Revisiting Cournot and Bertrand in the presence of income effects

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Abstract

TBD

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1 Introduction: TBD

2 The model and preliminary results

2.1 Firms and consumers

The economy involves one sector supplying a horizontally differentiated good and one production factor - labor. There is a continuum of unit mass of identical consumers endowed with one unit of labor. The labor market is perfectly competitive and labor is chosen as the numéraire. The differentiated good is made available under the form of a finite and discrete number $n \geq 2$ of varieties. Each variety is produced by a single firm and each firm produces a single variety. Thus, n is also the number of firms. Each firm needs $c > 0$ units of labor to produce one unit of its variety. Since wage is normalized to 1, the cost of producing q_i units of variety $i = 1, ..., n$ is equal to cq_i.

Consumers share the same additive preferences given by

$$U(\mathbf{x}) = \sum_{i=1}^{n} u(x_i) \tag{1}$$

where $u(x_i)$ is thrice continuously differentiable, strictly increasing, strictly concave, and such that $u(0) = 0$. The strict concavity of u means that a consumer has a love for variety: when the consumer is allowed to consume X units of the differentiated good, she strictly prefers the consumption profile $x_i = X/n$ to any other profile $\mathbf{x} = (x_1, ..., x_n)$ such that $\sum_i x_i = X$. Because consumers are identical, they consume the same quantity x_i of variety $i = 1, ..., n$.

Following Zhelobodko et al. (2012), we define the relative love for variety (RLV) as follows:

$$r_u(x) = -\frac{xu''(x)}{u'(x)}$$

which is strictly positive for all $x > 0$. Under the CES, we have $u(x) = x^\rho$ where ρ is a constant such that $0 < \rho \leq 1$, thus implying a constant RLV given by $1 - \rho$. Another example of additive preferences is provided by Behrens and Murata (2007) who consider the CARA utility $u(x) = 1 - \exp(-\alpha x)$ where $\alpha > 0$ is the absolute love for variety; the RLV is now given by αx. Very much like the Arrow-Pratt's relative risk-aversion, the RLV measures the intensity of consumers' variety-seeking behavior.

A consumer's income is equal to her wage plus her share in total profits. Since we focus on symmetric equilibria, consumers must have the same income, which means that profits have to be uniformly distributed across consumers. In this case, a consumer's income y is given by

$$y = 1 + \sum_{i=1}^{n} \Pi_i \geq 1 \tag{2}$$

where the profit made by the firm selling variety i is given by

$$\Pi_i = (p_i - c)q_i \tag{3}$$

p_i being the price of variety i. Evidently, the income level varies with firms' strategies.

A consumer's budget constraint is given by

$$\sum_{i=1}^{n} p_i x_i = y \tag{4}$$

where x_i stands for the consumption of variety i.

The first-order condition for utility maximization yields

$$u'(x_i) = \lambda p_i \tag{5}$$

where λ is the Lagrange multiplier

$$\lambda(\mathbf{x}, y) = \frac{\sum_{j=1}^{n} u'(x_j)x_j}{y} \geq 0. \tag{6}$$

Therefore, a consumer's inverse demand for variety $i = 1, ..., n$ is as follows:

$$p_i(\mathbf{x}, y) = \frac{yu'(x_i)}{\sum_{j=1}^{n} u'(x_j)x_j}. \tag{7}$$

2.2 Market equilibrium

The market equilibrium is defined by the following conditions:

(i) each consumer maximizes her utility (1) subject to her budget constraint (4),

(ii) each firm i maximizes its profit (3) with respect to q_i (Cournot) or p_i (Bertrand),

(iii) product market clearing:

$$x_i = q_i \qquad \text{for all } i = 1, ..., n,$$

(iv) labor market clearing:

$$c \sum_{i=1}^{n} q_i = 1.$$

The last two equilibrium conditions thus imply that

$$\bar{x} \equiv \bar{q} \equiv \frac{1}{cn} \tag{8}$$

is the only candidate symmetric equilibrium output. As a consequence, Cournot competition and Bertrand competition are equally efficient. Observe that this result holds true for any symmetric and convex preferences. It is also independent of how profits are redistributed. Therefore, the widely-accepted property in oligopoly theory, which says that price-setters produce more than quantity-setters at a symmetric equilibrium stems from the absence of labor market considerations. To be precise, oligopoly models assume implicitly that the labor supply is perfectly elastic. By contrast, labor supply is perfectly inelastic in our setting.

Between these two extreme cases, there is a continuum of possibilities. For example, when the labor supply curve has a positive and finite elasticity, firms must pay a higher wage to the workers they need to produce more. This implies a strictly increasing marginal cost $\gamma(q_i)$. In this case, the equilibrium consumption and output are given by

$$x^* \equiv q^* \equiv \gamma^{-1}\left(\frac{1}{n}\right).$$

The following proposition summarizes the above discussion.

Proposition 1. *Assume that the number of firms is exogenous and that all firms have access to the same technologies. If the labor supply curve has a positive elasticity and if a symmetric equilibrium exists, then the equilibrium output is the same under Cournot and Bertrand competition.*

The expression (8) has another far-reaching implication. When it is recognized that the income is endogenous in consumers' budget constraint, Cournot competition and Bertrand competition always deliver the first-best outcome when the number of firms is exogenous and the same. Indeed, $\bar{x}$ is the equilibrium consumption of a variety when all varieties are priced at their marginal cost. Note, however, that firms produce the same output under the two competitive regimes does not imply that firms charge the same price. Therefore, the total income in the economy (GDP) need not be the same.

Since the product market clearing condition implies that $q_i = x_i$ for all i, from now on we write all expressions in terms of x_i only. Let

$$m \equiv \frac{p-c}{p}$$

be the markup at any symmetric outcome. Then, (2) can be rewritten as follows:

$$y = 1 + \sum_{j=1}^{n} \frac{p_j - c}{p_j} p_j x_j,$$

which, after symmetrization, amounts to

$$y = 1 + nmpx = 1 + my,$$

where we have used the budget constraint. Therefore, the corresponding income is given by

$$y = \frac{1}{1-m}. \tag{9}$$

3 When Bertrand and Cournot meet Ford

As shown by (5) and (6), the income level influences firms' demands, whence their profits. As a result, firms must anticipate accurately what the total income will be. In addition, firms should be aware that they can manipulate the income level, whence their "true" demands, through their own strategies with the aim of maximizing profits (Gabszewicz and Vial, 1972). This feedback effect is known as the *Ford effect* (d'Aspremont et al., 1996). Unfortunately, as will be shown, proving the existence and uniqueness of an equilibrium in such a context appears to be a hard task (Roberts and Sonnenschein, 1977).

3.1 Bertrand

Let $\mathbf{p} = (p_1, ..., p_n)$ be a price profile. In this case, consumers' demand functions $x_i(\mathbf{p})$ are obtained by solving the system of equations (7) where consumers' income y is now defined as follows:

$$y^B(\mathbf{p}) = 1 + \sum_{j=1}^{n}(p_j - c)x_j(\mathbf{p}).$$

It follows from (6) that the marginal utility of income λ is a market aggregate that depends on the price profile $\mathbf{p}$. Indeed, the budget constraint

$$\sum_{j=1}^{n} p_j x_j(\mathbf{p}) = y^B(\mathbf{p})$$

implies that

$$\lambda(\mathbf{p}) = \frac{1}{y^B(\mathbf{p})}\sum_{j=1}^{n} x_j(\mathbf{p})u'\left(x_j(\mathbf{p})\right). \tag{10}$$

Since $u'(x)$ is strictly decreasing, the demand function for variety i is thus given by

$$x_i(\mathbf{p}) = \xi(\lambda(\mathbf{p})p_i), \tag{11}$$

where ξ is the inverse function of u'. Thus, firm i's profits can be rewritten as follows:

$$\Pi_i^B(\mathbf{p}) = (p_i - c)x_i(\mathbf{p}) = (p_i - c)\xi(\lambda(\mathbf{p})p_i). \tag{12}$$

For any given $n \geq 2$, a *Bertrand equilibrium* is a vector $\mathbf{p}^* = (p_1^*, ..., p_n^*)$ such that p_i^* maximizes $\Pi_i^B(p_i, \mathbf{p}_{-i}^*)$ for all $i = 1, ..., n$. This equilibrium is symmetric if $p_i^* = p^B$ for all i.

Applying the first-order condition to (12) yields

$$\frac{p_i - c}{p_i} = -\frac{\xi(\lambda p_i)}{\xi'(\lambda p_i)p_i\left(\lambda + p_i \frac{\partial \lambda}{\partial p_i}\right)}, \tag{13}$$

which involves $\partial\lambda/\partial p_i$ because λ depends on $\mathbf{p}$. Unlike what is assumed in partial equilibrium models of oligopoly, λ is here a function of $\mathbf{p}$, so that the markup depends on $\partial\lambda/\partial p_i \neq 0$. But how does firm i determine $\partial\lambda/\partial p_i$?

Since firm i is aware that λ is endogenous and depends on $\mathbf{p}$, it understands that the demand functions (11) must satisfy the budget constant as an identity. The consumer budget constraint can be rewritten as follows:

$$\sum_{j=1}^{n} p_j \xi(\lambda(\mathbf{p})p_j) = 1 + \sum_{j=1}^{n}(p_j - c)\xi(\lambda(\mathbf{p})p_j),$$

which boils down to

$$\sum_{j=1}^{n} \xi(\lambda(\mathbf{p})p_j) = 1/c. \tag{14}$$

Differentiating (14) with respect to p_i yields

$$\xi'(\lambda p_i)\lambda + \frac{\partial \lambda}{\partial p_i}\sum_{j=1}^{n} p_j \xi'(\lambda p_j) = 0$$

or, equivalently,

$$\frac{\partial \lambda}{\partial p_i} = -\frac{\xi'(\lambda p_i)\lambda}{\sum_{j=1}^{n} \xi'(\lambda p_j)p_j}. \tag{15}$$

Substituting (15) into (13) and symmetrizing the resulting expression yields the candidate equilibrium markup:

$$\bar{m}^{BF} = -\frac{\xi(\lambda p)}{\xi'(\lambda p)\lambda p\left(1 - \frac{1}{n}\right)} = \frac{n}{n-1} r_u\left(\frac{1}{cn}\right) \tag{16}$$

where we have used the identity

$$r_u(x) \equiv -\frac{\xi(\lambda p)}{\xi'(\lambda p)\lambda p}.$$

Proposition 2. *Assume that firms account for the Ford effect and that a symmetric equilibrium* *exists under Bertrand competition. Then, the equilibrium markup is given by*

$$\bar{m}^{BF} = \frac{n}{n-1} r_u \left(\frac{1}{cn} \right).$$

Note that $r_u(1/cn)$ must be smaller than 1 for $\bar{m}^{BF} < 1$ to hold. Since $1/(cn)$ can take on any positive value, it must be

$$r_u(x) < 1 \quad \text{for all } x > 0. \tag{17}$$

This condition means that the elasticity of a monopolist's inverse demand is smaller than 1 or, equivalently, the elasticity of the demand exceeds 1. In other words, the marginal revenue is positive. However, (17) is not sufficient for $\bar{m}^{BF}$ to be smaller than 1. Here, a condition somewhat more demanding than (17) is required for the markup to be smaller than 1, that is, $r_u(1/cn) < (n-1)/n$. Otherwise, there exists no symmetric price equilibrium. For example, in the CES case, $r_u(x) = 1 - \rho$ so that

$$\bar{m}^{BF} = \frac{n}{n-1}(1-\rho) < 1$$

which means that ρ must be larger than $1/n$. This condition is likely to hold because econometric estimations of the elasticity of substitution $\sigma = 1/(1-\rho)$ exceeds 3 (Anderson and van Wincoop, 2004).

Using (16) yields the equilibrium price

$$\bar{p}^{BF} = c \frac{n-1}{n(1 - r_u\left(\frac{1}{cn}\right)) - 1} \tag{18}$$

which decreases with n when r_u is increasing. Using (9) yields the equilibrium income

$$\bar{y}^{BF} = \frac{n-1}{n(1 - r_u\left(\frac{1}{cn}\right)) - 1}.$$

It can be shown that, for each firm, the second-order condition holds in a neighborhood of (18).[1] In other words, $\bar{p}^{BF}$ is always a "local" Bertrand equilibrium. However, we have not been able to prove that the second-order condition is satisfied globally, and thus the existence of a symmetric Bertrand equilibrium under the Ford effect remains an open question.

3.2 Cournot

Firm i's profit may be expressed as follows:

$$\Pi_i^C(\mathbf{x}) = \left(y^C \frac{u'(x_i)}{\sum_{j=1}^n x_j u'(x_j)} - c \right) x_i \tag{19}$$

[1] The proof can be obtained from the authors upon request.

where

$$y^C = 1 + \sum_{j=1}^{n} (p_j(\mathbf{x}) - c)x_j$$

depends on $\mathbf{x}$. For any given $n \geq 2$, a *Cournot equilibrium* is a vector $\mathbf{x}^* = (x_1^*, \ldots, x_n^*)$ such that x_i^* maximizes $\Pi_i^C(x_i, \mathbf{x}_{-i}^*)$ for all $i = 1, ..., n$. This equilibrium is symmetric if $x_i^* = x^C$ for all i.

Accounting for the Ford effect under Cournot competition gives rise to unsuspected implications. Although we have seen that there is a single symmetric equilibrium output $\bar{q}^C = 1/cn$, the approach followed above can no longer be applied. Indeed, plugging

$$y^C = 1 + \sum_{j=1}^{n} (p_j - c)x_j$$

into the budget constraint (4) implies that

$$\sum_{j=1}^{n} p_j x_j = 1 + \sum_{j=1}^{n} (p_j - c)x_j \Longleftrightarrow 1 = c \sum_{j=1}^{n} x_j$$

which yields an expression independent of the price profile $\mathbf{p}$. As a result, any variation in consumers' expenditure generated by a price change is offset by the same variation in consumers' income. Therefore, the individual consumption is unaffected by a price change. Therefore, the equilibrium markup, price and profits are not uniqually determined. To put it differently, there exists a continuum $[0, 1]$ of equilibrium markups, which generates a continuum of equilibrium prices, which implies that we do not know at which market price the quantity $\bar{x}^C$ is sold. As a consequence, we are unable to find the equilibrium income, and thus the marginal utility of income is undetermined.

4 Income-taking firms

Since accounting for the Ford effect seems to be a dead-end, we may assume that, although firms are aware that consumers' income is endogenous, firms treat this income as a parameter. In other words, firms behave like *income-takers*. This approach is in the spirit of Hart (1985) for whom firms should take into account only some effects of their policy on the whole economy. Note that the income-taking assumption does not mean that profits have no impact on the market outcome. It means only that no firm seeks to manipulate its own demand through the income level.

4.1 Cournot

Since firms are income-takers, we have

$$\frac{\partial y^C}{\partial x_i} = 0 \qquad \text{for all } i. \tag{20}$$

However, firm i manipulates the other terms of (6), which accounts for the strategic interactions among firms.

Using (20), applying the first-order condition to (19) and using (7) yields

$$\frac{\partial \Pi_i^C}{\partial x_i} = p_i - c - \left[r_u(x_i) + \frac{x_i u'(x_i)\,(1 - r_u(x_i))}{\sum_{j=1}^n x_j u'(x_j)} \right] p_i = 0. \tag{21}$$

Lemma 1 in Appendix implies that the first-order conditions are sufficient for the existence of a Cournot equilibrium, while Lemma 2 implies that any equilibrium is symmetric. Furthermore, it follows from Proposition 1 that the symmetric equilibrium is unique and given by $\bar{x}^C = 1/(nc)$. Therefore, symmetrizing (21) shows that the only equilibrium markup is given by

$$\bar{m}^C \equiv \frac{p^C - c}{p^C} = \frac{1}{n} + \frac{n-1}{n} r_u \left(\frac{1}{cn} \right). \tag{22}$$

It follows from (17) that $\bar{m}^C$ is always smaller than 1. Furthermore, Lemma 1 in Appendix shows that $\Pi_i^C(x_i, \mathbf{x}_{-i})$ is strictly concave in x_i if

$$r_{u'}(x) = -\frac{x u'''(x)}{u''(x)} = -\frac{x p''(x)}{p'(x)} < 2. \tag{23}$$

This amounts to assuming that the elasticity of the slope of the inverse demand cannot be large, which rules out the case of inverse demands that are "too" convex (Seade, 1980). The condition (23) highlights the need to impose restrictions on the third derivative of the utility u to prove the existence and uniqueness of an equilibrium.

Thus, we have shown:

Proposition 3. *Assume that firms are income-takers. If* (17) *and* (23) *hold, then* $\bar{x}^C = 1/(nc)$ *is the unique Cournot equilibrium while the corresponding markup is given by* (22).

It follows from (22) that the market price is given by

$$\bar{p}^C = \frac{cn}{(n-1)(1 - r_u(1/cn))}.$$

Using (9) shows that the equilibrium income

$$\bar{y}^C = \frac{n}{(n-1)(1 - r_u(1/cn))}$$

is well defined. Then, we may use (6) to determine the equilibrium value of the marginal utility of income, which is now univocally defined.

Finally, both the equilibrium markup, price and income decrease with n when r_u is increasing, which corresponds to the pro-competitive case when the market is governed by monopolistic competition (Zhelobodko et al., 2012). This suggests that the market outcome behaves in a similar way under these two types of market structure.

4.2 Bertrand

Since firms are income-takers, we have

$$\frac{\partial y^B}{\partial x_i} = 0 \qquad \text{for all } i.$$

Differentiating both sides of the budget constraint

$$\sum_{j=1}^{n} p_j \xi(\lambda(\mathbf{p}) p_j) = y^B \tag{24}$$

with respect to p_i, where y^B is treated as a parameter, yields the following equation

$$\xi(\lambda p_i) + p_i \xi'(\lambda p_i)\lambda + \frac{\partial \lambda}{\partial p_i} \sum_{j=1}^{n} p_j^2 \xi'(\lambda p_j) = 0.$$

Solving this equation with respect to $\partial\lambda/\partial p_i$, we obtain

$$\frac{\partial \lambda}{\partial p_i} = -\frac{\xi(\lambda p_i) + p_i \xi'(\lambda p_i)\lambda}{\sum_{j=1}^{n} p_j^2 \xi'(\lambda p_j)}. \tag{25}$$

Since the first-order condition is still given by (13), we substitute (25) into (13). After symmetrization, we then get the candidate equilibrium markup:

$$\bar{m}^B = -\frac{\xi(\lambda p)}{\xi'(\lambda p)\lambda p \left(1 - \frac{1}{n} + r_u(\bar{x})\right)} = \frac{n}{n - 1 + r_u\left(\frac{1}{cn}\right)} r_u\left(\frac{1}{cn}\right) < 1. \tag{26}$$

Hence, the following result holds true.

Proposition 4. *Assume that firms are income-takers. If* (17) *holds and if a symmetric equilibrium exists under Bertrand competition, then the equilibrium markup is given by*

$$\bar{m}^B(n) = \frac{n}{n - 1 + r_u\left(\frac{1}{cn}\right)} r_u\left(\frac{1}{cn}\right).$$

Note that $\bar{m}^B(n) < 1$ when $r_u < 1$. In Appendix, we show that an equilibrium exists under the CES (Lemma 3). Therefore, the class of additive preferences for which Proposition 4 holds is non-empty. Furthermore, it can be shown that, for each firm, the second-order condition holds in a neighborhood of the symmetric market outcome given by (26),[2] and thus our solution is always a local Bertrand equilibrium.

Using (26) yields the equilibrium price

$$\bar{p}^B = c\frac{n-1+r_u\left(\frac{1}{cn}\right)}{(n-1)(1-r_u\left(\frac{1}{cn}\right))}$$

which need not decrease with n even when r_u is increasing. Moreover, the equilibrium income is given by

$$\bar{y}^B = \frac{n-1+r_u\left(\frac{1}{cn}\right)}{(n-1)(1-r_u\left(\frac{1}{cn}\right))}.$$

Using (10) thus yields the equilibrium value of the marginal utility of income.

5 Comparing Cournot and Bertrand

Using (22) and (26), we have the following proposition.

Proposition 5. *Assume that firms are income-takers. Then, the equilibrium markups are such that*

$$\bar{m}^C(n) > \bar{m}^B(n).$$

Furthermore, we have:

$$\lim_{n\to\infty} \bar{m}^C(n) = \lim_{n\to\infty} \bar{m}^B(n) = r_u(0).$$

Thus, when the number of income-taking firms is given and the same, Cournot competition always generates a higher markup than Bertrand competition. This reflects the folk wisdom according to which Cournot competition is "softer" than Bertrand competition (Vives, 1985, 1999). Furthermore, as the number of competitors gets very large, both types of oligopolistic competition delivers very close market outcomes.

Whether the limit of Cournot and/or Bertrand competition is perfect competition (firms price at marginal cost) or monopolistic competition (firms price above marginal cost) when n is arbitrarily large depends on the value of $r_u(0)$. More precisely, when $r_u(0) > 0$, a very large number of firms whose size is small relative to the market size is consistent with a positive markup. This agrees with Chamberlin (1933). On the contrary, when $r_u(0) = 0$, a growing number of firms always leads to the perfectly competitive outcome, as maintained by Robinson (1934). To illustrate, consider the CARA utility given by $u(x) = 1 - \exp(-\alpha x)$. In this case, we have $r_u(0) = 0$, and thus the CARA model of monopolistic competition is not the limit of a large group of firms. By

[2] The proof can be obtained from the authors upon request.

contrast, under CES preferences, $r_u(0) = 1 - \rho > 0$. Therefore, the CES model of monopolistic competition is the limit of a large group of firms.

Using (9) and Proposition 5 yields

$$\lim_{n \to \infty} \bar{y}^B(n) = \lim_{n \to \infty} \bar{y}^C(n) = \frac{1}{1 - r_u(0)} > 1.$$

Therefore, when the number of firms becomes arbitrarily large, total profits are given by

$$\lim_{n \to \infty} n\bar{\Pi}(n) = \lim_{n \to \infty} \bar{y} - 1 = \frac{r_u(0)}{1 - r_u(0)}.$$

In words, because markups need not tend to zero when n goes to infinity, *total profits do not necessarily vanish when the supply side of the market involves a great many firms*. More precisely, total profits are positive if and only if $r_u(0) > 0$.

To be complete, it remains to discuss Bertrand competition with and without the Ford effect. Comparing $\bar{p}^{BF}$ and $\bar{p}^B$ reveals that the market price is higher when firms take the Ford effect into account than when firms are income-takers. Firms' profits are higher in the former case than in the latter. Since profits are redistributed to consumers, the demand functions (11) are shifted upward when firms account for the Ford effect, thus allowing them to sell the same amount of their varieties at a higher price, thus giving rise to a higher total income in the economy.

Conclusion:TBD

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Appendix

Lemma 1. Assume that $r_{u'}(x) < 2$. If firms are income-takers, then $\Pi_i^C(x_i, \mathbf{x}_{-i})$ is strictly concave in x_i.

Proof. Setting $S \equiv \sum_{j=1}^n u'(x_j)x_j$, the first-order condition for profit maximization is given by

$$y^C \frac{u'(x_i) + u''(x_i)x_i}{S^2} \left[S - u'(x_i)x_i\right] - c = 0. \tag{A.1}$$

Differentiating S twice,

$$\frac{\partial^2 S}{\partial x_i^2} = 2u''(x_i) + u'''(x_i)x_i = u''(x_i)\left(2 - r_{u'}(x_i)\right) < 0$$

because $r_{u'}(x) < 2$. Therefore, we have

$$\frac{\partial^2 \Pi_i^C}{\partial x_i^2} = y^C \frac{S - u'(x_i)x_i}{S^2} \left[\frac{\partial^2 S}{\partial x_i^2} - \frac{2}{S}\left(\frac{\partial S}{\partial x_i}\right)^2\right] < 0$$

because $S - u'(x_i)x_i > 0$, which means that Π_i^C is strictly concave in x_i. Q.E.D.

Lemma 2. If $r_{u'}(x) < 2$, then there exists no asymetric Cournot equilibrium when firms are income-takers.

Proof. Assume that the exists a Cournot equilibrium such that $x_i > x_j$. It follows from (A.1) that

$$\frac{u'(x_i)+u''(x_i)x_i}{u'(x_j)+u''(x_j)x_j} = \frac{S-u'(x_j)x_j}{S-u'(x_i)x_i}. \tag{A.2}$$

Since $r_{u'}(x) < 2$, the function $u'(x)+u''(x)x$ is decreasing. As a result, the LHS of (A.2) is smaller than 1. Furthermore, it follows from $r_u(x) < 1$ that $u'(x)x$ is increasing. Therefore, the RHS of (A.2) is larger than 1, a contradiction. Q.E.D.

Lemma 3. Under the CES, Π_i^B is strictly concave in p_i.

Proof. The statement is proven if the second derivative of Π_i^B with respect to p_i is negative:

$$2\frac{\partial x_i}{\partial p_i} + (p_i - c)\frac{\partial^2 x_i}{\partial p_i^2} < 0 \iff -2\frac{p_i}{x_i}\frac{\partial x_i}{\partial p_i} + \frac{p_i}{\frac{\partial x_i}{\partial p_i}}\frac{\partial}{\partial p_i}\left(\frac{\partial x_i}{\partial p_i}\right) = 2\varepsilon_i + \eta_i > 0$$

where

$$\varepsilon_i = -\frac{p_i}{x_i}\frac{\partial x_i}{\partial p_i}$$

is the price elasticity of $x_i(\mathbf{p})$, while

$$\eta_i = \frac{p_i}{\frac{\partial x_i}{\partial p_i}}\frac{\partial}{\partial p_i}\left(\frac{\partial x_i}{\partial p_i}\right)$$

is the elasticity of the derivative of $x_i(p)$.

Under CES preferences, consumers' demand is given by by

$$x_i(\mathbf{p}) = \frac{y^B p_i^{-\sigma}}{P},$$

where $\sigma > 1$ and P is the price index given by

$$P = \sum_{j=1}^{n} p_j^{1-\sigma}.$$

It is then readily verified that

$$\varepsilon_i = \frac{\sigma P + (1-\sigma)p_i^{1-\sigma}}{P},$$

$$\eta_i = -\frac{(\sigma+1)\sigma P^2 - 3(\sigma-1)P p_i^{1-\sigma} + 2(\sigma-1)^2 p_i^{2(1-\sigma)}}{(\sigma P + (1-\sigma)p_i^{1-\sigma})P},$$

which implies

$$2\varepsilon_i + \eta_i = \frac{\sigma(\sigma-1)\left(\sum_{j\neq i} p_j^{1-\sigma}\right)}{\sigma\left(\sum_{j\neq i} p_j^{1-\sigma}\right) + p_i^{1-\sigma}} > 0.$$

Q.E.D.