments, socialist or not, is whether selling their capital can maximize their net revenues.

For example, in the transition to a market economy, a socialist government must sell assets -- in principle, in order to increase efficiency (Kikeri et al., 1992). But in practice, a government in the transition often has trouble collecting taxes, so the revenues from privatization sales become crucial to its budget (Bolton and Roland, 1992). This paper analyzes the financial implications to the government of selling durable capital rather than continuing to collect tax revenues in exchange for the services generated by the capital.

Even as a monopolist of durable capital, the government cannot always simply set its price above marginal cost, since potential buyers may wait for it to lower its price, compelling it to equate its price to marginal cost immediately (Coase, 1972). For

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1 Send correspondence to ltaylor@kimep.kz. I thank Shahnoza Seidmedova for capable research assistance as well as Murat Alikhanov and participants in a seminar of the KIMEP International Research Conference. Antonio Estache suggested to me the application of the durable-goods monopoly to the government’s sale of its enterprises, and he commented with insight on early drafts.
example, suppose that it auctions off an enterprise. When potential buyers are few, they may form a cartel that agrees on a maximum bid.

In a similar predicament is the government that privatizes in order to raise quick cash to pay down its debt and avoid default. In 2015, Kazakhstan sought British buyers of state-owned enterprises after low prices for oil exports reduced annual government revenues by 40%.

Creditors of governments deep in debt often recommend to them capital sales as part of an austerity program. But the net public revenues are usually small, partly because the firm sold is small, or – if the enterprise is large -- because the government must settle the firm’s debt as well as absorb its unpaid taxes and provide credit for the private purchase (Kikeri et al., 1992).

This paper focuses on a less-explored reason for the loss of revenue: Buyers may insist on a price so low that the sales revenues are less than the stream of discounted net tax revenues that the government would have received in exchange for the services generated by the capital. (That is, the government could have continued to lease out the capital rather than sell it.) In that case, privatization could strap the government in the short run, even when neither it nor the capital buyer is corrupt.

This problem may have several solutions. One is that circumstances may preclude the problem. Suppose that all agents know that potential buyers are impatient to consume but that the government is patient for its money. Then the buyers’ threat to wait for lower prices may not be credible. If they and the government realize this, all may accept immediately a monopoly price on public capital.

Or the government may exploit market power by committing to an intertemporal price schedule, convincing the prospective buyer of the futility of waiting for the price to fall. Stokey (1979) finds that the monopolist can profit from selling a durable good when the subjective rate of time preference differs among buyers. Some would buy only at a low price and are willing to wait for it.

Finally, the government may remove the root of its problem – the durability of its capital, which enables buyers to wait for a more favorable price. In this approach, the government transforms its public capital into a nondurable good by leasing it out for short periods rather than selling it. By committing to the lease, the government may sustain the monopoly lease price over time. It may either lease the state-owned enterprise to an entrepreneur or continue to provide the enterprise’s services to the public in exchange for a tax payment, which is in the nature of a lease.²

Of these three solutions, the government may favor the first. But if it must choose between the latter two, then it may wish to know which would generate the higher net revenues. This paper addresses that question. It focuses on the most likely case for the short run, that in which the government does not turn over to the private sector all capital at its disposal.

The paper attempts two contributions. First, it applies the durable-goods monopoly to the literature on privatization. This enables rigorous analysis of a financial problem that preoccupies governments making the transition to markets – how to make it pay off for them.³ A key element of this problem is that the government’s discount rate

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² In the early 1990s, the Hungarian government auctioned off leasing rights to state-owned enterprises (Bolton and Roland, 1992; Kikeri et al., 1992).
³ The government making this transition may be, but need not be, socialist.
may differ from that of the public. Second, to the literature on the durable-goods monopoly, the paper provides a continuous-time model. This may generate insights about optimal timing that are not available in the two-period model that has become standard.

Almost by definition, a privatizing government lacks market information about the value of its assets, so it is unsure of their sales prices. The analysis below will try to provide rough indications of what those prices might be.

2. Literature review

Coase (1972) suggested that the monopolist of a durable good may prefer to lease it rather than sell it, since otherwise a patient consumer may wait for the sales price to fall. Much of the consequent literature concerns the conditions under which this conjecture holds.

Production costs determine the results of selling rather than leasing. With high costs, a monopolist that sells can persuade potential customers that it must limit supply, which will protect high prices. If costs are low relative to price, the producer may restrict supply (in terms of services generated by the products) by introducing obsolescence, perhaps by innovating. On the other hand, “as the cost becomes a larger fraction of price, durability will be a virtue – but not as much of a virtue to a seller as a renter,” writes Bulow (1982, p. 324).

Reputation may enable the monopolist to convince the public that it will limit supply (Bulow, 1982; Kreps and Wilson, 1980). If the government has long spent little on goods and services, as a share of gross domestic product, then potential buyers of its capital may infer that it will not spend much to create more of it.

The government’s most powerful commitment to limits on supply may be the decision itself to privatize, since a return to socialism would involve costly institutional changes. In the Commonwealth of Independent States, no government has chosen to fully reverse its transition to markets.

The definition of the time period may affect results. Suppose that the modeler characterizes the period as the amount of time that must pass before the firm can produce again. Lengthening the period may confer credibility on the producer that vows to restrict supply. The Coase conjecture, which assumes that the monopolist cannot commit credibly to a limit on supply, may be most likely to hold in continuous-time models. But assuming a discrete period may be reasonable because of the cost in changing the pace of production. A power plant may need time to boost electrical generation in order to satisfy a sudden surge in peak demand. On the other hand, long periods pressure the monopolist to sell in order to avoid inventory costs, which include foregone interest that could have been earned on sales revenues (Bulow, 1982).

Most early papers on the durable-goods monopoly used a continuous demand function. But according to Bagnoli et al. (1989), the assumption of a discrete number of buyers may invalidate three key results: That the monopolist may lose market power, since buyers will wait for it to cut its price (Coase, 1972); that committing to an intertemporal price schedule may create profits, by convincing buyers that the sales price will not fall to the competitive level (Stokey, 1979); and that leasing out is more profitable than selling, since the lease removes the durability that causes buyers to
anticipate a fall in price (Bulow, 1982). In this paper, I consider privatization with many potential buyers. Perhaps the government distributes to the public stock shares of state-owned enterprises, or it resorts to an initial public offering (Kikeri et al., 1992; Bolton and Roland, 1992). So, I use a continuous demand function.

In addition to its impact on public revenues, the choice between leasing and selling — e.g., between government ownership and private ownership — may have real consequences. Consider spillover costs. Leasing may pollute less than selling if it induces government officials to create capital that is unusually durable, but this depends on whether they restrict leasing in order to protect the market for sold goods or services (Agrawal et al., 2009). Selling may also affect the manager’s efforts to decrease production costs or increase quality. If a private manager would reap the gains from cost reduction but not from quality improvements, he will cut costs rather than raise quality. If cost cutting reduces quality, privatization may deliver an inferior product (Hart, Schleifer and Vishny, 1997). These results assume that the government turns over production facilities to the private sector but continues to offer the services produced. In contrast, my paper will assume that the private sector takes over the state-owned enterprise entirely, controlling both production and produced services.

The model below adopts some standard assumptions in the literature on the durable-goods monopolist: Capital markets enable people to borrow and lend; buyers and sellers have all relevant information; the unit cost is constant; no competitor threatens to enter the monopolized market; and all potential buyers have the same rate of time preference (but this may differ from the seller’s rate). Stokey (1979) discusses consequences of relaxing some of these assumptions.

3. Intertemporal pricing

To do its job, the price schedule must be credible. This rules out a schedule of rising prices, since the potential buyer knows that the government will never price below marginal cost and hence will have room to lower it later. The schedule would be more credible if it left the potential buyer indifferent as to when to purchase the capital. In drawing up such a schedule, the government might ensure that the present value of the price falls over time at the buyer’s subjective rate of time preference so that she is indifferent between a current or future purchase. The advantage of buying now — immediate consumption — would be exactly offset by the advantage of buying later at a lower price.

This reasoning suggests the price schedule

\[ P(t) = P(0)e^{-rt}, \]

for all \( t \) in \([0, T]\). The public’s subjective rate of time preference is \( r \).

The government assumes that the following function measures demand for public capital:

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4 This short-run analysis takes the endtime \( T \) as fixed, but I will relax the assumption later.
\[ Q(t) = f[P(t); x] \]

where \( x \) is a set of exogenous factors. The government’s capital stock is \( \bar{Q} \):

**Equation 2**

\[ \bar{Q} \geq \int_0^T Q(t) \, dt. \]

The cost of marketing \( Q \) is

\[ C(t) = g[Q(P(t))] \geq 0. \]

This includes “settlement of enterprise debt, unpaid taxes, and transaction fees” (Kikeri et al., 1992, p. 30) as well as depreciation and corruption costs.

The government seeks to maximize net sales revenue, subject to the constraint of fixed supply and discounted at rate \( r_1 \). It chooses the initial price in Equation 1, \( P(0) \), to maximize

**Equation 3**

\[ \int_0^T e^{-r_1 t} \left[ P(t)Q(P(t); x) - C(Q(P(t))] \right] dt \]

subject to Equation 2.

The nature of the problem stated by Equation 3 depends on whether the capacity constraint binds. For now, I assume that it does not. While the government may recognize the theoretical advantages of selling facilities producing private goods, it often retains some facilities producing public goods, such as a laboratory researching infectious diseases (Andersson and Söderberg, 2011; Hart et al., 1997). For example, Mexico began privatizing in 1985 and by June 1992 had sold just 62% of its 586 state-owned enterprises (López-de-Silanes, 1997, p. 978). In any event, if the government retains some capital, then the problem in Equation 3 is static, since the sale of some \( Q \) at a given moment does not force a reduction in the amount to be sold later. If the constraint does bind, then a change in the present amount of \( Q \) sold affects amounts sold later, and the problem becomes dynamic. That restriction on supply might also support high prices (Stokey, 1979).

The choice variables are \( P(0) \) and \( \lambda \), the Lagrangian multiplier associated with the capacity constraint. In an interior solution, the following first-order condition equals zero:
From Equation 1,
\[
\frac{\partial P(t)}{\partial P(0)} = e^{-rt}.
\]

In Equation 4, the term $e^{rt}$ may take on any non-negative finite value. This implies that the bracketed term equals zero for any value of $t$ in $[0, T]$. Thus
\[e^{-rt} Q(t) + P(t) \frac{\partial Q(t)}{\partial P(t)} e^{-rt} - \frac{\partial C(t)}{\partial Q(t)} \frac{\partial Q(t)}{\partial P(t)} e^{-rt} = 0.\]

Equation 5

For simplicity, assume a linear demand function:

Equation 6
\[Q(t) = A - kP(t).\]

Substitute Equation 6 into Equation 5 and solve for $P(t)$:
\[P(t) = \frac{A}{2k} + \frac{\partial C / \partial Q}{2} - \frac{\lambda}{2r} \left[1 - e^{rt}\right].\]

Specifically,

Equation 7
\[P(0) = \frac{A}{2k} + \frac{\partial C / \partial Q}{2}.\]

Equations 1 and 7 imply that any optimal price path must take the form

Equation 6
\[P(t) = \left[\frac{A}{2k} + \frac{\partial C / \partial Q}{2}\right] e^{-rt}.\]

Equation 8 suggests several policy implications:
(1) A higher subjective rate of time preference $r$ lowers $P(t)$. The public prefers more strongly than at lower values of $r$ to consume today rather than tomorrow, so it must be lulled by a lower price into a current purchase.

The parameter $r$ may rise because of political or economic uncertainty. For example, creditors of the government may demand immediate repayment. Under these circumstances, sudden privatization may raise less money than had been anticipated.

(2) A higher marginal cost must raise the capital price so that the government can recover costs. Marginal cost might rise over time because the government postpones selling enterprises that are expensive to prepare for auction.

(3) A higher value of $k$ lowers the capital price since potential buyers become more sensitive to price. But a higher demand-intercept, $A$, raises the price since buyers now want more capital at the old given price. Again, these results might relate to the pattern of privatization. The government may sell last those enterprises that provide necessary services, such as water treatment. Demand for these services entails a low value of $k$ and a high value of $A$, so the sales price may be unusually high.

Total discounted net revenues are

**Equation 7**

$$NR_s = \int_0^T \left[ P(t)Q(t) - C(t) \right] e^{-rt} dt,$$

where the cost function is

**Equation 10**

$$C(t) = mQ(t).$$

Then, if $T$ is finite,

**Equation 11**

$$NR_s = \frac{A + mk}{2(r_1 + r)k} \left[ A + m \right] \left[ 1 - e^{-(\eta + r)T} \right] - \frac{(A + mk)^2}{4k(r_1 + 2r)} \left[ 1 - e^{-(\eta + 2r)T} \right] - \frac{Am}{r_1} \left[ 1 - e^{-\eta T} \right].$$

2.1 Finite time period. The government may set a deadline $T$ for privatization if, for example, it is pressured by debts. A change in $T$ may affect net sales revenues in a complex way (Appendix 2).

If buyers are not sensitive to the price, increasing the time period will raise revenues, since the government need not worry that buyers will take advantage of the additional time by holding out for a lower price. This might apply to privatization by a
government that owns commanding-heights industries but not small enterprises.\(^5\)
Compared to other nations, they have longer deadlines; they take their time selling
enterprises that are large and vital to national production.

Strikingly, if all demand is sensitive to the price, lengthening the time period will
again raise revenues since it enables the government to offer a low price in the endtime
that attracts buyers. One thinks of a nonsocialist nation which owns only small
enterprises.

If the seller’s costs are low, increasing the time period will raise revenue, because
the new, low endtime price can still cover them. An increase in \(T\) will also raise revenue
when either discount rate is close to zero. This case is closer to leasing than to selling.
The government is in no hurry to privatize, or people are in no hurry to buy its
enterprises.

If the time span \(T\) is short, increasing it will slightly boost revenues by creating a
few more opportunities for sales. Thus a government that has recently settled overdue
debts may profit by decelerating privatization. By similar reasoning, if the time span is
long, increasing it slightly will not affect revenues (except in the cases, already described,
of extremely inelastic or elastic demand with respect to price). A government that is
privatizing slowly should not slow down even more.

2.2 Indefinite time period. The government might not specify a period for privatization,
perhaps because it lacks experience with economic transitions or because it owes too
little to have to sell assets for quick cash. In that case, \(T\) goes to infinity. In the limit,

\[
NR_s = \left[ \frac{A + mk}{2(r_1 + r)} \right] \left[ \frac{A}{k} + m \right] - \frac{(A + mk)^2}{4k(r_1 + 2r)} - \frac{Am}{r_1}. 
\]

When a government is not under pressure to raise money, it typically first sells
capital that it need not prepare, such as small enterprises (Kikeri et al., 1992). So consider
the subcase of zero costs.

2.2.1 Zero marginal cost \((m = 0)\). Equation 12 becomes

\[
NR_s = \frac{A^2}{4k} \left[ \frac{r_1 + 3r}{(r_1 + r)(r_1 + 2r)} \right].
\]

In Equation 13:

(1) An increase in either discount rate, \(r\) or \(r_1\), reduces net sales revenues
(Appendix 2). A high discount rate of the government \(r_1\) may connote

\(^5\) Scandinavian nations come to mind.
impatience for revenues that leads officials to decrease prices in the near
future by more than would otherwise maximize net revenues over time.

(2) A rise in $A$ or a fall in $k$ will raise net sales revenues, since a growing
insensitivity to price enables the government to increase its price for capital at
any given moment (Appendix 2).

These results imply that in selling small enterprises the government would begin
with those least essential to the economy.

2.2.2 Positive marginal cost ($m > 0$). A small increase in low costs reduces net sales
revenues. As the government begins to sell larger and costlier enterprises over time, its
proceeds from the most recent sale might fall. Perversely, when initial costs are high, a
cost increase might raise net sales revenues (Appendix 2).

Some results are straightforward to interpret. An increase in price sensitivity $k$ lowers
revenues since buyers are less eager now to pay a monopoly price. An increase in $r_1$ also
lowers revenues since the government now discounts them at a higher rate. Finally, a rise
in $r$ cuts revenues because it raises the public’s discount of the sales price at a given time
(Appendix 2).

In summary, the profit in privatization depends on real characteristics of the product
produced by the public capital as well as on subjective characteristics of the capital’s
buyers and sellers.

4. **Leasing**

If the government can commit credibly to a constant lease price $R$ over time, then it will
seek $R$ to maximize

\[
\int_{0}^{T} e^{-rt} \left[ RQ(R) - C[Q(R)] \right] dt + \lambda \left[ Q - Q(R) \right].
\]

The first-order condition is

\[
\int_{0}^{T} e^{-rt} \left[ Q(R) + \left( R - \frac{\partial C}{\partial Q} \right) \frac{\partial Q}{\partial R} \right] dt - \lambda \frac{\partial Q}{\partial R} = 0.
\]

When the constraint in Equation 14 does not bind, the optimal value of $\lambda$ is 0, and
Equation 15 is satisfied if

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6 Other results are more involved. A rise in the demand intercept $A$ increases net sales revenues if marginal
cost $m$ is low, if price sensitivity $k$ is small, or if the government discount rate $r_1$ is high. But the rise in $A$
decreases revenues if the public’s discount rate $r$ is high, if $k$ is high, or if $r_1$ is small.
Equation 16
\[ Q(R) + \left( R - \frac{\partial C}{\partial Q} \right) \frac{\partial Q}{\partial R} = 0 \]
for all \( t \in [0, T] \). Rearrangement of Equation 16 expresses the lease price:

Equation 17
\[ R = \frac{\partial C}{\partial Q} - \frac{Q(R)}{\frac{\partial Q}{\partial R}}. \]

In Equation 17, the optimal lease price exceeds marginal cost unless potential buyers are infinitely responsive to price; in that case, the lease equals marginal cost.\(^7\)

For sharp results, again specify linear demand and cost functions (Equations 6 and 10). These render the optimal lease price (Equation 17) as

Equation 18
\[ R = \frac{m}{2} + \frac{A}{2k}. \]

Insert Equation 18 into Equation 14 to obtain maximum net revenues from the lease when the capacity constraint does not bind:

Equation 19
\[ \frac{1}{4r_1 k} \left[ A - mk \right]^2 \left[ 1 - e^{-r_1 T} \right] \]

As expected, an increase in the time span \( T \) or in price-independent demand \( A \) increases lease revenues (Appendix 2). An increase in the seller’s cost \( m \) or in the buyer’s price sensitivity \( k \) decreases revenues. The subjective rate of time preference \( r \)

\(^7\) Another arrangement of Equation 16 yields
\[ \frac{\partial C}{\partial Q} - \frac{Q(R)}{\frac{\partial Q}{\partial R}} = 1. \]

The price elasticity of demand relates inversely to marginal cost – a familiar result for monopolists.
does not affect revenues since the consumer pays the lease over the entire time horizon. The government’s discount rate does affect the government’s revenues, but not in a clear-cut way. If the time horizon is long, then an increase in \( r_1 \) will raise revenues (Appendix 2).

If we interpret the lease price as a tax price, then a socialist government can increase its tax revenues – and, presumably, its durability – if it postpones privatization, particularly by hanging on to the commanding-heights industries. This may help explain why governments prefer to privatize their small, price-sensitive enterprises first.

*Comparing leasing to selling.* Comparison of Equation 19 to the maximum of net revenues from intertemporal sales (Equation 11) makes clear that a high discount rate among buyers \( (r) \) should dispose the government toward leasing rather than selling, since in that case impatient buyers could not pressure the government into a once-and-for-all sale later at a lower price. In such a case, a socialist government might balk at privatizing. This might occur when the subjective rate of time preference is high because the population is aged.

In general, excess net revenues from selling rather than leasing are:

**Equation 20**

\[
ENR = \frac{(A + mk)^2}{2k} \left[ \frac{1 - e^{-(r_1 + r)T}}{r_1 + r} - \frac{1 - e^{-(r_1 + 2r)T}}{2(r_1 + 2r)} \right] - \frac{Am}{r_1} \left[ 1 - e^{-r_1 T} \right] - \frac{1}{4r_1 k} \left[ A - mk \right]^2 \left[ 1 - e^{-r_1 T} \right]
\]

For example, if sales revenues net of costs are $100 million, and lease revenues net of costs are $60 million, then ENR is $40 million.

We can use the model to analyze austerity programs in which governments privatize in order to earn revenues for paying off debts. Since the government needs money right away, its discount rate \( r_1 \) probably exceeds the subjective rate of time preference \( r \); for simplicity, set \( r \) to zero. If the government is beginning an austerity program, then it probably does not already have a privatization schedule, so let \( T \) go to infinity. Table 1 summarizes comparative statics under these two assumptions (Appendix 2). Given that potential buyers of state-owned enterprises can threaten to delay their purchases, the government would raise more money by leasing rather than selling – that is, by *not* privatizing -- although the difference tends to disappear when \( r_1 \) is extremely high (Appendix 2). Of course, raising revenues is not the only goal of an austerity program.

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8 In Equation (11), as \( r \) goes to infinity, in the limit net sales revenues go to \( [(Am)/r_1][1 - e^{r_1 T}] \), which is negative.

9 The government also would receive more in net revenues by leasing rather than selling when its discount rate \( r_1 \) equals zero and the public’s discount rate \( r \) is positive.
Table 1: Comparative statics for selling rather than leasing ok

<table>
<thead>
<tr>
<th>Variable</th>
<th>Net revenues -- sales</th>
<th>Net revenues -- leases</th>
<th>Net revenues of sales minus those of leases (ENR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ increases</td>
<td>Positive</td>
<td>Positive</td>
<td>Zero</td>
</tr>
<tr>
<td>$k$ increases</td>
<td>Negative</td>
<td>Negative</td>
<td>Zero</td>
</tr>
<tr>
<td>$m$ increases</td>
<td>Negative</td>
<td>Negative</td>
<td>Zero</td>
</tr>
<tr>
<td>$r_1$ increases</td>
<td>Negative</td>
<td>Negative</td>
<td>Zero</td>
</tr>
</tbody>
</table>

Results for an infinite time horizon include:

(1) Net revenues rise for either sales or leasing when the demand for capital increases independently of its price. (This demand is $A$.) The relative importance to demand of price sensitivity dwindles, so the government need not cater to buyers by lowering its prices at a given moment. It might gain, then, by selling commanding-heights enterprises before selling small enterprises, given preparation costs.

(2) A rise in unit costs $m$ nibbles away at net sales revenues since public expectations that the sales price would fall over time constrain the government when charging a price high enough to cover costs. Similarly, the cost hike also reduces net leasing revenues. Since the two effects are symmetric, a cost increase does not affect the government’s preference for selling or leasing. A similar interpretation holds for a change in price sensitivity. These results imply that operating costs, which may include costs of corruption, may reduce revenues from privatizing – or, for that matter, from refusing to privatize.

(3) When the government becomes more impatient for its money (i.e., $r_1$ rises), its discounted revenues from sales or leases fall, but the relative advantage to it of leasing does not change, as long as $r_1$ remains finite. Consider a government – like Greece in 2015 – that suddenly must dispose of its capital quickly in order to raise money to pay off debts. This scenario may also suit transition economies in Asia, where high savings rates indicate a low subjective rate of time preference.

Conclusions

A government seeking revenue may prefer leasing (not selling) capital-intensive products that buyers view as vital. In addition to banks, examples may include plants producing such essential inputs as electricity and treated water. A paradox is that leasing
of such enterprises conspicuously slows the transition to a market economy. A
government intent on a quick transition may find that it raises less money from sales of
capital than it had anticipated, because buyers hold out for low prices. One thinks of
Yeltsin’s Russia.

State capital need not be physical. The model may apply to dispensing
environmental rights to polluters, in the form of a renewable permit (a lease) or a
deregulatory policy that transfers environmental rights permanently (a sale).

The main difficulty in the analysis is the assumption that agents are well-
informed. The analysis may be more normative than positive.

Future research might consider the impact on privatization of competition among
jurisdictions for buyers of public capital. Swan (1970) concludes that competitive
producers will choose the same degree of durability for their products as does the
monopolist. However, Bulow (1982) argues that if the monopolist decides to sell but
cannot control the resale market, it will make its product less durable than competitive
firms would, in order to maintain market power. These results might apply to
jurisdictions that compete for mobile private capital, such as foreign direct investment, by
building infrastructure. Such competition might produce more durable infrastructure than
would be provided by a jurisdiction whose infrastructure is unique. For example, consider
governments that compete for entrepreneurs by developing industrial parks. If the parks
are generic, then the competition may produce more durable roads than would have been
built by a government designing a park for a location-specific industry.
Bibliography


Appendix 1

Optimality conditions for selling and leasing

For the problem stated in Equation 3, the first-order condition with respect to $P_0$ is equivalent to

\begin{equation}
\int_{0}^{T} e^{-rt} \left[ e^{-rt} Q(t) + P(t) \frac{\partial Q}{\partial P(t)} e^{-rt} - \frac{\partial C}{\partial Q} \frac{\partial Q}{\partial P(t)} e^{-rt} \right] dt - \lambda \left[ \int_{0}^{T} \frac{\partial Q}{\partial P} e^{-rt} dt \right] = 0.
\end{equation}

where $r_g = r_1$, the government discount rate. Since $\lambda = 0$, Equation 21 boils down to

\begin{equation}
\int_{0}^{T} e^{-rt} \left[ e^{-rt} Q(t) + P(t) \frac{\partial Q}{\partial P(t)} e^{-rt} - \frac{\partial C}{\partial Q} \frac{\partial Q}{\partial P(t)} e^{-rt} \right] dt = 0.
\end{equation}

Equation 22 is zero if the bracketed expression is 0 for every value of $t$. Thus a choice of $P_0$ satisfies Equation 21 if

\begin{equation}
e^{-rt} Q(t) + P(t) \frac{\partial Q}{\partial P(t)} e^{-rt} - \frac{\partial C}{\partial Q} \frac{\partial Q}{\partial P(t)} e^{-rt} = 0.
\end{equation}

for all $t$ in $[0, T]$. This is a current-value problem.

The other first-order condition concerns the multiplier:

\begin{equation}
\frac{\partial L}{\partial \lambda} = \overline{Q} - \int_{0}^{T} Q(t) dt.
\end{equation}

By the Kuhn-Tucker conditions, the optimal value of $\lambda$, given slack capacity, is 0 (Henderson and Quandt 1980: 18). This simplifies the first-order condition in Equation 4 as

\begin{equation}
\frac{\partial L}{\partial P(0)} = e^{-rt} Q(t) + P(t) \frac{\partial Q}{\partial P(t)} e^{-rt} - \frac{\partial C}{\partial Q} \frac{\partial Q}{\partial P(t)} e^{-rt} = 0.
\end{equation}
The second-order condition is

\[ \frac{\partial^2 L}{\partial P^2} = e^{-2rt} \left[ 2 \frac{\partial Q}{\partial P} + \left( P[t] - \frac{\partial C}{\partial Q} \right) \frac{\partial^2 Q}{\partial P^2} - \frac{\partial^2 C}{\partial Q^2} \left( \frac{\partial Q}{\partial P} \right)^2 \right] - \lambda \int_0^t \frac{\partial^2 Q}{\partial P^2} e^{-2rt} \, dt. \]

The condition is negative if

\[ \frac{\partial Q}{\partial P} \leq 0, \]
\[ \frac{\partial^2 Q}{\partial P^2} = 0, \]
\[ \frac{\partial^2 C}{\partial Q^2} \geq 0, \]

and either \( \frac{\partial Q}{\partial P} \) or \( \frac{\partial^2 C}{\partial Q^2} \) (or both) is nonzero. The sufficiency conditions require a linear \( Q(t) \).

**Leasing.** The second-order condition can be expressed as

**Equation 25**

\[ 2 \frac{\partial Q}{\partial R} + \left[ R - \frac{\partial C}{\partial Q} \right] \frac{\partial^2 Q}{\partial R^2} - \frac{\partial^2 C}{\partial Q^2} \left( \frac{\partial Q}{\partial R} \right)^2 \]

Equation 25 is negative, satisfying sufficiency, if

\[ \frac{\partial Q}{\partial R} \leq 0, \]
\[ \frac{\partial^2 Q}{\partial R^2} \leq 0, \]
\[ \frac{\partial^2 C}{\partial Q^2} \geq 0, \]

and not all three terms are zero.
Appendix 2
Comparative statics under a finite or infinite time horizon

Sales revenues in a finite time period. The impact of the time horizon $T$ on net sales revenues is

\[ \frac{\partial NR_S}{\partial T} = \frac{(A+mk)^2}{4k} e^{-(r_1+r)rT} \left[ 2 - e^{-rt} \right] - Am e^{-r_1T}. \]

In Equation 26, if $T$ is close to zero, the impact on net sales revenues of lengthening the period slightly is

\[ \frac{\partial NR_S}{\partial T} = \frac{1}{4k} (A + mk)(A - mk) > 0. \]

If $T$ goes to infinity, then the impact on revenues of lengthening the period still more is small:

\[ \lim_{T \to \infty} \frac{\partial NR_S}{\partial T} = \frac{(A+mk)^2}{4k} e^{-(r_1+r)rT} \left[ 2 - e^{-rt} \right] - Am e^{-r_1T} = 0. \]

If $A$ is zero, then the impact on revenues of increasing the time period slightly is positive:

\[ \frac{\partial NR_S}{\partial T} = \frac{m^2k}{4} e^{-(r_1+r)rT} \left[ 2 - e^{-rt} \right] > 0. \]

The impact is also positive when $k$ is zero. And likewise when $m$ is zero:

\[ \frac{\partial NR_S}{\partial T} = \frac{A^2}{4k} e^{-(r_1+r)rT} \left[ 2 - e^{-rt} \right] > 0. \]

When $r_1$ is zero,

\[ \frac{\partial NR_S}{\partial T} = \frac{e^{-rT}}{4k} (A + mk)(A - mk) > 0. \]
When $r$ is zero,

$$\frac{\partial NR_s}{\partial T} = \frac{e^{-r_1T}}{4k} (A + mk)(A - mk) > 0.$$ 

_Sales revenues: The case of an infinite time horizon._ When $T$ goes to infinity and $m$ equals zero,

**Equation 27**

$$NR_s = \frac{A^2}{4k} \left[ \frac{r_1 + 3r}{(r_1 + r)(r_1 + 2r)} \right].$$

The equation makes clear that the derivative of net revenues is positive with respect to $A$ and negative with respect to $k$.

Also,

$$\frac{\partial NR_s}{\partial r} = -\frac{A^2r(2r + 3r)}{2k(r_1 + r)^2(r_1 + 2r)^2} < 0,$$

$$\frac{\partial NR_s}{\partial r_1} = -\frac{A^2(r_1^2 + 6rr_1 + 7r^2)}{4k(r_1 + r)^2(r_1 + 2r)^2} < 0.$$

From Equation 11,

$$\frac{\partial NR_s}{\partial m} = \frac{1}{2(r_1 + r)} \left[ A + mk \right] - \frac{A}{r_1}.$$ 

When $m = 0$,

$$\frac{\partial NR_s}{\partial m} = \frac{A(r_1 + 2r)}{2r_1(r_1 + r)} < 0.$$ 

When marginal costs are positive, Equation 12 gives net sales revenues. The derivative with respect to the demand intercept $A$ is
\[
\frac{\partial NR_s}{\partial A} = \left[\frac{A}{k} + m\right] \left[\frac{r_1 + 3r}{2(r_1 + r)(r_1 + 2r)}\right] \frac{m}{r_1}.
\]

The sign of this derivative depends on the parameters. The derivative with respect to price sensitivity \(k\) is

\[
\frac{\partial NR_s}{\partial k} = -\frac{(A + mk)(A - mk)(r_1 + 3r)}{4k^2(r_1 + 2r)(r_1 + r)} < 0.
\]

To sign this derivative, note that \(A - mk = Q > 0\) when the sales price equals marginal cost.

The derivative with respect to the government discount rate is

\[
\frac{\partial NR_s}{\partial r_1} = -\frac{1}{4kr_1^2} (A - mk)^2 < 0.
\]

And the one with respect to the public’s discount rate is

\[
\frac{\partial NR_s}{\partial r} = -\frac{3(A + mk)^2}{8kr^2} < 0.
\]

**Leasing revenues.** Comparative statics include:

\[
\frac{\partial NR_L}{\partial T} = \frac{e^{-r_1T}}{4k} (A - mk)^2 > 0.
\]

\[
\frac{\partial NR_L}{\partial A} = \frac{1}{2r_1k} (A - mk)(1 - e^{-r_1T}) > 0.
\]

\[
\frac{\partial NR_L}{\partial m} = -\frac{1}{2r_1} (A - mk)(1 - e^{-r_1T}) < 0.
\]

\[
\frac{\partial NR_L}{\partial r} = 0.
\]
\[ \frac{\partial NR_L}{\partial r_1} = \frac{1}{4 r_1 k^2} [A - mk]^2 \left[ T - \frac{1 - e^{-r_1 T}}{r_1} \right] \]

\[ \frac{\partial NR_i}{\partial k} = -\frac{1}{4 r_1 k^2} [A - mk][A + mk][1 - e^{-r_1 T}] \leq 0. \]

Sales revenues net of leasing revenues (ENR). As \( T \) goes to infinity,

Equation 28

\[ ENR = \frac{(A + mk)^2}{4 k} \left[\frac{r_1 + 3r}{(r_1 + r)(r_1 + 2r)}\right] - \frac{Am}{r_1} - \frac{1}{4 r_1 k} (A - mk)^2. \]

For the impact of price-insensitive demand \( (A) \), consider the derivative

\[ \frac{\partial ENR}{\partial A} = \frac{(A + mk)}{2k} \left[\frac{r_1 + 3r}{(r_1 + r)(r_1 + 2r)}\right] - \frac{m}{r_1} - \frac{1}{2 r_1 k} (A - mk). \]

When \( r = 0 \),

\[ \frac{\partial ENR}{\partial A} = 0. \]

Now consider how price-insensitive demand affects sales revenue only. For the impact on this derivative of a rise in cost, consider

\[ \frac{\partial^2 NR_L}{\partial A \partial m} = \frac{1}{2} \left[\frac{r_1 + 3r}{(r_1 + r)(r_1 + 2r)}\right] - \frac{1}{r_1} < 0. \]

Unit costs. The impact of \( m \) on net lease revenues is

\[ \frac{\partial NR_L}{\partial m} = -\frac{1}{2 r_1} [A - mk] < 0. \]
The impact on net sales revenues is

$$\frac{\partial NR_S}{\partial m} = \frac{(A + mk)}{2} \left[ \frac{r_1 - 3r}{(r_1 + r)(r_1 + 2r)} \right] - \frac{A}{r_1}$$

When \( r = 0 \),

$$\frac{\partial NR_S}{\partial m} = \frac{(A - mk)}{2r_1} < 0.$$ 

The impact on net revenues from sales minus those from leasing is

$$\frac{\partial ENR}{\partial m} = \frac{\partial NR_S}{\partial m} - \frac{\partial NR_L}{\partial m} = -\frac{(A - mk)}{2r_1} + \frac{(A - mk)}{2r_1} = 0.$$ 

*Price sensitivity.* The impact of price sensitivity on net leasing revenues is

$$\frac{\partial NR_L}{\partial k} = -\frac{(A - mk)(A + mk)}{4r_1k^2} < 0.$$ 

The impact on net sales revenues is

$$\frac{\partial NR_S}{\partial k} = -\left[ \frac{r_1 - 3r}{(r_1 + r)(r_1 + 2r)} \right] \frac{(A + mk)(A - mk)}{4k^2}.$$ 

When \( r = 0 \),

$$\frac{\partial NR_S}{\partial k} = -\frac{(A + mk)(A - mk)}{4r_1k^2} < 0.$$ 

Since the effects on sales and revenues are the same in magnitude, changes in price sensitivity do not affect the government’s relative preference for sales or leasing.

*The government’s discount rate.* The impact of this rate on net sales revenues when \( r = 0 \) is

$$\frac{\partial NR_S}{\partial r_1} = -\frac{(A - mk)^2}{4kr_1^2} < 0.$$
As for net leasing revenues,

\[ \frac{\partial NR_2}{\partial r_1} = -\frac{(A - mk)^2}{4kr_1^2} < 0. \]

The marginal impact on sales revenues net of leasing revenues is of course zero.

When \( T \) goes to infinity in the limit, \( r \) equals zero and \( r_1 \) is positive, then the excess net revenues due to selling rather than leasing are

\[ ENR = -\frac{(A^2 + m^2 k^2)}{2r_1 k} - \frac{Am}{r_1}. \]

This is negative but tends towards zero when the government’s discount rate, \( r_1 \), goes to infinity in the limit.