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Abstract

This paper proposes a cultural evolutionary model in which the assortativity level of matching is endogenously determined. We consider a population consisting of two cultural groups. Each group has a leader who can actively exert effort to enhance social connections among group members. Social connections increase the agents' probabilities of matching with one another among the same group in economic activities and thus increase the assortativity of matching in the population. We find that the endogenous process by which the assortativity level is determined can lead to cultural heterogeneity. While cultural homogeneity is the only prediction when the assortativity level is constant.

Keywords: Cultural evolution, social connections, cultural heterogeneity, assortative matching, evolutionary game theory.

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1 Introduction

A large literature of social capital starting from Coleman (1990) and Putnam et al (1993) is devoted to exploring how social connections among individuals affect interpersonal interactions.¹ It is natural to conjecture that social connections have positive effects on preserving cultural traits of certain cultural groups over time. Empirical evidence has shown that in some ethnic or religious groups, this is indeed the case. For example, Maghribi traders before the modern era of trade persisted even when they emigrated from North Africa to other trade centers; for generations, the descendents of Maghribis continued to cooperate with one another and their Arabic-Jewish traditions were preserved in different host countries (see Grief (1993, 1994, 1997)).

To our limited knowledge, there are not many theoretical works examining how endogenous formation of social connections shapes the trajectory of cultural evolution. This paper serves as an attempt to investigate this question.

We propose a cultural evolutionary model on a population of agents with two cultural groups. Each group has a leader who can exert costly effort to increase social connections among group members. Social connections among a group's members increase their probability of matching with one another in economic activities. Hence, social connections increase the assortativity level of matching in the population.

There are several possible channels through which the leaders can enhance social connections within their own groups. For example, they can organize social events such as picnics, parties, festivities, recreational activities and religious instruction for their own members so that the members have a higher probability to find jobs in firms owned by their own group members, or engage in trade and partnership opportunities with their own group members. Another possibility is that the the leaders can create some unique ethnic or religious markers such as dress codes that distinguish themselves from other cultural groups. These markers enable the members to easily identify

¹For example, Granovetter (1975) and Montgomery (1991) demonstrate that social connections may facilitate information sharing in the job market and increase the efficiency of the job matching process. On the other hand, Bigsten et al (2000) and Fafchamps and Minten (2002) find that in developing countries where laws and courts are insufficient to protect transactions and respect commercial contracts, social connections plays a crucial role minimizing the business owners' exposure to contractual risk. Bernstein (1992) finds that in the diamond industry, because there is a great need for credit, closely connected clubs are formed to provide implicit capital markets for their club members. In addition, experimental evidence such as Dawes and Taler (1988), Bohnet and Frey (1999) and Glaeser et al. (2000), among many others, shows that interpersonal relationships enhance trust and cooperation. See Durlauf and Fafchamps (2006) for a survey.

one another in the matching process. The leaders can also geographically segregate their groups' members. Historical examples include the settlement of Salt Lake Valley by the Mormon Pioneers.

After the agents are matched in pairs, each pair engages in economic activities which are described by some pairwise interactions. We focus on two specific payoff structures of the pairwise interactions resembling 1) coordination game and 2) prisoner's dilemmas. Under both payoff structures, cultural homogeneity is the only prediction of the cultural evolutionary dynamic if the assortativity level of matching in the population is constant. However, with social connections, cultural heterogeneous states emerge.

There are two key factors for the emergence of cultural heterogeneity. Both of them depend on the group leaders' cost of effort to increase social connections. First, when a cultural group is sufficiently small, its leader's marginal cost of effort is sufficiently low, such that the leader can effectively increase social connections among the group members to a certain level to ensure the survival of the group. Second, the marginal cost of effort can not stay low as the group expands. Otherwise, this group would expand until it dominates the whole population. Our results provide a possible explanation for the existence of certain "persistent" minorities of small sizes but with high levels of social connections.²

This paper is closely related to the literature of evolution in population with assortative matching. A classic example of assortative matching is interaction between relatives in the same generation of a sexually reproducing population, in which the probability of matching is determined by Wright's coefficient of relatedness (Wright (1921, 1922)). Important works include Hamilton (1964 a,b), Bergstrom (1995, 2003, 2013), Van Veelen (2006), Alger and Weibull (2010, 2011, 2013), among many others. However, most of these works focus on exogenous level of assortativity. The main contribution of this paper is that the assortativity of matching is endogenized by the effort exerted by the groups' leaders to increase social connections.

The paper is organized as follows. Section 2 describes the model. Section 3 and 4 conduct analysis on payoff structures resembling coordination games and prisoner's dilemmas, respectively. Section 5 concludes.

2 The Model

In each generation, a continuum of agents constitutes a population. Each agent either carries a cultural trait θ or another cultural trait τ , dividing the population into two cultural groups. Each

²For example, see Hirschman and Wong (1986) for a discussion on Asian immigrants in the United States.

group has one leader. In this paper, the cultural traits refer to preferences, beliefs and norms that govern the behaviors of individuals in interactions with other agents. Let $x \in [0,1]$ denote the proportion of agents with trait τ and 1 - x the proportion of agents with trait θ . x can also be regarded as the population state.

All agents enter a random matching process, in which the probabilities of matching are determined by the levels of social connections in both groups. Social connections within a group is determined by the effort exerted by its group leader. The agents are matched in pairs and engage in some identical pairwise interaction.

In the next generation, the distribution of cultural traits in the population is determined by the comparison of the average material payoffs of the two cultural groups in the previous generation. This results in an evolutionary process that is governed by an imitative dynamic.

In Section 2.1, we specify the matching process and the pairwise interaction. In Section 2.2, we illustrate how social connections are formed endogenously and how the assortativity level of matching in the population is determined by social connections. In Section 2.3, we derive the evolutionary dynamic.

2.1 Matching and Pairwise Interaction

Let $\Pr[\theta|\tau, x]$ denote the probability that a τ agent is matched with a θ agent. Let $\sigma(x) \in [0, 1]$ be the index of assortativity of the matching, which is the difference between the probability that a θ agent is matched with a θ agent and the probability that a τ agent is matched with a θ agent:

$$\sigma(x) = \Pr[\theta|\theta, x] - \Pr[\theta|\tau, x].$$
(1)

The higher $\sigma(x)$ is, the higher the probability that an agent is matched with one of his own group members. $\sigma(x)$ was first introduced by Bergstrom (2003) as an exogenous parameter to study strategy evolution, known as algebraic assortative encounter, and later adopted by Alger and Weibull (2012, 2013) to study preference evolution. For now we assume that $\sigma(x)$ is exogenous.

To ensure that the expected number of agents with trait θ matched with agents with trait τ equals the expected number of agents with trait τ matched with agents with trait θ , the following balancing condition is needed:

$$(1-x)(1-\Pr[\theta|\theta,x]) = x\Pr[\theta|\tau,x].$$
(2)

From (1) and (2), we get

$$\Pr[\theta|\theta, x] = (1-x)(1-\sigma(x)) + \sigma(x); \tag{3}$$

$$\Pr[\tau|\theta, x] = x(1 - \sigma(x)); \tag{4}$$

$$\Pr[\theta|\tau, x] = (1 - x)(1 - \sigma(x)); \tag{5}$$

$$\Pr[\tau|\tau, x] = x(1 - \sigma(x)) + \sigma(x).$$
(6)

For $i, j \in \{\theta, \tau\}$, let V(i, j) denote the equilibrium material payoff of a *i* agent against a *j* agent.³ For notational convenience, denote $V(\theta, \theta) = a, V(\theta, \tau) = b, V(\tau, \theta) = c, V(\tau, \tau) = d$. Table 1 tabulates the material payoffs that the agents can have:



Table 1

The average material payoffs of θ group is:

$$F_{\theta}(x,\sigma(x)) = \Pr[\theta|\theta,x]V(\theta,\theta) + \Pr[\tau|\theta,x]V(\theta,\tau)$$

= $[(1-x)(1-\sigma(x)) + \sigma(x)]a + x(1-\sigma(x))b;$ (7)

The average material payoffs of τ group is:

$$F_{\tau}(x,\sigma(x)) = \Pr[\theta|\tau,x]V(\tau,\theta) + \Pr[\tau|\tau,x]V(\tau,\tau)$$

= $(1-x)(1-\sigma(x))c + [x(1-\sigma(x))+\sigma(x)]d.$ (8)

³Note that we adopt two common assumptions in the literature of preference evolution. First, we assume that the pairwise interaction has a unique equilibrium for each pair of agents with any cultural traits. Methods of handling the potential problem of multiplicity of equilibria in specific contexts have been discussed in the literature (see for example, Alger and Weibull (2013)). Nevertheless, since we seek general results that can hold across a variety of contexts, we maintain our assumption of uniqueness. Second, we assume that the agents have complete information (see Eswaran and Neary (2014) for a discussion on the justification of observability of preferences by appealing to the psychology of deception).

2.2 Endogenous Formation of Social Connections

In this section, we endogenize the assortativity of matching. A group leader can exert costly effort to increase social connections among group members through organizing social events. Social connection increase the probability that these members match with one another other in pairwise interactions.

We focus on the case in which the group leaders' interests are aligned with the common interests of their group members. In other words, they benefit directly from an increase in their own groups' average material payoffs.⁴ Therefore a group leader exerts effort to maximize the average material payoff of the group minus the cost of effort.

Let $e_{\theta}(x) \in [0, 1)$ denote the effort exerted by the θ group's leader and $e_{\tau}(x) \in [0, 1)$ denote the effort exerted by the τ group's leader at state x. The assortativity of matching is now determined by the efforts of both leaders, and can be written as a function of efforts: $\sigma(\cdot, \cdot) : [0, 1) \times [0, 1) \rightarrow [0, 1)$. The assortativity level $\sigma(e_{\theta}(x), e_{\tau}(x))$ is increasing in both group leaders' efforts $e_{\theta}(x), e_{\tau}(x)$.

For purposes of tractability, we consider a particular functional form for the leaders' cost of effort. Let the cost functions be quadratic functions of the exerted effort.

The maximization problems of the group leaders are given as follows:

$$e_{\theta}^{*}(x)$$
 solves $\max_{e} F_{\theta}(x, \sigma(e, e_{\tau}^{*}(x))) - \frac{1}{2}k(1-x)e^{2};$ (9)

$$e_{\tau}^*(x)$$
 solves $\max_e F_{\tau}(x, \sigma(e_{\theta}^*(x), e)) - \frac{1}{2}k(x)e^2.$ (10)

In the cost function, k captures the marginal cost of effort for a leader and it is a differentiable function of the group size. One can impose certain properties on function k based on the applications in hand. In this paper, we focus on two common types of marginal cost of effort. 1) k is strictly increasing. Social events such as picnics, parties, festivities may be increasing costly to organize for the leaders as a group expands. 2) k is strictly decreasing. Social events such as religious instruction may be easier to hold for the leaders as a group expands.

For notational convenience, we use $\sigma^*(x)$ to denote the optimal level of assortativity $\sigma(e^*_{\theta}(x), e^*_{\tau}(x))$.

⁴It is also possible that the leaders may want to expand their own groups' memberships because they can benefit from group expansion. Nevertheless, given that the evolutionary process we model in Section 2.3 is determined by the difference between average material payoffs of the two groups, maximizing the average material payoffs of their own groups' members indirectly help the leaders to achieve their goals of expansion.

2.3 The Cultural Evolutionary Dynamic

In this section, we specify the evolutionary process of cultural traits across generations over time: When the average material payoff of θ group is higher than that of τ group in the current generation, θ group expands and τ group shrinks in the next generation. On the other hand, when the average material payoff of θ group is lower than that of τ group in the current generation, θ group shrinks and τ group expands in the next generation. Such an evolutionary process can be captured by a discrete version of the imitative dynamic⁵ (from now on, we add time indexes to the population state x):

$$x_{t+1} = x_t + (1 - x_t)x_t h(F_\tau(x_t, \sigma^*(x_t)) - F_\theta(x_t, \sigma^*(x_t))), \text{ with initial condition } x_0 \in [0, 1], \quad (11)$$

where $h(\cdot): \mathbb{R} \to [-1, 1]$ is a differentiable and strictly increasing function that satisfies:

$$h(a) > 0, \text{ if } a > 0;$$

 $h(a) = 0, \text{ if } a = 0;$
 $h(a) < 0, \text{ if } a < 0.$ (12)

The two culturally homogeneous states (x = 0 and x = 1) are steady states. When there exists an interior state $x^* \in (0,1)$ such that the two groups have equal average material payoffs, i.e. $F_{\theta}(x^*, \sigma^*(x^*)) = F_{\tau}(x^*, \sigma^*(x^*)), x^*$ is a steady state in which both cultural traits coexist.

3 Payoff Structures Resembling Coordination Games

In this section, we study the payoff structures shown in Table 1 that resemble coordination games. In other words, a θ agent does at least weakly better by interacting with another θ agent than a τ agent does. And a τ agent does at least weakly better by interacting with another τ agent than a θ agent does. This requires $a \ge c$ and $d \ge b$.

This form of payoff structures is of interest because the cultural evolutionary dynamic converges to either one of the two culturally homogeneous states when the assortativity level of matching is constant. In what follows, we explore if cultural heterogeneity arises in this case.

Without loss of generality, let a < d. Given that c < a, we have c < d, which means that the τ type agents gain more from self-matching than from cross-matching. Hence, the τ group's leader always has an incentive to exert effort to increase social connections for τ group members.

⁵See Sandholm (2010).

There are two possibilities for the θ type agents: 1) we discuss $a \leq b$ in Section 3.1, and 2) we discuss a > b in Section 3.2.

3.1 An Inward Looking Group vs An Outward Looking Group

We first consider $a \leq b$. That is, the θ agents gains more from cross-matching than from selfmatching. Hence, the θ group's leader has no incentive to exert effort. The property $a \leq b$ also implies that all agents gain more from matching with τ type agents rather than from matching with θ type agents.

We call the θ agents "outward looking" because they prefer to interact with other type agents and call the τ type agents "inward looking" because they prefer to interact with agents of the same type.

For tractability, we adopt a particular functional form for assortativity: $\sigma^*(x) = \sigma(0, e_{\tau}^*(x)) = e_{\tau}^*(x)$. That is, the optimal level of assortativity is equal to the equilibrium effort exerted by the τ group leader when θ group leader does not exert any effort. We focus on the case that an interior solution exists to the maximization problem of the τ group's leader in (10). This requires $k(0) \geq d - c$.

By solving (10), the equilibrium effort exerted by the τ group's leader at time t (equivalently, the corresponding level of assortativity of matching) is given by

$$\sigma^*(x_t) = e^*_{\tau}(x_t) = \frac{(d-c)(1-x_t)}{k(x_t)}.$$
(13)

We first assess the locally asymptotic stability of the culturally homogeneous states (x = 0, 1). For the benchmark case in which assortativity level is constant, we have

Lemma 1

When $\sigma(x) = \sigma$ for any $x \in [0, 1]$, where $\sigma \in [0, 1]$ is a constant,

- (1) if $\sigma < \frac{a-c}{d-c}$, both x = 0 and x = 1 are locally asymptotically stable;
- (2) if $\sigma \geq \frac{a-c}{d-c}$, x = 1 is globally asymptotically stable.

Proof: See Appendix.

Lemma 1 states when the assortativity level is sufficiently low, a small group of "inward looking" agents would get assimilated in a population dominated by "outward looking" agents, unless the

"inward looking" group is sufficiently large in the population at the first place. On the other hand, when the assortativity level is sufficiently high, regardless of the size of the "inward looking" group, it always expand and eventually dominates the whole population.

When $\sigma^*(x_t)$ is determined by the effort exerted by the τ group's leader as in (13), we have

Proposition 1

(a) The state x = 1 is locally asymptotically stable.
(b) If k(0) > (d-c)²/(a-c), then x = 0 is locally asymptotically stable, if the inequality is reversed, then x = 0 is not locally asymptotically stable. *Proof:* See Appendix.

Proposition 1(a) states that cultural heterogeneity cannot arise from a homogeneous population populated by the "inward looking" agents (τ agents). The rationale is as follows: When the size of the "inward looking" group exceeds $\frac{a-c}{a+d-b-c}$ (where $x = \frac{a-c}{a+d-b-c}$ is the population state in which the average material payoffs of the two groups are equal given that the matching is uniformly random), the "inward looking" type agents have a higher average material payoff than the "outward looking" type agents even when their leader does not exert any effort to increase their social connections.

On the other hand, Proposition 1 (b) states that cultural heterogeneity can arise from a homogeneous population dominated by "outward looking" type agents if the marginal cost of effort of the "inward looking" group's leader is sufficiently low $(k(0) < \frac{(d-c)^2}{(a-c)})$ when the group is small. In this case, the "inward looking" group's members have strong social connections and thus earn a higher average material payoff than the "outward looking" group's members.

Next, we consider the locally asymptotic stability of the culturally heterogeneous states if exist. If a culturally heterogeneous state x^* is locally asymptotically stable, it must be a steady state. Hence, it satisfies $F_{\theta}(x^*, \sigma^*(x^*)) = F_{\tau}(x^*, \sigma^*(x^*))$. This implies that

$$k(x^*) = \frac{(d-c)^2(1-x^*)^2 + (b-a)(d-c)x^*(1-x^*)}{(a-c)(1-x^*) - (d-b)x^*}.$$
(14)

Let us define the following function, which can be regarded as the marginal benefit of increasing social connections for the "inward looking" group members:

$$l(x) = \frac{(d-c)^2(1-x)^2 + (b-a)(d-c)x(1-x)}{(a-c)(1-x) - (d-b)x}.$$
(15)

Intuitively, when the marginal cost of effort for the "inward looking" group's leader is below the

marginal effect of increasing social connections for own group members, the optimal effort exerted by the group leader is sufficiently high to make sure that the group has a higher average material payoff than the "outward looking" group. As a result, the "inward looking" group expands. On the other hand, when the marginal cost of effort for the "inward looking" group's leader is above the marginal effect of increasing social connections for its members, the optimal effort exerted by the leader is no longer sufficient to make the group have a higher average material payoff than the "outward looking" group. Hence, the "inward looking" group shrinks. We have the following result:

Proposition 2

For a steady state $x^* \in (0,1)$, if $k'(x^*) > l'(x^*)$ and there exists a constant m > 0 such that h'(0) < m, then x^* is locally asymptotically stable. Moreover, $k'(x^*) > l'(x^*)$ is a necessary condition for x^* to be locally asymptotically stable.

Proof: See Appendix.

Proposition 2 states that when the marginal cost of effort crosses the marginal benefit of increasing social connections from below at x^* , and the dynamic moves smoothly (h'(0) < m) around x^* , then given small perturbations to the culturally heterogeneous state x^* , the dynamic converges back to x^* .

The rationale is as follows. k'(x) > l'(x) ensures that when the "inward looking" group's size is smaller than but close to x^* , the group expands. When the "inward looking" group's size is larger than but close to x^* , the group shrinks.

Next, we investigate the graphical relationship between functions k and l and study how their shapes affect the trajectory of the cultural evolutionary dynamic.

Corollary 1

(A) If $k(0) > \frac{(d-c)^2}{(a-c)}$, there is a unique interior steady state $x^* \in (0, \frac{a-c}{a+d-b-c}]$, such that $\lim_{t\to\infty} x_t = 0$, for any $x_0 \in (0, x^*)$ and $\lim_{t\to\infty} x_t = 1$, for any $x_0 \in (x^*, 1)$. (B) If $k(0) < \frac{(d-c)^2}{(a-c)}$ and k(x) and l(x) intersect at most once, then $\lim_{t\to\infty} x_t = 1$, for any $x_0 \in (0, 1)$. (C) If $k(0) < \frac{(d-c)^2}{(a-c)}$ and k(x) and l(x) intersect at least twice, then there exist two steady states $x^*, x^{**} \in (0, \frac{a-c}{a+d-b-c})$ such that $x^* < x^{**}$. The cultural evolutionary dynamic is stabilizing around x^* , that is: (1) when $0 < x_t < x^*, x_t < x_{t+1}$; (2) when $x^* < x^{**}, x_t > x_{t+1}$. *Proof:* See Appendix. Corollary 1(A) states that if the marginal cost of effort for the "inward looking" group's leader is sufficiently high $(k(0) > l(0) = \frac{(d-c)^2}{(a-c)})$, the cultural evolutionary dynamic converges to either one of the two culturally homogeneous states as in the benchmark case.

On the other hand, if the marginal cost of effort for the "inward looking" group's leader is sufficiently low $(k(0) < l(0) = \frac{(d-c)^2}{(a-c)})$, k(x) cannot cross l(x) only once because when x approaches $\frac{a-c}{a+d-b-c}$, $l'(x) \to \infty$. Hence, there are three possible cases. First, k(x) and l(x) do not intersect. This happens when k(x) is always below l(x) on $[0, \frac{a-c}{a+d-b-c})$. Second, k(x) is tangent to l(x). Third, k(x) intersects l(x) at least twice. Corollary 1 (B) states that in the former two cases, the marginal cost of effort of the "inward looking" group's leader is sufficiently low, such that even if initially the "inward looking" group is very small, it eventually dominates the whole society. Corollary 1 (C) describes the scenario in which, although an initially small "inward looking" group is able to expand because of the low initial marginal cost of effort for its leader, it stops growing due to either the fast-increasing marginal cost of effort or the slow-decreasing marginal cost of effort.

We graphically illustrate how different specifications of marginal cost of effort affect the stable steady states of the cultural evolutionary dynamic in the following numerical example. Let a = 2, b = 2, c = 0, d = 3 and marginal cost of effort k is linear in x. In Figure 1 to Figure 3, all the l functions pictured are derived from this payoff structure, Δx is the change in the size of the "inward looking" group by eliminating the time indexes, stable steady states are marked with solid squares and unstable steady states are marked with empty squares.



Figure 1: High initial marginal cost of effort

Figure 1 depicts the scenario described in Corollary 1(A). The high initial marginal cost of effort may due to the institutional environment of the society. For example, In the beginning of 20th century, President Woodrow Wilson legitimized conformist pressures on Americans who clung to their identities as members of ethnic groups (see a discussion of "Americanization" policies in Kuran and Sandholm (2008)). These pressures might make it costly for the new immigrant groups' members to increase social connections. Another example would be the case of Maghribi traders. As noted by Grief (1994), towards the end of the twelfth century, the Maghribi traders were forced by the rulers of Egypt to cease trading, at which point they integrated with Jewish communities and vanished from history.



Figure 2: Low marginal cost of effort

Figure 2 illustrates the scenario described in Corollary 1 (B). Chinese minorities in South-East Asia serve as a good example. As discussed in Landes (1998), for centuries, Chinese who wished to engage in entrepreneurship were thwarted by "bad government" at home and emigrated to South-East Asia where the institutional environment was less tight. In their new countries, they were able to build strong business networks, bringing them economic success. More importantly, their industrious values spread through the incumbent societies.



Figure 3: Initially low but fast-increasing marginal cost of effort

Figure 3 illustrates the scenario described in Corollary 1 (C). The phenomenon of "persistent" minorities may be explained by Figure 3. Certain ethnic minorities or religious groups stay in

small sizes in certain societies for centuries. This is because even though there are people who do not belong to these groups but appeal to the cultural traits (excluding ethnic or religious aspects of these traits) of these groups, it is hard for them to have social connections with these groups' members because social connections in these groups are religiously and ethnically based. Thus a prerequisite for the leaders to increase social connections among their own group members and those non-group members having the similar cultural traits is to convert those non-group members. This conversion often proves to be costly.⁶

3.2 Two "Inward Looking" Groups

Next, consider the case a > b. In this case, both cultural groups' leaders have an incentive to exert effort to increase social connections for their own group members because all the agents benefit more from cross-matching than self-matching. Such a payoff structure captures many real life phenomena. For example, due to cultural barriers such as languages, habits and etc, agents from different cultural groups may not be able to work together efficiently. In this case, all agents are better off by matching with their own members.

We still maintain the assumption that $\sigma(0, e_{\tau}^*(x)) = e_{\tau}^*(x)$ for any $x \in [0, 1]$ as in Section 3.1. We show that Proposition 1 remains unchanged in the case of a > b:

Proposition 3

Proposition 1 holds in the case of a > b.

Proof:

1) Claim: x = 1 is locally asymptotically stable.

To prove this claim, observe that $F_{\tau}(x_t, \sigma^*(x_t)) - F_{\theta}(x_t, \sigma^*(x_t))$ is a linear function of $\sigma^*(x)$. Moreover, $F_{\tau}(x_t, 0) - F_{\theta}(x_t, 0) > 0$ for $x_t \in (\frac{a-c}{a+d-b-c}, 1)$ and $F_{\tau}(x_t, 1) - F_{\theta}(x_t, 1) = d - a > 0$. Hence, the dynamic converges monotonically to x = 1 for any $x_t \in (\frac{a-c}{a+d-b-c}, 1)$ regardless of the efforts exerted by the group leaders.

2) Claim: If $k(0) > \frac{(d-c)^2}{(a-c)}$, then x = 0 is locally asymptotically stable, if the inequality is reversed, then x = 0 is not locally asymptotically stable.

We first want to show that $e_{\theta}^*(0) = 0$ for any $e \in [0,1]$. When x = 0, we have $Pr[\theta|\theta,0] = 1$

⁶For example, as noted in Lamb and Bryant (1999), "anyone converting to Judaism has to satisfy the Beth Din as to the integrity of his or her motivation, adequacy of knowledge about Jewish law, custom and practice, and has to fulfill certain ritual requirements."

and $Pr[\tau|\theta, 0] = 0$, for any $\sigma(0) \in [0, 1]$. Therefore, the average material payoff of the θ group $F_{\theta}(0, \sigma^*(0)) = a$. As a result, the θ group's leader does not want to exert any effort. Given this, we have $\sigma^*(0) = \sigma^*(0, e_{\tau}^*(0)) = e_{\tau}^*(0) = \frac{(d-c)}{k(0)}$.

When $k(0) > \frac{(d-c)^2}{(a-c)}$, we have $F_{\tau}(0, \sigma^*(0)) < F_{\theta}(0, \sigma^*(0))$. By continuity, there exists a $\delta > 0$, such that for any $x_t \in (0, \delta)$, $F_{\tau}(x_t, \sigma^*(x_t)) < F_{\theta}(x_t, \sigma^*(x_t))$. In this case, the dynamic converges to x = 0. The necessary part follows a similar argument. *Q.E.D.*

Proposition 3 demonstrates that the main results we obtained in Section 3.1 are preserved to the case a > b. That is, if the marginal cost of effort for the τ group's leader is sufficiently low when the τ group is small, the τ group expands, indicating that cultural heterogeneity may arise.

4 Payoff Structures Resembling Prisoner's Dilemmas

In this section, we focus on payoff structures shown in Table 1 that resemble prisoner's dilemmas. This requires c < a < d < b.

This form of payoff structures is of interest because although a society populated with τ type agents has the highest average material payoff, the cultural evolutionary dynamic eventually converges to the culturally homogeneous state with only θ agents if the matching process is uniformly random matching.

We first investigate the benchmark case with constant assortativity level:

Lemma 2

Suppose $\sigma(x) = \sigma$, where $\sigma \in [0, 1]$ is a constant for any $x \in [0, 1]$. When $a + d \ge b + c$,

- (1) if $\sigma \leq \frac{b-d}{b-a}$, x = 0 is globally asymptotically stable;
- (2) if $\sigma \geq \frac{a-c}{d-c}$, x = 1 is globally asymptotically stable;
- (3) if $\frac{b-d}{b-a} < \sigma < \frac{a-c}{d-c}$, both x = 0 and x = 1 are locally asymptotically stable.

Proof: See Appendix.

Lemma 2 states that when the total equilibrium material payoff of self-matching (a + d) exceeds the total equilibrium material payoff of cross-matching (b + c), only culturally homogeneous states are stable.

Lemma 3

Suppose $\sigma(x) = \sigma$, where $\sigma \in [0, 1]$ is a constant for any $x \in [0, 1]$. When a + d < b + c,

- (1) if $\sigma \leq \frac{a-c}{d-c}$, x = 0 is globally asymptotically stable;
- (2) if $\sigma \geq \frac{b-d}{b-a}$, x = 1 is globally asymptotically stable;
- (3) if $\frac{a-c}{d-c} < \sigma < \frac{b-d}{b-a}$, both x = 0 and x = 1 are not stable.

Proof: See Appendix.

Lemma 3 shows that when the total equilibrium material payoff of self-matching (a + d) is smaller than the total equilibrium material payoff of cross-matching (b + c), cultural heterogeneity can emerge when the constant assortativity level is in an intermediate range.

Next, we focus on the case $a + d \ge b + c$ because cultural homogeneity is the only prediction when assortativity level is constant. We explore if social connections can give rise to cultural heterogeneity. We call an agent with cultural trait θ a "defecting" type agent, and one with cultural trait τ a "collaborating" type agent. Since a < b and c < d, the "defecting" group's leader has no incentive to exert effort to increase social connections, while the "collaborating" group's leader has an incentive to exert effort. Again, for tractability, assume $\sigma^*(x) = \sigma(0, e^*_{\tau}(x)) = e^*_{\tau}(x)$ and the equilibrium effort level exerted by the τ group's leader is $e^*_{\tau}(x_t) = \frac{(d-c)(1-x_t)}{k(x_t)}$, given $k(0) \ge d - c$.

Most results we obtained in Section 3 can be directly applied here. However, there is a new phenomenon that can only occur in payoff structures resembling prisoner dilemmas, which is stated in the following proposition:

Proposition 4

If $k(0) < \frac{(d-c)^2}{(a-c)}$, k(x) is strictly increasing, and there exists a constant m > 0 such that h'(0) < m, then there exists a uniquely locally asymptotically stable state $x^* \in (0, 1)$. *Proof:* See Appendix.

Proposition 4 indicates that when the cultural evolutionary dynamic is sufficiently smooth, cultural heterogeneity can be the unique stable prediction of the dynamic, provided that the marginal cost of effort for the "collaborating" group's leader is initially low and increases as the group expands. In Figure 4, we graphically illustrate Proposition 4 in the following numerical example: a = 5, b = 12, c = 0, d = 10. The *l* functions pictured in Figure 4 is derived from this payoff structure.



Figure 4: Initially low and non-decreasing marginal cost of effort

Figure 4 shows that a unique culturally heterogeneous stable state emerges in our model because when the "collaborating" group is small, a high level of social connections protects the group members from being "exploited". However, when the group becomes dominant, social connections are no longer able to protect the group members.

Note that a unique culturally heterogeneous stable state can also emerge in Bisin and Verdier's (2001) cultural transmission model. In their model, cultural traits are transmitted from parents to children. Parents are assumed to have cultural biases. Hence, when a group is small, the parents from that group have a strong enough incentive to inculcate their cultural traits to their children such that the group can resist the pressure of assimilation.

5 Conclusion

In this paper, we propose a cultural evolutionary model in which each cultural group's leader can exert effort to increase social connections among group members. Social connections among a group's members in turn induces a higher probability for them to match with one another in pairwise interactions. Hence, social connections increase the assortativity level of matching in the population. We consider two interesting scenarios that arise from payoff structures resembling 1) coordination games and 2) prisoner's dilemmas. We show that social connections can lead to cultural heterogeneity while cultural homogeneity is the only prediction without endogenous formation of social connections.

There are many possible directions for future research. First, we consider a population consisting of only two cultural traits. It would be interesting to generalize our study to an assortative matching framework for a population with multiple cultural groups. Second, we only consider pairwise interactions. The role of social connections in a context with multiple-player interactions is yet to be explored. Third, formal institutions in a society, such as political institutions and legal institutions, also shape the cultural evolutionary dynamic (See Tabellini (2008), Wu (2015)). It would be of interest to explore the interaction between social connections and different formal institutions.

Appendix

Proof of Lemma 1

When $\sigma < \frac{a-c}{d-c}$, we have $F_{\theta}(0,\sigma) > F_{\tau}(0,\sigma)$. By continuity, there exists a $\delta > 0$, such that for any $x_t \in (0,\delta)$, $F_{\theta}(x_t,\sigma^*(x_t)) > F_{\tau}(x_t,\sigma^*(x_t))$. Therefore, x = 0 is locally asymptotically stable. On the other hand, $F_{\theta}(1,\sigma) < F_{\tau}(1,\sigma)$ for any $\sigma \in [0,1]$. Therefore, x = 1 is also locally asymptotically asymptotically stable by continuity. When $\sigma \geq \frac{a-c}{d-c}$, $F_{\theta}(0,\sigma) \leq F_{\tau}(0,\sigma)$, and $F_{\theta}(x_t,\sigma) < F_{\tau}(x_t,\sigma)$ for any $x_t \in (0,1)$. Therefore, x = 1 is globally asymptotically stable. Q.E.D.

Proof of Proposition 1

We have

$$F_{\tau}(x_t, \sigma^*(x_t)) - F_{\theta}(x_t, \sigma^*(x_t)) = (d-b)x_t - (a-c)(1-x_t) + \frac{(d-c)^2(1-x)^2 + (b-a)(d-c)x(1-x)}{k(x_t)}.$$
 (16)

Therefore, when $x_t > \frac{a-c}{a+d-b-c}$, the average material payoff of the τ group is always higher than that of the θ group. This implies that the cultural evolutionary dynamic converges to x = 1 for any initial condition $x_0 \in (\frac{a-c}{a+d-b-c}, 1)$.

On the other hand, x = 0 is locally asymptotically stable if and only if there exists a $\delta > 0$, such that for any $x_t \in (0, \delta)$, $F_{\tau}(x_t, \sigma^*(x_t)) < F_{\theta}(x_t, \sigma^*(x_t))$. In this case, the dynamic converges to x = 0.

By continuity, the sufficient condition is equivalent to $F_{\tau}(0, \sigma^*(0)) < F_{\theta}(0, \sigma^*(0))$, which is equivalent to $k(0) > \frac{(d-c)^2}{(a-c)}$. On the other hand, the necessary condition is $F_{\tau}(0, \sigma^*(0)) \leq F_{\theta}(0, \sigma^*(0))$, which is equivalent to $k(0) \geq \frac{(d-c)^2}{(a-c)}$. Q.E.D.

Proof of Proposition 2

We first prove sufficiency. Let x^* satisfy $F_{\theta}(x^*, \sigma^*(x^*)) = F_{\tau}(x^*, \sigma^*(x^*))$. If $k'(x^*) > l'(x^*)$, then

there must exist a $\delta > 0$, such that $F_{\theta}(x_t, \sigma^*(x_t)) < F_{\tau}(x_t, \sigma^*(x_t))$ for $x_t \in (x^* - \delta, x^*)$ and $F_{\theta}(x_t, \sigma^*(x_t)) > F_{\tau}(x_t, \sigma^*(x_t))$ for $x_t \in (x^*, x^* + \delta)$.

For notational convenience, we define

$$f(x_t) = x_t + (1 - x_t)x_t h(F_{\tau}(x_t, \sigma^*(x_t)) - F_{\theta}(x_t, \sigma^*(x_t)))$$

= $x_t + (1 - x_t)x_t h(-p(x_t) + \frac{q(x_t)}{k(x_t)}),$ (17)

where

$$p(x_t) = (a-c)(1-x_t) - (d-b)x_t,$$
(18)

$$q(x_t) = (d-c)^2 (1-x_t)^2 + (b-a)(d-c)x_t(1-x_t).$$
(19)

Note that $l(x_t) = \frac{q(x_t)}{p(x_t)}$. In addition, at the steady state x^* , $k(x^*) = l(x^*) = \frac{q(x^*)}{p(x^*)}$. We have the following:

$$f'(x^*) = 1 + (1 - 2x^*)h(0) + x^*(1 - x^*)h'(0)(-p'(x^*) + \frac{q'(x^*)k(x^*) - k'(x^*)q(x^*)}{k^2(x^*)})$$

$$< 1 + x^*(1 - x^*)h'(0)(-p'(x^*) + \frac{q'(x^*)k(x^*) - l'(x^*)q(x^*)}{k^2(x^*)})$$

$$= 1 + x^*(1 - x^*)h'(0)(-p'(x^*) + \frac{q'(x^*)\frac{q(x^*)}{p(x^*)} - \frac{q'(x^*)p(x^*) - p'(x^*)q(x^*)}{p^2(x^*)}q(x^*)}{\frac{q(x^*)^2}{p^*(x)^2}})$$

$$= 1.$$
(20)

The above inequality ensure that $f'(x^*)$ is less that 1. When h'(0) is sufficiently small, we have $f'(x^*) > -1$. Therefore, we have $|f'(x^*)| < 1$, implying that x^* is locally asymptotically stable.

Next, we show that $k(x^*) > l(x^*)$ is a necessary condition for x^* to be asymptotically stable by proving the contrapositive statement. When $k(x^*) < l(x^*)$, we have $f'(x^*) > 1$, which implies that x^* is not locally asymptotically stable. When $k(x^*) = l(x^*)$, we know that k and l are tangent at x^* . Therefore, there exists a $\delta > 0$, such that for any $x \in B(x^*, \delta)$, either 1) $k(x) \ge l(x)$ or 2) $k(x) \le l(x)$. In the former case, the cultural evolutionary dynamic fails to converge to x^* for any $x_0 \in (x^* - \delta, x^*)$. In the latter case, the cultural evolutionary dynamic fails to converge to x^* for any $x_0 \in (x^*, x^* + \delta)$. Q.E.D.

Proof of Corollary 1

First note that when $x_t \in (\frac{a-c}{a+d-b-c}, 1]$, $F_{\tau}(x_t, \sigma^*(x_t)) > F_{\theta}(x_t, \sigma^*(x_t))$. Hence, any interior steady state of the dynamic must fall in the range $(0, \frac{a-c}{a+d-b-c}]$.

(A) When $k(0) > \frac{(d-c)^2}{(a-c)} = l(0)$, we have two scenarios: (1) If k(x) > l(x) for any $x \in [0, \frac{a-c}{a+d-b-c}]$, then the unique interior steady state is $x^* = \frac{a-c}{a+d-b-c}$, which is not stable. If k(x) intersects with l(x) for some $x^* \in (0, \frac{a-c}{a+d-b-c})$, then $k'(x^*) < l'(x^*)$, which implies that x^* is not stable. In both scenarios, $F_{\tau}(x_t, \sigma^*(x_t)) > F_{\theta}(x_t, \sigma^*(x_t))$ for any $x_t \in (x^*, 1)$ and $F_{\tau}(x_t, \sigma^*(x_t)) < F_{\theta}(x_t, \sigma^*(x_t))$ for any $x_t \in (0, x^*)$.

(B) When $k(0) < \frac{(d-c)^2}{(a-c)} = l(0)$ and k(x) and l(x) fail to intersect at least twice, we have two scenarios: 1) $k(x_t) < l(x_t)$ for any $x_t \in (0, \frac{a-c}{a+d-b-c})$, which implies that $F_{\tau}(x_t, \sigma^*(x_t)) > F_{\theta}(x_t, \sigma^*(x_t))$ for any $x_t \in (0, 1)$. Hence, x = 1 is globally asymptotically stable. 2) There exists a unique $x^* \in (0, \frac{a-c}{a+d-b-c})$ such that $k(x^*) = l(x^*)$. However, for any $x_t \in (0, \frac{a-c}{a+d-b-c}) \setminus \{x^*\}$, $k(x_t) < l(x_t)$. Hence, x^* is not locally asymptotically stable and x = 1 is globally asymptotically stable..

(C) When $k(0) < \frac{(d-c)^2}{(a-c)} = l(0)$ and k(x) and l(x) intersect at least twice, let x^* be the first intersection and x^{**} be the second intersection. We then have that $F_{\tau}(x_t, \sigma^*(x_t) > F_{\theta}(x_t, \sigma^*(x_t)))$ for $x_t \in (0, x^*)$ and $F_{\tau}(x_t, \sigma^*(x_t)) < F_{\theta}(x_t, \sigma^*(x_t))$ for $x \in (x^*, x^{**})$. Hence, we have (1) when $0 < x_t < x^*, x_t < x_{t+1}$; (2) when $x^* < x_t < x^{**}, x_t > x_{t+1}$. Q.E.D.

Proof of Lemma 2

When $\sigma \leq \frac{b-d}{b-a}$, $F_{\theta}(1,\sigma) \geq F_{\tau}(1,\sigma)$ and $F_{\theta}(x_t,\sigma) > F_{\tau}(x_t,\sigma)$ for any $x_t \in [0,1)$. Hence, x = 0 is globally asymptotically stable. When $\sigma \geq \frac{a-c}{d-c}$, we have $F_{\theta}(0,\sigma) \leq F_{\tau}(0,\sigma)$ and $F_{\theta}(x_t,\sigma) < F_{\tau}(x_t,\sigma)$ for any $x_t \in (0,1]$. Hence, x = 1 is globally asymptotically stable. When $\sigma \in (\frac{b-d}{b-a}, \frac{a-c}{d-c})$ $((\frac{b-d}{b-a}, \frac{a-c}{d-c}) = \emptyset$ if a + d = b + c), there exists a $x^* = \frac{(a-c)-\sigma(d-c)}{(1-\sigma)(a+d-b-c)} \in (0,1)$, such that $F_{\theta}(x_t,\sigma) > F_{\tau}(x_t,\sigma)$ for $x_t \in [0,x^*)$ and $F_{\tau}(x_t,\sigma) > F_{\theta}(x_t,\sigma)$ for $x_t \in (x^*,1]$. Hence, both x = 0 and x = 1 are locally asymptotically stable. Q.E.D.

Proof of Lemma 3

When $\sigma \leq \frac{a-c}{d-c}$, $F_{\theta}(0,\sigma) \geq F_{\tau}(0,\sigma)$ and $F_{\theta}(x_t,\sigma) > F_{\tau}(x_t,\sigma)$ for any $x_t \in (0,1]$. Hence, x = 0 is globally asymptotically stable. When $\sigma \geq \frac{b-d}{b-a}$, we have $F_{\theta}(1,\sigma) \leq F_{\tau}(1,\sigma)$ and $F_{\theta}(x_t,\sigma) < F_{\tau}(x_t,\sigma)$ for any $x_t \in [0,1)$. Hence, x = 1 is globally asymptotically stable. When $\sigma \in (\frac{a-c}{d-c}, \frac{b-d}{b-a})$, there exists a $x^* = \frac{(a-c)-\sigma(d-c)}{(1-\sigma)(a+d-b-c)} \in (0,1)$, such that $F_{\theta}(x_t,\sigma) < F_{\tau}(x_t,\sigma)$ for $x_t \in [0,x^*)$ and $F_{\tau}(x_t,\sigma) < F_{\theta}(x_t,\sigma)$ for $x_t \in (x^*,1]$. Hence, both x = 0 and x = 1 are not stable. Q.E.D.

Proof of Proposition 4

First, similar to Proposition 1, we have that when $\frac{(d-c)^2}{(a-c)}$, x = 0 is not stable.

Second, when k(x) is strictly increasing, k(1) > 0. This implies that $\sigma^*(1) = \frac{(d-c)(1-1)}{k(1)} = 0$. Therefore, we have

$$F_{\tau}(1, \sigma^*(1)) = d < F_{\theta}(1, \sigma^*(1)) = b.$$
(21)

Hence, x = 1 is not locally asymptotically stable.

Third, we find that

$$l'(x) < \frac{(d-c)(d-b)(d-a)}{((a-c)(1-x) + (b-d)x)^2}.$$
(22)

Given that c < a < d < b, the RHS of the above inequality is strictly negative. Therefore, l(x) is strictly decreasing. When $k(0) < \frac{(d-c)^2}{(a-c)}$ and k(x) is a strictly increasing function in x, k(x) crosses l(x) exactly once from below. Hence the interior steady state $x^* \in (0, 1)$ is locally asymptotically stable as we have proved in Proposition 2. *Q.E.D.*

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