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December 2015

Online at https://mpra.ub.uni-muenchen.de/69620/
MPRA Paper No. 69620, posted 21 Feb 2016 16:03 UTC
An Economic Rationale for
Dismissing Low-Quality Experts in Trial*

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Abstract
The history of the admissibility standard for expert testimony in American courtrooms reveals that the standard has gradually increased to a high level since a series of important decisions by the Supreme Court. Whether such a stringent standard for expert testimony is beneficial or detrimental to the American justice system is still under fierce debate, but there has been scant economic analysis of this issue. This paper attempts to fill the gap by presenting a game-theoretic argument showing that a stringent admissibility standard operates to increase the judicial decision’s accuracy under certain situations. More precisely, when the judge faces uncertainty regarding an expert’s quality, the admissibility standard may provide the judge with information about the quality of expert testimony, thereby increasing the accuracy of the judicial decision by mitigating the judge’s inference problem. I show the ways in which this effect dominates at trial and discuss related issues.

Keywords: expert testimony; admissibility standard; persuasion game; evidence distortion.

JEL: C72; D82; K41.

1 Introduction
An important feature of American tort law is that not all experts can testify in courtrooms. Since a series of decisions by the Supreme Court regarding the admissibility of expert testimony, experts are required to pass a stringent admissibility standard to provide testimony on a dispute at trial. In particular, Federal Rule of Evidence 702 provides that expert testimony that would otherwise be helpful to the jury is admissible only when (i) the testimony is based

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*I am grateful to Eric Maskin and participants at various seminars and conferences for their valuable comments. All remaining errors are mine.

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on sufficient facts or data, (ii) the testimony is the product of reliable principles and methods, and (iii) the witness has applied the principles and methods reliably to the facts of the case.

Legal scholars usually find a rationale behind such a legal institution from bounded rationality: triers of fact, especially lay juries, are not sufficiently sophisticated and hence are vulnerable to expert bias. Although experts are presumably “neutral” to the case under consideration and willing to provide honest testimony without intentionally withholding relevant evidence, they can be “biased” toward a cause. Such a situation may occur in reality because of litigants’ competition in providing expert testimony favorable to their own causes. Even when the judge appoints his own expert, such bias can still exist if the expert belongs to a group that advocates one of the causes. For example, a cardiologist testifying in a medical lawsuit involving a heart surgery may have a natural inclination to provide testimony favorable to the doctor who performed the surgery. Critics argue that because triers of fact are inadequately prepared to evaluate biased expert testimony, to protect the triers of fact from the *ipse dixit* of experts with low quality, the admissibility standard is needed to screen experts.

This bounded rationality argument might not be satisfactory to those scholars who are inclined to explain legal institutions within the rational choice framework. What if judges are sufficiently sophisticated to understand experts’ incentive and behavior? Is there still a rationale for the stringent admissibility requirements for expert testimony? Although the answer to this question is important in light of fierce debate in the legal community, law and economics literature has been silent on this issue. To the best of my knowledge, this paper is the first attempt to fill the gap by presenting a game-theoretic model for answering this question.

If judges are rational and able to use Bayesian reasoning, can we still find a rationale for a stringent admissibility standard? In a model with Bayesian judges, an immediate question is...
of the following form: if a judge can use Bayesian reasoning, she can extract useful information even from low-quality experts, so why not let the judge interact with any expert regardless of the expert’s quality? Isn’t it the case that dismissing low-quality experts only increases error costs in litigation processes because the judge loses valuable information possessed by those experts? The answer to this question is not straightforward in an actual litigation environment in which an expert’s quality is unknown. In such a situation, the judge is ill-prepared to interact with a testifying expert because the uncertainty about the expert’s quality imposes an additional constraint on the judge’s Bayesian reasoning, and therefore the judge may value some information about the testifying expert’s quality, which is the primary benefit of a stringent admissibility standard as suggested in this paper.

The judge could obtain such information about the expert’s quality from the opposing counsel’s motion to dismiss the expert, where the party collects and presents verifiable evidence proving its allegation that the expert cannot satisfy the admissibility standard. This may bring forth the other party’s counter-evidence and so forth, in which process the judge can assess the expert’s quality more precisely. Thus, on one hand, by dismissing low-quality experts from the court, the judge cannot utilize the information possessed by them, in which case the judge should make a decision without that information. This situation raises error costs by reducing the judicial decision’s accuracy. On the other hand, the judge could obtain better estimates of the testifying expert’s quality if the expert passes the admissibility standard. Thus, a stringent admissibility standard generates benefits in that situation by raising the judicial decision’s accuracy. Because there exist two countervailing forces, if the latter effect is dominant, a stringent admissibility standard could increase ex ante accuracy of the judge’s decision.

Within a standard litigation model, I show that the latter effect could be quite strong under certain litigation environments. In particular, I show that a stringent admissibility standard could increase ex ante accuracy of the judge’s decision even when the latter effect is taken to be minimal in the sense that the only information available to the judge about the testifying expert’s quality is the very fact that the expert passed the admissibility standard. Thus, if the judge can obtain more information about the testifying expert’s quality as in real litigation situations, it would strengthen my claim that a stringent admissibility standard could increase ex ante accuracy of the judge’s decision, which provides a rationale for its use in litigation processes.

As previously mentioned, although law and economics scholars have extensively studied various topics about decision-making in courtrooms, no economic analysis has been performed on the admissibility standard. Somewhat related is a study by Kim (2015a) which demonstrates that client-expert relationships do not exhibit adversarial bias under certain circumstances. Another work by Kim (2015b) is a study of whether it is preferable to require judges to select their own neutral experts rather than to have litigants present their own biased ex-
That study shows that there exists a trade-off and that the cost of using expert advice is an important factor in the evaluation of the reform. Tomlin and Cooper (2006) also analyze the effect of the reform and provide a rationale for using court-appointed experts. Their main intuition behind the benefit of using court-appointed experts comes from reputation effects: when litigants perceive that the judge is more likely to appoint a neutral expert, they are less likely to present biased expert testimony, which increases the accuracy of the final decision.

The key feature of the model employed in the current paper is the evidence distortion and inference problem. A suitable economic model with which to analyze this problem is the persuasion-game framework presented by Milgrom (1981). Using a seller-buyer example, he studies the ways in which a buyer draws inferences about a product’s quality in the face of the seller’s incentive to conceal evidence detrimental to the sale of the product. He shows that the equilibrium is characterized by full revelation, in which the seller reveals all relevant evidence about the product. Extending his analysis, Milgrom and Roberts (1986) study decision-making under evidence distortion by competing litigants and confirm the robustness of the full revelation phenomenon.

The key reason for the full revelation of relevant information in these models is that the informed party always possesses some information. If this assumption is relaxed, that is, if the informed party may not possess relevant information, the party with the information advantage may distort the evidence in equilibrium, inducing the uninformed party to draw an inference about the hidden evidence. See Shin (1998), Demougin and Fluet (2008), Kim (2014a), and references therein for this line of research. An important assumption in these papers is that the uninformed party knows the quality of information possessed by the informed party. More precisely, the uninformed party knows that the informed party observes hidden information with probability $q$ that is common knowledge in these models. This assumption can be quite strong in many applications because $q$ may also be private information to the informed party. The current paper is different from these papers in that I make a more realistic assumption that the uninformed party (a judge) cannot observe the quality of information possessed by the informed party (an expert).

The remainder of the paper is organized as follows. Section 2 presents a basic game-theoretic model in which a judge interacts with a biased expert. Section 3 analyzes the model

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6There have been numerous reform proposals suggesting that the judge appoint his own experts, thereby enhancing the inquisitorial component in the American legal system. For example, see Runkle (2001), who discusses the structure of the Court Appointed Scientific Experts program created by American Association for the Advancement of Science in order to help judges obtain independent experts. See also Hillman (2002), Adrogue and Ratliff (2003), and Kaplan (2006), among others. Based on his experience as Judge Richard Posner’s court-appointed economic expert, Sidak (2013) argues for court-appointed, neutral economic experts.

7In a related vein, researchers study the problem of an uninformed principal who should design a contract with an informed agent. For this line of research, see Dewatripont and Tirole (1999), Palumbo (2001, 2006), Iossa and Palumbo (2007), Defaux and Demougin (2008), and Kim (2014b).

8Kim (2014a) adopts a slightly different approach in that the informed party chooses, rather than takes as given, the quality of information.
and provides a numerical example. Finally, Section 4 concludes with discussion. Proofs, which do not appear in the main text, can be found in the Appendix.

2 Model

I develop a stylized model of the litigation process in which an expert may testify before a judge who must make a decision about a dispute. Formally, I model the litigation process as a game in which there are two players, the expert and the judge. The truth, indicated by the variable $t$, takes one of two possible values, $a$ and $b$, in which $t = a$ indicates that cause $A$ should prevail and $t = b$ indicates that cause $B$ should prevail in the litigation process. The judge, as a trier of fact aspiring to render a correct verdict, seeks to make a binary decision, $d \in \{A, B\}$, accurately reflecting the truth: the judge wants to rule in favor of cause $A$, $d = A$, if the truth supports that cause (i.e., $t = a$) and in favor of cause $B$, $d = B$, otherwise (i.e., when $t = b$). To be more precise, I assume that the judge obtains a payoff of 1 if he makes a correct decision and a payoff of 0 otherwise.

As the judge is not perfectly informed about the truth in reality, I assume that the judge faces uncertainty regarding the truth in the model. This is captured by the judge’s prior belief about the truth, $\mu \equiv P(t = a)$, which is strictly between 0 and 1. A high value of $\mu$ indicates that the judge believes that the truth supports the cause $A$ with a high probability. A low value of $\mu$ carries the opposite meaning. When additional information is presented during the litigation process, the judge forms a posterior belief about the truth using Bayes’ rule. In this model, such additional information could be provided by the expert testimony. In general, expert testimony is an important form of information at trial. As an expert is someone who is better equipped than a layperson through his “knowledge, skill, experience, training, or education (Federal Rule of Evidence 702)” to perceive the truth in his specialized domains, his testimony can provide the judge with valuable information about the dispute.\(^9\)

Although experts are presumably “neutral” to the case under consideration and willing to provide honest testimony without intentionally withholding any relevant evidence, an expert can be “biased” toward a cause. Such a situation may occur in reality because of litigants’ competition in providing expert testimony favorable to their own causes. Even when the judge appoints his own expert, such bias can still exist if the expert belongs to a group advocating one of the causes. As this paper’s main issues are evidence distortion and inference problems arising from expert bias, I assume that the expert is biased toward one of the causes. Without loss of generality, I assume that the expert is biased toward cause $A$: the expert obtains a payoff of 1 if $d = A$ and a payoff of 0 otherwise.

The expert can supply the judge with a valuable piece of information for decision-making.

Formally, there exists one hidden piece of evidence $x$ that takes one of two possible values, $x \in \{\alpha, \beta\}$, where $x = \alpha$ is a piece of evidence supporting cause $A$ (i.e., $t = a$) and $x = \beta$ supports the other cause (i.e., $t = b$). I assume that the evidence $x$ is realized according to the conditional density, $P(\alpha|a) = P(\beta|b) \equiv p > \frac{1}{2}$. I also assume that this evidence is influential in the judge’s decision making: $\mu \in (1 - p, p)$. If this assumption is not satisfied, the judge’s prior belief is so strong that the judge’s decision is the same regardless of the hidden evidence. This hidden evidence can be uncovered by the expert and revealed to the judge. As the expert is knowledgeable but not perfectly informed about the dispute in reality, I assume that the expert can observe the hidden evidence only with a positive probability. To be more precise, the expert observes $x$ with probability $q \in (0, 1)$. This probability can be interpreted as the expert’s level of “skill” or “experience.” As this probability increases toward 1, the expert is more likely to possess a relevant piece of information about the truth. Using $\tilde{\phi}$, I denote the event in which the expert could not observe the hidden evidence. The judge cannot observe $q$ and only knows the probability distribution $F(q)$ with the support over the open unit interval $(0, 1)$.

When allowed in the courtroom, the expert’s testimony, denoted by $r$, takes the following form. When the expert observed the hidden evidence, he can reveal it truthfully to the judge, $r(x) = x$, or he can suppress the evidence and not reveal any evidence to the judge, $r(x) = \phi$, where $\phi$ indicates the event in which the expert reveals no evidence before the judge. In the latter, the expert is intentionally withholding valuable evidence. If the expert could not observe the evidence, he cannot provide evidence to the judge, $r(\tilde{\phi}) = \phi$, as fabricating evidence is not allowed in the courtroom. If the expert does not reveal any evidence at trial, the judge cannot obtain direct evidence from the expert. However, the judge can still obtain indirect evidence about the truth: if the judge employs Bayesian reasoning in assessing the expert’s testimony, he can infer that the expert does not reveal any evidence under two possible circumstances, i.e., either when the expert could not observe the hidden evidence or when the expert is suppressing a relevant piece of evidence. Thus, accounting for the expert’s behavior, the judge can extract a certain amount of information even when the expert reveals no hard evidence. This observation will be clarified in the subsequent analysis.

An important feature of American tort law is that not all experts can testify in courtrooms, as Federal Rule of Evidence 702 imposes a stringent standard regarding expert testimony. To model this institutional feature formally, I assume that there exists an admissibility standard $\bar{q} \in [0, 1]$ such that only those experts with $q \geq \bar{q}$ are allowed to testify before the judge. In reality, a party may collect and present verifiable evidence to prove its allegation that the opponent’s expert cannot satisfy the admissibility standard. This may bring forth the other

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10. Thus, according to economics jargon, I assume that the information provided to the judge is hard. Models with hard information seem reasonable in a trial setting in which falsifying evidence imposes large penalties on the party. For the soft-information approach to the court decision-making, see Emons and Fluet (2009a,b).

11. This motion is called a Daubert motion, which is a special case of motion in limine raised before or during
party’s counter-evidence and so forth, in which process the expert’s quality is assessed and those experts with \( q < \bar{q} \) are dismissed from the court.

A brief description of the expert-screening procedure above suggests that the actual process is quite complex, involving many players in intricately strategic situations. To effectively convey the intuition behind my results and illustrate the relationship between the admissibility standard and the judicial decision’s accuracy, I abstract from the details and treat the expert-screening procedure as a black box: the expert goes through the procedure and is allowed to testify before the judge only when \( q \geq \bar{q} \). In particular, I abstract from the details about how information about \( q \) is revealed through motions in the court, which indicates that my model is not intended as a thorough analysis of motions to dismiss experts. Rather, the focus of my model is to illustrate the intuition behind the ways in which a higher level of admissibility standard may increase the judicial decision’s accuracy within a simple game-theoretic framework.

If the expert is dismissed, the judge cannot utilize the expert’s information, and therefore the extent of the judge’s knowledge about \( q \) does not matter in this case. In contrast, if the expert passes the admissibility standard, how much the judge knows about \( q \) is important because it influences the judge’s belief about the truth and thereby her final decision. In practice, the expert-screening procedure may reveal some information about \( q \), and the extent of this information may vary across cases: the judge may obtain very accurate estimates of \( q \) in some tort cases but not in others, depending on the characteristics and details of the underlying suits. To demonstrate that my results are robust to these varying factors, I assume that the judge’s knowledge about \( q \) is minimal in the sense that if the expert passes the admissibility standard, the judge only knows the very fact that the expert’s quality \( q \) is above the threshold \( \bar{q} \). As the main results show that a stringent admissibility standard could increase the judicial decision’s accuracy, if the judge obtains more information about \( q \) during the procedure, it will strengthen my results.

If \( \bar{q} = 0 \), any expert can freely testify before the judge regardless of his quality; as \( \bar{q} \) rises above 0, a more stringent standard is required for expert testimony. The history of the admissibility standard for expert testimony in American courtrooms reveals that the standard has gradually increased from virtually \( \bar{q} = 0 \)\(^{12}\) to a high level \( \bar{q} > 0 \) since a series of important decisions by the Supreme Court. Whether such a stringent standard for expert testimony is beneficial or detrimental to the American justice system is still under fierce debate, but there is scant economic analysis of this issue. This paper attempts to fill the gap by investigating the ways in which \( \bar{q} \) influences the judicial decision’s accuracy. In particular, the main result

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\(^{12}\)The original Rule 702, enacted in 1975, required a very forgiving test, providing that “if scientific, technical, or other specialized knowledge will assist the trier of fact to understand the evidence or to determine a fact in issue, a witness qualified as an expert by knowledge, skill, experience, training, or education, may testify thereto in the form of an opinion.”
demonstrates that the optimal admissibility standard that maximizes ex ante accuracy is positive, i.e., $\bar{q}^* > 0$, under certain situations; in other words, dismissing low-quality experts via a stringent admissibility standard may help the judge make a more accurate decision ex ante.

To conclude, the sequence of events is summarized below.

- **Period 1**: The expert goes through the expert-screening procedure. If $q < \bar{q}$, he is dismissed from the court, and the judge makes a decision based on the prior probability. If $q \geq \bar{q}$, the expert is allowed to testify before the judge and the game proceeds to the next stage.

- **Period 2**: After observing the hidden evidence with probability $q$, the expert reports $r$ to the judge.

- **Period 3**: Only knowing $q \geq \bar{q}$, the judge makes a decision based on the posterior probability influenced by $r$.\(^{13}\)

In the next section, I find the perfect Bayesian equilibrium of the game, which is simply referred to as equilibrium, and study whether dismissing low-quality experts from the court (i.e., setting $\bar{q} > 0$) is beneficial in increasing ex ante accuracy of the judge’s decision.

3 Analysis

The judge in my model may make two types of errors: despite that $t = a$, cause $B$ prevails, and despite that $t = b$, cause $A$ prevails. For future reference, let us call the first type of error the Type-I error, and the second type of error the Type-II error. Thus, the expected error from the judge’s decision-making, $\eta$, is given by

$$\eta(\bar{q}) = \mu \cdot \text{(Type I error)} + (1 - \mu) \cdot \text{(Type II error)}$$

$$\eta(\bar{q}) = \mu P(d = B | t = a) + (1 - \mu) P(d = A | t = b)$$

where $\eta$ is a function of the admissibility standard $\bar{q}$, which will be clarified in the subsequent analysis.

If the expert is of low quality, $q < \bar{q}$, the expert is dismissed as unreliable during the expert-screening process and cannot therefore testify before the judge. In such a situation, the judge has no additional evidence toward his decision, and he thus makes a decision based on his prior belief regarding the truth. Thus, the judge rules in favor of cause $A$ if $\mu \geq \frac{1}{2}$ and in favor of cause $B$ otherwise.

\(^{13}\)Thus, the judge’s decision is a function of $r$, i.e., $d(r) \in \{A, B\}$. 
Instead, suppose that the expert’s quality is above the admissibility standard, \( q \geq \bar{q} \). Then, the expert is allowed to testify before the judge, and three events are possible at trial: \( x = \alpha \) is revealed, \( x = \beta \) is revealed, or no evidence is revealed. If the expert reveals \( x = \alpha \) at trial, the judge’s posterior belief becomes

\[
\lambda(\alpha) \equiv P(t = a| x = \alpha) = \frac{\mu p}{\mu p + (1 - \mu)(1 - p)} > \frac{1}{2}
\]

(1)

where the inequality holds because \( \mu \in (1 - p, p) \). In (1), \( \lambda(\alpha) \) represents the judge’s posterior belief conditional on the evidence \( x = \alpha \) presented by the expert. As \( t = a \) is more likely than \( t = b \) conditional on \( x = \alpha \), the revelation of \( x = \alpha \) induces the judge to rule in favor of cause \( A \) in any equilibrium, i.e., \( d^*(\alpha) = A \). Anticipating this outcome, the expert always reveals \( x = \alpha \) in any equilibrium whenever possible, i.e., \( r^*(\alpha) = \alpha \).

If the expert reveals \( x = \beta \) at trial, the judge’s posterior belief becomes

\[
\lambda(\beta) \equiv P(t = a| x = \beta) = \frac{\mu(1 - p)}{\mu(1 - p) + (1 - \mu)p} < \frac{1}{2}
\]

(2)

where the inequality holds because \( \mu \in (1 - p, p) \). In (2), \( \lambda(\beta) \) represents the judge’s posterior belief conditional on the evidence \( x = \beta \) presented by the expert. As \( t = a \) is less likely than \( t = b \) conditional on \( x = \beta \), the revelation of \( x = \beta \) induces the judge to rule in favor of cause \( B \) in any equilibrium, i.e., \( d^*(\beta) = B \). Anticipating this outcome, the expert with \( x = \beta \) must choose whether to reveal or conceal his evidence. On one hand, if the expert believes that the concealment of evidence (which leads to \( \phi \)) induces the judge to rule in favor of cause \( A \), the expert suppresses the evidence. On the other hand, if the expert believes that the concealment of evidence also leads to cause \( B \)’s winning, he is indifferent between revealing and suppressing \( x = \beta \). To avoid unnecessary complication, I assume that the expert adopts a pure strategy in this situation and that the expert suppresses evidence if he is indifferent between revealing and suppressing the evidence.\(^{14}\) Thus, if the expert observed \( x = \beta \), he suppresses the evidence and does not reveal evidence before the judge in any equilibrium, leading to the event \( \phi \), i.e., \( r^*(\beta) = \phi \). As the expert also does not reveal evidence when he could not observe the hidden evidence, i.e., \( r^*(\bar{\phi}) = \phi \), the no-evidence event \( \phi \) occurs under two situations: either the expert suppressed \( x = \beta \) or could not observe the hidden evidence.

Supposing for a moment that the judge knows the expert’s quality \( q \in (0, 1) \), the judge’s posterior belief regarding the truth under \( \phi \) becomes

\[
\lambda(\phi; q) \equiv P(t = a| \phi; q) = \frac{\mu(q(1 - p) + 1 - q)}{\mu(q(1 - p) + 1 - q) + (1 - \mu)(qp + 1 - q)} < \mu
\]

(3)

\(^{14}\)For example, introducing a small amount of evidence revelation cost can rationalize this assumption.
with its derivative with respect to $q$ given by

$$\frac{\partial \lambda(\phi; q)}{\partial q} = \frac{\mu(1 - 2p)(1 - \mu)}{(\mu(q(1 - p) + 1 - q) + (1 - \mu)(qp + 1 - q))^2} < 0$$

where the last inequality holds because $p > \frac{1}{2}$. The sign of the derivative is intuitive. If $q$ is high, the expert is highly likely to have observed the hidden evidence, and therefore the chances are high that $\phi$ occurs from the expert’s evidence distortion, i.e., the hidden evidence is $x = \beta$. Thus, the judge’s posterior becomes “lower,” with a lower probability that the truth is in favor of cause $A$.

Returning to my modelling assumption, the judge must calculate the expectation of $\lambda(\phi; q)$ over $q$ because he cannot directly observe the expert’s quality $q$; he only knows that the expert’s quality is above the threshold $\bar{q}$. Thus, the judge’s correct posterior belief under $\phi$ is given by

$$\Lambda(\bar{q}) \equiv E_q[\lambda(\phi; q)|q \geq \bar{q}] = \int_{\bar{q}}^{1} \lambda(\phi; q) \frac{f(q)}{1 - F(\bar{q})} dq < \mu$$

where $\Lambda$ is a function of $\bar{q}$. Thus, under $\phi$, the judge rules in favor of cause $A$ if $\Lambda(\bar{q}) \geq \frac{1}{2}$ and in favor of cause $B$ otherwise. For example, suppose $\bar{q} = 0.3$ and $\Lambda(0.3) = 0.4$. This example means that (i) only those experts with quality higher than 0.3 can testify before the judge, (ii) when an expert testifies at trial and reveals $x = \alpha$, the judge rules in favor of cause $A$ according to (1), and (iii) when an expert testifies at trial but does not reveal any evidence, the judge believes that the truth is $t = a$ with only 40%, thereby ruling in favor of cause $B$. Furthermore, $\Lambda(\bar{q}) < \mu$ for all $\bar{q} \in [0, 1]$ because $\Lambda(\bar{q})$ is an average of $\lambda(\phi; q)$ where $\lambda(\phi; q) < \mu$ for all $q \in (0, 1)$ as shown in (3).

Intuitively, $\Lambda$ must be a decreasing function of $\bar{q}$. As society imposes a more stringent standard for expert testimony (leading to a higher value of $\bar{q}$), the judge reasons that those experts testifying at trial must be of high quality on average. Therefore, the no-evidence event $\phi$ is indicative of evidence distortion (i.e., the hidden evidence is $x = \beta$), lowering the judge’s posterior belief regarding $t = a$. Taking the derivative of $\Lambda$, I indeed confirm that this posterior belief decreases as $\bar{q}$ increases:

$$\Lambda'(\bar{q}) = -\lambda(\phi; \bar{q}) \frac{f(\bar{q})}{1 - F(\bar{q})} + \int_{\bar{q}}^{1} \lambda(\phi; q) \frac{f(q)f(\bar{q})}{(1 - F(\bar{q}))^2} dq$$

$$= \frac{f(\bar{q})}{(1 - F(\bar{q}))^2} \int_{\bar{q}}^{1} (\lambda(\phi; q) - \lambda(\phi; \bar{q})) f(q) dq$$

$$< 0$$

where the last inequality holds because $\frac{\partial \Lambda}{\partial \bar{q}} < 0$ as shown in (4). These results are summarized in the following lemma:
Lemma 1. There exists a unique equilibrium in the subgame following $q \geq \bar{q}$ such that (i) the expert’s equilibrium strategy is given by

$$r^*(\alpha) = \alpha$$
$$r^*(\beta) = \phi$$
$$r^*(\hat{\phi}) = \phi$$

and (ii) the judge’s equilibrium strategy is given by

$$d^*(\alpha) = A$$
$$d^*(\beta) = B$$
$$d^*(\phi) = \begin{cases} A & \text{if } \Lambda(\bar{q}) \geq \frac{1}{2} \\ B & \text{if } \Lambda(\bar{q}) < \frac{1}{2} \end{cases}$$

where $\Lambda$ is given by (5).

As the expert is biased toward cause $A$, he distorts evidence for that cause: when the expert observed the hidden evidence, he reveals only favorable evidence, $x = \alpha$, and suppresses unfavorable evidence, $x = \beta$. Although the expert is biased, he cannot forge favorable evidence, because the evidence is assumed to be verifiable. This verifiability assumption also implies that the expert cannot reveal any information when he could not observe the hidden evidence. Thus, there are two possibilities at trial: the expert either reveals $x = \alpha$ or he does not reveal any evidence.\(^{15}\) As a Bayesian decision-maker, the judge updates his belief based on the evidence presented by the expert at trial. In particular, even when the expert reveals no evidence, the judge can extract a certain amount of information from the situation $\phi$ by considering the expert’s evidence-distorting behavior and the underlying information structure. The amount of indirect information obtained under $\phi$ can be captured by $|\Lambda(\bar{q}) - \mu|$. If this quantity is large, the judge obtains a large amount of indirect information under the no-evidence event, and vice versa.

**Favorable Cases**

To calculate the expected error of the judge’s decision-making, first consider those cases favorable toward the cause that the expert favors, i.e., $\mu \geq \frac{1}{2}$. I classify this case into two subcases: (1) $\frac{1}{2} \leq \Lambda(0) < \mu$ and (2) $\Lambda(0) < \frac{1}{2} \leq \mu$. In the first subcase, if $\bar{q} = 0$, cause $A$ always prevails. Because there is no admissibility requirement for expert testimony, any expert can freely testify before the judge regardless of his quality. If the expert presents evidence in support of cause $A$, $x = \alpha$, the judge rules in favor of cause $A$ according to (1).

\(^{15}\)Therefore, the revelation of $x = \beta$ occurs in the off-equilibrium path.
If the expert does not reveal evidence before the judge, the posterior belief of the judge in such a situation is given by \( \Lambda(0) \geq \frac{1}{2} \), which induces the judge to also rule in favor of cause \( A \). Thus, cause \( A \) always prevails; therefore, there is no Type-I error, and the Type-II error is maximized. More precisely, the expected error in this case is given by

\[
\eta(0) = \mu P(d = B|t = a) + (1 - \mu)P(d = A|t = b) = 1 - \mu
\]

As society begins to impose an admissibility standard for expert testimony, \( \bar{q} \) begins to increase above 0. As \( \bar{q} \) increases, \( \Lambda(\bar{q}) \) decreases because \( \Lambda'(\bar{q}) < 0 \) as shown previously, but \( \eta(\bar{q}) \) stays at \( 1 - \mu \) as long as \( \Lambda(\bar{q}) \geq \frac{1}{2} \), because cause \( A \) still always prevails. However, \( \eta(\bar{q}) \) cannot stay at the same value for all possible values of \( \bar{q} \), because \( \Lambda(\bar{q}) \) falls below \( \frac{1}{2} \) as \( \bar{q} \) nears 1.\(^{16}\) Thus, there exists \( \bar{q}' \in (0, 1) \) such that \( \Lambda(\bar{q}) \geq \frac{1}{2} \) for all \( \bar{q} \leq \bar{q}' \) and \( \Lambda(\bar{q}) < \frac{1}{2} \) for all \( \bar{q} > \bar{q}' \).

Now consider \( \bar{q} \) such that \( \Lambda(\bar{q}) < \frac{1}{2} \).\(^{17}\) For such \( \bar{q} \), there are two possibilities:

(i) If \( q < \bar{q} \), the expert cannot testify before the judge. Thus, the judge makes a decision based on his prior belief, \( \mu \geq \frac{1}{2} \), and cause \( A \) prevails.

(ii) If \( q \geq \bar{q} \), the expert can testify before the judge. If the expert reveals \( x = \alpha \), cause \( A \) prevails, and if the expert does not reveal evidence, the judge rules in favor of cause \( B \) because \( \Lambda(\bar{q}) < \frac{1}{2} \).

Thus, for such \( \bar{q} \), the expected error \( \eta \) is given by

\[
\eta(\bar{q}) = \mu P(d = B|t = a) + (1 - \mu)P(d = A|t = b) = \mu \gamma E_q[1 - q + q(1 - p)|q \geq \bar{q}] + (1 - \mu)(1 - \gamma + \gamma E_q[q(1 - p)|q \geq \bar{q}])
\]

\[
\leq \mu \gamma (1 - \bar{q} + \hat{q}(1 - p)) + (1 - \mu) \left( 1 - \gamma + \gamma \hat{q}(1 - p) \right)
\]

where \( \gamma = P(q \geq \bar{q}) \) and \( \hat{q} = E(q|q \geq \bar{q}) \), which shows that \( \eta \) is a function of \( \bar{q} \). Note that \( \gamma \) is the probability that the expert is of high quality and testifies before the judge, and \( \hat{q} \) is the average quality of the testifying expert. First, consider (*), the probability that cause \( B \) prevails under \( t = a \). Inspecting the two possibilities in the previous paragraph (i.e., (i) and (ii)), cause \( B \) prevails only under (ii) (with probability \( \gamma \)); given (ii), cause \( B \) prevails under \( \phi \), which occurs either when the expert could not observe the hidden evidence (with

\(^{16}\)Consider the off-equilibrium path situation in which the judge observes \( x = \beta \). In such a situation, the judge’s posterior is given by (2) which is lower than \( \frac{1}{2} \). As \( \bar{q} \) approaches 1, the judge becomes more confident that the testifying expert is of high quality on average and that \( \phi \) occurs from evidence distortion, i.e., the expert is hiding \( x = \beta \). Thus, \( \Lambda(\bar{q}) \) approaches \( \lambda(\beta) < \frac{1}{2} \) as \( \bar{q} \) approaches 1. By continuity, \( \Lambda(\bar{q}) \) becomes lower than \( \frac{1}{2} \) for a sufficiently high value of \( \bar{q} \).

\(^{17}\)The analysis for the second subcase, \( \Lambda(0) < \frac{1}{2} \leq \mu \), is identical to this part.
probability $1 - \hat{q}$) or when the expert observed $x = \beta$ (with probability $\hat{q}(1 - p)$). Combining these probabilities provides us with ($\ast$). The other part, ($\ast\ast$), can be similarly understood.

This expression can be rearranged to

$$\eta(\bar{q}) = \gamma(\hat{q}(1 - p) + (1 - \hat{q})\mu) + (1 - \gamma)(1 - \mu)$$

(6)

where ($\ast$) is the expected error when the expert passes the admissibility standard and testifies before the judge, and ($\ast\ast$) is the expected error when the expert is dismissed. To understand the expected error in this view, observe that there are two possibilities when the expert passes the admissibility standard and testifies before the judge (the ($\ast$) part): (i) the expert observed the hidden evidence (with probability $\hat{q}$) but the evidence does not match the truth (with probability $1 - p$),\(^{18}\) and (ii) the expert could not observe the evidence (with probability $1 - \hat{q}$), leading to $\phi$ and the judge’s ruling in favor of cause $B$, but the truth is $t = a$ (with probability $\mu$). When the expert fails to pass the admissibility standard (the ($\ast\ast$) part), cause $A$ prevails based on the judge’s prior belief, and such a decision can be erroneous if the truth is $t = b$ (with probability $1 - \mu$).

The expression (6) shows that the admissibility standard $\bar{q}$ influences the expected error $\eta$ through two channels, $\hat{q}$ and $\gamma$. To better understand the total effect of $\bar{q}$ on $\eta$, let us inspect the partial effect of $\bar{q}$ on $\eta$ through each channel. For future reference, I denote the partial effect of $\bar{q}$ on $\eta$ through $\hat{q}$ as the quality effect, and the other partial effect through $\gamma$ as the dismissal effect.

First, let us inspect the quality effect. Observe that as $\bar{q}$ increases, $\hat{q}$ increases as well, which in turn reduces $\eta$. This effect is intuitive. As society imposes a more stringent standard for expert testimony (higher $\bar{q}$), the average quality of the testifying expert increases (higher $\hat{q}$). This alleviates the judge’s inference problem, because the judge becomes more confident about the hidden evidence under $\phi$, which reduces the expected error (lower $\eta$). Thus, the quality effect is always beneficial for accuracy.

Second, let us inspect the dismissal effect. Observe that as $\bar{q}$ increases, $\gamma$ decreases: more experts are dismissed as it becomes more difficult for low-quality experts to pass the admissibility standard. However, the effect of $\gamma$ on $\eta$ is ambiguous and depends on the magnitude of $\hat{q}$. As we can observe from (6), if ($\ast$) is larger than ($\ast\ast$), $\eta$ decreases as $\gamma$ decreases, and if ($\ast$) is smaller than ($\ast\ast$), $\eta$ increases as $\gamma$ decreases. In other words, there exists $\bar{q}^o$ such that

- if $\hat{q} < \bar{q}^o$, $\eta$ decreases as $\gamma$ decreases and

\(^{18}\)Either the truth is $t = a$ and the evidence is $x = \beta$ (with probability $\mu(1 - p)$) or the truth is $t = b$ and the evidence is $x = \alpha$ (with probability $(1 - \mu)(1 - p)$). Adding these probabilities provides us with $1 - p$. 

13
• if $\hat{q} > \hat{q}^{o}$, $\eta$ increases as $\gamma$ decreases, where\(^{19}\)

$$\hat{q}^{o} = \frac{2\mu - 1}{\mu - (1 - p)} \in (0, 1).$$

To better understand this dismissal effect, consider the first case in which $\hat{q} < \hat{q}^{o}$. In this case, the average quality of those experts passing the admissibility standard is relatively poor, which exacerbates the judge’s inference problem. If the expert passes the standard and testifies at trial, the chances are high that the expert cannot observe the hidden evidence, which leads to $\phi$ and the judge’s ruling against cause $A$. As the prior belief is in favor of cause $A$, such a ruling based on the lack of evidence is expected to generate more mistakes than a decision solely based on the prior belief. Therefore, it is beneficial to induce the judge to make a decision based only on the prior belief more often; that is, dismissal of more experts reduces the expected error. In contrast, when the average quality of those experts passing the admissibility standard is relatively high, i.e., $\hat{q} > \hat{q}^{o}$, the judge can make a relatively accurate decision with the help of expert testimony. Therefore, in this case it is preferable to supply the judge with more experts; that is, dismissal of fewer experts reduces the expected error. Thus, in contrast to the quality effect, the dismissal effect could be beneficial or detrimental for accuracy, depending on underlying litigation environments.\(^{20}\)

Inspection of the two partial effects demonstrates that the total effect of $\bar{q}$ on $\eta$ is unambiguous under certain situations; but under other situations, it is ambiguous and depends on the relative magnitude of the two partial effects. If $\hat{q} < \hat{q}^{o}$, a higher level of $\bar{q}$ unambiguously reduces $\eta$ because both the quality effect and the dismissal effect are beneficial for accuracy. If $\hat{q} > \hat{q}^{o}$, the effect of a higher level of $\bar{q}$ on $\eta$ is ambiguous. As $\hat{q}$ increases, $\eta$ decreases when the quality effect dominates the dismissal effect but $\eta$ increases otherwise.

**Adverse Cases**

To complete the analysis, consider the remaining case, $\mu < \frac{1}{2}$. Then, it must be true that $\Lambda(\bar{q}) < \frac{1}{2}$ for all $\bar{q} \in [0, 1]$ because $\Lambda(\bar{q})$ is always smaller than $\mu$ as shown previously. In other words, if the expert passes the admissibility standard and testifies before the judge, the

\(^{19}\)The numerator of $\hat{q}^{o}$ is positive and smaller than its denominator:

$$0 < 2\mu - 1 = \mu - (1 - \mu) < \mu - (1 - p)$$

where the last inequality holds because $\mu < p$.

\(^{20}\)If $\hat{q}^{o}$ is very small, it is possible that $\hat{q}$ is always larger than $\hat{q}^{o}$ for all $\bar{q} \in [0, 1]$. To see this, observe that the minimum of $\bar{q}$ is achieved when $\bar{q} = 0$, that is, for all $\bar{q} \in [0, 1],

$$\bar{q}^{*} = E(q|q \geq \bar{q}) \geq E(q|q \geq 0) = E(q) > 0$$

which implies that $\bar{q}$ cannot fall below $E(q)$ that is a positive number. Thus, if $\hat{q}^{o} < E(q)$, $\bar{q}$ is always larger than $\hat{q}^{o}$ for all $\bar{q} \in [0, 1]$, and in such a situation the dismissal effect is always detrimental for accuracy.
no-evidence event $\phi$ always induces the judge to rule in favor of cause $B$ regardless of the admissibility standard $\bar{q}$.

Thus, for $\bar{q} \in [0, 1]$, there are two possibilities:

(i) If $q < \bar{q}$, the expert cannot testify before the judge. Thus, the judge makes a decision based on his prior belief, $\mu < \frac{1}{2}$, and cause $B$ therefore prevails.

(ii) If $q \geq \bar{q}$, the expert can testify before the judge. If the expert reveals $x = \alpha$, cause $A$ prevails, and if the expert does not reveal any evidence, the judge rules in favor of cause $B$ because $\Lambda(\bar{q}) < \frac{1}{2}$.

Therefore, for $\bar{q} \in [0, 1]$, the expected error $\eta$ is given by

$$
\eta(\bar{q}) = \mu P(d = B|t = a) + (1 - \mu)P(d = A|t = b)
= \mu(1 - \gamma + \gamma(1 - \hat{q} + \hat{q}(1 - p))) + (1 - \mu)\gamma \hat{q}(1 - p)
= \gamma(\hat{q}(1 - p) + (1 - \hat{q})\mu) + (1 - \gamma)\mu
$$

which again shows that the admissibility standard $\bar{q}$ influences the expected error $\eta$ through two channels, $\hat{q}$ and $\gamma$. It is routine to verify that the quality effect is always beneficial for accuracy in adverse cases as well. However, contrary to the favorable cases, the dismissal effect is always detrimental for accuracy and dominates the quality effect, as the following lemma shows:

**Lemma 2.** $\eta'(\bar{q}) > 0$ for all $\bar{q} \in (0, 1)$.

*Proof.* See the Appendix.

To understand this result, remember that there are two possibilities when the expert passes the admissibility standard and testifies before the judge: the expert either reports favorable evidence $x = \alpha$ or reveals nothing, leading to $\phi$. In the former case, the judge “correctly” —in the sense that the judge’s decision is based on all available evidence—rules in favor of cause $A$. In the latter case, there are two possibilities. First, if $\phi$ occurs owing to the expert’s lack of information, the judge’s correct posterior must be equal to his prior belief because there is no additional information for decision-making. Thus, the judge should rule in favor of cause $B$ because $\mu < \frac{1}{2}$. Second, if $\phi$ occurs because of the expert’s evidence distortion, it means that the hidden evidence is $x = \beta$ and therefore the judge should rule in favor of cause $B$. Observe that both possibilities require the judge to rule in favor of cause $B$, which is exactly what the judge does in equilibrium under the no-information event $\phi$. This finding shows that evidence distortion is not necessarily detrimental for Bayesian decision-making under certain parameter values. Therefore, it is detrimental for accuracy to dismiss any expert from the courtroom, which explains the intuition behind the lemma.
Summary and Example

The following proposition summarizes the results from the main analysis:

**Proposition 1.** If $\mu < \frac{1}{2}$, $\eta'(\tilde{q}) > 0$ for all $\tilde{q} \in (0,1)$. If $\mu \geq \frac{1}{2}$, there are two cases:

(i) Suppose $\frac{1}{2} \leq \Lambda(0) < \mu$. There exist $\tilde{q}' \in (0,1)$ and $\hat{q}^o \in (0,1)$ such that

- for $\tilde{q} \leq \tilde{q}'$, $\eta(\tilde{q}) = 1 - \mu$, and
- for $\tilde{q} > \tilde{q}'$, (i) if $\tilde{q} < \hat{q}^o$ then $\eta'(\tilde{q}) < 0$, and (ii) if $\tilde{q} > \hat{q}^o$ then $\eta'(\tilde{q}) < 0$ when the quality effect dominates the dismissal effect and $\eta'(\tilde{q}) > 0$ otherwise.

(ii) Suppose $\Lambda(0) < \frac{1}{2} \leq \mu$. There exists $\hat{q}^o \in [0,1)$ such that, for $\tilde{q} \in [0,1]$, (i) if $\tilde{q} < \hat{q}^o$ then $\eta'(\tilde{q}) < 0$, and (ii) if $\tilde{q} > \hat{q}^o$ then $\eta'(\tilde{q}) < 0$ when the quality effect dominates the dismissal effect and $\eta'(\tilde{q}) > 0$ otherwise.

To get a feel for the implications from this proposition, let us consider a numerical example. Suppose the parameter values are such that $\mu = \frac{2}{3}$ and $p = \frac{3}{4}$. Also assume that $F(q)$ is a uniform distribution over the open unit interval. Figure 1 demonstrates the shape of $\eta(q)$ under these parameter values. See the Appendix for calculations behind this example.

Because $\Lambda(0.566) = 0.5$, $\eta$ is constant at $1 - \mu = \frac{1}{3}$ until $\tilde{q}$ reaches 0.566; as long as $\tilde{q} < 0.566$, $\Lambda(\tilde{q})$ is higher than $\frac{1}{2}$, which induces cause $A$ always to prevail. As $\tilde{q}$ increases above 0.566, $\eta$ begins to decline. For the range of $\tilde{q} \in (0.566,1]$, $\eta$ takes the form of a quadratic function that is convex and achieves the minimum at $\tilde{q}^* = 0.8$. Thus, the decision-making error can be minimized by dismissing as much as 80% of experts as unreliable.
4 Concluding Remarks and Discussion

Employing a stylized model of litigation, I study the conditions under which a stringent admissibility standard for expert testimony is beneficial to the American justice system. My main results demonstrate that one can increase \textit{ex ante} accuracy of the judge’s decision by requiring all testifying experts to pass a stringent admissibility standard in certain favorable cases but not in adverse cases. I conclude with a discussion of the implications and extensions of the basic model.

Neutral Experts and Burden of Proof

Observe that the main results hold only when the expert is biased. The main reason for discarding valuable information from low-quality experts is to mitigate the judge’s inference problem with an evidence-distorting expert; therefore, if the expert is neutral and truth-telling, there is no gain from increasing the admissibility standard in the basic model. Combining this observation with the results from Kim (2015a), one can derive the following implication. Kim (2015a) demonstrates that a litigant is more likely to present unbiased expert testimony when he bears the burden of proof at trial. Thus, application of a stringent admissibility standard to the expert from the party with the burden of proof could be detrimental to accuracy because the chances are high that the expert is unbiased. An implication of this discussion is that, in order to raise its decision-making accuracy, the court could take into consideration the allocation of the burden of proof when choosing the degree of admissibility requirement for expert testimony.

Battle of the Experts in Litigation

How does the main result change in a “battle of the experts” situation? Consider two competing parties, a defendant and a plaintiff, where each party wants the judge to rule in favor of his own cause. To influence the judge’s decision, each party presents an expert who is biased toward his client. For simplicity, I assume that both experts have access to the same information source \( x \in \{D, P\} \), where \( x = D \) (\( x = P \)) is favorable evidence for the defendant (the plaintiff).\(^{21}\) Let us denote by \( q_D \) (\( q_P \)) the probability with which the defendant’s (the plaintiff’s) expert observes the hidden evidence. These probabilities can be interpreted as experts’ quality, which the judge cannot observe. Instead, the judge knows only that \( q_D \) and \( q_P \) are \textit{i.i.d.} random variables following a distribution function \( F(q) \) with a support over \((0, 1)\). Furthermore, assume that the same admissibility standard \( \bar{q} \) is applied to both experts.

\(^{21}\)Thus, there are four possibilities: (i) both experts do not observe \( x \), (ii) only the defendant’s expert observes \( x \), (iii) only the plaintiff’s expert observes \( x \), and (iv) both experts observe the same piece of evidence \( x \). This is a standard modeling approach in the literature studying the competition between litigants. For example, see Shin (1998), Demougin and Fluet (2008), Kim (2014a).
Under $\bar{q} = 0$, both experts can freely testify before the judge. Thus, three events are possible: (i) $x = D$ is revealed by the defendant’s expert, leading to the prevalence of the defendant’s cause; (ii) $x = P$ is revealed by the plaintiff’s expert, leading to the prevalence of the plaintiff’s cause; (iii) no evidence is revealed by any expert, leading to the no-evidence event $\phi$, under which the judge makes a decision after calculating his posterior belief incorporating the experts’ strategies and the underlying information structure. As $q_D$ and $q_P$ are i.i.d. random variables, the judge believes that the experts’ qualities must be the same on average. This implies that the judge must exercise the same degree of skepticism toward both experts under $\phi$. Therefore, indirect evidence from each expert’s silence exactly cancels out, leading to the judge’s posterior equal to his prior belief.

Under $\bar{q} > 0$, there are three cases to analyze:

1. First, both experts are dismissed as unreliable. Then, there is no additional information for the judge’s decision-making, and the judge makes a decision based on his prior belief. This case generates an information loss.

2. Second, only one of the experts is dismissed. Although the judge loses one expert, he can interact with an expert with high quality on average. This case is analyzed in the basic model because the judge interacts with only one expert. Thus, the main results from the basic model apply to this case, and an information gain may exist as $\bar{q}$ increases.

3. Third, it is possible that both experts pass the admissibility requirement and they are admitted to testify before the judge. In this case, the outcome is the same as that under $\bar{q} = 0$: (i) $x = D$ is revealed by the defendant’s expert, leading to the prevalence of the defendant’s cause; (ii) $x = P$ is revealed by the plaintiff’s expert, leading to the prevalence of the plaintiff’s cause; (iii) no evidence is revealed by any expert, leading to the no-evidence event $\phi$, under which the judge makes a decision after calculating his posterior belief incorporating the experts’ strategies and the underlying information structure. As the same admissibility standard is applied to both experts, the judge believes that their qualities must be the same on average. Therefore, as under $\bar{q} = 0$, the judge must exercise the same degree of skepticism toward both experts under $\phi$, leading to the judge’s posterior equal to his prior belief. Because the outcome is the same and the judge observes hard evidence with a higher probability (because experts’ qualities are high on average), accuracy in this case must be higher than that under $\bar{q} = 0$. Thus, there exists an information gain.

This discussion suggests that the qualitative result from the basic model is still expected to hold in this extended formulation. On one hand, there is an information loss from dismissing some of the experts. On the other hand, there exists an information gain because the judge’s...
inference problem is mitigated. Thus, it is possible that a higher admissibility standard increases accuracy in this extended formulation as well.

5 Appendix

5.1 Proof for Lemma 2

To prove Lemma 2, taking derivative of $\eta$ gives

$$
\eta'(\bar{q}) = \frac{d\gamma}{d\bar{q}}(\dot{q}(1-p) + (1-\dot{q})\mu) + \gamma(1-p-\mu) \frac{d\dot{q}}{d\bar{q}} - \mu \frac{d\gamma}{d\bar{q}}
$$

$$
= \frac{d\gamma}{d\bar{q}}(\dot{q}(1-p) + (1-\dot{q})\mu - \mu) + \gamma(1-p-\mu) \frac{d\dot{q}}{d\bar{q}}
$$

$$
= -f(\dot{q})(\dot{q}(1-p) + (1-\dot{q})\mu - \mu)
$$

$$
+ (1-F(\dot{q}))(1-p-\mu) \left( \frac{-\dot{q}f(\dot{q})}{1-F(\dot{q})} + \int_{\dot{q}}^{1} q f(q) f(\dot{q})^2 dq \right)
$$

where $\gamma = P(q \geq \dot{q}) = 1 - F(\dot{q})$ and $\dot{q} = E(q|q \geq \dot{q}) = \int_{\dot{q}}^{1} q f(q) f(\dot{q})^2 dq$. Simplifying the algebra gives

$$
\eta'(\dot{q}) = f(\dot{q})\dot{q}(\mu - (1-p)) > 0
$$

where the inequality holds because $\mu \in (1-p, p)$. This completes the proof.

5.2 Uniform Distribution Example

It is straightforward to calculate the following quantities under the given parameter values:

$$
\dot{q}^o = \frac{4}{5}
$$

$$
\lambda(\phi; q) = \frac{\frac{2}{3} - \frac{q}{1 - \frac{7q}{12}}}{1 - \frac{7q}{12}}
$$

$$
\Lambda(0) = E(\lambda(\phi; q)|q \geq 0) = \int_{0}^{\frac{2}{3} - \frac{q}{1 - \frac{7q}{12}}} \lambda(\phi; q) dq \approx 0.57128
$$

$$
\Lambda(0.566) = 0.5
$$

$$
E(q|q > \frac{3}{5}) = \hat{q}^o = \frac{4}{5}
$$

$$
\hat{q} = \frac{1 + \hat{q}}{2}
$$

$$
\gamma = 1 - \hat{q}
$$

$$
\eta(\hat{q}) = \frac{5}{24}\hat{q}^2 - \frac{8}{24}\hat{q} + \frac{11}{24}
$$

$$
\hat{q}^* = \arg\min \eta(\hat{q}) = \frac{4}{5}
$$

$$
\Lambda(\hat{q}^*) = 0.45411
$$
References


