Fertility, Retirement Age, and PAYG Pensions

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ABSTRACT

This paper develops an overlapping generations (OLG) model with exogenous and endogenous retirement age to examine the effects of fertility on long-run pay-as-you-go (PAYG) pensions. We find that in both cases, pensions may not necessarily increase with the fertility rate. In the case with exogenous retirement age, an increase in the fertility rate (retirement age) may raise pensions when the output elasticity of capital is low. When the output elasticity of capital is high, an increase in the fertility rate (retirement age) will reduce (raise) pensions if the tax rate is sufficiently high. In the case with endogenous retirement age, a higher fertility rate will reduce pensions if the fertility rate is sufficiently high, but such a change will raise pensions if the output elasticity of capital and the tax rate are sufficiently low. Our results indicate that raising the fertility rate is more likely to reduce pensions in developing countries than in developed countries, while such a change tends to raise pensions for countries with sufficiently low output elasticity of capital and tax rate.

Keywords: Fertility; Retirement; OLG, PAYG pensions.

JEL Classification: H55, J13, J26.

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1. INTRODUCTION

Over the past century, many developed countries have witnessed steady declines in their fertility rate and mortality rate, leading to concerns about the sustainability of social security systems, particularly the pay-as-you-go (PAYG) pension system. There is a huge amount of literature devoted to possible solutions to the problem of the increasing pension burden caused by aging populations. One possible solution is to reform the pension system from unfunded schemes to funded schemes. Since such a reform would cause a dramatic effect on individuals during the transition,[1] the provision for child allowances and the postponement of retirement have become popular governmental policies in recent decades, with the common belief that raising the fertility rate or delaying retirement would lead to a situation where more workers support fewer old people, thus reducing the burden placed upon government budgets. The provision for child allowances is quite common nowadays in many developed countries such as Australia, Japan and Sweden. The provision of child allowances reduces the cost of raising children, and parents then have stronger incentives to have more children. With more workers to support old people, the burden placed upon government budgets will be reduced. The effects of child allowances on fertility and pensions are examined by van Groezen, Leers and Meijdam (2003) based on an overlapping generations (OLG) model with a PAYG pension system and endogenous fertility decision.

However, the consensus that an increase in the fertility rate leads to a positive impact on pensions has been questioned by Cigno (2007), who argues that "The combined effect of fewer births, longer lives and sluggish retirement age is putting public pension system, all essentially pay-as-you-go, under increasing strain." Recently, Fanti and Gori (2012) analytically examine the effect of fertility on pensions based on a model with a PAYG pension system and exogenous fertility rate and find that pensions do not necessarily increase with the fertility rate. Although increasing fertility raises the labor force and generates a positive effect on pensions, it also causes a negative effect due to the general equilibrium feedback of the wage rate.

Previous studies concerning the effects of fertility on pensions tend to ignore the role of retirement which is also an important determinant to pensions. The postponement of retirement means that workers work longer and benefit from pensions for a shorter length of time. By assuming that the population is constant, Miyazaki (2014a) and in Miyazaki (2014b) respectively examine the influences of retirement on pensions in a model with exogenous and endogenous retirement age.[2] Thus, in prior pension crisis

[1] See Zhang and Zhang (2003) for a comparison of unfunded and funded social security systems.

[2] Michel and Pestieau (2013) also consider a model with exogenous and endogenous retirement age. Unlike this paper which focuses on the interaction between fertility and retirement age and how this interaction affects pensions, both Michel and Pestieau (2013) and Miyazaki (2014b) focus on the retirement age and optimal PAYG social security policy. Thus, fertility is not considered in their studies.

studies there has been a tendency to study the effects of fertility and retirement time on pensions separately, thereby ignoring the interaction between fertility and retirement.

This paper develops an OLG model with retirement to examine the effect of fertility on long-run PAYG pensions. Two types of retirement age are considered: exogenous and endogenous retirement age. Since our focus is the influence of fertility on pensions, we follow Fanti and Gori (2012) and assume that the fertility rate is exogenously given. Workers are composed of adults and old agents. Adults need to work in order to consume and to raise children. Adults supply labor inelastically while old agents need to work for a fraction of time in old age. After they retire, they enjoy pension benefits. Government implements the PAYG social security system. All workers need to pay income tax to support the pensions.

In the first case with exogenous retirement age, since we do not distinguish between retirement age and official pension age and assume that old agents are eligible for pensions after they retire as regulated by law, our model allows us to study the effects of an increase in the official pension age on pensions. The postponement of official pension age is also a commonly used policy to solve the problem of pension crisis. For examine, in Australia, men and women currently aged 65 are eligible for the age pension. The Australian government announced in 2009 that the Age Pension age will start increasing from 2017 and reach 76 years of age by 2023.

We find that increasing the fertility rate or official pension age does not necessarily raise pensions. Such a change in fertility or official pension age causes a positive direct effect due to more labor contributing to pensions and a negative effect due to decreases in the equilibrium wage and savings. Our results indicate that the output elasticity of capital plays an important role in determining the effects of fertility and official pension age on pension benefits since it governs the general equilibrium feedback of the wage rate. Besides the output elasticity of capital, other important determinants include the level of the fertility rate, the costs of rearing children and the tax rate. When the output elasticity of capital is low, an increase in the fertility rate will reduce pensions if the fertility rate (or the cost of rearing children) is sufficiently high and vice versa. Similar results also can be found when the output elasticity of capital is high and the tax rate is low. However, when both the output elasticity of capital and the tax rate are high, an increase in the fertility rate will cause a negative effect on pensions. Regarding the effects of official pension age, we find that when the output elasticity of capital is low, there will be a positive relationship between official pension age and pensions. When the output elasticity of capital is high, an increase in official pension age will reduce pensions if tax rate is low, but such a change will raise pensions if the tax rate is high.

In the second case with endogenous retirement, old agents can choose how long they want to work; that is, they can choose the time to retire. The endogeneity of retirement age allows the change in fertility to affect retirement and this will in turn affect pensions. We also find that pensions do not necessarily increase with the fertility rate in this case. Although an increase in fertility causes a positive direct effect due to more workers contributing to pensions, it also induces two negative effects due to decreases in the elderly's working time and the equilibrium wage. An increase in the fertility rate (tax rate) reduces the elderly's working time. A higher fertility rate will reduce pensions if the fertility rate is sufficiently high, but such a change will raise pensions if the output elasticity of capital and the tax rate are sufficiently low.

Comparing results in the cases with exogenous and endogenous retirement age, we find that an increase in the fertility rate will tend to reduce pensions if the fertility rate is sufficiently high in both cases. This implies that raising the fertility rate is more likely to reduce pensions in developing countries than in developed countries since developing countries usually have higher fertility rates. Furthermore, in both cases, a positive relationship between fertility and pensions is more likely to happen for countries with sufficiently low output elasticity of capital and tax rate. If costs of rearing children are sufficiently low, an increase in the fertility rate will tend to raise pensions, regardless of the type of retirement.

The remainder of this paper is organized as follows. Section 2 develops a model with exogenous retirement time. The effects of fertility and the official pension age on long-run pension benefits are also examined in this section. Section 3 develops a model with endogenous retirement time and analyzes the effects of fertility on long-run pension benefits. Section 4 concludes.

2. THE MODEL WITH EXOGENOUS RETIREMENT AGE

We consider an overlapping generations model with PAYG pensions. Agents live for three periods - childhood, adulthood (parenthood), and old age. In each period, agents are endowed with one unit of time. Adults with population N_t spend all their time working to earn wages w_t. When they become old, they work at a fraction of time $l_{t+1} \in (0,1)$ to earn wages w_{t+1}. After they retire, they receive pensions provided by the government. Agents care about their consumption in adulthood (c_t) and in old age (c_{t+1}) as well as leisure in old age $(1 - l_{t+1})$. The utility function, which is identical for all agents, is defined as:

$$u_t = \ln c_t + \beta \ln c_{t+1} + +\gamma \ln(1 - l_{t+1}), \qquad (1)$$

where $\beta \in (0,1)$ is the discount factor, and $\gamma \in (0,1)$ is the weight of leisure in the utility function.

All decisions are made in adulthood. Following Fanti and Gori (2012), we assume that the number of children (n) is exogenous. It takes a fixed proportion ($q \in (0,1)$) of each adult's wage to raise a child (see, e.g., Wigger 1999; Boldrin and Jones 2002; Fanti and Gori 2012). In order to pay for pensions for the elderly, the government levies an income tax with the rate $\tau \in (0,1)$. Let s_t represent savings in the adulthood, then the budget constraint for an adult is:

$$c_t + s_t + nqw_t = (1-\tau)w_t. \tag{2}$$

The budget constraint for an old agent is:

$$c_{t+1} = R_{t+1}s_t + (1-\tau)\theta l_{t+1}w_{t+1} + (1-l_{t+1})P_{t+1}, \tag{3}$$

where R_{t+1} is the gross rate of return of savings, P_{t+1} is the pensions per unit of time and $\theta > 0$ is the productivity of an old agent.[3]

The government runs a balanced budget. Given that the tax revenue is used to provide pensions for the old, this implies that:

$$(1-l_{t+1})N_t P_{t+1} = \tau w_{t+1}(N_{t+1} + \theta l_{t+1}N_t). \tag{4}$$

Output is produced by using capital (K_t) and effective labor ($L_t = N_t + \theta l_t N_{t-1}$) and is based on the Cobb-Douglas production function $Y_t = AK_t^{\alpha}L_t^{1-\alpha}$, where $A > 0$ and $\alpha \in (0,1)$ is the output elasticity of capital. We define $k_t = K_t/L_t$. The gross rate of the return on capital and the real wage rate are respectively:

$$R_t = \alpha A k_t^{\alpha-1}, \tag{5}$$

$$w_t = (1-\alpha)Ak_t^{\alpha}. \tag{6}$$

2.1 Equilibrium and steady state

We first consider a model with exogenous retirement age; that is, $l_{t+1} = l$. We do not distinguish between retirement age and official pension age and assume that old agents are eligible for pensions at age $(2 + l)$ as regulated by law. Adults decide how much to consume, and how much they should save for their old age. The optimal decision of s_t is:

$$s_t = \frac{\beta(1-\tau-qn)w_t}{1+\beta} - \frac{(1-\tau)\theta l w_{t+1} + (1-l)P_{t+1}}{(1+\beta)R_{t+1}}. \tag{7}$$

Market clearing in the capital market indicates that $(n + \theta l)k_{t+1} = s_t$. Using (4)-(7), the law of motion of k_t is:

$$k_{t+1} = \frac{\alpha(1-\alpha)A\beta(1-\tau-qn)}{(n+\theta l)\alpha(1+\beta) + (1-\alpha)(\tau n + \theta l)}k_t^{\alpha}. \tag{8}$$

Equation (8) implies that there exists a unique, globally stable steady state k^*:

$$k^* = k^*(n,l) = \left[\frac{\alpha(1-\alpha)A\beta(1-\tau-qn)}{\alpha(1+\beta)(n+\theta l) + (1-\alpha)(\tau n + \theta l)}\right]^{\frac{1}{1-\alpha}}. \tag{9}$$

To ensure $k^* > 0$, we assume that $1 - \tau < qn$.

2.2 Pensions

[3] Sala-i-Martin (1996) argues that old workers are less productive than young workers due to the depreciation of human capital.

Let w^* represent the steady-state wage rate; that is, $w^* = (1-\alpha)(k^*)^\alpha$. Equation (4) implies that the steady-state pensions per efficiency unit of labor (p^*) can be re-written as:

$$p^* = \frac{\tau w^*(n+\theta l)}{1-l} = p^*(n, l, k^*(n, l)). \qquad (10)$$

From (10), we derive:

$$\frac{dp^*}{dx} = \underbrace{\frac{\partial p^*}{\partial x}}_{+} + \underbrace{\underbrace{\frac{\partial p^*}{\partial w^*}}_{+}\underbrace{\frac{\partial w^*}{\partial k^*}}_{+}\underbrace{\frac{\partial k^*}{\partial x}}_{-}}_{-}, \qquad (11)$$

where $x = n, l$. Although a higher fertility rate (retirement age) means more labor contributing to pensions (direct effect), the lower equilibrium wage rate reduces savings and capital accumulation, causing pensions per efficiency unit of labor to decrease (indirect effect). Fertility or retirement age and pensions are positively (negatively) correlated if the direct (indirect) effect dominates.

Combining (4), (6) and (9), we can derive long-run PAYG pensions per efficiency unit of labor as:

$$p^* = \frac{\tau(n+\theta l)}{1-l}\left\{A(1-\alpha)\left[\frac{\alpha\beta(1-\tau-qn)}{\alpha(1+\beta)(n+\theta l)+(1-\alpha)(\tau n+\theta l)}\right]^\alpha\right\}^{\frac{1}{1-\alpha}}. \qquad (12)$$

Proposition 1. (1) For $\alpha \in (0, \frac{1}{2}]$, then $\frac{dp^*}{dn} < 0$ ($\frac{dp^*}{dn} > 0$) if the fertility rate is sufficiently large (small). (2) For $\alpha \in (\frac{1}{2}, 1)$, there exists a unique τ_n such that if $\tau < \tau_n$, then $\frac{dp^*}{dn} < 0$ ($\frac{dp^*}{dn} > 0$) if the fertility rate is sufficiently large (small). If $\tau > \tau_n$, then $\frac{dp^*}{dn} < 0$.

Proof: From (12), we have:

$$\frac{dp^*}{dn} = \frac{\tau[-q\xi_1 + (1-\tau)\xi_2]}{1-l}\left\{\frac{A[(1-\alpha)\alpha\beta]^\alpha(1-\tau-qn)^{2\alpha-1}}{\alpha(1+\beta)(n+\theta l)+(1-\alpha)(\tau n+\theta l)}\right\}^{\frac{1}{1-\alpha}},$$

where

$$\xi_1 = (1-\alpha)n^2[\alpha(1+\beta)+\tau(1-\alpha)] + (n+\alpha\theta l)(1+\alpha\beta)\theta l > 0,$$

$$\xi_2 = (1-2\alpha)n[\alpha(1+\beta)+\tau(1-\alpha)] + (1-\alpha)(1+\alpha\beta)\theta l.$$

Note that $\xi_2 > 0$ if $\alpha \in (0, \frac{1}{2}]$. The sign of $\frac{dp^*}{dn}$ is determined by $[-q\xi_1 + (1-\tau)\xi_2]$. We can re-write $[-q\xi_1 + (1-\tau)\xi_2]$ as:

$$[-q\xi_1 + (1-\tau)\xi_2] = n[\alpha(1+\beta) + \tau(1-\alpha)[-q(1-\alpha)n + (1-\tau)(1-2\alpha)]$$
$$+(1+\alpha\beta)\theta l[-q(n+\alpha\theta l) + (1-\tau)(1-\alpha)].$$

Define $n_1 = \frac{(1-\tau)(1-2\alpha)}{q(1-\alpha)}$ and $n_2 = \frac{(1-\tau)(1-\alpha)-q\alpha\theta l}{q}$. Note that $[-q(1-\alpha)n + (1-\tau)(1-2\alpha)] < 0$ if $n > n_1$ and vice versa. Moreover, $[-q(n+\alpha\theta l) + (1-\tau)(1-\alpha)] < 0$ if $n > n_2$ and vice versa. Then $\frac{dp^*}{dn} < 0$ if the fertility rate is sufficiently large such that $n > max\{n_1, n_2\}$ and $\frac{dp^*}{dn} > 0$ if the fertility rate is sufficiently small such that $n < min\{n_1, n_2\}$.

If $\alpha \in (\frac{1}{2}, 1)$, then $\xi_2 > 0$ if $\tau < \tau_n = \frac{(1-\alpha)(1+\alpha\beta)\theta l-(2\alpha-1)n\alpha(1+\beta)}{(2\alpha-1)n(1-\alpha)}$. This implies that $\frac{dp^*}{dn} < 0$ if the fertility rate is sufficiently large such that $n > max\{n_1, n_2\}$ and $\frac{dp^*}{dn} > 0$ if the fertility rate is sufficiently small such that $n < min\{n_1, n_2\}$. If $\tau > \tau_n$, then $\xi_2 < 0$, implying $\frac{dp^*}{dn} < 0$.

QED.

Recall that an increase in fertility generates two opposite effects on p^*. When the output elasticity of capital is lower than $\frac{1}{2}$, the effect of fertility on pensions per efficiency unit of labor depends on the level of fertility rate. When the fertility rate is sufficiently large, the contribution of more labor (due to an increase in the fertility rate) to p^* is small. Therefore, the negative effect caused by an increase in the fertility rate on pensions outweighs its positive effect, causing a reduction in p^*. This result will be reversed if the fertility rate is sufficiently small. If the fertility rate is between n_1 and n_2, then we need to consider the costs of rearing children (q) in order to determine the sign of $\frac{dp^*}{dn}$. Defining $q_n = \frac{(1-\tau)\xi_2}{\xi_1}$, then $[-q\xi_1 + (1-\tau)\xi_2] < 0$ if $q > q_n$ and vice versa. This implies that there exists a unique $q_n > 0$ such that $\frac{dp^*}{dn} < 0$ if $q > q_n$ and vice versa. The reason is that the higher costs of rearing children induce lower savings and capital accumulation and result in a negative effect of n on p^*.

When $\alpha > \frac{1}{2}$, the effect of fertility on pensions per efficiency unit of labor depends on the tax rate. On the one hand, a higher tax rate directly increases contributions to p^*. On the other hand, it reduces p^* indirectly due to lower after-tax income and savings. We find that if the tax rate is sufficiently low such that $\tau < \tau_n$, then the effect of an increase in fertility on p^* is similar to the case of $\alpha \leq \frac{1}{2}$ and that an increase in the fertility rate may increase or decrease p^*, depending on the values of n and q. However, if the tax rate is sufficiently high such that $\tau > \tau_n$, then an increase in fertility reduces p^*, regardless of the values of n and q. Notice that $\frac{\partial \tau_n}{\partial l} > 0$ and $\frac{\partial \tau_n}{\partial \theta} > 0$, impliying that earlier retirement age and lower productivity of the old decreases the critical value of the tax rate (τ_n). Since developing countries tend to have higher fertility rates, a higher value of α and a lower value of τ_n (due to early retirement age and low productivity of old workers), then an increase in fertility is very likely to reduce p^* in these countries.

The following proposition concerns the effect of retirement age (official pension age) on pensions.

Proposition 2. There exists a unique $\alpha_l \in (0,1)$ such that: (1) for $\alpha \in (0, \alpha_l]$, then $\frac{dp^*}{dl} > 0$; (2) for $\alpha \in (\alpha_l, 1)$, then there exists a unique τ_l such that $\frac{dp^*}{dl} < 0$ if $\tau < \tau_l$ and vice versa.

Proof: From (12), we have:

$$\frac{dp^*}{dl} = \frac{\tau(\xi_3 + \xi_4)}{(1-l)^2(1-\alpha)} \left[\frac{A(1-\alpha)[\alpha\beta(1-\tau-qn)]^{\alpha}}{\alpha(1+\beta)(n+\theta l) + (1-\alpha)(\tau n + \theta l)}\right]^{\frac{1}{1-\alpha}},$$

where

$$\xi_3 = (1-\alpha)n(\theta + n)[\alpha(1+\beta) + \tau(1-\alpha)] > 0,$$

$$\xi_4 = (1+\alpha\beta)\theta\{l[\theta(1+l) + n] - \alpha(2\theta l + n)\}.$$

Define $\alpha_l = \frac{l[\theta(1+l)+n]}{2\theta l + n}$. Note that $0 < \alpha_l < 1$. If $\alpha \leq \alpha_l$, then $\xi_4 \geq 0$, implying $\frac{dp^*}{dl} > 0$.

If $\alpha \in (\alpha_l, 1)$, then $\xi_4 < 0$. Define $\tau_l = \frac{(1+\alpha\beta)\theta\{-l[\theta(1+l)+n]+\alpha(2\theta l+n)\} - (1-\alpha)n(\theta+n)\alpha(1+\beta)}{n(\theta+n)(1-\alpha)^2}$.

Thus, $\frac{dp^*}{dl} < 0$ if $\tau < \tau_l$ and vice versa.

QED.

Because the lower the output elasticity of capital is, the smaller the reduction in wages caused by the postponement of retirement age (official pension age), then $\frac{dp^*}{dl} > 0$ if $\alpha < \alpha_l$. If $\alpha > \alpha_l$, then the postponement of retirement age will cause a larger reduction in wages and the results will depend on τ. Our results are different from Miyazaki (2014a) who finds that p^* will increase in l if

l is sufficiently high, regardless of the value of α. Note that α_l depends on n, l, and θ with $\frac{\partial \alpha_l}{\partial n} < 0$, $\frac{\partial \alpha_l}{\partial l} > 0$, and $\frac{\partial \alpha_l}{\partial \theta} > 0$. Since the critical value α_l tends to be higher in developed countries due to lower fertility, higher retirement age and higher productivity of old workers, it is very likely that postponing the official pension age would increase p^* in developed countries.

3. THE MODEL WITH ENDOGENOUS RETIREMENT

In this section, we consider a model where retirement age is endogenously determined by agents. We now allow agents to choose to work at a fraction of time $l_{t+1} \in (0,1)$ to earn wages w_{t+1} in the old age. After they retire, they receive pensions provided by the government. The budget constraint for an adult is the same as (2) while the budget constraint for an old agent is the same as (3). The government budget is given by (4). Factor prices are given by (5) and (6).

3.1 Equilibrium and steady state

The optimal decisions of c_t and l_{t+1} are:

$$c_t = \frac{(1-\tau-qn)w_t}{1+\beta} + \frac{(1-\tau)\theta l_{t+1} w_{t+1} + (1-l_{t+1})P_{t+1}}{(1+\beta)R_{t+1}}, \tag{13}$$

$$l_{t+1} = \frac{\theta\beta(1-\tau)(1-\alpha) - [\alpha\gamma + \tau(1-\alpha)(\beta+\gamma)]n}{\theta[(1-\alpha)\beta+\gamma]}. \tag{14}$$

Equation (14) indicates that working time for the elderly is constant; that is, $l_{t+1} = \tilde{l}$. Note that $\tilde{l} < 1$. In order to guarantee that $\tilde{l} > 0$, we assume that $\theta > \frac{[\alpha\gamma+\tau(1-\alpha)(\beta+\gamma)]n}{\beta(1-\tau)(1-\alpha)}$. From (14), we have:

$$\frac{d\tilde{l}}{dn} = -\frac{\alpha\gamma + \tau(1-\alpha)(\beta+\gamma)}{\theta[(1-\alpha)\beta+\gamma]} < 0. \tag{15}$$

$$\frac{d\tilde{l}}{d\tau} = -\frac{(1-\alpha)[\theta\beta + (\beta+\gamma)n]}{\theta[(1-\alpha)\beta+\gamma]} < 0. \tag{16}$$

Equation (15) indicates that an increase in fertility reduces working time in old age. On the one hand, the increase in the fertility rate will result, *ceteris paribus*, in a higher expenditure on raising children and lower savings. This will motivate old agents to work longer. On the other hand, the increase in the fertility rate means higher pensions due to more workers contributing to pension funds, which will cause old agents to retire earlier. As the fertility rate increases, the second effect will dominate the first effect and the overall working time for the elderly will decrease. Concerning the effect of the tax rate on the elderly's working time, we find that although an increase in the tax rate lowers the motivation to work and reduces the working time for the elderly, the lower after-tax income induces the elderly to work longer. Equation (16) shows that an increase in the tax rate makes old agents retire earlier.

Using (2), (4)-(6), (13) and (14), we can derive savings as:

$$s_t = \frac{1-\alpha}{1+\beta+\gamma}\left[(\beta+\gamma)(1-\tau-qn)Ak_t^{\alpha} - \frac{(1-\tau)\theta}{\alpha}k_{t+1}\right]. \tag{17}$$

Market clearing in the capital market indicates that $(n+\theta l_{t+1})k_{t+1} = s_t$. This implies the following law of motion of k_t:

$$k_{t+1} = \frac{\alpha A(1-\tau-qn)[(1-\alpha)\beta+\gamma]}{(1-\tau)[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)]}k_t^{\alpha}. \tag{18}$$

Equation (18) implies that there exists a unique, globally stable-steady state k^*:

$$k^* = \left[\frac{\alpha A(1-\tau-qn)[(1-\alpha)\beta+\gamma]}{(1-\tau)[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)]}\right]^{\frac{1}{1-\alpha}}. \tag{19}$$

Under the assumption that $1-\tau < qn$, we ensure $k^* > 0$. Differentiating (19) with respect to the fertility rate yields:

$$\frac{dk^*}{dn} = -\frac{[\alpha(1-\tau)(1+\beta+\gamma)+q\theta(1+\alpha\beta)]k^*}{(1-\alpha)(1-\tau-qn)[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)]} < 0. \tag{20}$$

Equation (20) indicates that an increase in the fertility rate reduces savings and the accumulation of capital, resulting in a lower k^*.

3.2 Pensions

Equation (4) implies that the steady-state pensions per efficiency unit of labor can be re-written as:

$$p^* = \frac{\tau w^*(n+\theta\tilde{l})}{1-\tilde{l}}. \tag{21}$$

Substituting (14) and (19) into (21) yields:

$$p^* = p^*(n, \tilde{l}(n), k^*(n)). \tag{22}$$

Differentiating (22) with respect to n gives:

$$\frac{dp^*}{dn} = \underbrace{\frac{\partial p^*}{\partial n}}_{+} + \underbrace{\underbrace{\frac{\partial p^*}{\partial \tilde{l}}}_{+}\underbrace{\frac{\partial \tilde{l}}{\partial n}}_{-}}_{-} + \underbrace{\underbrace{\frac{\partial p^*}{\partial w^*}}_{+}\underbrace{\frac{\partial w^*}{\partial k^*}}_{+}\underbrace{\frac{\partial k^*}{\partial n}}_{-}}_{-}. \tag{23}$$

Equation (23) shows that an increase in the fertility rate will cause three effects on pensions: one positive direct effect and two negative indirect effects. First, a higher fertility rate means that more workers contribute to p^* (direct effect). Second, the higher fertility rate causes old agents to retire earlier and reduces p^* (indirect effect). Third, the lower equilibrium wage rate caused by the increasing fertility rate reduces p^* (indirect effect). Depending on the magnitude of these effects, the fertility rate may increase or decrease p^*. The following proposition provides a sufficient condition for $\frac{dp^*}{dn} < 0$.

Proposition 3. An increase in the fertility rate will reduce pensions per efficiency unit of labor if the fertility rate is sufficiently large.

Proof: Substituting (15) and (20) into (23) yields:

$$\frac{dp^*}{dn} = \frac{\tau w^*[\alpha(1+\beta+\gamma)\xi_3 + \theta(1+\alpha\beta)\xi_4]}{(1-l)^2(1-\alpha)(1-\tau-qn)[\alpha n(1+\beta+\gamma)+\theta(1+\alpha\beta)][(1-\alpha)\beta+\gamma]}, \quad (24)$$

where

$$\xi_3 = (1-\tau)\{-(1-\tilde{l})\alpha(n+\theta\tilde{l})[(1-\alpha)\beta+\gamma] + (1-\alpha)^2\gamma(1-\tau-qn)n\},$$

and

$$\xi_4 = -(1-\tilde{l})\alpha q(n+\theta\tilde{l})[(1-\alpha)\beta+\gamma] + (1-\alpha)^2(1-\tau)\gamma(1-\tau-qn).$$

Define $\chi = \tau[(1-\alpha)\beta+\gamma] + (1-\tau)\alpha\gamma > 0$. Using (14) to substitute $\tilde{l}$ in ξ_3 and ξ_4, we have:

$$\xi_3 = (1-\tau)(1-\alpha)\left\{\frac{-\theta\{[(1-\alpha)\tau\beta+\gamma]+\chi n\}\alpha(1-\tau)[n(\beta+\gamma)+\theta\beta]}{\theta[(1-\alpha)\beta+\gamma]} + \gamma(1-\alpha)n(1-\tau-qn)\right\},$$

and

$$\xi_4 = (1-\tau)(1-\alpha)\left\{\frac{-\{[(1-\alpha)\tau\beta+\gamma]+\chi n\}\alpha q[n(\beta+\gamma)+\theta\beta]}{(1-\alpha)\beta+\gamma} + (1-\alpha)\gamma(1-\tau-qn)\right\}.$$

We re-write ξ_3 as:

$$\xi_3 = (1-\tau)(1-\alpha)\left\{n\left[\gamma(1-\alpha)(1-\tau-qn) - \frac{\chi\alpha(1-\tau)[n(\beta+\gamma)+\theta\beta]}{\theta[(1-\alpha)\beta+\gamma]}\right] - \frac{\theta[(1-\alpha)\tau\beta+\gamma]\alpha(1-\tau)[n(\beta+\gamma)+\theta\beta]}{\theta[(1-\alpha)\beta+\gamma]}\right\}.$$

Therefore, $\xi_3 < 0$ if $\gamma(1-\alpha)(1-\tau-qn) < \frac{\chi\alpha(1-\tau)[n(\beta+\gamma)+\theta\beta]}{\theta[(1-\alpha)\beta+\gamma]}$. This implies that $\xi_3 < 0$ if

$$n > n_3 = \frac{\gamma(1-\tau)\{(1-\alpha)\theta[(1-\alpha)\beta+\gamma] - \chi\alpha\theta\beta\}}{\gamma(1-\alpha)q\theta[(1-\alpha)\beta+\gamma] + \chi\alpha(1-\tau)(\beta+\gamma)}.$$

Note that

$$\xi_4 < (1-\tau)(1-\alpha)\left\{\frac{-[(1-\alpha)\tau\beta+\gamma]\alpha q[n(\beta+\gamma)+\theta\beta]}{(1-\alpha)\beta+\gamma} + (1-\alpha)\gamma(1-\tau-qn)\right\}.$$

Therefore, $\xi_4 < 0$ if $\frac{-[(1-\alpha)\tau\beta+\gamma]\alpha q[n(\beta+\gamma)+\theta\beta]}{(1-\alpha)\beta+\gamma} + (1-\alpha)\gamma(1-\tau-qn) < 0$. This implies that $\xi_4 < 0$ if

$$n > n_4 = \frac{(1-\alpha)\gamma(1-\tau)[(1-\alpha)\beta+\gamma] - [(1-\alpha)\tau\beta+\gamma]\alpha q\theta\beta}{q\{\beta(1-\alpha)\gamma\theta[(1-\alpha)\beta+\gamma] + [(1-\alpha)\tau\beta+\gamma]\alpha(\beta+\gamma)\}}.$$

Therefore, if n is sufficiently large such that $n > max\{n_3, n_4\}$, then $\xi_3 < 0$ and $\xi_4 < 0$. From (24), we have $\frac{dp^*}{dn} < 0$ if n is sufficiently large such that such that $n > max\{n_3, n_4\}$.

QED.

Recall that an increase in fertility generates three effects on p^*. When the fertility rate is high, an increase in fertility will induce a small positive direct effect on p^*. However, it will cause large reductions in the elderly's working time and wages, resulting in a large decrease in p^*. Proposition 3 demonstrates that if the fertility rate is sufficiently large, then an increase in the fertility rate will reduce p^*. Comparing Proposition 3 with Proposition 1, we find that an increase in the fertility rate is very likely to reduce p^* if the fertility rate is sufficiently large, regardless how retirement age is determined. Because developing countries usually have higher fertility rates, an increase in the fertility rate is more likely to reduce p^* in these countries than in developed countries.

The following proposition provides a sufficient condition for $\frac{dp^*}{dn} > 0$. In particular, we concern how the tax rate affects the influence of the fertility rate on pensions.

Proposition 4. An increase in the fertility rate will raise pensions per efficiency unit of labor if the output elasticity of capital and the tax rate are sufficiently low.

Proof: Since $0 < \tilde{l} < 1$, we have:

$$\xi_3 > (1-\tau)\left\{-\alpha\left(n+\theta\tilde{l}\right)[(1-\alpha)\beta+\gamma] + (1-\alpha)^2\gamma(1-\tau-qn)n\right\},$$

and

$$\xi_4 > -\alpha q\left(n+\theta\tilde{l}\right)[(1-\alpha)\beta+\gamma] + (1-\alpha)^2(1-\tau)\gamma(1-\tau-qn).$$

Using (14) to substitute $\tilde{l}$ in ξ_3 and ξ_4, we have:

$$\begin{aligned}
&\alpha(1+\beta+\gamma)\xi_3 + \theta(1+\alpha\beta)\xi_4 \\
&> (1-\tau)(1-\alpha)\{-\alpha[\alpha(1-\tau)(1+\beta+\gamma) + \theta q(1+\alpha\beta)][n(\beta+\gamma)+\theta\beta] \\
&\quad +(1-\alpha)\gamma(1-\tau-qn)[\alpha n(1+\beta+\gamma) + \theta(1+\alpha\beta)]\} \\
&= (1-\tau)(1-\alpha)(-\tau\xi_5 + \xi_6),
\end{aligned}$$

where

$$\xi_5 = (1-\alpha)\gamma[\alpha n(1+\beta+\gamma) + \theta(1+\alpha\beta)] - \alpha^2(1+\beta+\gamma)[n(\beta+\gamma)+\theta\beta],$$

and

$$\begin{aligned}
\xi_6 = {} & (1-\alpha)\gamma(1-qn)[\alpha n(1+\beta+\gamma) + \theta(1+\alpha\beta)] \\
& -\alpha[\alpha(1+\beta+\gamma) + \theta q(1+\alpha\beta)][n(\beta+\gamma)+\theta\beta].
\end{aligned}$$

Note that $\xi_5 > \xi_6$.

We can re-write ξ_6 as:

$$\xi_6 = \alpha(1+\beta+\gamma)\{(1-\alpha)\gamma(1-qn)n - \alpha[n(\beta+\gamma)+\theta\beta]\}$$
$$+\theta(1+\alpha\beta)\{(1-\alpha)\gamma(1-qn) - \alpha q[n(\beta+\gamma)+\theta\beta]\}.$$

Thus, $\xi_6 > 0$ if $(1-\alpha)\gamma(1-qn)n > \alpha[n(\beta+\gamma)+\theta\beta]$ and $(1-\alpha)\gamma(1-qn) > \alpha q[n(\beta+\gamma)+\theta\beta]$. Define $\alpha_1 = \frac{\gamma(1-qn)n}{\gamma(1-qn)n+n(\beta+\gamma)+\theta\beta} < 1$ and $\alpha_2 = \frac{\gamma(1-qn)}{\gamma(1-qn)+q[n(\beta+\gamma)+\theta\beta]} < 1$. Then $\xi_6 > 0$ if α is sufficiently small such that $\alpha < min(\alpha_1, \alpha_2)$. Since $\xi_5 > \xi_6$, then $\xi_5 > 0$ if $\xi_6 > 0$. It also implies that $\frac{\xi_6}{\xi_5} < 1$. Thus, $\frac{dp^*}{dn} > 0$ if $\alpha < min(\alpha_1, \alpha_2)$ and $\tau < \frac{\xi_6}{\xi_5}$.

QED.

When the output elasticity of capital is low, an increase in the fertility rate causes a small reduction in wages, generating a small reduction in p^*. Concerning the effect of the tax rate, although a lower tax rate directly decreases contributions to p^*, such a change also induces two indirect effects on p^*. First, it raises p^* due to higher after-tax income and savings. Second, as indicated by (16), a lower tax rate makes old agents delay their retirement, leading to an increase in p^* due to more contributions to pensions. We find that if the output elasticity of capital and the tax rate are sufficiently low, then an increase in the fertility rate causes an overall increase in p^*. Notice that $\frac{\partial\tau_n}{\partial q} < 0$. Since a lower q implies a higher threshold τ_n, then for those countries with low costs of raising children, an increase in the fertility rate tends to raise p^*. Proposition 1 demonstrates that when retirement age is exogenously given, then $\frac{dp^*}{dn} < 0$ if the output elasticity of capital and the tax rate are sufficiently high. Then a comparison between Proposition 4 and Proposition 1 indicates that regardless of the type of retirement, a positively relationship between the fertility rate and p^* is more likely to be found in an economy with sufficiently low output elasticity of capital and tax rate. Furthermore, if costs of rearing children are sufficiently low, it is very likely that higher fertility rate will generate higher p^*, regardless of the type of retirement .

4. CONCLUSIONS

In this paper we examine the effect of the fertility rate on PAYG pensions based on an overlapping generations model with exogenous and endogenous retirement age. Although most people believe that increasing fertility (or delaying retirement age) would raise pensions, our results suggest that governments should also take economic variables such as the level of the fertility rate, the costs of rearing children, the output elasticity of capital, and the tax rate into consideration when looking into the problem of a pension crisis.

REFERENCES

Boldrin, M., Jones, L.E., 2002. Mortality, fertility, and saving in a Malthusian economy. Review of Economic Dynamics 5, 775-814.

Cigno, A., 2007. Low fertility in Europe: is the pension system the victim or the culprit? In: Europe and the demographic challenge. CESio Forum 8, 7-11.

Fanti, L., Gori, L., 2012. Fertility and PAYG pensions in the overlapping generations model. Journal of Population Economics 25, 955-961.

Michel, P., Pestieau, P., 2013. Social security and early retirement in an overlapping-generations growth model. Annals of Economics and Finance 14, 705-719.

Miyazaki, K., 2014a. The effects of the raising-the-official-pension-age policy in an overlapping generations economy. Economics Letters 123, 329-332.

Miyazaki, K., 2014b. Optimal pay-as-you-go social security when retirement is endogenous and labor productivity depreciates. Mimeo, Kagawa University.

Sala-i-Martin, X. X., 1996. A positive theory of social security. Journal of Economic Growth 1, 277-304.

van Groezen, B., Leers, T., Meijdam, L., 2003. Social security and endogenous fertility: pensions and child allowances as siamese twins. Journal of Public Economics 87, 233-251.

Wigger, B.U., 1999. Pay-as-you-go financed public pensions in a model of endogenous growth and fertility. Journal of Population Economics 12, 625-640.

Zhang, J., Zhang, J., 2003. Long-run effects of unfunded social security with earnings-dependent benefits. Journal of Economic Dynamics and Control 28, 617-641.