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Abstract

We analyze the relation between policy mixture and equilibrium determinacy in an economy where money and government expenditures are used for production. We find that an adequate mix of income tax and interest-rate control is important to realize a stable economy, as well as the relation between contribution of government expenditures to production and the basic tax rate as a source of the revenue for these expenditure.

Keywords: equilibrium determinacy, progressive income tax, Taylor rule, productive government expenditure and money.

JEL Classification Numbers: E52, E62.

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1 Introduction

We investigate the effect of mixture of fiscal and monetary policies, on macroeconomic stability in terms of equilibrium determinacy, which implies that one stable equilibrium path is determined. In this study, we focus on the role of money and government spending in production.

Under economic models including interest-rate control type of monetary policy suggested by Taylor (1993), such as those in the studies by Benhabib, Schmitt-Grohé and Uribe (2001) and Meng and Yip (2004), a standard result is called the "Taylor principle". This implies that an aggressive response of nominal interest rate to inflation tends to generate a unique path to a determinate equilibrium that is unaffected by expectation, and thus macroeconomy is stable. These studies using the Taylor rule to analyze equilibrium determinacy assume the economy with two types of money; in the utility function as in Sidrauski (1967) or in the production function proposed by Sinai and Stokes (1989). These studies reveal that productive money can drastically change the results for an appropriate policy to realize a stable economy. Fujisaki (2012b) expands this framework into a two-country economy.

Fiscal policy is another means for stabilizing an economy. The tax rate is often assumed to be constant in macroeconomic models for simplicity. However, Guo and Lansing (1998) formulate the realistic tax rate that is progressive (or regressive) to income, and display aggregate stability in the economy with increasing returns in production. Fujisaki and Mino (2008) apply this tax rule and Taylor-type interest-rate control to the standard real business cycle model to analyze the relation between equilibrium determinacy and fiscal and monetary policy rules. They find that monetary policy does not affect macroeconomic stability under enough progressive tax regime.

Following Barro (1990), many studies such as Guo and Harrison (2008) have been published in which utility or production includes government spending generated by fiscal policy. Kamiguchi and Tamai (2011 and 2012), Fujisaki (2012a) and Tamai (2008) assume productive government expenditures. In particular, Kamiguchi and Tamai (2011) reveal that an important factor of determinacy is a revenue source for providing public services rather than the presence of productive government spending. As we see, these studies neglect the monetary aspect.

In addition, no study so far has investigated a stability effect of a policy interaction on the economy that includes government spending and money beneficial to production or utility, although there have been many studies about policy mixture of interest-rate control and fiscal policy, such as Benhabib and Eusepi (2005), Edge and Rudd (2007), Guo and Harrison (2004), Leeper (1991), and Schmitt-Grohe and Uribe (1997). Xue and Yip (2013) show that equilibrium may not be determinate even if utility is additively separable between consumption and labor. They assume consumption tax under the conditions of fixed government spending, a constant monetary growth rate, and a cash-in-advance constraint to consumption.

We assume the tax rate varied with income as in Guo and Lansing (1998) and Taylor-type interest-rate control, to investigate the stability effect of policy mixture on the economy with money and government spending beneficial to output.

2 Analysis and Results

2.1 Model

The representative agents solve the following dynamic optimization problem;

$$\max \quad \int_0^\infty \frac{c^{1-\sigma}}{1-\sigma} e^{-\rho t} dt, \quad \rho > 0, \quad \sigma > 0, \tag{1}$$

subject to

$$\dot{a} = (R-\pi)a - Rm - (R-\pi)k + (1-\tau_y(y))y - c, \tag{2}$$

and the no-Ponzi condition, where ρ denotes the time discount rate, $a \equiv b+m+k$ total asset summing bonds b, money for production m, and capital k, $\tau_y(y)$ the tax rate on income which depends on income level y as below, and g government expenditure. The production function is assumed as

$$y = k^\alpha m^{1-\alpha} g^\beta, \quad 0 < \alpha < 1, \quad 0 < \beta < 1. \tag{3}$$

The conditions for this optimization are the followings:

$$c^{-\sigma} = \lambda, \tag{4}$$

$$[1 - \tau_y(y) - \tau_y'(y)y]\frac{(1-\alpha)y}{m} = R, \tag{5}$$

$$[1 - \tau_y(y) - \tau_y'(y)y]\frac{\alpha y}{k} = R - \pi, \tag{6}$$

$$\dot{\lambda} = [\rho - (R-\pi)]\lambda, \tag{7}$$

with transversality condition, where λ is a shadow value of capital.

The budget constraint for government is

$$g = \tau_y(y)y, \tag{8}$$

and the tax rate varies with income,

$$\tau_y(y) = 1 - \eta\left(\frac{\bar{y}}{y}\right)^{\phi}, \quad \eta \in (0,1], \quad \phi < 1. \tag{9}$$

where η the rate of the disposable income around the steady-state level of income $\bar{y}$. [1] This progressive tax is as in Guo and Lansing (1998), and then the marginal tax rate is

$$\tau_y(y) + \tau_y'(y)y = 1 - \eta(1-\phi)\left(\frac{\bar{y}}{y}\right)^{\phi},$$

and thus

$$1 - \tau_y(y) - \tau_y'(y)y = \eta(1-\phi)\left(\frac{\bar{y}}{y}\right)^{\phi}. \tag{10}$$

On the other hand, we formulate interest-rate control as in Taylor (1993),

$$r(R) = \bar{R}\left(\frac{R}{\bar{R}}\right)^{\frac{\psi}{1+\psi}} = R - \pi, \quad -1 < \psi \neq 0. \tag{11}$$

This is not a regulative formulation, but we use this for simplicity in solving the problem. [2] The basic property of Taylor rule in which nominal interest rate R should be higher as inflation π increases is satisfied. Around the steady-state level $R = \bar{R}$, if ψ is negative (resp. positive), real interest rate $r = R - \pi$ decreases (resp. increases) as the inflation rate becomes higher so that the policy is called passive (resp. active). [3]

2.2 Dynamic System

From Eqs. (5), (6) and (11),

$$\frac{1-\alpha}{\alpha}\frac{k}{m} = \frac{R}{r(R)} = \left(\frac{R}{\bar{R}}\right)^{\frac{1}{1+\psi}}. \tag{12}$$

[1] Guo and Lansing (1998) assume $\phi \in \left(-\frac{1-\alpha}{\alpha}, 1\right)$, because the steady-state values of economic variables should not be negative under positive depraciation rate of capital.

[2] This formulation implies that the target rate of inflation is zero, $\bar{\pi} = 0$, since the nominal interest rate equals to the real one.

[3] Around the steady state $R = \bar{R}$, $\left.\frac{dR}{d\pi}\right|_{ss} = 1 + \psi$ and thus $\left.\frac{dr}{d\pi}\right|_{ss} = \psi$.

Combining Eqs. (3), (8), (9) and (12), we obtain

$$R = \bar{R}\left\{\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}\frac{y^{1-\beta}}{k}[1-\eta\bar{y}^{\phi}y^{-\phi}]^{-\beta}\right\}^{-\frac{1+\psi}{1-\alpha}}. \tag{13}$$

Substituting this into Eq. (6), we acquire the relation between capital and income, $y = y(k)$:

$$k = \left(\frac{\bar{R}}{\eta\alpha(1-\phi)\bar{y}^{\phi}}\right)^{-\frac{1-\alpha}{\psi+1-\alpha}} y^{\frac{(1-\beta)\psi+(1-\phi)(1-\alpha)}{\psi+1-\alpha}}\left\{\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}[1-\eta\bar{y}^{\phi}y^{-\phi}]^{-\beta}\right\}^{\frac{\psi}{\psi+1-\alpha}}, \tag{14}$$

satisfying

$$y'(\bar{k}) = \left.\frac{dy}{dk}\right|_{ss} = \frac{(1-\eta)(\psi+1-\alpha)}{\psi[1-\eta-\beta+\eta\beta(1-\phi)]+(1-\phi)(1-\eta)(1-\alpha)}\frac{\bar{y}}{\bar{k}}. \tag{15}$$

In addition, consumption is $c = c(\lambda) = \lambda^{-\frac{1}{\sigma}}$ from Eq. (4).

The following equations comprise the dynamic system, which implies the goods-market equilibrium condition and the Euler equation: [4]

$$\dot{k} = \eta\bar{y}^{\phi}y(k)^{1-\phi} - c(\lambda), \tag{16}$$

$$\dot{\lambda} = \left[\rho - \eta\alpha(1-\phi)\bar{y}^{\phi}\frac{y(k)^{1-\phi}}{k}\right]\lambda. \tag{17}$$

The coefficient matrix of the linearized system of the original one (16)−(17) around the steady state is

$$J = \begin{bmatrix}\dot{k}_k & \dot{k}_\lambda \\ \dot{\lambda}_k & \dot{\lambda}_\lambda\end{bmatrix},$$

where

$$\dot{k}_k = \left.\frac{\partial\dot{k}}{\partial k}\right|_{ss} = \eta(1-\phi)y'(\bar{k}), \quad \dot{k}_\lambda = \left.\frac{\partial\dot{k}}{\partial\lambda}\right|_{ss} = \frac{1}{\sigma}\frac{\bar{c}}{\bar{\lambda}},$$

$$\dot{\lambda}_k = \left.\frac{\partial\dot{\lambda}}{\partial k}\right|_{ss} = -\eta\alpha(1-\phi)\bar{\lambda}\frac{(1-\phi)\bar{k}y'(\bar{k})-\bar{y}}{\bar{k}^2}, \quad \dot{\lambda}_\lambda = \left.\frac{\partial\dot{\lambda}}{\partial\lambda}\right|_{ss} = 0.$$

[4] The equilibrium condition for money and bond is

$$\dot{b} + \dot{m} = (R-\pi)b - \pi m.$$

$(i)\beta < 1-\eta$	$\phi < \dfrac{\beta(1-\eta)}{1-\eta-\eta\beta}$	$\dfrac{\beta(1-\eta)}{1-\eta-\eta\beta} < \phi < 1$
$\psi > 0$	NS	D
$-(1-\alpha) < \psi < 0$	D	D, NS
$-1 < \psi < -(1-\alpha)$	D, NS	D

Table 1. Equilibrium Determinacy when $\beta < 1-\eta$

$(ii)\beta > 1-\eta$	$\phi < \dfrac{(1-\beta)(1-\eta)}{\eta\beta}$	$\dfrac{(1-\beta)(1-\eta)}{\eta\beta} < \phi < 1$
$\psi > 0$	NS	D, NS
$-(1-\alpha) < \psi < 0$	D	D
$-1 < \psi < -(1-\alpha)$	D, NS	D

Table 2. Equilibrium Determinacy when $\beta > 1-\eta$

D=Determinate, NS=Non-stationary.

Therefore,

$$DetJ = \mu_1\mu_2 = \dot{k}_k \cdot \dot{\lambda}_\lambda - \dot{k}_\lambda \cdot \dot{\lambda}_k = \\ -\frac{\rho^2}{\sigma\alpha(1-\phi)} \frac{\psi[(1-\eta-\eta\beta)\phi - \beta(1-\eta)]}{\psi[1-\eta-\beta+\eta\beta(1-\phi)] + (1-\phi)(1-\eta)(1-\alpha)}, \quad (18)$$

$$TraceJ = \mu_1 + \mu_2 = \dot{k}_k + \dot{\lambda}_\lambda \\ = \frac{\rho}{\alpha} \frac{(1-\eta)(\psi+1-\alpha)}{\psi[1-\eta-\beta+\eta\beta(1-\phi)] + (1-\phi)(1-\eta)(1-\alpha)}, \quad (19)$$

because

$$\frac{\eta\bar{y}}{\bar{k}} = \frac{\bar{c}}{\bar{k}} = \frac{\rho}{\alpha(1-\phi)},$$

and

$$DetJ = -\frac{\rho}{\sigma(1-\phi)} \frac{\psi[(1-\eta-\eta\beta)\phi - \beta(1-\eta)]}{(1-\eta)(\psi+1-\alpha)} TraceJ. \quad (20)$$

There is one jump variable, λ, and one predetermined variable, k, in the dynamic system so that the steady state satisfies local determinacy, if one eigenvalue is positive, that is, $DetJ = \mu_1\mu_2 < 0$. When all eigenvalues μ_1 and μ_2 are positive, non-stationary holds ($TraceJ = \mu_1 + \mu_2 > 0$), that is, equilibrium paths always diverge. Otherwise, equilibrium is indeterminate

($TraceJ < 0$), which implies that there are multiple paths which converge equilibrium. However, as we see from Tables 1 and 2, indeterminacy does not occur in our economy.

2.3 Intuitive Interpretation

From the previous subsection, we find that the condition of determinate equilibrium implies that the self-fulfilling expectation is not realized. That is, increasing capital accumulation based on the positive anticipation for the economy decreases the rate of return from capital.

In addition, an adequate mix of fiscal and monetary policies is important for stable economy, as well as the relation between the contribution of government expenditures (β) to production and the basic tax rate ($1-\eta$) used for the expenditures.

For instance, whether β is higher than $1-\eta$ or not, surprisingly, passive Taylor rule can be a good stabilizer. In this economy, a more flat tax, which is usually considered to stabilize an economy under the standard economic models, does not work for a brake on increasing productive government expenditures, and thus the equilibrium paths tend to diverge, particularly under the combination of an active interest-rate control and a flat tax. A passive monetary policy is effective to stabilize an economy in that higher nominal interest rate generates lower real one, and then productive money is depressed so that self-fulfilling positive expectation is harder to realize.

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