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Hammad, Siddiqi

The University of Queensland

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Hammad Siddiqi

The University of Queensland

Australia

h.siddiqi@uq.edu.au

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Abstract

What happens when the capital asset pricing model (CAPM) is adjusted for the anchoring and adjustment heuristic of Tversky and Kahneman (1974)? The surprising finding is that adjusting CAPM for anchoring provides a plausible unified framework for understanding almost all of the key asset pricing anomalies. The anomalies captured in the theoretical framework include the well-known size, and value effects, high-alpha-of-low-beta-stocks, accruals, low volatility, momentum effect, stock-splits, and reverse stock-splits. The equity premium is also larger with anchoring. This suggest that the anchoring adjusted capital asset pricing model (ACAPM) may provide the needed unifying structure to behavioral finance.

Keywords: *Size Premium, Value Premium, Behavioral Finance, Stock Splits, Equity Premium Puzzle, Anchoring and Adjustment Heuristic, CAPM, Asset Pricing, Accrual Anomaly, Low Volatility Anomaly, Low-beta-high-alpha, Momentum Effect*

JEL Classification: *G12, G11, G02*

Anchoring Adjusted Capital Asset Pricing Model

What happens when the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965) is adjusted for the anchoring heuristic of Tversky and Kahneman (1974)? The surprising finding is that it provides a unified theoretical framework for understanding key asset pricing anomalies. The anchoring model converges to CAPM if there is no anchoring bias. Hence, CAPM is obtained as a special case of the anchoring adjusted model.

This article develops the theoretical framework without asserting that the proposed framework completely explains all of the anomalies discussed here. Such an assertion demands a comprehensive empirical examination of a large number of anomalies, which no single article can hope to achieve. The primary aim of this article is to demonstrate that the anchoring and adjustment heuristic must be considered as an alternative explanation that demands comprehensive empirical scrutiny. An often mentioned weakness of behavioral finance is that it is a collection of ad hoc models without a unifying structure (Shefrin (2010)). By potentially capturing almost all of the key asset pricing anomalies in a single framework, this article shows that the anchoring adjusted capital asset pricing model (ACAPM) may provide the needed unifying structure to behavioral finance.

Starting from Tversky and Kahneman (1974), over 40 years of research has demonstrated that while forming estimates, people tend to start from what they know and then make adjustments to their starting points. However, adjustments typically remain biased towards the starting value known as the anchor (see Furnham and Boo (2011) for a general review of the literature). Describing the anchoring heuristic, Epley and Gilovich write (2001), "*People may spontaneously anchor on information that readily comes to mind and adjust their response in a direction that seems appropriate, using what Tversky and Kahneman (1974) called the anchoring and adjustment heuristic. Although this heuristic is often helpful, the adjustments tend to be insufficient, leaving people's final estimates biased towards the initial anchor value*." (Epley and Gilovich (2001) page. 1).

A few examples illustrate this heuristic quite well. When respondents were asked which year George Washington became the first US President, most would start from the year the US became a country (in 1776). They would reason that it might have taken a few years after that to elect the first president so they add a few years to 1776 to work it out, coming to an answer of 1778 or 1779. George Washington actually became president in 1789. Similarly, most people would know the freezing point of water (0 degrees Celsius) as compared to vodka. So if they were asked what the freezing temperature of vodka is, they would tend to start from 0 and adjust downwards. The freezing temperature of vodka is around -24 degrees Celsius, much lower than what people usually answer. What is the fair price of a 3-bedroom house in a particular neighbourhood of Chicago? If you know that price of a 4-bedroom house in the same neighbourhood but in a slightly better location, you would probably start from that price and adjust for differences between the two properties. Anchoring bias implies that such adjustments tend to be insufficient.

The initial experiments of Tversky and Kahneman (1974) have led to numerous studies in which robust influence of anchoring in various decision-making situations is found. The situations tested are diverse and include price estimation (Amado et al (2007)), probability calculations (Chapman and Johnson (1999)), as well as factual knowledge (Blankenship et al (2008), Wegener et al (2001)). Expertise does not seem to reduce the anchoring bias supporting the notion that experts and novices alike are prone to anchoring (Englich and Soder (2009)). Overall, the anchoring bias is "exceptionally robust, pervasive and ubiquitous" (Furnham and Boo 2011, p. 41) regarding experimental variations.

Any financial asset is just a label given to a future payoff stream. So, knowing the future payoff distribution is crucial for valuing assets. In a standard application of CAPM, rational expectations are assumed about every asset irrespective of whether it belongs to a well-established firm or a relatively new firm (ex-ante expectations are assumed to coincide with ex-post realizations on average). In reality, stocks differ in terms of analyst and media coverage, history, and data availability. Well-established stocks, known as blue-chips get a lion's share of coverage, and have large comprehensive datasets available about them. The coverage bias is substantial with 83% of full time analysts only covering the blue-chip stocks (4% of the firms)[1]. Prominent blue-chips also remain in focus because stock analysts typically analyse a given firm in comparison with the sector leader (typically a prominent blue-chip company). MBA and CFA texts teach the same approach[2]. Furthermore, industry analysis, which is a universal part of almost every analyst research note also brings prominent blue-chips into focus as shapers of the particular industry under consideration.

Given the key role of prominent blue-chips in stock analysis, and the exceptional pervasiveness and ubiquity of anchoring, it is plausible that a typical investor uses the payoffs of

[1] http://punchinvest.com/wp-content/uploads/2011/11/The-Blind-Spot-Impact-of-Analyst-Coverage1.pdf

[2] One example is the popular textbook on financial management of Petty and Titman (2012).

prominent blue-chips as starting points, which are then adjusted to form judgments about other firms. Consider the following firms: Apple, Applied Materials, and Auto Desk. All three are large information technology firms included in the S&P 500 index. However, Apple is a household name that clearly stands out. Apple is the largest information technology firm in the world, and gets a lion's share of analysts and media coverage when compared with the other two firms. Given its prominence, a typical investor may start from Apple and then attempt to make appropriate adjustments to form judgments about the other two firms.

A key stylized fact noted in the anchoring literature is that higher the task complexity or cognitive load involved in a judgment task, larger is the error caused by anchoring (Kudryavtsev and Cohen (2010), Meub and Proeger (2014), references therein). From a given starting distribution, adjusting for volatility perhaps requires more effort than adjusting for expected payoffs. In principle, the only adjustment required for expected payoff is "size adjustment"; however, in order to estimate volatility, many other differences such as the differential response to external shocks must also be accounted for. Plausibly, due to the higher cognitive load involved, the anchoring bias is larger in volatility estimation. For simplicity, I assume that expected payoffs are correctly estimated and the anchoring bias is displayed in volatility estimation only. Alternatively, without changing results, one could assume that the anchoring bias is present in both expected payoff and volatility estimation with the bias being larger in volatility estimation. This focus on volatility instead of expected payoffs is also typical of the Bayesian learning literature (see Weitzman (2007) and references therein). This is in sharp contrast with other asset pricing literature that allows for errors in expected payoffs while assuming that volatilities are correctly estimated (partly because of convenience).

Another key stylized fact from the anchoring literature is that greater the distance between the starting point and the correct value, larger is the error due to anchoring (Epley and Gilovich (2006) (2001) and references therein). When considering payoff volatilities, one simple way of capturing this is to use the following formulation:

$$\sigma_A^2(X_{Target}) = (1-m)\sigma^2(X_{BlueChip}) + m\sigma^2(X_{Target}) \text{ with } 0 \leq m \leq 1$$

In the above formulation, if $m = 1$, there is no anchoring bias and the anchoring influenced value, $\sigma_A^2(X_{Target})$, is equal to the correct value, $\sigma^2(X_{Target})$. In this case, ACAPM would converge to CAPM. If $m = 0$, the anchoring bias is maximal and the anchoring influenced value is equal to the starting value. Consistent with the anchoring literature, for a given positive m, greater the distance between the starting value, $\sigma^2(X_{BlueChip})$, and the correct value, $\sigma^2(X_{Target})$, larger is the error due to the anchoring bias.

To sum up, the notion that the payoff distributions of prominent blue-chips may be starting points for forming judgments about the payoff distributions of other firms can be made precise by using the following stylized facts: Stylized fact #1) Error due to anchoring is larger if the cognitive load involved in the judgment task is higher => focus on payoff volatilities instead of expected payoffs. Stylized fact #2) Error due to anchoring is larger if the distance between the starting value and the correct value is larger. A simple formulation capturing this is: $\sigma_A^2(X_{Target}) = (1-m)\sigma^2(X_{BlueChip}) + m\sigma^2(X_{Target})$.

Section 1 develops the anchoring adjusted model (ACAPM) by adjusting the standard CAPM for anchoring in the light of the above mentioned stylized facts. ACAPM converges to CAPM if the adjustments are correct. Hence, CAPM is obtained as a special case of ACAPM. Section 2 shows that ACAPM provides a unified explanation for key asset pricing anomalies. Section 3 concludes.

I complete a brief literature review of the role of anchoring in financial decision making in the remaining part of this section. Hirshleifer (2001) considers anchoring to be an *"important part of psychology based dynamic asset pricing theory in its infancy"* (p. 1535). Shiller (1999) argues that anchoring appears to be an important concept for financial markets. This argument has been supported quite strongly by recent empirical research on financial markets. Anchoring has been found to matter for credit spreads that banks charge to firms (Douglas et al (2015), it matters in determining the price of target firms in mergers and acquisitions (Baker et al (2012), and it also affects the earnings forecasts made by analysts in the stock markets (Cen et al (2013)). Furthermore, Siddiqi (2015) shows that anchoring provides a unified explanation for a number of key puzzles in options market. Given the importance of this bias in financial decision making, it is natural to see what happens when a canonical asset pricing model of the stature of CAPM is adjusted for anchoring. This is the contribution of this article.

1. Anchoring Adjusted CAPM

A typical derivation of CAPM via utility maximization starts by considering an overlapping generations (OLG) economy with agents having identical beliefs. Each agent lives for two periods. Agents that are born at t aim to maximize their utility of wealth at $t+1$. Their utility functions are identical and exhibit mean-variance preferences. They trade securities $s = 1, \dots, S$ where security s pays dividends δ_t^s and has x^{*s} shares outstanding, and invest the rest of their wealth in a risk-free asset that offers a rate of r_F.

Market dynamics can be described by a representative agent who maximizes:

$$\max x'\{E_t(P_{t+1} + \delta_{t+1}) - (1 + r_F)P_t\} - \frac{\gamma}{2}x'\Omega_t x$$

where P_t is the vector of prices, Ω_t is the variance-covariance matrix of $P_{t+1} + \delta_{t+1}$, and γ is the risk-aversion parameter.

The maximization problem described above has three decision primitives, which are as follows: 1) Vector of expected payoffs in the next period, $E_t(P_{t+1} + \delta_{t+1})$. 2) Expected variances associated with these payoffs, that is, the diagonal elements in the variance-covariance matrix Ω_t. 3) Expected covariances associated with these payoffs, which are the off-diagonal elements in Ω_t.

Once the above three decision primitives are specified, CAPM follows via a series of logical steps, with applications usually assuming that expectations coincide with reality.

How does anchoring influence this picture? I assume that the representative agent forms correct judgments about expected payoffs and displays the anchoring bias in estimating volatilities. This is line with the stylized fact#1 discussed in the introduction. The variance-covariance matrix includes variances as well as covariances. Assuming that they have similar cognitive loads, one can allow the anchoring bias to matter equally for both variances as well as covariances. However, without changing results, a much simpler way is to allow for the bias in variances while assuming that covariances are also correctly estimated. This is the approach taken here.

Another modelling choice is whether to use a representative agent who is anchoring prone or to mix the two types of agents with one type anchoring prone and the other type holding rational expectations with limits to arbitrage ensuring survival. The results are identical in both cases; however, the former approach is mathematically simpler. Furthermore, it is more reasonable to think that ex-ante nobody knows the exact future payoff volatility of a typical firm. Hence, I choose the former approach for reasons of simplicity and realism.

To illustrate the implications of anchoring, I start by considering the simplest case of one prominent blue-chip stock and one normal or typical stock in section 1.1. Section 1.2 considers the case of one prominent blue-chip and many typical stocks, and section 1.3 adds a large number of prominent blue-chip stocks to the picture with one blue-chip corresponding to each sector of the economy.

1.1 Anchoring Adjusted CAPM: The Simplest Case

Consider an overlapping-generations (OLG) economy in which agents are born each period and live for two periods. For simplicity, in the beginning, I assume that they trade in the stocks of two firms and invest in a risk-free asset. One firm is well-established with large payoffs (the prominent blue-chip or leader firm), and the second firm is a relative new-comer with much smaller payoffs (typical or normal firm). The next period payoff per share of the leader firm is denoted by $\delta_{Lt+1} = P_{Lt+1} + d_{Lt+1}$ where P_{Lt+1} is the next period share price and d_{Lt+1} is the per share dividend of the large firm. Similarly, the next period payoff per share of the normal firm is defined by $\delta_{St+1} = P_{St+1} + d_{St+1}$. The risk-free rate of return is r_F. At time t, each agent chooses a portfolio of stocks and the risk-free asset to maximize his utility of wealth at $t+1$. There are no transaction costs, taxes, or borrowing constraints.

The market dynamics are described by a representative agent who maximizes utility:

$$n_L\{E_t(\delta_{Lt+1}) - (1+r_F)P_{Lt}\} + n_S\{E_t(\delta_{St+1}) - (1+r_F)P_{St}\} \\ - \frac{\gamma}{2}\{n_L^2\sigma_L^2 + n_S^2\sigma_S^2 + 2n_L n_S \sigma_{LS}\}$$

where $n_L, n_S, and\ \gamma$ denote the number of shares of the leader firm, the number of shares of the normal firm, and the risk aversion parameter respectively. Next period variances of the leader firm and the normal firm payoffs per share are $\sigma_L^2 = Var(\delta_{Lt+1})$ and $\sigma_S^2 = Var(\delta_{St+1})$ respectively with $\sigma_L^2 > \sigma_S^2$, and σ_{LS} denotes their covariance. Note, that payoff variances are different from return variances. The payoff variance of the normal firm's stock, σ_S^2, is smaller than the payoff variance of the leader firm's stock, σ_L^2, because of the much smaller size of its payoffs. In contrast, the return variance of the normal firm may be much larger than the return variance of the large firm's stock because of the smaller share price of the normal firm. To see this clearly, consider an example. Suppose the possible payoffs of the leader firm stock, in the next period, are 300, 350, and 400 with equal chance of each. The variance of these payoffs can be calculated easily and is equal to 1666.667. In a risk-neutral world, with zero risk-free interest rate, the price must be 350, so corresponding (gross) returns are: 0.857, 1, 1.143. The return variance is 0.010. Assume that the next period payoffs of the normal firm are 0, 35, and 70. The variance of these payoffs is 816.667. The risk neural price (with zero risk-free rate) is 35 leading to possible returns of 0, 1, and 2. The corresponding return variance is 0.66. As can be seen in this example, the payoff variance of the normal firm stock is smaller than the payoff variance of the leader firm stock, whereas the return variance of the normal firm is much larger.

The first order conditions of the maximization problem are:

$$E_t(\delta_{Lt+1}) - (1 + r_F)P_{Lt} - \gamma n_L \sigma_L^2 - \gamma n_S \sigma_{LS} = 0 \tag{1}$$

$$E_t(\delta_{St+1}) - (1 + r_F)P_{St} - \gamma n_S \sigma_S^2 - \gamma n_L \sigma_{LS} = 0 \tag{2}$$

Solving (1) and (2) for prices yields:

$$P_{Lt} = \frac{E_t(\delta_{Lt+1}) - \gamma n_L \sigma_L^2 - \gamma n_S \sigma_{LS}}{1 + r_F}$$

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma n_S \sigma_L^2 - \gamma n_L \sigma_{LS}}{1 + r_F}$$

If the number of shares of the leader firm outstanding is n_L', and the number of shares of the normal firm outstanding is n_S', then the equilibrium prices are:

$$P_{Lt} = \frac{E_t(\delta_{Lt+1}) - \gamma n_L' \sigma_L^2 - \gamma n_S' \sigma_{LS}}{1 + r_F} \tag{3}$$

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma n_S' \sigma_S^2 - \gamma n_L' \sigma_{LS}}{1 + r_F} \tag{4}$$

Next, I show how anchoring alters the above equilibrium. I assume that the representative agent is unsure about the variance of the normal firm's payoffs, and to form his judgment, he starts from the variance of the leader firm and subtracts from it.

The agent knows that as the normal firm has smaller payoffs, its payoff variance must also be smaller. So, he starts from the variance of the leader firm and subtracts from it to form his judgment about the normal firm's variance: $\hat{\sigma}_S^2 = \sigma_L^2 - A$. If he makes the correct adjustment, then $A = \sigma_L^2 - \sigma_S^2$. Anchoring bias implies that the adjustment falls short. That is, $A = m(\sigma_L^2 - \sigma_S^2)$ with $0 \leq m < 1$. Hence, $\hat{\sigma}_S^2 = (1-m)\sigma_L^2 + m\sigma_S^2$. If the adjustment is correct then $m = 1$. Note, that this formulation is consistent with the stylized fact#2 discussed in the introduction which says that greater the distance between the starting value and the correct value, larger is the error due to anchoring.

With anchoring, the equilibrium price of the normal firm falls, however, the equilibrium price of the leader firm remains unchanged.

The equilibrium price of the normal firm with anchoring is:

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma n_S' m \sigma_S^2 - \gamma n_S'(1-m)\sigma_L^2 - \gamma n_L' \sigma_{LS}}{1 + r_F} \tag{5}$$

Adding and subtracting $\gamma n_S' \sigma_S^2$ to the numerator of the above equation and using $cov(\delta_{St+1}, n_L'\delta_{Lt+1} + n_S'\delta_{St+1}) = n_L'\sigma_{LS} + n_S'\sigma_S^2$ leads to:

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma[cov(\delta_{St+1}, n_L'\delta_{Lt+1} + n_S'\delta_{St+1}) + n_S'(1-m)(\sigma_L^2 - \sigma_S^2)]}{1 + r_F} \tag{6}$$

With correct adjustment, that is, with $m = 1$, there is no anchoring bias and (6) reduces to (4).

Expressing (6) in terms of the expected stock return leads to:

$$E_t(r_S) = r_F + \frac{\gamma}{P_{St}}[cov(\delta_{St+1}, n_L'\delta_{Lt+1} + n_S'\delta_{St+1}) + n_S'(1-m)(\sigma_L^2 - \sigma_S^2)] \tag{7}$$

Anchoring does not change the share price of the leader firm. By re-arranging (3), the expected return expression for the stock price of the leader firm is obtained:

$$E_t(r_L) = r_F + \frac{\gamma}{P_{Lt}}[cov(\delta_{Lt+1}, n_L'\delta_{Lt+1} + n_S'\delta_{St+1})] \tag{8}$$

Expected return on the total market portfolio is obtained by multiplying (7) by $\frac{n_S' P_{St}}{n_L' P_{Lt} + n_S' P_{ST}}$

and (8) by $\frac{n_L' P_{St}}{n_L' P_{Lt} + n_S' P_{ST}}$ and adding them:

$$E_t[r_M] = r_F + \frac{\gamma}{n_L' P_{Lt} + n_S' P_{St}} [Var(n_L' \delta_{Lt+1} + n_S' \delta_{St+1}) + n_S'^2 (1 - m)(\sigma_L^2 - \sigma_S^2)] \tag{9}$$

Proposition 1 ***The expected return on the market portfolio with anchoring is larger than the expected return on the market portfolio without anchoring.***

Proof.

Follows directly from (9) by realizing that with anchoring P_{St} is smaller than what it would be without anchoring, and the second term, $n_S'^2 (1 - m)(\sigma_L^2 - \sigma_S^2)$, which is positive with anchoring is equal to zero without anchoring.

■

From (9), one can obtain an expression for the risk aversion coefficient, γ, as follows:

$$\gamma = \frac{(E_t[r_M] - r_F) \cdot (n_L' P_{Lt} + n_S' P_{St})}{Var(n_L' \delta_{Lt+1} + n_S' \delta_{St+1}) + n_S'^2 (1 - m)(\sigma_L^2 - \sigma_S^2)} \tag{10}$$

Substituting (10) in (7) and (8) and using $P_{Mt} = n_L' P_{Lt} + n_S' P_{St}$ leads to:

$$E_t(r_S) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_S, r_M) + \frac{n_S'(1 - m)(\sigma_L^2 - \sigma_S^2)}{P_{St} P_{Mt}}}{Var(r_M) + \frac{n_S'^2 (1 - m)(\sigma_L^2 - \sigma_S^2)}{P_{Mt}^2}} \tag{11}$$

$$E_t(r_L) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_L, r_M)}{Var(r_M) + \frac{n_S'^2 (1 - m)(\sigma_L^2 - \sigma_S^2)}{P_{Mt}^2}} \tag{12}$$

Equations (11) and (12) are the expected return expressions for the normal stock and the leader stock respectively with anchoring. They give the expected return under the anchoring adjusted

CAPM (ACAPM). It is straightforward to see that substituting $m = 1$ in (11) and (12) leads to the classic CAPM expressions. That is, without anchoring ACAPM converges to CAPM, with beta being the only priced risk factor, $\beta_S = \frac{Cov(r_S, r_M)}{Var(r_M)}$, and $\beta_L = \frac{Cov(r_L, r_M)}{Var(r_M)}$.

Proposition 2 shows that anchoring implies that the normal firm has a larger beta-adjusted return than the leader firm.

Proposition 2 ***The beta-adjusted excess return on the normal stock is larger than the beta-adjusted excess return on the leader stock.***

Proof.

From (12):

$$\frac{E_t[r_L - r_F]}{\frac{Cov(r_L, r_M)}{Var(r_M)}} < E_t[r_M - r_F]$$

and from (11)

$$\frac{E_t[r_S - r_F]}{\frac{Cov(r_S, r_M)}{Var(r_M)}} > E_t[r_M - r_F]$$

Hence, the beta-adjusted excess return on the normal stock must be larger than the beta-adjusted excess return on the leader stock.

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In the next two sub-sections, the above results are generalized. In section 1.2, the results are generalized to include a large number of normal firms while keeping the number of leader firm at one. In section 1.3, the results are generalized to include a large number of leader firms as well.

1.2 Anchoring adjusted CAPM with many normal firms

It is straightforward to extend the anchoring approach to a situation in which there are a large number of normal firms. Equation (6) remains unchanged. However, equation (9) changes slightly to the following:

$$E_t[r_M] = r_F + \frac{\gamma}{P_{Mt}}\left[Var(\delta_{Mt+1}) + \sum_{i=1}^{k} n_{Si}'^2(1-m)(\sigma_L^2 - \sigma_{Si}^2)\right] \tag{13}$$

where δ_{Mt+1} is the payoff associated with the aggregate market portfolio in the next period, and k is the number of normal firms in the market.

From (13), it follows that:

$$\gamma = \frac{(E_t[r_M] - r_F)\cdot(P_{Mt})}{Var(\delta_{Mt+1}) + \sum_{i=1}^{k} n_{Si}'^2(1-m)(\sigma_L^2 - \sigma_{Si}^2)} \tag{14}$$

The corresponding expression for a normal firm j's expected return can be obtained by substituting (14) in (7):

$$E_t(r_{Sj}) = r_F + E_t[r_M - r_F]\cdot\frac{Cov(r_{Sj}, r_M) + \frac{n_{Sj}'(1-m)(\sigma_L^2 - \sigma_{Sj}^2)}{P_{Sjt}P_{Mt}}}{Var(r_M) + \sum_{i=1}^{k}\frac{n_{Si}'^2(1-m)(\sigma_L^2 - \sigma_{Si}^2)}{P_{Mt}^2}} \tag{15}$$

The corresponding expression for the leader firm is obtained by substituting (14) in (8):

$$E_t(r_L) = r_F + E_t[r_M - r_F]\cdot\frac{Cov(r_L, r_M)}{Var(r_M) + \sum_{i=1}^{k}\frac{n_{Si}'^2(1-m)(\sigma_L^2 - \sigma_{Si}^2)}{P_{Mt}^2}} \tag{16}$$

(15) and (16) provide the expected return expressions corresponding to a situation in which there are a large number of normal firms and one leader firm. It is straightforward to check that in the absence of the anchoring bias, that is, when $m = 1$, the anchoring model converges to the classic CAPM expressions of expected returns. In the next section, I generalize the model to include a large number of leader firms as well.

1.3 ACAPM with Q leader firms and $Q \times k$ normal firms

It is natural to expect that every sector has its own leader firm whose stock is used as a starting point to form judgments about other firms in the same sector. I assume that there are Q sectors and every sector has one leader firm. I assume that the number of normal firms in every sector is k. That is, the total number of normal firms in the market is $Q \times k$. As the total number of leader firms is Q. The total number of all firms (both leader and normal) in the market is $Q + (Q \times k)$.

Following a similar set of steps as in the previous two sections, the expected return expression for a normal firm j in sector $q \in Q$ is given by:

$$E_t(r_{qSj}) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_{qSj}, r_M) + \frac{n'_{qSj}(1-m)(\sigma_{qL}^2 - \sigma_{qSj}^2)}{P_{qSjt}P_{Mt}}}{Var(r_M) + \sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{n'^2_{qSi}(1-m)(\sigma_{qL}^2 - \sigma_{qSi}^2)}{P_{Mt}^2}} \tag{17}$$

The corresponding expression for the leader firm in sector q is given by:

$$E_t(r_{qL}) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_{qL}, r_M)}{Var(r_M) + \sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{n'^2_{qSi}(1-m)(\sigma_{qL}^2 - \sigma_{qSi}^2)}{P_{Mt}^2}} \tag{18}$$

(17) and (18) are the relevant expected return expressions under ACAPM. As before, it is easy to see that, in the absence of the anchoring bias, that is, if $m = 1$, ACAPM expressions converge to the classic CAPM expressions.

Section 1.4 presents an alternate derivation of ACAPM by using portfolio expected returns and standard deviations.

1.4 ACAPM: A derivation based on portfolio expected return and standard deviation

In this section, I show that ACAPM can also be derived by directly considering returns and variances of returns instead of starting from the decisions primitives of expected payoffs and payoff variances as needed for utility maximization. With anchoring in judgments regarding payoff variances, the return variance of a normal firm stock i in sector q is:

$$\hat{\sigma}_{Rqi}^2 = \frac{(1-m)\sigma_{Lq}^2 + m\sigma_{qi}^2}{P_{qi}^2}$$

where $\hat{\sigma}_{Rqi}^2$ is the estimated return variance of a normal stock i in sector q, σ_{Lq}^2 is the payoff variance of the leader stock, σ_{qi}^2 is the payoff variance of the normal stock, and P_{qi} is the price of the normal stock. It is easy to see that the estimated return variance with anchoring is larger than the actual return variance:

$$\hat{\sigma}_{Rqi}^2 = \frac{(1-m)\sigma_{Lq}^2}{P_{qi}^2} + m\sigma_{Rqi}^2$$

As σ_{Lq}^2 is larger than σ_{qi}^2, it follows:

$$\hat{\sigma}_{Rqi}^2 > \sigma_{Rqi}^2$$

ACAPM can be derived by minimizing portfolio standard deviation for a given level of expected return.

I illustrate the two asset case below with one leader stock and one normal stock. Generalizations to large number of normal and leader stocks are straightforward. The portfolio standard deviation is:

$$\sigma_0 = \sqrt{w_L^2\sigma_{RL}^2 + w_S^2\hat{\sigma}_{RS}^2 + 2w_L w_S Cov(r_L, r_S)}$$

where w_L and w_S are portfolio weights of leader and normal stock respectively. $\sigma_{RL}^2$is the variance of leader stock's return. $\hat{\sigma}_{RS}^2$ is the judgment of the anchoring prone investor regarding the return variance of normal stock.

Anchoring bias implies the following:

$$\hat{\sigma}_{RS}^2 = \frac{(1-m)\sigma_L^2}{P_S^2} + m\sigma_{RS}^2 > \sigma_{RS}^2$$

where σ_L^2 is the payoff variance of the leader firm's stock, and P_S is the normal stock's price.

The optimization problem of the representative agent is as follows:

$$L = \sigma_0 + \lambda(E(r_0) - w_L E(r_L) - w_S E(r_S) - (1 - w_L - w_S) r_F)$$

The first order conditions are:

$$\frac{1}{\sigma_0}(w_L \sigma_{RL}^2 + w_S Cov(r_L, r_S)) = \lambda(E(r_L) - r_F)$$

$$\frac{1}{\sigma_0}\left(w_S m \sigma_{RS}^2 + \frac{w_S(1-m)\sigma_L^2}{P_S^2} + w_L Cov(r_L, r_S)\right) = \lambda(E(r_S) - r_F)$$

Multiply the first equation with w_L, multiply the second equation with w_S, add them together and carry out a little algebraic manipulation to arrive at the following result at the point where $w_L + w_S = 1$:

$$\frac{1}{\lambda} = \frac{[E[r_M] - r_F]\sigma_M}{\sigma_M^2 + \frac{w_S^2(1-m)(\sigma_L^2 - \sigma_S^2)}{P_S^2}}$$

where $E[r_M]$ and σ_M identify the portfolio expected return and the portfolio standard deviation at the point $w_L + w_S = 1$. That is, they correspond to the market portfolio. σ_S^2 is the payoff variance of the normal stock. Note, if there is no anchoring bias, then the expression corresponding to the classical CAPM is obtained: $\frac{1}{\lambda} = \frac{[E[r_M] - r_F]}{\sigma_M}$.

As $w_S = \frac{n_S' P_S}{P_M}$, it follows that:

$$\frac{1}{\lambda} = \frac{[E[r_M] - r_F]\sigma_M}{\sigma_M^2 + \frac{n_S'^2(1-m)(\sigma_L^2 - \sigma_S^2)}{P_M^2}}$$

Substituting $\frac{1}{\lambda}$ in the first order conditions and carrying out a little algebraic manipulation leads to:

$$E_t(r_S) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_S, r_M) + \frac{n_S'(1-m)(\sigma_L^2 - \sigma_S^2)}{P_{St} P_{Mt}}}{Var(r_M) + \frac{n_S'^2(1-m)(\sigma_L^2 - \sigma_S^2)}{P_{Mt}^2}}$$

$$E_t(r_L) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_L, r_M)}{Var(r_M) + \frac{n_S'^2(1-m)(\sigma_L^2 - \sigma_S^2)}{P_{Mt}^2}}$$

The above two equations are identical to equations (11) and (12) respectively.

It is straightforward to generalize to the case of Q leader firms and $Q \times k$ normal firms to obtain expressions identical to 17 and 18.

2. Anchoring Adjusted CAPM and Asset Return Anomalies

Finance theory predicts that risk adjusted returns from all stocks must be equal to each other. The starting point for thinking about the relationship between risk and return is the Capital Asset Pricing Model (CAPM) developed in Sharpe (1964), and Lintner (1965). CAPM proposes that beta is the sole measure of priced risk. If CAPM is correct then the beta-adjusted returns from all stocks must be equal to each other. A large body of empirical evidence shows that beta-adjusted stock returns are not equal but vary systematically with factors such as "size" and "value". Size premium means that small-cap stocks tend to earn higher beta-adjusted returns than large-cap stocks.[3] Value premium means that value stocks tend to outperform growth stocks.[4] Value stocks are those with high book-to-market value. They typically have stable dividend yields. Growth stocks have low book-to-market value and tend to reinvest a lot of their earnings. Value stocks are typically less volatile than growth stocks. Fama-French (FF) value and growth indices (monthly returns data from July 1963 to April 2002) show the following standard deviations: FF small value: 19.20%, FF small growth: 24.60%, FF large value: 15.39%, and FF large growth: 16.65%. That is, among both small-cap and large-cap stocks, value stocks are less volatile than growth stocks.

Intuitively, the value premium is even more surprising than the size premium as it is plausible to argue that small size means higher risk (such as business cycle or liquidity risk) with

[3] Size effect is documented in Banz (1981), Reinganum (1981), Blume and Stambaugh (1983), Brown et al (1983), and more recently in Hou, Xue, and Zhang (2014), and Fama and French (2015) among many others.

[4] Value premium is documented in Fama and French (1998) among many others.

size premium being compensation for higher risk; however, how can less volatility be more risky as the value premium seems to suggest?[5]

The existence of size and value premiums has led to a growing body of research that attempts to explain them. In particular, there is the empirical asset pricing approach of Fama and French (1993) in which these factors are taken as proxies for risks with the assumption that all risks are correctly priced.[6] The task then falls to the asset pricing branch of theory to explain the sources of these risks. Fama and French (1997) argue that value firms are financially distressed, and the existence of the value premium is a compensation for bearing this risk. However, the empirical evidence is largely inconsistent with this view as distressed firms are found to have low returns rather than high returns (see Dichev (1998), Griffin and Lemmon (2002), and Campbell et al (2008) among others). Behavioral explanations of the value premium have also been put forward. In particular, Debondt and Thaler (1987), Lakonishok et al (1994), and Haugen (1995) argue that traders overreact to news, and overprice "hot" stocks which tend to be growth firms, and underprice out of favor stocks which tend to be value firms. When this overreaction is eventually corrected, value premium arises.

Apart from size and value, there also exists, what is known as, the momentum effect in the stock market. Momentum effect refers to the tendency of "winning stocks" in recent past to continue to outperform "losing stocks" for an intermediate horizon in the future. Momentum effect has been found to be a robust phenomenon, and can be demonstrated with a number of related definitions of "winning" and "losing" stocks. Jegadeesh and Titman (1993) show that stock returns exhibit momentum behavior at intermediate horizons. A self-financing strategy that buys the top 10% and sells the bottom 10% of stocks ranked by returns during the past 6 months, and holds the positions for 6 months, produces profits of 1% per month. George and Hwang (2004) define "winning' stocks as having price levels close to their 52-week high, and "losing" stocks as those with price levels that are farthest from their 52-week high, and show that a self-financing strategy that shorts "losing" stocks and buys "winning" stocks earns abnormal profits over an intermediate horizon (up to 12 months) consistent with the momentum effect.

[5] Researchers appeal to other dimensions of risk different from volatility in attempts to explain value premium. However, no consensus explanation exists as to the source of value premium.
[6] Recently Fama and French (2015) show that value premium is also captured by adding "investment" and "profitability" factors to size and beta factors.

Other well documented anomalies include:

Low β High α

It is well known that the relation between univariate market β and average stock return is flatter than predicted by the CAPM of Sharpe (1964) and Lintner (1965) (see Black, Jensen, and Scholes (1972), Fama and MacBeth (1973), Frazzini and Pedersen (2014) among others). This is known as the low-beta-high-alpha anomaly.

Low Volatility Anomaly

Stocks with highly volatile returns tend to have low average returns irrespective of whether volatility is measured as the variance of daily returns or as the variance of the residuals from the FF three-factor model (see Baker and Haugen (2012), Wurgler et al (2010), and Ang et al (2006) among others).

Accruals

Sloan (1996) is the first paper to find that low returns are associated with high accruals. Accruals arise because accounting decisions cause book earnings to differ from cash earnings. The puzzle is that equity in firms that have a large accrual component of earnings performs worse than the equity in firms that have a lower accrual component.

Stock-Splits and Reverse Stock-Splits

Using data from 1975 to 1990, Ikenberry et al (1996) shows that stock-splits are associated with 8% positive abnormal returns after one year, and 16% abnormal returns over three years. Ikenberry et al (2003) uses data from 1990 to 1997 and confirms the earlier findings. Ohlson and Penman (1985) are the first to show that return volatility increases by about 30% following a stock split. Kim et al (2008) examine the long-run performance of 1600 firms with reverse stock-splits and reports negative abnormal returns. Koski (2007) shows a decrease in return volatility of 25% subsequent to a reverse stock-split.

2.1 Anchoring Adjusted CAPM as a Unified Framework for Asset Return Anomalies

Without asserting that the anchoring adjusted CAPM provides a complete explanation for the puzzles mentioned above, which requires a comprehensive empirical examination beyond the scope of any one article, I show that it provides a plausible unified explanation for the puzzles. I am not aware of any other approach, either traditional or behavioral, that provides such a unified explanation. Given the large number of puzzles addressed, careful empirical evaluation of each puzzle is beyond the scope of this article or any one article. As this article is purely theoretical, the aim is modest, which is to lay the theoretical groundwork for future comprehensive empirical work that checks the anchoring based explanation against other proposed explanations.

2.1.1 ACAPM and the Size Effect

The size effect with anchoring can be easily seen after a little algebraic manipulation of (17). Beta-adjusted excess return on a normal firm's stock is:

$$BetaAdjusted\ excess\ return = \frac{E(r_{qSj}) - r_F}{\frac{Cov(r_{qSj}, r_M)}{Var(r_M)}}$$

Beta-adjusted excess return on a normal firm's stock can be written as:

$$BetaAdjusted\ excess\ return\ (normal\ stock) = [h] \cdot [1 + g]$$

where $g = \frac{n'_{qSj}(1-m)\left(\sigma_{qL}^2 - \sigma_{qSj}^2\right)}{P_{qSjt}P_{Mt} \cdot Cov(r_{qSj}, r_M)} = \frac{n'_{qSj}(1-m)\left(\sigma_{qL}^2 - \sigma_{qSj}^2\right)}{Cov(stock's\ payoff, market\ payoff)}$

and $h = \frac{Var(r_M) \cdot E_t[r_M - r_F]}{Var(r_M) + \sum_{q=1}^{Q} \sum_{i=1}^{k} \frac{n'^2_{qSi}(1-m)\left(\sigma_{qL}^2 - \sigma_{qSi}^2\right)}{P_{Mt}^2}} = constant$

In a given cross-section of stocks, h is a constant. So, in order to make predictions about what happens to beta-adjusted return when size varies, we need to look at how $g = \frac{n'_{qSj}(1-m)\left(\sigma_{qL}^2 - \sigma_{qSj}^2\right)}{Cov(stock's\ payoff, market\ payoff)}$ changes with size.

Proposition 3 (The Size Premium):

Beta-adjusted excess returns with anchoring fall as payoff size increases. In the absence of anchoring, beta-adjusted excess returns do not vary with size and are always equal to the market risk premium.

Proof.

$$BetaAdjusted\ excess\ return = \frac{E(r_{qSj}) - r_F}{\frac{Cov(r_{qSj}, r_M)}{Var(r_M)}}$$

Substituting from (17) and re-arranging leads to:

$$BetaAdjusted\ excess\ return$$
$$= \left[\frac{Var(r_M) \cdot E_t[r_M - r_F]}{Var(r_M) + \sum_{q=1}^{Q} \sum_{i=1}^{k} \frac{n'^2_{qSi}(1-m)(\sigma^2_{qL} - \sigma^2_{qSi})}{P^2_{Mt}}}\right]$$
$$\cdot \left[1 + \frac{n'_{qSj}(1-m)(\sigma^2_{qL} - \sigma^2_{qSj})}{P_{qSjt} P_{Mt} \cdot Cov(r_{qSj}, r_M)}\right]$$

That is, beta-adjusted excess return can be written in the form:

$$BetaAdjusted\ excess\ return = [h] \cdot [1 + g]$$

where $g = \frac{n'_{qSj}(1-m)(\sigma^2_{qL} - \sigma^2_{qSj})}{P_{qSjt} P_{Mt} \cdot Cov(r_{qSj}, r_M)} = \frac{n'_{qSj}(1-m)(\sigma^2_{qL} - \sigma^2_{qSj})}{Cov(stock's\ payoff, market\ payoff)}$

and $h = \frac{Var(r_M) \cdot E_t[r_M - r_F]}{Var(r_M) + \sum_{q=1}^{Q} \sum_{i=1}^{k} \frac{n'^2_{qSi}(1-m)(\sigma^2_{qL} - \sigma^2_{qSi})}{P^2_{Mt}}} = constant$

Clearly, as payoff variance and covariance with the market increase, g falls. Payoff variance and covariance with the market increase with size. It follows that beta-adjusted excess returns must fall as payoff size increases when there is anchoring bias. In the absence of anchoring bias, that is, with $m = 1$, it follows that $g = 0$ and $h = E[r_M - r_F]$. Hence, in the absence of anchoring, beta-adjusted excess return does not change with payoff covariance and payoff variance, and remains equal to the market risk premium.

■

2.1.2 ACAPM and the Value Effect

Value stocks have less volatile returns than growth stocks. Fama-French (FF) value and growth indices (monthly returns data from July 1963 to April 2002) show the following standard deviations: FF small value: 19.20%, FF small growth: 24.60%, FF large value: 15.39%, and FF large growth: 16.65%. That is, among both small-cap and large-cap stocks, value stocks returns are less volatile than growth stocks. By definition, value stocks have high book-to-market ratio when compared with growth stocks. It follows that, for a given book value of equity, value stocks have lower market prices when compared with growth stocks. Hence, the value stocks not only have smaller return volatility, but must also have smaller payoff volatility.

Proposition 4: (The Value Premium):

Beta-adjusted excess return on stocks with smaller payoff volatility is larger than the beta-adjusted excess returns on stocks with higher payoff volatility. In the absence of anchoring, beta-adjusted returns do not vary with payoff volatility and are always equal to the market risk premium.

Proof.

From the proof of proposition 3, we know that:

$$BetaAdjusted\ excess\ return = [h] \cdot [1 + g]$$

where $g = \frac{n'_{qSj}(1-m)\left(\sigma^2_{qL}-\sigma^2_{qSj}\right)}{P_{qSjt}P_{Mt} \cdot Cov(r_{qSj}, r_M)} = \frac{n'_{qSj}(1-m)\left(\sigma^2_{qL}-\sigma^2_{qSj}\right)}{Cov(stock's\ payoff, market\ payoff)}$

and $h = \frac{Var(r_M) \cdot E_t[r_M - r_F]}{Var(r_M) + \sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{n'^2_{qSi}(1-m)\left(\sigma^2_{qL}-\sigma^2_{qSi}\right)}{P^2_{Mt}}} = constant$

Note, that $g = \frac{n'_{qSj}(1-m)\left(\sigma^2_{qL}-\sigma^2_{qSj}\right)}{Cov(stock's\ payoff, market\ payoff)}$. As payoff volatility rises, g falls because numerator falls and denominator rises. It follows that beta-adjusted excess return falls as payoff volatility rises, holding all else constant.

■

Corollary 4.1: (The Value Premium Falls with Size):

At larger payoff sizes, the impact of an increase in payoff volatility (due to a given change in payoffs) on beta-adjusted excess returns is smaller.

2.1.3 ACAPM and the Low-β-High-α Anomaly

Stocks with low betas tend to have low return volatilities and stocks with high betas tend to have high return volatilities (Baker et al (2011), Chow et al (2014)). Also, it is well known that earnings volatility is positively related to stock return volatility (see Beaver et al (1970), Ryan (1997) among others). One can safely conclude that low beta stocks also have low earnings volatility, and high beta stocks have high earnings volatility.

From (17):

$$E_t(r_{qSj}) = r_F + E_t[r_M - r_F] \cdot \frac{Cov(r_{qSj}, r_M) + \frac{n'_{qSj}(1-m)(\sigma^2_{qL} - \sigma^2_{qSj})}{P_{qSjt}P_{Mt}}}{Var(r_M) + \sum_{q=1}^{Q}\sum_{i=1}^{k}\frac{n'^2_{qSi}(1-m)(\sigma^2_{qL} - \sigma^2_{qSi})}{P^2_{Mt}}}$$

As can be seen from above, in a given cross-section, stocks with higher value of the anchoring term $\frac{n'_{qSj}(1-m)(\sigma^2_{qL}-\sigma^2_{qSj})}{P_{qSjt}P_{Mt}}$ outperform stocks with a lower value of the anchoring term on beta-adjusted basis. The anchoring term is larger for stocks with less volatile earnings. As low beta stocks tend to have low earnings volatility, the anchoring term is higher for them. Hence, with anchoring, stocks with low CAPM betas must have high CAPM alphas leading to a flatter relationship between CAPM beta and average returns.

2.1.4. ACAPM and the Low Volatility Anomaly

As stocks with less volatile stock returns typically belong to firms with less volatile earnings (see Beaver et al (1970), Ryan (1997) among others), the anchoring term in (17) is larger for them. Hence, the anchoring explanation for the low volatility anomaly is very similar to the anchoring explanation for low-beta-high-alpha anomaly.

2.1.5 ACAPM and the Accruals Anomaly

Sloan (1996) finds that low returns are associated with high accruals. Accruals arise because accounting decisions cause book earnings to differ from cash earnings. One expects that firms with large accrual component of earnings will have less persistent (more volatile earnings) when compared with firms with smaller accrual component of earnings. Sloan (1996) explicitly tests for this and finds strong support. The anchoring based explanation is straightforward. As high accruals imply high earnings volatility, and high earnings volatility reduces the anchoring term, average returns from high accrual firms must be lower. Note, that the explanation for accruals has the same underlying logic as the explanations for low-beta-high-alpha and low volatility anomalies. Hence, the anchoring approach suggests that these seemingly different anomalies are much the same phenomena.

2.1.6 ACAPM, Stock-Splits, and Reverse Stock-Splits

A stock-split increases the number of shares proportionally. In a 2-for-1 split, a person holding one share now holds two shares. In a 3-for-1 split, a person holding one share ends up with three shares and so on. A reverse stock-split is the exact opposite of a stock-split. Stock-splits and reverse stock- splits appear to be merely changes in denomination, that is, they seem to be accounting changes only that should not have any real impact on returns. With CAPM, stock-splits and reverse stock-splits do not change expected returns. To see this clearly, consider equation (4), which is reproduced below:

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma n_S' \sigma_S^2 - \gamma n_L' \sigma_{LS}}{1 + r_F}$$

A 2-for-1 split in the small firm's stock divides the expected payoff by 2, divides the variance by 4 and covariance by 2, while multiplying the number of shares outstanding by 2. That is, a 2-for-1 split leads to:

$$P_{St}{}^{Split} = \frac{\frac{E_t(\delta_{St+1})}{2} - \gamma \frac{2n_S' \sigma_S^2}{4} - \frac{\gamma n_L' \sigma_{LS}}{2}}{1 + r_F}$$

It follows that $P_{St}{}^{Split} = \frac{P_{St}}{2}$.

That is, the price with split is exactly half of what the price would have been without the split. As both the expected payoff and the price are divided by two, there is no change in expected returns associated with a stock-split under CAPM. An equivalent conclusion follows for a reverse stock-split as well. For a reverse split, both the expected payoffs and the price increase by the same factor.

With anchoring, the price given in (6) is as follows:

$$P_{St} = \frac{E_t(\delta_{St+1}) - \gamma[cov(\delta_{St+1}, n_L'\delta_{Lt+1} + n_S'\delta_{St+1}) + n_S'(1-m)(\sigma_L^2 - \sigma_S^2)]}{1 + r_F}$$

As can be seen from the above expression, a 2-for-1 stock-split reduces the price by more than half. It follows that, with anchoring, stock-split increases returns as well volatility of returns. A reverse stock-split of 1-for-2, on the other hand, more than doubles the price, reducing returns as well as volatility of returns.

2.1.7 ACAPM and the Momentum Effect

From (17), one can see that, in a given cross-section of stocks, keeping everything else the same, low "m" stock do better than high "m" stocks. But, how can we identify low vs high "m" stocks? Plausibly, we can identify them by looking at their recent performances. Winning stocks are likely to get more strongly anchored to the leader stock as their recent success makes them more like the leader. For losing stocks, their recent bad spell makes them less like the leader. That is, "m" falls for winning stocks and rises for losing stocks. So, winning stocks continue to outperform losing stocks till the effect of differential news on "m" dissipates, and "m" returns to its normal level. Of course, there could be multiple ways of identifying "low m" vs. "high m" stocks. Plausibly, stocks with prices at or closest to their 52-week high can be considered as stocks with low "m" values, and stocks with prices at or near their 52-week low, can be considered as high "m" stocks. It takes a series of positive news to get to the 52-week high, and a series of negative news to get to the 52-week low. Alternatively, recent high returns is another way of separating the winners.

The ACAPM based explanation for the momentum effect is related to the explanations offered in Barberis, Shleifer, and Vishny (1998), and Daniel, Hirshleifer, and Subrahmanyam (1998). The explanation in Barberis et al (1998) is based on the idea that traders are slow to update their priors when new information arrives. That is, traders under-react to new information. In contrast, Daniel et al (1998) argue that momentum occurs because traders overreact to prior information if the new information confirms it. With ACAPM, the payoff volatility judgment about a normal firm's stock is anchored to the payoff volatility of a prominent blue-chip stock in the same sector:

$$\hat{\sigma}_S^2 = (1-m)\sigma_L^2 + m\sigma_S^2$$

One can see both under-reaction and overreaction in the above expression. Firm specific news, that is, news specific to σ_S^2 is under-reacted to. However, an anchoring prone trader also reacts to unrelated news. He overreacts by also responding to news only related to the leader firm. That is, he overreacts by responding to news only related to σ_L^2. Even though ACAPM explanation of the momentum effect is different from previous explanations, it has aspects of under-reaction and overreaction that other explanations appeal to.

2.1.8 ACAPM and the High Equity Premium

Since its identification in Mehra and Prescott (1985), a large body of research has explored what is known in the literature as the equity premium puzzle. It refers to the fact that historical average return on equities (around 7%) is so large when compared with the historical average risk-free rate (around 1%) that it implies an implausibly large value of the risk aversion parameter. Mehra and Prescott (1985) estimate that a risk-aversion parameter of more than 30 is required, whereas a much smaller value of only about 1 seems reasonable.

If one is unaware of the phenomenon of anchoring, and uses CAPM to estimate risk-aversion then the following equation is appropriate:

$$E_t[r_M] = r_F + \frac{\gamma}{P_{Mt}}[Var(\delta_{Mt+1})] \quad (19)$$

One may recover the corresponding value of γ, the risk-aversion parameter from (19) by substituting for all other variables and parameters in (19). The equity premium puzzle, when translated in the CAPM context, is that the recovered value of risk-aversion parameter is implausibly large.

With CAPM adjusted for anchoring, the expected return on the market portfolio is given by:

$$E_t[r_M] = r_F + \frac{\gamma}{P_{Mt}}\left[Var(\delta_{Mt+1}) + \sum_{q=1}^{Q}\sum_{i=1}^{k} n'^2_{qSi}(1-m)\left(\sigma^2_{qL} - \sigma^2_{qSi}\right)\right] \quad (20)$$

A comparison of (20) and (19) shows that, with anchoring accounted for, a much smaller value of the risk-aversion parameter is required to justify the observed equity premium. Hence, ACAPM may be relevant for the equity premium puzzle.

2.2. ACAPM: A Numerical Example

In this section, a numerical example is presented, which considers the implications of ACAMP and CAPM when there is one leader firm and three normal firms of similar size in the market. It is shown that under CAPM, beta-adjusted excess returns of all four firms are equal to each other, whereas, under ACAPM beta-adjusted excess returns are larger for normal firms when compared with the leader firm. The three normal firms, although of similar size (similar expected payoffs and market capitalizations) vary in one crucial way. Their payoff variances are different with S1 having the highest payoff variance, followed by S2, and then by S3. We will see that, in line with the value premium, less volatile payoffs lead to higher beta-adjusted excess returns among similar size firms.

Suppose there are four types of stocks with next period payoffs as shown in Panel A of Table 1. Type "Large" belongs to a large well-established firm with large cash flows. Types S1, S2, and S3 are smaller firms with equal expected payoffs, however, their payoff volatilities are 416.667, 216.667, and 66.667 respectively. That is, among the small firms, S1 is the most volatile, followed by S2, and then S3. Panel B of Table 1 shows the associated covariance matrix. The risk aversion parameter is assumed to be 0.001, and the one period risk-free rate is 0.01. Every type is assumed to have exactly one share outstanding. Another way of seeing the difference between S1, S2, and S3 is to calculate the quantity: $\frac{Variance\ (stock\ payoff)}{Covariance\ (stock\ payoff, market\ payoff)}$. The values for S1, S2, and S3 are 0.238, 0.161, and 0.095 respectively. S1 is akin to growth stock due to high payoff volatility, whereas S3 is similar to a value stock due to low payoff volatility. One can verify

Table 1

CAPM Returns and Prices

$\boldsymbol{\gamma = 0.001, and\ r_F = 0.01}$

Panel A: Payoffs

	Large	S1	S2	S3
	100	5	10	20
	150	30	30	30
	200	55	50	40
Expected Payoff	150	30	30	30

Panel B: The Covariance Matrix

	Large	S1	S2	S3
Large	1666.667	833.333	666.667	333.333
S1	833.333	416.667	333.333	166.667
S2	666.667	333.333	216.667	133.333
S3	333.333	166.667	133.333	66.667

Panel C: CAPM Prices

	Large	S1	S2	S3	Mkt Portfolio Value
Price	145.0495	27.9703	28.3168	29.0099	230.3465
Expected Returns	0.03413	0.0726	0.0594	0.03413	0.0419
Variance of Mkt Portfolio's Return	0.1385				
Covariance with Mkt Portfolio's Return	0.10475	0.2716	0.2146	0.10475	0.1385

Panel D: CAPM Beta and Beta-Adjusted Excess Returns

	Large	S1	S2	S3	Mkt Portfolio Value
CAPM Beta	0.75622	1.96081	1.54945	0.75622	1
Beta Adjusted Excess Returns	0.03191	0.03191	0.03191	0.03191	0.03191

that g is smallest for S1, and largest for S3. So, beta-adjusted excess returns on S3 must be larger than the beta-adjusted excess returns on S1 with anchoring, in line with the value premium.

Prices implied by CAPM can be calculated for each stock from (3) and (4) and are shown in Panel C of Table 1. Panel C also shows expected returns, the value of the aggregate market portfolio, the variance of the market portfolio's return, and the covariance of each stock's return with the market portfolio's return. Panel D shows each stocks beta and the corresponding beta-

Table 2

ACAPM Returns and Prices

$\boldsymbol{\gamma = 0.001, r_F = 0.01, and\ m = 0.90}$

Panel A: ACAPM Prices					
	Large	S1	S2	S3	Mkt Portfolio
Price	145.0495	27.84653	28.18722	28.85149	229.9257
Expected Return	0.03413	0.0773	0.0647	0.0398	0.0438
Variance of Mkt Portfolio's Return	0.139031				
Covariance with Mkt Portfolio's Return	0.1049	0.2733	0.2161	0.1055	0.139031
Panel B: CAPM Beta and Beta Adjusted Returns under ACAPM					
CAPM Beta	0.7548	1.9659	1.5542	0.75898	1
Beta Adjusted Excess Returns	0.031967	0.03425	0.035164	0.039274	0.0338

adjusted excess return. It can be seen that all stocks have the same beta-adjusted excess return, which is equal to the excess return on the market portfolio.

The key prediction of CAPM can be seen in the last line of Table 1. That is, beta-adjusted excess returns of all assets must be equal. In other words, beta is the only measure of priced risk in CAPM. And, investors are rewarded based on their exposure to beta-risk. Once beta-risk has been accounted for, there is no additional return.

Next, we will see what happens with anchoring. Table 2 shows the results from ACAPM. Everything is kept the same except that now anchoring is allowed in variance judgments. The anchoring prone marginal investor starts from the variance of the large firm and subtracts from it to form variance judgments about the small stocks. For the purpose of this illustration, I assume that he goes 90% of the way. That is, $m = 0.90$. As can be seen, price of the large firm does not change, however, the prices of small firms change, and can be calculated from (6). As expected, expected return on the large firm's stock does not change. However, as the market portfolio changes, all betas change. Expected returns on small firms can be calculated from (15).

As can be seen from Table 2, beta-adjusted excess returns on normal stocks are larger than the beta-adjusted excess return on the leader stock. Furthermore, the value premium can be seen in Table 2 among normal firms. Highest payoff volatility S1 has the smallest excess return of 0.03425, whereas the lowest volatility S3 has the highest excess return of 0.039274. As value stocks typically have lower payoff volatility than growth stocks, in this example, S3 is like a value stock, and S1 is like a growth stock.

3. Conclusions and Discussion

A challenge for asset pricing theory is to modify the basic framework while maintaining its elegance in order to capture the large number of anomalies that empirical literature has uncovered over the years. Anchoring bias in volatility estimates is one such modification. Arguably, it is simpler than Fama-French three or Five factor models, and is able to capture anomalies which are difficult to explain in other frameworks (one example: momentum effect). It is also theoretically satisfying as one can see how anomalies arise from the first principles. One criticism of behavioral finance is that it seems to be a collection of ad hoc models without a unifying structure. This article shows that adjusting CAPM for anchoring potentially provides a unifying framework in which almost all key asset return anomalies can be seen as arising from the first principles. CAPM is obtained as a special case corresponding to correct adjustments.

The essential idea is that the volatility judgment about an average stock is influenced by the payoff volatilities of prominent blue-chip stocks as they are the starting points. An interesting question is, which are the most prominent blue-chip stocks? Oil, energy, and commodity stocks are among the most prominent blue-chips, and plausible rest of the market may be anchored to them. A way to test for this is to look at periods of unusually high volatility in oil and energy, and see how the rest of the market is impacted. The market behavior in early 2016 is a case in point as it corresponds to unusually high uncertainty about oil. If the anchoring approach is correct, unusually high uncertainty about oil should impact the rest of the market negatively. This appears to be the case with stock markets around the world closely following oil dynamics. This is quite puzzling from a purely fundamentals based economic perspective.[7]

This article develops the basic theoretical framework. Comprehensive empirical evaluation of anchoring vs other explanations is the subject of future research.

[7] http://blogs.piie.com/realtime/?p=5341

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