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1. Foreword

Production prices and labour values can be compared and some connections have already been detected (see this author's article (2009)). Classical production prices and labour values are, nevertheless, not related in a simple way and their coincidence is only a matter of chance.

Given that according to the labour theory of value, the rewarding of embodied labour or capital - as a consequence of the monopoly of the owners of the means of production - can only come from the new work performed by the working population, all the subsequent magnitudes, such as profits and prices, must in the end depend on the living value generated. This is the basis for considering labour values to be better at describing the associated costs and profits of the economic system (See this author (2011)). Therefore, facts must be identified and consequences drawn with regards to their imbalances.

We begin the article with a probe of commensurability of production prices and labour values and by describing useful new concepts such as total or cumulated profits and surplus value. We then compare the results of a capitalist economic system through balances and imbalances of the data in production prices and labour values. We detect new relations between the two ways of measurement with reference to the values that the rate of profit in prices may take.

We relate the concepts employed with those of other visions of the transformation of values in prices: the use of a monetary expression of labour time (MELT) and the establishment of the value added of one period as the only subject of transformation.

2. Previous concepts

2.1. Commensurability of values and prices of production

It has been said both by some Ricardians and some Marxians that the Sraffian system of prices and the system of labour values are not comparable. Some of the former say that the set of prices are relative and labour values are not only absolute but also useless¹. Similarly, some Marxians believe that values are only a useful theoretical, non-measurable instrument or abstraction², and besides this, values relate to socially necessary time, not to real time employed. We assume here that all the labour time employed is socially necessary.

¹ Vegara. P 141 (translated) *"It lacks theoretical importance to relate surplus value with profits"*

² Benetti. P 232 (translated) *"We have seen that the relation between profits and surplus value cannot be demonstrated departing from the transformation of labour values into price schema"*

It is nevertheless true that prices, when measured in wage units, are higher (both individually and added) (*see Vegara, p. 56*) than their corresponding labour values and, hence, they have to be comparable:

$$P_{i,w} > \Lambda_i$$

(*See annex for the meaning of symbols*)

whatever the units implied, which we do not discuss here³.

The expression of prices in wage units is as follows:

$$P'_w = L.(1+r) + L.A.(1+r)^2 + L.A^2.(1+r)^3 + \dots(1)$$

or equivalently

$$P'_w = (1+r).L.(I - (1+r).A)^{-1}$$

In relation to the presentation of the Sraffian reduction to dated labour (*see Sraffa p. 58, point 46*), it can be seen that the wage is not present in the above expressions because it is the unit used in this specific case.

We can go one step further in relating prices and labour values, and convert prices measured in wage units into prices calculated with a wage expressed, or normalized, in labour time. In other words, prices where wages are measured in paid labour time (in this case production prices are no longer necessarily higher than their corresponding labour values)⁴:

$$P' = (1+r).\Lambda' B.L.(I - (1+r).A)^{-1} (2)$$

where $\Lambda' B$ is a scalar corresponding to the wage in labour terms that is between 0 and 1.

1 would correspond to the total labour time worked by one unit of labour. The meaning of the scalar can be better seen in the following formula:

$$\Sigma L = \Sigma(1 - \Lambda'.B).\Sigma L + \Lambda'.B.\Sigma L$$

With regards to the graph in the annex, equation (2) will produce a shift downwards in the line of the sums of prices, calculated in relation to abscises representing the rate of profit, and create a parallel line of these sums of prices. This is a linear shift.

As for the possibility that the sum of prices and labour values coinciding, the graph shows the intersection of the total prices curve and the total labour flat value line, which is a straight line since it is not dependent on the rate of profit. This intersection only occurs when production prices incorporate wages – in formula (2), and not formula (1) where the wage is not present because it is used as a unit – so as their sum is able to become equal to the sum of values (otherwise, as stated, prices are strictly higher than labour values and there is no possibility of intersection).

Setting aside the commensurability issue, with regard to the accuracy of the data, labour time must prevail over production prices as a better representation of costs and profits since the

³ The unit of prices measured in wages could be hours, provided that the formula finally relates them with labour

⁴ The unit of prices where wages are counted in paid labour time could also be hours

specific values are not mediated by a common rate of profit as in the calculation of the prices of production (which have to be modified in relation to the labour values in order to achieve the equal remuneration of capitals). The rate of profit in terms of labour time is, therefore, better or real than the corresponding rate in prices. In addition, it is calculated from actual values and depends on the current sectorial composition.

2.2. Concept of cumulated value and profits

The concepts of total profits and total surplus value of a system (of prices and of values respectively) used in this article are, needless to say, higher than those of the current period.

For instance in expression (3) below:

$$\Sigma\Lambda = \Sigma(1 - \Lambda'.B). \Sigma\Lambda + \Lambda'.B. \Sigma\Lambda \quad (3)$$

the sum of labour values is equal to the sum of total surplus value, or non-paid value, plus the sum of paid value (this is not the same as constant capital, which is the sum of the vector components $\Lambda'.A$)

Total profits come from the subtraction of total prices in wage units and total labour values:

$$\Sigma P_w - \Sigma\Lambda$$

This includes profits of all periods.

The cumulated version of the surplus value and the one of the current period are as follows:

$$(1 - \Lambda'.B). \Sigma\Lambda$$

$$(1 - \Lambda'.B). \Sigma L$$

There is, also, a specific rate of profit at which the total surplus value and the total profits match.

$$r_{SPL=profits} = [\Sigma\Lambda - \Sigma\Lambda'.A - \Sigma(L/(2 - \Lambda'.B))]/[\Sigma\Lambda'.A + \Sigma(L/(2 - \Lambda'.B))] \quad (4)$$

Where $(2 - \Lambda'.B) = 1 + (1 - \Lambda'.B)$ reduces the quantity of labour time effectively employed into a labour time magnitude that would grow at a rate: $(1 - \Lambda'.B)$ per unit of product:

$$\Sigma L_{generator}. (1 + (1 - \Lambda'.B)) = \Sigma L$$

At this rate: $r_{SPL=profits}$, the sum of total labour values and the total surplus value that equals the profits, is identical to the sum of production prices in wage units.

This rate does not coincide exactly with the rate at which total prices and total labour values match:

$$r_{\Sigma P = \Sigma\Lambda} = [\Sigma\Lambda - \Sigma\Lambda'.A - \Sigma\Lambda'.B.L]/[\Sigma\Lambda'.A + \Sigma\Lambda'.B.L] \quad (5)$$

which uses the sum of wages effectively employed in production. They can be expressed as follows:

$$\left[\Lambda'.B. \Sigma L = \Sigma L / \left(1 + \frac{(1 - \Lambda'.B)}{\Lambda'.B} \right) \right]$$

unlike the previous formula.

3. Joint analysis of production prices and values

(See graph and example data in annex)

3.1. Order of rates of profit

In relation with the order of rates of profit: the one corresponding to the eigenvalue: $r_{Eigenvalue}$, the one corresponding to the prices that make total surplus value equal to total profits: $r_{SPL=profits}$ and the one corresponding to the r that makes total profits equal to total values: $r_{\Sigma P=\Sigma \Lambda}$, the highest one corresponds to the eigenvalue by definition. The position of the other two rates is as follows. The $r_{\Sigma P=\Sigma \Lambda}$ is the higher of the two, since, looking at equations (4) and (5), $\sum(\frac{L}{2-\Lambda'.B})$, that can be expressed also as: $\sum(\frac{L}{(1+(1-\Lambda'.B))})$, is higher than $\sum L/(1 + \frac{(1-\Lambda'.B)}{\Lambda'.B})$ and that implies a lower rate of profit.

3.2. Condition of equality of the sum of prices and values and the sum of surplus value and profits

A way to find the rate of profit that makes it possible that at the same time, both the sum of values and the sum of prices be equal and the sum of (the total) surplus value and the sum of (the total) profits is first to equate:

$$\sum P_w \cdot \Lambda'.B = (1 - \Lambda'.B) \cdot \sum \Lambda'$$

The meaning of this is that the multiplication of the prices in wage units by the wage rate has to coincide with the total surplus value, so that the total prices curve cuts the total labour values. This can be seen better looking at the graph in the annex. This can also be expressed as the relation of prices in wage units and labour values being equal to the rate of surplus value:

$$\sum P_w / \sum \Lambda' = (1 - \Lambda'.B) / \Lambda'.B$$

In addition to this, we know that at this rate of profit the following expression must also hold:

$$\sum P_w = \Sigma \Lambda + \Sigma Pl = \Sigma \Lambda \cdot (1 + (1 - \Lambda'.B))$$

This is the condition of equality between total surplus value and total profits.

Therefore, the above equation can be written as:

$$\Sigma \Lambda \cdot (1 + (1 - \Lambda'.B)) / \sum \Lambda' = (1 - \Lambda'.B) / \Lambda'.B \quad (6)$$

Solving the equation (6) to find the unknown wage in labour terms $\Lambda'.B$, we obtain a value for it (and therefore for r) where $(1 - \Lambda'.B) / \Lambda'.B$ maintains the *Golden ratio* (1,61)

Thus, only when the Golden ratio is applied to surplus value and wage in labour terms, may the total surplus value be equal to the total profits and the total prices and total labours be equal. Nothing ensures, however, that such a wage is viable.

The other situation where there is coincidence between the two equalities is the Sraffian composite commodity (in the production of the Sraffian composite commodity, individual inputs and outputs are proportional, and the price or value that measures its component commodities does not make any difference).

3.3. Prices and values unbalanced

When prices are measured in wage units, they are always greater than labour values except when $r=0$, in which case they are equal. They grow as a consequence of the rate of profit growth or, equivalently, as a consequence of wage rate decreases in the absence of productivity increases.

From zero to $r_{SPL=profits}$, the total surplus value is greater than the total profits and this condition means that the profits can progressively buy less new labour or command (mobilise) fewer new resources.

Similarly, from zero to $r_{\Sigma P=\Sigma \Lambda}$, the sum of prices at a fixed wage measured in labour values are lower than the sum of values. From this point on, the sum of prices begin to be greater than the labour values and progressively, as in the previous case, less capable to commanding resources.

When the rate of profit is $r_{SPL=profits}$, i.e., at the point where profits are represented by an equal amount of surplus value, the existing sum of prices are lower than the sum of labour values and, hence, more capable of commanding resources and are more valued with regard to total profits versus total surplus value. Moving from this point towards the $r_{\Sigma P=\Sigma \Lambda}$ point, means an increase in the rate of profit but profits are inflated beyond their real labour value.

As the rate of profit becomes higher than both rates, prices move to have a lesser capacity to mobilise resources, and are progressively devaluated in relation to the labour value units.

In any case, provided that the capitalist economy is situated at $r_{SPL=profits}$ point, and once the whole capital invested is recovered, it can be said that there has been a complete transformation of labour values into production prices.⁵

4. Conclusions

Relative prices of production and labour values are two independent ways of looking at the economy that can be analysed conjointly because they refer to and describe one common reality. Production prices are a schematic representation of the market in which capitalists operate, and labour values are a deeper account of costs, exploitation and profits. The reduction of prices to dated labour from P. Sraffa is an example of this connection or conjoint analysis.

There is another significant specific connection between these two systems when production prices are measured in wage units: at the Marxian rate of profit a production price set in wage units is determined, which makes total profits equal to total surplus value while the sum of labour values plus the total surplus value is equal to the sum of these prices. At this rate of profit, the Marxian assumption of distribution of surplus value among the different sectors is thus achieved. There is, however, no reason to think that there is a normal rate of profit in a capitalist

⁵ But in this as well as in any other circumstance, labour values end up being translated into prices due to their definition as values, so price deviations should only be temporary and should finally return to these values.

economic system; nothing ensures, for instance, the existence of a level of wage that fixes a rate of profit along the abscise axis between the points of the axis above mentioned in section 3.3 where total surplus value equals total profits and where total value equals the sum of prices with the wage expressed in paid labour values. (*See graph in the annex*)

Furthermore, the sum of these production prices (growing with the increase in the rate of profit) intersects with the flat line representing the total labour value at some point (at some rate of profit), which is not normally coincident with the one that equals total surplus value to total profits. A specific wage (in labour value terms) could, nevertheless, be found that allows the coincidence of both Marxian equalities: total prices and total labour values and total surplus value and total profits, which corresponds to a Golden ratio between the surplus value and the wage in labour units.

In Dumenil&Foley's "new interpretation" or "Single System Labour Theory of Value" (SS-LTV), the transformation refers only to the value added generated in one period and a unique relation is made between the value added in prices equal to the value added in labour value terms.

In this framework, in contrast with this interpretation, there are connections between two systems (prices and values) which are shown along the evolution of the abscise axis of the rate of profit in price terms, allowing a theoretical variety of MELT ratios in the absence of changes in the techniques used. Moreover, the total surplus value concept is used – so as to equate it to total profits at a specific rate of profit - instead of the current-period value added and ensures that the initial capital is recovered and all subsequent sums of prices originated are dependent on labour values.

We envisage an open production prices system with no reasons for Marxian equalities to be automatically achieved but which distributes surplus value, and does so in ways that have unknown economic consequences. In our opinion, production prices as defined by Marx in chapter 9 of Capital volume III are not plausible for the representation of real production prices, which could, however, be implied in chapter 50 of Capital volume III.

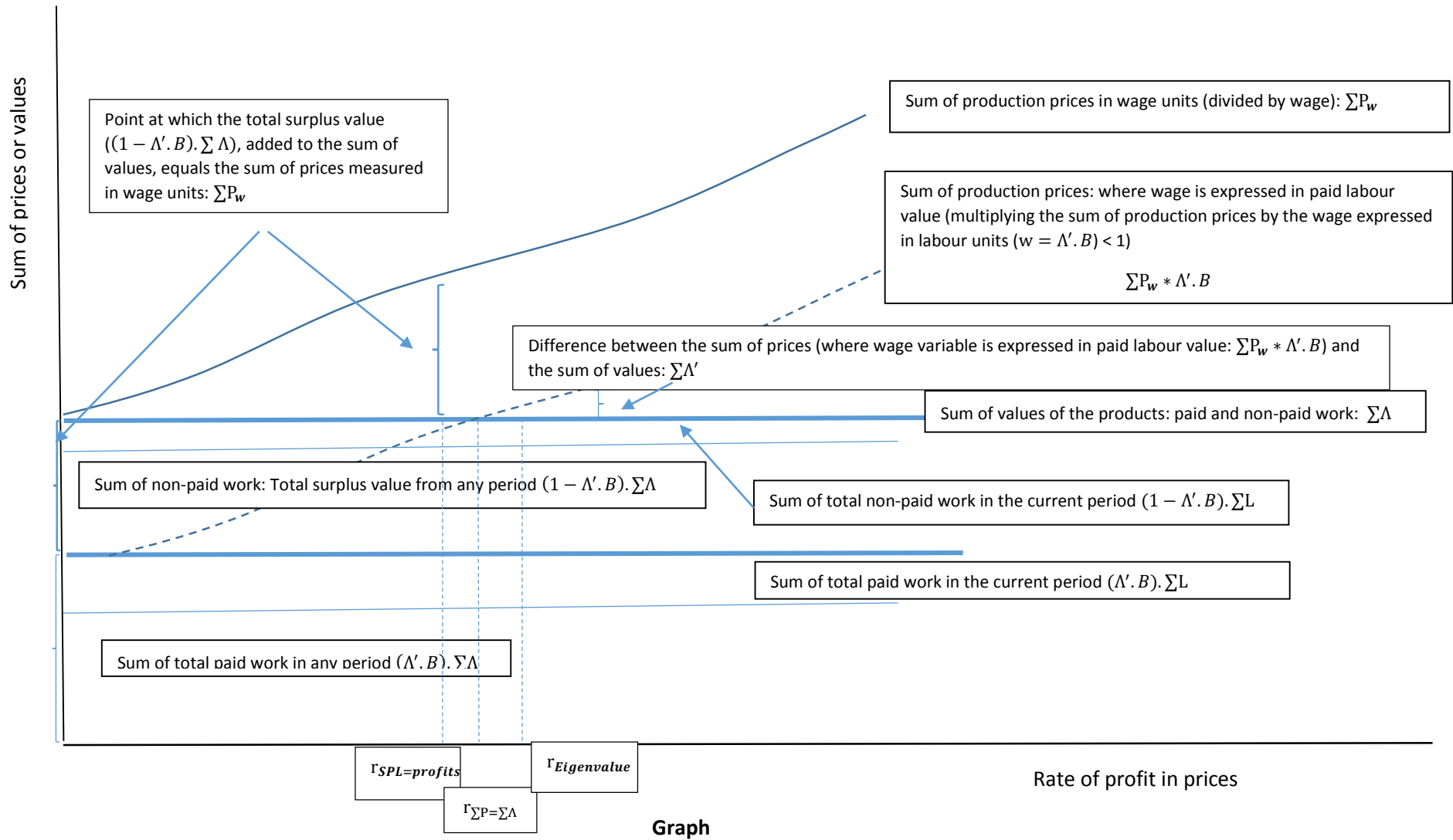
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Annex

Data

Initial data			
Leontief I/O matrix, A			
	0.30	0.20	0.20
	0.10	0.10	0.00
	0.10	0.20	0.05
L Labour employed in each industry (per unit of product)	1.00	0.10	2.00
B Wage goods per unit of labour	0.10	0.10	0.25
Sum of inputs in value	5.3561		
Variable capital	2.8938		
Constant capital	2.4624		
Sum of outputs in value	5.5624		
Individual values Λ			
Λ_1	1.9452		
Λ_2	1.1023		
Λ_3	2.5149		
Rate of profit in value (Marxian)	0.0384		
Wage (in value, per unit of labour) $\Lambda'. B$	0.9335		
Total paid value	5.1924		
Total non-paid value	0.3700		
Rate of surplus value $(1 - \Lambda'. B)$	(0.0665=1-0.9335)		
Surplus value (period)	0.2062		
Total surplus value	0.3700		
Individual rates of profit in value			
r_1	0.0354		
r_2	0.0061		
r_3	0.0559		
Rate of profit that equals Total profits and Total surplus value $r_{SPL=profits}$	0.0360		
Sum of new prices vector (2.08+1.20+2.64)	5.9284		
$\Sigma\Lambda + \Sigma PL = \Sigma\Lambda. (1 + (1 - \Lambda'. B))$	5.92		
Rate of profit that equals Total prices and Total values $r_{\Sigma P=\Sigma\Lambda}$	0.0384		
Rate of profit (eigenvalue) $r_{Eigenvalue}$	0.047		
Sum of production prices vector -in wage units (ΣP_w) (2.11+1.23+2.66)	6		
Rate of surplus value $(1 - \Lambda'. B)$ allowing double Marxian equality: Total profits and Total surplus value, and Total prices and Total values	(0.618=1-0.382)		
Rate of profit for double equality	0.27		
Sum of production prices vector -in wage units (ΣP_w) (3.225+2.127+3.587) at r=0.27	8.939		



Relations between production prices, labour values and rates of profit

Production of one unit per sector assumed