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3 March 2016

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March, 2016

Abstract

We analyze aggregate stability of a monetary economy with an interest-rate control type of monetary policy and endogenous consumption tax rate under balanced-budget rule, in terms of equilibrium determinacy. We find the effect of the response to income in monetary policy on macroeconomic stability depends on whether the consumption tax rate is adequately high.

Keywords: aggregate stability, endogenous consumption tax rate, Taylor rule.

JEL Classification Numbers: E52, E62.

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1 Introduction

We investigate the effect of mixture of fiscal and monetary policies on macroeconomic stability in terms of equilibrium determinacy, which implies that one stable equilibrium path is determined.

In this study, we use the basic monetary model in Xue and Yip (2013), who assume the constant-return production technology by capital and labor; the cash-in-advance (CIA) constraint whereby money balances bind to the expenditures of consumption including tax; and the endogenous rate of consumption tax to satisfy a government's budget. Instead of the constant growth rate of nominal monetary stocks, we suppose a Taylor-type monetary policy rule which implies the positive response of a nominal interest rate to inflation and to income as suggested by Taylor (1993). A standard result is said to adhere to the "Taylor principle", which suggest that an aggressive response of a nominal interest rate to inflation tends to generate a unique path to a determinate equilibrium that is unaffected by expectation, thereby rendering the macroeconomy stable.

Aggregate stability under taxation is often discussed in view of the externality of government expenditure in utility or production; this argument is evident in studies by Guo and Harrison (2004, 2008), Hori and Maebayashi (2013), and Kamiguchi and Tamai (2011). One of the main results of these papers is that consumption tax is beneficial for macroeconomic stability since it may depress the externality of government expenditure [1]. Our purpose is to examine the result using the model with monetary aspect. [2]

Xue and Yip (2013) demonstrate that equilibrium may not be determinate even if utility is additively separable between consumption and labor. We reconsider the condition of aggregate stability under another form of monetary policy rule. The main conclusions are similar, but the degree of policy response to economic variables may be more significant in our model since the results are more complicated. In particular, we show that the effect of the response to income in monetary policy on macroeconomic stability depends on whether the consumption tax rate is high enough.

[1] Meng and Xue (2015) demonstrate that the economy is saddle-path stable under the small-open economy with balanced-budget rule in which consumption tax is the only revenue of government expenditure.

[2] Assuming the economy with increasing returns, which is externality in production, Guo and Lansing (1998) formulate a tax rate that is progressive (or regressive) to income. Fujisaki and Mino (2008) apply this tax rule and a Taylor-type interest-rate control to the standard real business cycle model to analyze the relation between equilibrium determinacy and fiscal and monetary policy rules. They find that monetary policy does not affect macroeconomic stability under enough progressive tax regime.

2 Model

Our economy is close to Xue and Yip's (2013) basic model shown in Sections 3 and 4 of their paper, except that we assume a Taylor-type interest-control rule as monetary policy rule, instead of a constant growth rate of nominal monetary stock.

Representative agents obtain utility from both consumption and leisure (disutility from labor). The dynamic optimization problem is

$$\max \quad \int_0^\infty \left[\frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\chi}}{1+\chi}\right] e^{-\rho t} dt, \quad \rho > 0, \quad \sigma > 0, \quad \chi \geq 0, \tag{1}$$

subject to

$$\dot{a} = (R-\pi)a + y - (1+\tau_c)c - (R-\pi)k - Rm, \tag{2}$$

$$m = (1+\tau_c)c, \tag{3}$$

and the no-Ponzi condition, where ρ is the time discount rate, depreciation rate δ, consumption c, labor l, capital k, total asset $a \equiv b+m$, bond b, money m, and the tax rate on consumption τ_c. Equation (3) is the CIA constraint. In addition, the function of output y is [3]

$$y = k^\alpha l^{1-\alpha}, \quad 0 < \alpha < 1. \tag{4}$$

Defining λ and λ' for the costate variables of capital and CIA constraint respectively, we acquire the conditions for the optimization:

$$c^{-\sigma} = (1+\tau_c)(\lambda + \lambda'), \tag{5}$$

$$l^\chi = \frac{\lambda(1-\alpha)y}{l}, \tag{6}$$

$$\lambda R = \lambda', \tag{7}$$

$$\dot{\lambda} = [\rho - (R-\pi)]\lambda, \tag{8}$$

$$R - \pi = \frac{\alpha y}{k}, \tag{9}$$

with a transversality condition.

The budget constraint for the government is

$$\bar{g} = \tau_c c, \tag{10}$$

[3] Recently, some papers such as Ghilardi and Rossi (2014) and Xue and Yip (2015) have analyzed the effect of labor income tax under the balanced-budget rule on macroeconomic stability under the constant elasticity of substitution (CES) production.

where $\bar{g}$ is the fixed level of government expenditure. Under the balanced-budget rule as in Schmitt-Grohé and Uribe (1997), consumption tax is endogenously determined such that

$$\tau_c = \frac{\bar{g}}{c}.$$

We formulate interest-rate control as in Taylor (1993),

$$R(\pi, y) = \pi + \bar{r}\left(\frac{\pi}{\bar{\pi}}\right)^{\theta_\pi}\left(\frac{y}{\bar{y}}\right)^{\theta_y}, \quad -1 < \theta_\pi < 1, \quad 0 \leq \theta_y < 1, \tag{11}$$

where $\bar{r}(=\rho)$, $\bar{\pi}(>\rho)$ and $\bar{y}$ are the steady-state levels of real interest rate, inflation rate and output, respectively. This is not for regulative formulation but for simplicity of solving the problem. The basic property of the Taylor rule in which nominal interest rate R should be higher as inflation π (or output y) increases is satisfied. If θ_π is positive (resp. negative), real interest rate $r = R - \pi$ increases (resp. decreases) as inflation rate becomes higher so that the policy is called active (resp. passive).

Since $l = [(1-\alpha)\lambda k^\alpha]^{\frac{1}{\alpha+\chi}}$ from Eq. (6), output y is a function of k and λ:

$$y = y(k, \lambda) = k^{\frac{\alpha(1+\chi)}{\alpha+\chi}}[(1-\alpha)\lambda]^{\frac{1-\alpha}{\alpha+\chi}}. \tag{12}$$

Combining Eqs. (9), (11) and (12), we obtain

$$\pi(k, \lambda) = \bar{\pi}\left[\alpha\frac{\bar{y}^{\theta_y}}{\bar{r}}k^{-\frac{\alpha(1+\chi)\theta_y+\chi(1-\alpha)}{\alpha+\chi}}[(1-\alpha)\lambda]^{\frac{(1-\alpha)(1-\theta_y)}{\alpha+\chi}}\right]^{\frac{1}{\theta_\pi}}. \tag{13}$$

From Eqs. (5), (7), (9) and (10), optimal consumption $c = c(k, \lambda)$ satisfies

$$\frac{c^{1-\sigma}}{c+\bar{g}} = \left(1 + \pi(k, \lambda) + \frac{\alpha y(k, \lambda)}{k}\right)\lambda. \tag{14}$$

3 Aggregate Stability of Dynamic System

The dynamic system is summarized as follows [4]:

$$\dot{k} = y(k, \lambda) - c(k, \lambda) - \bar{g}. \tag{15}$$

[4] The equilibrium condition for money and bond is

$$\dot{b} + \dot{m} = (R - \pi)b - \pi m.$$

$$\dot{\lambda} = \left[\rho - \frac{\alpha y(k,\lambda)}{k}\right]\lambda, \tag{16}$$

We investigate the aggregate stability of this economy around the steady state in view of equilibrium determinacy. The coefficient matrix of the linearized system of the original Eqs. (15)−(16) around the unique non-trivial steady state is

$$J = \begin{bmatrix} \dot{k}_k & \dot{k}_\lambda \\ \dot{\lambda}_k & \dot{\lambda}_\lambda \end{bmatrix},$$

where

$$\dot{k}_k = \frac{\partial \dot{k}}{\partial k}\bigg|_{ss} = y_k - c_k, \quad \dot{k}_\lambda = \frac{\partial \dot{k}}{\partial \lambda}\bigg|_{ss} = y_\lambda - c_\lambda,$$

$$\dot{\lambda}_k = \frac{\partial \dot{\lambda}}{\partial k}\bigg|_{ss} = -\frac{\bar{\lambda}}{\bar{k}}(\alpha y_k - \rho), \quad \dot{\lambda}_\lambda = \frac{\partial \dot{\lambda}}{\partial \lambda}\bigg|_{ss} = -\frac{\alpha\bar{\lambda}}{\bar{k}} y_\lambda,$$

and thus

$$\begin{aligned} DetJ &= \mu_1\mu_2 = \dot{k}_k \cdot \dot{\lambda}_\lambda - \dot{k}_\lambda \cdot \dot{\lambda}_k = \frac{\bar{\lambda}}{\bar{k}}[\alpha(y_\lambda c_k - y_k c_\lambda) - \rho(y_\lambda - c_\lambda)] \\ &= \frac{\bar{y}}{\bar{k}}\frac{\rho(1-\alpha)}{\alpha+\chi}\left[\frac{\chi}{(-\sigma+(1-\sigma)\bar{\tau}_c)} - 1 - \frac{\theta_y\bar{\pi}}{(1+\bar{\pi}+\rho)\theta_\pi(-\sigma+(1-\sigma)\bar{\tau}_c)}\right], \end{aligned} \tag{17}$$

$$\begin{aligned} TraceJ &= \mu_1 + \mu_2 = \dot{k}_k + \dot{\lambda}_\lambda \\ &= \rho + \frac{\bar{y}}{(-\sigma+(1-\sigma)\bar{\tau}_c)(1+\bar{\pi}+\rho)}\left(\frac{\alpha(1+\chi)\theta_y + \chi(1-\alpha)}{(\alpha+\chi)\theta_\pi}\frac{\bar{\pi}}{\bar{k}} + \frac{\chi(1-\alpha)}{\alpha+\chi}\frac{\rho}{\bar{k}}\right), \end{aligned} \tag{18}$$

because

$$y_k = \frac{\partial y}{\partial k}\bigg|_{ss} = \frac{\rho(1+\chi)}{\alpha+\chi}, \quad y_\lambda = \frac{\partial y}{\partial \lambda}\bigg|_{ss} = \frac{1-\alpha}{\alpha+\chi}\frac{\bar{y}}{\bar{\lambda}},$$

$$\pi_k = \frac{\partial \pi}{\partial k}\bigg|_{ss} = -\frac{\alpha(1+\chi)\theta_y + \chi(1-\alpha)}{(\alpha+\chi)\theta_\pi}\frac{\bar{\pi}}{\bar{k}}, \quad \pi_\lambda = \frac{\partial \pi}{\partial \lambda}\bigg|_{ss} = \frac{(1-\alpha)(1-\theta_y)}{(\alpha+\chi)\theta_\pi}\frac{\bar{\pi}}{\bar{\lambda}},$$

$$c_k = \frac{\partial c}{\partial k}\bigg|_{ss} = \frac{\bar{y}}{(-\sigma+(1-\sigma)\bar{\tau}_c)(1+\bar{\pi}+\rho)}\left(\pi_k - \frac{\chi(1-\alpha)}{\alpha+\chi}\frac{\rho}{\bar{k}}\right),$$

$$\begin{aligned} c_\lambda &= \frac{\partial c}{\partial \lambda}\bigg|_{ss} = \frac{\bar{y}}{\bar{\lambda}(-\sigma+(1-\sigma)\bar{\tau}_c)(1+\bar{\pi}+\rho)}\left(1+\bar{\pi}+\rho+\bar{\lambda}\left[\pi_\lambda + \frac{\alpha y_\lambda}{\bar{k}}\right]\right) \\ &= \frac{1}{(-\sigma+(1-\sigma)\bar{\tau}_c)}\frac{\bar{y}}{\bar{\lambda}}\left(1 + \frac{(1-\alpha)(1-\theta_y)\bar{\pi}}{(\alpha+\chi)(1+\bar{\pi}+\rho)\theta_\pi} + \frac{\rho(1-\alpha)}{(\alpha+\chi)(1+\bar{\pi}+\rho)}\right). \end{aligned}$$

Table 1: Determinacy of Equilibrium

	$-\sigma + (1-\sigma)\bar{\tau}_c < 0$ (low $\bar{\tau}_c$ or $\sigma \geq 1$)	$-\sigma + (1-\sigma)\bar{\tau}_c > 0$ (high $\bar{\tau}_c$ or $\sigma < 1$)
$\theta_\pi > 0$ (active)	D D, NS, I if $\theta_y > 0$	D, NS
$\theta_\pi < 0$ (passive)	D	D, NS, I

D = Determinate, NS = Non-Stationary, I = Indeterminate

There is one jump variable, λ, and one predetermined variable, k, in the dynamic system so that the steady state satisfies local determinacy, if one eigenvalue is positive, that is, $DetJ < 0$. Then, macroeconomy is stable in that unique equilibrium path exists. When all eigenvalues μ_1 and μ_2 are positive, non-stationary holds ($DetJ > 0$ and $TraceJ > 0$). Otherwise, equilibrium is indeterminate ($DetJ > 0$ and $TraceJ < 0$).

From equation (17), the results of equilibrium determinacy are displayed in Table 1. To consider the economic intuition, suppose that an agent anticipates a positive future and thus accelerates capital accumulation today. Then, the real rate of return from capital falls, and then nominal interest rate becomes lower if monetary policy is active from a non-arbitrage condition. On the other hand, the shadow value of household's budget as the marginal utility of income increases.

Under the CIA constraint, marginal utility from consumption discounted by consumption tax rate also decreases when the opportunity cost of holding money in terms of the marginal utility of income becomes lower. Usually, higher consumption reduces the marginal utility, but it also implies a lower tax rate under the balanced-budget rule and thus the marginal utility may decrease with small consumption when the tax rate is high enough. If consumption is much lower, the positive expectation is not self-fulfilling.

When the interest rate also responds to income ($\theta_y > 0$), the effect of lowering the real interest rate (and thus the marginal utility of consumption) under an active Taylor-rule is stronger so that the economy tends to be stable (resp. instable) under the higher (resp. lower) consumption tax rate, because the self-fulfilling larger consumption becomes hard (resp. easy) to emerge. [5] This may be a reason why determinate (resp. indeterminate) equilibrium can emerge under $\sigma < 1$ (resp. $\sigma > 1$) in this economy, in contrast to

[5] As seen from Eq. (17), $DetJ$ becomes negative more easily when $-\sigma + (1-\sigma)\bar{\tau}_c > 0$.

Xue and Yip's (2013) standard monetary economy. The result implies that an expansion of the role of monetary policy can be one of the means for macroeconomic stability under higher consumption tax rate.

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