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A Review of Some Postwar Economic Growth Theories and Empirics

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Abstract

The evolution of growth theories from the 1956 seminal work of Solow and Swan to Aghion and Howitt's 1992 Schumpeterian model is traced herein. How growth empirics helped improve some existing theories is also presented. As a matter of fact, the empirical evidence that countries were not converging as the Solow-Swan model predicted led to the development of endogenous growth theories pioneered by Romer (1986) and Lucas Jr (1988). Thereafter, semi-endogenous growth models originated from the observation that growth rate across countries was not proportional to the size of skilled labor as endogenous growth theories predicted.

I also present my own empirical assessment of some predictions from growth theories and find supporting evidence of (1) convergence of GDP across Canada and the countries of the West African Economic and Monetary Union and (2) a positive relationship between output and the accumulation of knowledge through R&D across Canada. I also find, in Canada, the evidence of a positive relationship between economic growth and skilled labor, as some model predicted.

Keywords: Economic Growth, endogenous growth, exogenous growth, growth empirics

JEL: B22, N01, O40

Contents

1 Introduction

The main purpose of (economic) growth models is to explain the determinants of the long-run growth in per capita output. On the contrary, real business-cycle models deal with the determinants of the short-run fluctuations in output. In terms of duration, *i.e.*, number of months, quarters, or years, there is not a clear-cut definition of the concepts of long and short-runs. According to the National Bureau of Economic Research (NBER), the average duration of the thirty-three cycles the United Sates (US) experienced between 1854 and 2009 is roughly fifty-six months. [1] Baxter and King (1999), following the NBER's investigations on the US economy,defined the business cycle as lasting between eighteen months and eight years. The long-run *can* therefore be defined as a period of time of at least eight years.

Growth models are made up of both theories and cross-country empirical investigations also called growth regressions. The essential of the post World War II (WWII) [2] growth models takes its inspiration from the neoclassical growth theory simultaneously and independently developed by the American and Austrian economists Robert Solow and Trevor Swan in 1956. Solow will be awarded the Nobel Prize in 1987 for his contribution. [3] Before Solow and Swan's contributions, the prevailing growth model was Roy Harrod's 1939 and Evsey Domar's 1946 AK model. The Harrod-Domar model assumes capital and labor are used in fixed proportions by firms. It then predicts that long-run growth equilibrium is unstable. A slight deviation of an economy from its long-run growth equilibrium path would result either in a growing unemployment or in a perpetual growth of idle machinery. Solow and Swan questioned the lack of substitutability between capital and labor in the Harrod-Domar model deeming the assumption implausible. They therefore replaced the fixed proportion production function (also called Leontief technology) in the Harrod-Domar model with a *neoclassical production function.* A neoclassical production function is a function that assumes *inter alia* that both capital and labor are needed to produce output, if the quantity used of both inputs are simultaneously, say, doubled, output also doubles, the marginal product of each of these inputs is positive but decreasing. A Cobb-Douglass production function is an example. In the Solow-Swan model, only firms behave optimally maximizing their profits. Households' saving rate is constant and exogenous. In 1965, Cass and Koopmans following Ramsey's 1928 paper endogenized saving in the Solow-Swan model assuming households also behave optimally choosing their consumption and saving so as to maximize their lifetime utility.

The Solow-Swan and the Cass-Koopmans-Ramsey models and their many extensions make up what is called the neoclassical or exogenous growth models in the sense they assume *technological progress*, the main determinant of long-run growth, takes place outside the model. Technological progress is the improvement in the existing means and methods of production or the invention of new ones. [4] Exogenous

[1] http://www.nber.org/cycles.html

[2] WWII opposing, among others, Germany, Italy, and Japan (the Axis) to Poland, France, the United Kingdom, the United States, and Canada (the Allied) broke out on September 1, 1939 with the German invasion of Poland and ended on September 2, 1945 with Japan formally surrendering.

[3] On the fiftieth anniversary of his seminal work, Solow (2007, pp 3-4) humbly gave his account of why his contribution and not Swan's became the most popular. In short, he deemed his approach simpler than Swan's.

[4] The terminologies technological progress and technical progress are interchangeably used in the

growth models also share a common feature: they assume the aggregate production function exhibits diminishing returns. Because of the diminishing returns in capital and labor, these models predict, in the long-run, per capita output across countries would *converge*. That is, countries with low per capita output would grow faster to catch up with countries with higher per capita output.

The 1980s witnessed the beginning of both the endogenous growth and the growth regressions literature. Endogenous growth theories, contrary to neoclassical growth theories, explain technological progress as the result of firms' research and development (R&D) activities and *human capital* as the abilities acquired by households through schooling and learning-by-doing. [5] Paul Romer (1986, 1990) and Robert Lucas Jr (1988) were the first to motivate and elaborate these theories. They were followed by Aghion and Howitt (1992). Romer (1990) dealt with what is called *horizontal innovations*. According to him, technological progress arises because firms engaged in R&D allocate resources to the design of new and distinct (producer) durables. As for Aghion and Howitt (1992), technological progress arises because R&D firms are concerned with designing improved quality durables that render previous ones out-of-date. That is what is called *vertical innovations*. Here are some examples of each of these two types of innovations. One can see the successive Microsoft Windows operating systems (Windows XP, Windows Vista, Windows 7, &c) and the successive Apple Macintosh operating systems (System, System Software, Mac OS, Mac OS X) as vertical innovations. On the other hand, the introduction in 2001 of both Microsolt tablet personal computer and Sharp Corporation's camera cellphone, the J-SH04, are horizontal innovations. Note that, earlier, Uzawa (1965) modeled technological change as an emanation from the education sector. Lucas Jr (1988) using Uzawa's framework introduced and endogenized human capital in the neoclassical growth framework. [6] He kept technological progress exogenous and presented human capital accumulation as an alternative growth engine increasing productivity and thus raising output.

Growth regressions mainly purport to (i) assess empirically the success of existing growth models using cross-country time series data, (ii) find out stylized facts across countries and over time in order to elaborate growth models consistent with observations. The growth regressions literature has enormously benefited from (i) the advances in econometrics particularly in the fields of cross-sectional time series analysis (also knows as panel data analysis) and nonparametric estimation along with (ii) the emergence of new and larger databases such as the Penn World Table. [7] Earlier growth regressions include Solow (1957) who, using US 1909-1949 data, estimated the contribution of capital and technological change to output growth. He attributed 87.5 % of the observed growth in ouptut to technological change. Then comes Romer (1986) who tested the convergence hypotheses.

literature. This paper holds on to the use of the former.

[5]Schultz (1960) and Becker (1964) introduced the concept of human capital. Becker was awarded the Nobel Prize in 1992 for integrating human behavior and interaction into microeconomics.

[6]Other well-known contributions of Uzawa to the growth literature include two-sector growth models, which are models attributing the production of consumption and investment goods to two different sectors: the consumption and the investment sectors. Each of these sectors combines its own capital and labor inputs to produce its output (Uzawa, 1961, 1963).

[7]The Penn World Table (http://pwt.econ.upenn.edu) is a database run at the University of Pennsylvania which contains, among other things, data on per capita gross domestic product and capital stock estimates for almost 190 countries over the whole or part of the sample period 1950-2004.

Barro (1991) used ninety-eight countries' data over the period 1960-1985 to test the convergence hypotheses and the contribution of human capital, goverment consumption, political stability, and market distortions to economic growth. He did not find strong evidence in the support of the convergence hypotheses.

There is a feedback relationship between growth theories and growth empirics. As an example, as said earlier, exogenous growth models predict that, in the long-run, per capita output across different countries will converge. This prediction was empirically tested by Romer (1986) and Lucas Jr (1988) among others who found no strong supporting evidence. This lack of empirical evidence called the *convergence controversy* led both of them to build alternative growth models consistent with their empirical findings.

Most growth theorists work with continuous time models. They solve optimal control problems and the tools often used are: differential calculus, the Pontryagin's maximum principle, and computer packages such as Maple and Matlab. The appendix on mathematical methods of Barro and Sala-i Martin's 2004 textbook goes through these tools. Goergen (2006) presents how both Maple and Matlab can be used to solve numerically growth models. Many growth theories assume no uncertainty. Agents thus have *perfect foresight*, *viz.* an accurate knowledge of the future. Some exceptions include Aghion and Howitt (1992), Azariadis (1981), and Cass and Shell (1983). Aghion and Howitt modeled the duration between two successive vertical innovations as a random variable following an exponential distribution. As for Azariadis and Cass and Shell, they modeled *extraneous* or *extrinsic* uncertainty, *i.e.*, an uncertainty arising from agents' subjective belief that prices are stochastic. This latter kind of uncertainty is also referred to as *self-fulfilling prophecies*.
As mentioned earlier, econometrics is the main tool used by growth empiricists. In the appendix of Aghion and Howitt's 2009 textbook, one can find some basic econometric techniques such as multiple regression and hypothesis testing used in the field. More advanced techniques include: cross-sectional time series regression to estimate models using samples made up of several countries' time series data, and nonparametric estimation to test hypotheses without assuming any functional relationship between the variables of interest.

Some concrete issues that growth economists investigate include fertility (Becker, Murphy, and Tamura, 1990), migration (Beine, Docquier, and Rapoport, 2001), education (Hartwick, 1992; Tran-Nam, Truong, and Van Tu, 1995; Shimomura and Tran-Nam, 1997; Ciriani, 2007), competition policy (Aghion, Harris, and Vickers, 1997; Aghion, Harris, Howitt, and Vickers, 2001; Acemoglu, Aghion, and Zilibotti, 2003; Aghion and Griffith, 2005), trade liberalization (Wacziarg and Welch, 2003), and democracy (Sirowy and Inkeles, 1990). The field has its own academic journal, the Journal of Economic Growth, established in 1996.

The rest of this paper is organized as follows. Section 2 presents some popular exogenous growth models whereas Sections 3 deals with endogenous growth models. Both theories and empirical evidence are presented. Most of the empirical evidence relates to the Canadian economy. Section 4 concludes by presenting some prospects for future research.
In Section 2, the evolution of growth theories from Solow and Swan's contribution to the convergence controversy is traced. The empirical investigations carried out in that section are: (1) the estimation of the contribution of capital, labor and

technological progress to growth in Canada [8] and (2) the assessment of the convergence hypothesis using the data of the ten provinces of Canada and those of eight countries making up the West African Monetary and Economic Union.
Section 3 presents the models of Lucas Jr, Romer, and Aghion and Howitt as well as some empirical evidence. After presenting the model of Romer, I have reviewed some contributions to that model by Jones (1995), Benassy (1998), and Alvarez-Pelaez and Groth (2005). The contribution of Jones followed the observation that growth rate and the size of skilled labor were not linearly related as endogenous growth models predicted. As for Benassy and Alvarez-Pelaez and Groth, they relaxed the assumptions on some parameters to show that it is possible the model of Romer generate excess R&D. The empirical investigations in that section consist of: (1) the break down and analysis of the share of labor income in the aggregate output by households' educational attainment, (2) the study of the relationship betwwn growth and skilled labor, and (3) the study of the relationship between output and R&D expenditure in Canada. It appears throughout this paper that each new growth theory is designed to amend or completement the prevailing one.

2 Some Exogenous Growth Models

Three exogenous growth theories are sketched in this section: the seminal Solow and Swan's model, the Cass-Koopmans-Ramsey and the overlapping generations models. Then follow two empirical investigations. In the first one, I have estimated the contribution of each of the three main inputs, which are capital, labor, and technological progress, to growth in Canada over 1976-2005. I then compare my estimates to two previous studies that covered the sample periods 1947-1973 and 1960-1995 in Canada. My second, investigations show convergence of GDP across the ten provinces of Canada and the eight countries making up the West African Monetary and Economic Union.

2.1 The Solow-Swan Model

The economy is composed of households and firms. Firms produce an aggregate output Y using as inputs both (physical) capital K rented from households and labor L. Each household inelastically supplies one unit of labor. Both inputs are essential in the production of Y and are fully employed. This implies L equates the population which grows exponentially at the constant rate n per time unit. As for the aggregate capital stock, it depreciates at the constant rate $0 \leq \delta \leq 1$ per time unit and its motion is described by the following relation

$$\dot{K} = I - \delta K$$

where the overdot denotes differentiation with respect to time $t \geq 0$, $\dot{K}$ is therefore the rate of change of capital or the net investment and I denotes gross investment.

The aggregate production function $F: \ Y = F(K, L)$ exhibits both *diminishing returns* in capital and labor and *constant returns to scale*. By diminishing returns, one means that the marginal products of both inputs, which are all positive, are decreasing. In mathematical terms, diminishing returns means the second derivatives of F with respect to K and L are negative, $F_{KK} < 0$ and $F_{LL} < 0$. The constant

[8] This exercise is called growth accounting.

returns to scale assumption means when it happens the quantities of capital and labor used in the production process are simultaneously, say, doubled, output also doubles, $\lambda Y = F(\lambda K, \lambda L),\ \lambda > 0$. The former assumption means the aggregate production function is concave and the latter assumption has two implications. First, with the optimizing firms paying capital and labor their marginal products, the aggregate output equals the sum of the gross capital and labor incomes received by households. [9] Second, the production function can be written in *intensive form*, *i.e* per capita output can be expressed as a function of per capita capital stock,

$$y = \frac{Y}{L} = F\left(\frac{K}{L}, 1\right) = f(k).$$

It is also assumed that the marginal products vanish when the quantities used of the inputs become extremely large and they explode when the inputs are more and more used in very small quantities. These last properties are called Inada conditions. [10]

At equilibrium, the aggregate supply of output equals its aggregate demand and, as a consequence, households' saving equates firms' investment. Assuming further that households save a constant fraction $0 < s < 1$ of their income, one, after some algebraic manipulations involving the law of motion of capital, ends up with what is called *the fundamental equation of the neoclassical growth theory* (henceforth, the fundamental equation) – for more details, see Appendix A.

$$\dot{k} = sf(k) - (n + \delta)k. \tag{2.1}$$

When $\dot{k}$ is positive this means per capita capital stock is rising over time because new saving (or gross investment) $sf(k)$ is greater than capital depreciation $(n+\delta)k$. A negative $\dot{k}$ means it per capita capital stock is falling due to insufficient saving. *Balanced growth* is defined as a situation characterized by a no growth in per capita capital stock, $\dot{k} = 0$; aggregate capital stock and output growing at the same rate n as population. Whatever the initial position of the economy – assume an initial per capita capital stock $k(0)$ lower than the balanced growth per capita capital stock $k^\star$ as in Figure 2.1, it will end up, in the long-run, at $k^\star$ due to the diminishing returns in capital. Assume now the economy is experiencing balanced growth, *i.e.* $\dot{k} = 0$, and the saving rate s has permanently increased. This will temporarily raise $\dot{k}$ and thus permanently raise the per capita capital stock and output. In the absence of technological progress, the growth in k fades out as the economy is reaching its new balanced growth path.

A further implication can be derived from the fundamental equation. The growth rate and the level of per capita capital stock are negatively related. This result is behind the *conditional* and *absolute convergence* hypotheses.
According to the conditional convergence hypothesis, an economy grows faster the further it is from its own balanced growth path (Barro and Sala-i Martin, 2004, pp 46-7). Consequently, if two economies are endowed with the same production function and have the same population growth, saving, and depreciation rates (and thus the same balanced growth per capita capital stock and output) but different initial per capita capital stock and output, the poorer economy, *i.e.*, the one with

[9] This is a result from the Euler theorem and the two first order conditions from firms' profit maximization program.

[10] After its author, the Japanese economist Ken-Ichi Inada (1963).

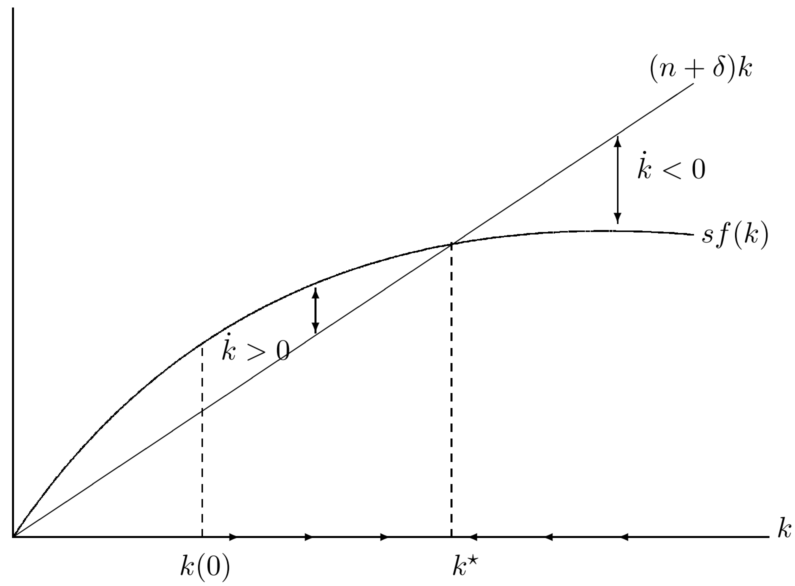

Figure 2.1: The Solow-Swan Model

the lowest per capita capital stock, will grow faster than the richer (Aghion and Howitt, 1998, pp 16-7). As for the absolute convergence hypothesis, it predicts that, in general, poor economies grow faster in per capita terms than rich ones.

On a balanced growth path, the only way to generate a sustainable growth in per capita variables is to continually have a *labor augmenting technological progress*, *i.e.*, technological knowledge appears in the production function as a multiple of labor. The production function thus takes the form $Y = F(K, A.L)$, where A denotes the state of technological knowledge (see, for details, Barro and Sala-i Martin, 2004, pp 51-5).

On the fiftieth anniversary of his seminal work, Solow (2007) attributed the unexpected success of his model to its simplicity, rightness, and plausibility. [11] Solow's 1956 model is not difficult to grasp or deal with. He assigned no functional form to the aggregate production function. Only its properties and shape are defined. Compared to the prevailing Harrod-Domar model, the model of Solow was plausible since it has matched some stylized facts as one will see in Subsection 2.5.

2.2 The Cass-Koopmans-Ramsey Model

The saving rate s in the Solow-Swan model is constant and exogenous. As a result, households do not necessarily behave optimally in that model. Cass (1965) and Koopmans (1965) inspired by Ramsey's 1928 paper amended the Solow-Swan model so as to explain households' saving. They allow the saving rate to change over time by assuming households choose their consumption and consequently their saving to maximize their lifetime utility subject to their budget constraint. Households are identical and infinitely lived. They derive utility from consumption. Their aggregate lifetime utility discounted back to time 0, their birth date, is defined

[11] Trevor Swan passed away in 1989.

by [12]

$$U = \int_0^{\infty} u\left[c(t)\right] \exp\left[(n-\rho)t\right] dt$$

where the parameter $0 < \rho < 1$ denotes the discount rate (also called time preference rate), u denotes the representative household's instantaneous utility function, and c his consumption function. The function u is concave, *i.e.*, the marginal utility $u^{'}(c)$ is positive but decreasing, $u^{'}(c) > 0$ and $u^{"}(c) < 0$. There is no disutility of labor and, each period, a household inelastically supplies one unit of labor in return of the competitive real wage w. They hold (financial) assets rented to firms in return of the real interest rate r. The aggregate budget constraint expressed in per capita terms is

$$\dot{a} = w + (r-n)a - c,$$

where the variables a and $\dot{a}$ denote respectively per capita assets and the accumulation of new assets by a household. Maximizing the discounted lifetime utility function subject to the budget constraint yields the following condition which describes the optimal path of consumption. The interested reader is referred to Appendix B for some details on solving a dynamic optimization problem in continuous time.

$$-\frac{u^{"}(c)c}{u^{'}(c)}.\frac{\dot{c}}{c} = r - \rho, \tag{2.2}$$

The left hand side element in the above condition is the inverse of the elasticity of inter-temporal substitution. Assuming a constant inter-temporal elasticity of substitution (CIES) utility function, [13]

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta}, \quad \theta > 0, \tag{2.3}$$

the optimal path of consumption described above becomes

$$\theta\frac{\dot{c}}{c} = r - \rho. \tag{2.4}$$

It appears that the pattern of per capita consumption depends on the magnitude of the difference between the interest and the discount rates. Consumption rises over time when this difference is positive, it falls when the difference is negative and is stable when it is nil.

At equilibrium, the assets held by households equal the capital stok rented by firms, $a(t) = k(t)$. Moreover, the real interest paid to households equal the marginal product of capital net of depreciation $r = f^{'}(k) - \delta$. Substituting this latter relation into the optimal path of consumption, on has

$$\theta\frac{\dot{c}}{c} = f^{'}(k) - \delta - \rho. \tag{2.5}$$

In the previous subsection, the fundamental equation was expressed as: $\dot{k} = sf(k) - (n+\delta)k$ — relation (2.1), where the first right hand side element denotes

[12] A household's birth date is the time he has become economically active.

[13] The shape of the CIES utility function depends on the value of the parameter θ. The utility function is linear when $\theta = 0$. When $\theta = 1$, its shape is like that of a Cobb-Douglas function and when $\theta = -\infty$, it looks like a Leontief utility function.

per capita saving. Now that saving is endogenous, the fundamental equation can be rewriten as follws:

$$\dot{k} = f(k) - c - (n + \delta)k. \tag{2.6}$$

In the Solow-Swan model, the saving rate s is constant and exogenous. But, in the Cass-Koopmans-Ramsey model, it is equal to $1 - c/f(k)$ and its dynamic behavior depends on two offsetting effects: the *substitution* and the *income effects*. When the capital stock rises, its marginal product $f^{'}(k)$ decreases and so does the real interest rate r, which in its turn causes a fall in the saving rate. [14] This induced effect is termed intertemporal substitution. As for the income effect, it causes the saving rate to rise when per capita capital stock increases. The behavior of the saving rate will therefore depend on the relative importance of these two effects. Unlike the Solow model, inefficient oversaving cannot occur in the Cass-Koopmans-Ramsey model because households are rational and are optimizing their welfare. The dynamics of per capita capital stock and consumption can be represented in a figure called phase diagram plotting relations (2.5) and (2.6) for when $\dot{c}$ and $\dot{k}$ are zeroh— for more details, see Barro and Sala-i Martin 2004, p 100 and Aghion and Howitt 1998, p 20.

The Cass-Koopmans-Ramsey model has subsequently been extended to include the government and foreign sectors, migration, desutility of labor, adjustment costs for investment. These contributions are presented in Barro and Sala-i Martin's 2004 textbook.

2.3 The Overlapping Generations Model

In the Cass-Koopmans-Ramsey model, households are identical and infinitely lived. Their age does not matter and no one passes away. Samuelson (1958), considering the undeniable fact that "we live in a world where new generations are always coming along" built a model in which new generations of households continually appear, coexist with older generations, grow old, procreate, and pass away. Diamond (1965) furthered Samuelson's 1958 work by including a neoclassical production function and capital as input into the overlapping generations model. Cass and Yaari (1967) following Samuelson and Diamond elaborated a continous time version of the overlapping generations model. A variant of their model is here presented.

All households live for one year. Thus at time $t \geq 0$, the population consists of households born at time $v \in [t-1,\ t]$. The size of the cohort born at time v — the generation v— is $\exp(nv)$. Assuming no one retires, the size of the labor force at time t is

$$L(t) = \int_{t-1}^{t} \exp(nv)dv = \frac{1 - \exp(-n)}{n} \exp(nt).$$

A household born at time v faces the following utility maximization problem

$$\begin{aligned} \max_{a(t,v),\ c(t,v)} \quad & \int_{v}^{v+1} u\left[c(t,v)\right] \exp\left[-\rho(t-v)\right] dt \\ \text{subject to} \quad & \dot{a}(t,v) = w(t) + r(t)a(t,v) - c(t,v), \\ & a(v,v) = a(v+1,v) = 0, \\ & c(t,v) \geq 0, \end{aligned}$$

[14] Recall $F_{KK} < 0$. So is $f^{''}(k)$.

where $a(t, v)$ and $c(t, v)$ respectively denote the asset holdings and the consumption at time t of a household born at time v. [15] According to the second constraint in the above optimization problem, a household does not inherit any asset at birth and does not either leave any. Assuming a logarithmic utility function, the optimal consumption path derived from the above program is

$$\frac{\dot{c}(t,v)}{c(t,v)} = r(t) - \rho. \tag{2.7}$$

The per capita aggregate consumption and assets at time t are defined as

$$\mathbf{c}(t) = \frac{1}{L(t)} \int_{t-1}^{t} c(t,v)\exp(nv)dv,$$
$$\mathbf{a}(t) = \frac{1}{L(t)} \int_{t-1}^{t} a(t,v)\exp(nv)dv,$$

where $c(t, v)$ and $a(t, v)$ are the closed form solutions obtained from solving both relation (2.7) and the budget constraint.

Assuming capital stock does not depreciate, the fundamental equation as it is specified in (2.6) becomes

$$\dot{k}(t) = f\left[k(t)\right] - \mathbf{c}(t) - nk(t),$$

which, after some substitution and rearrangement, yields as a necessary condition for a balanced growth equilibrium ($\dot{k}(t) = 0$)

$$\begin{aligned}\frac{f(k) - nk}{f(k) - f'(k)k} &= \frac{n}{1-\exp(-n)}\,\frac{\rho}{1-\exp(-\rho)}\,\frac{1-\exp(-r)}{r}\times \\ &\quad \frac{1-\exp\{-(n+\rho-r)\}}{n+\rho-r} \\ \psi(r) &= \phi(r).\end{aligned}$$

The per capita capital stock k such that

$$f'(k) = r = \rho = n$$

could be one of the solutions to the above equation provided the curves ψ and ϕ are tangent at $r = n$. On the existence of a balanced growth equilibrium, Cass and Yaari (1967) ended up with the conclusion that there may be none or several which are not necessarily efficient.

The overlapping generations framework has been used in the growth literature to investigate many policy issues including education financing, social security, public spending, and debt. For a revue, see de la Croix and Michel (2002) and Bewley (2007).

2.4 Growth Accounting

Because technological progress is not a measurable parameter, Solow (1957) initiated an indirect way of estimating, from known data, its contribution to growth.

[15] Apart from the fact that consumption and assets now depend on the household's birth date and the lifespan is finite, this optimization program looks like the one in Subsection 2.2.

Table 2.1: Growth Accounting Estimates for Canada

	Contribution from			
GDP growth rate	capital	labor	Solow residual	Capital share
Christensen et al (1980) sample period: 1947-1973				
.0517	.0254	.0088	.0175	.44
	(49%)	(17%)	(34%)	
Jorgenson and Yip (2001) sample period: 1960-1995				
.0369	.0186	.0123	.0057	.42
	(51%)	(33%)	(16%)	
My estimates, sample period: 1976-2005				
.0312	.0168	.0137	.0007	.4
	(54%)	(44%)	(2%)	

This technique received the name of *growth accounting* and the estimate it yields was named, obviously after its author, *Solow residual*. Growth accounting is an attempt to explain output growth rate in terms of the growth rates of all the inputs used in the production process. Let's use as an illustration the general specification of an aggregate production technology with labor-augmenting technological progress. Taking the natural logarithm of such a function, differencing it with respect to the time variable, and then rearranging yields

$$\frac{\dot{Y}}{Y} = \left(\frac{F_K K}{Y}\right)\frac{\dot{K}}{K} + \left(\frac{F_L L}{Y}\right)\frac{\dot{L}}{L} + \mathrm{g}, \tag{2.8}$$

where $F_K = \partial F/\partial K$, $F_L = \partial F/\partial L$, and $\mathrm{g} = (F_L L/Y)\,\dot{A}/A$.

In (2.8), $F_K K/Y$ and $F_L L/Y$, respectively the shares of aggregate output used to remunerate capital and labor, and $\dot{Y}/Y$, $\dot{K}/K$, and $\dot{L}/L$, respectively the rates of growth of output, capital, and labor, can be approximated using national account data. [16] The contribution of technological progress to output growth, g, is thereafter estimated as a residual.

Growth accounting estimates for Canada produced by Christensen, Cunnings, and Jorgenson (1980) for the sample period 1947-1973 and Jorgenson and Yip (1980) for the period 1960-1995 are reported in Table 2.1 along with my own estimates for the period ranging from 1976 to 2005. The annual data used in my investigations are from Statistics Canada and are the chained Fished quantity index of the business sector's GDP at basic price, the chained Fisher aggregation of hours worked by all workers, the chained Fisher aggregation of capital stock, and the income-based GDP. The income-based GDP data are used to compute the shares of aggregate output used to remunerate capital and labor (capital and labor shares, in short). The growth rate of output, $\dot{Y}/Y$ in (2.8), is approximated by the first difference $\ln Y_t - \ln Y_{t-1}$; the growth rates of the capital and labor inputs are computed along the same lines. The associated capital and labor shares are the average values between periods t and $t-1$.

[16] Cooley and Prescott (1995) and Gomme and Ruper (2005) detailed how to compute the shares of aggregate output used to remunerate capital and labor.

The sample average capital share I have reported is .4, which is almost the same as the ones over the two other sample periods in the table. This observation is in line with Kaldor's 1957 stylized fact according to which capital share is constant over time. Furthermore, the three empirical investigations indicate that capital stock accounts for about half of the observed growth in real output. However, there are some differences in the estimated rates of growth and the contributions from labor and technological progress over the three sample periods. Real output growth and the contribution of technological progress turns out to decrease over time. Output growth was 5.17% over the period 1947-1973, 3.69% over 1960-1995, and 3.12% over 1976-2005. As for the contribution of technological progress, it was 34% over the period 1947-1973, 16% over 1960-1995, and 2% over 1976-2005.

2.5 The Convergence Controversy

Equation (2.1), the fundamental equation, suggests a negative relationship between growth and per capita capital stock

$$\frac{\partial(\dot{k}/k)}{\partial k} = s\frac{kf'(k) - f(k)}{k^2} < 0, \tag{2.9}$$

with $f(k) - kf'(k) = F_L$, the marginal productivity of labor, which is positive.

Relations (2.1) and (2.9) thus lead to the following prediction called the conditional convergence hypothesis: (1) two economies endowed with the same production technology and the same saving, population, and depreciation rates have the same balanced growth per capita capital and output, (2) if the only difference between these two economies happens to be their starting level of per capita capital and output, then the economy with the lower per capita capital (the poorer economy) will grow faster to catch up with the other one (the richer economy). Relaxing the assumption on the underlying parameters leads to an alternative hypothesis called the absolute convergence hypothesis which simply predicts that poor economies tend to grow faster per capita than rich ones.

Romer (1986, pp 1008-9) observed a monotonic increase in the productivity growth rate across such successive leading countries as the Netherlands, the UK, and the US between 1700 and 1979. The productivity growth rate of the Netherlands, identified as a leader between 1700 and 1785, was nearly nil — -.07 %, more precisely. During two subsequent periods, which are 1785-1820 and 1820-1890, the UK was the leading country. It experienced a growth rate of .5 % and 1.4 %. Then, came the US with a growth rate of 2.3 % between 1890 and 1979. Focusing on the US, over five time periods ranging from 1800 to 1978, Romer also observed an increase in the per capita growth rate, which was .58 % over the period 1800-1840 and 2.47 % between 1960 and 1978. Comparing industrialized countries to less developed ones, Romer (1986, p 1012) concluded that per capita growth rate increased not only over time but also with countries' level of development, which meant industrialized countries grew faster than less developed ones. Lucas Jr (1988, p 4) also came up with similar findings. These observations, *i.e.* the positive trend in the per capita growth rate and the positive correlation between growth rate and the level of development, led Romer (1986) to question the convergence hypotheses derived from exogenous growth models. He sustained convergence did not occur because production function did not exhibit diminishing returns as exogenous growth models assumed. He then posited that the positive trend in the growth rate was due to increasing returns in the production function.

To test empirically for absolute convergence, one estimates econometric models of the form

$$\bar{\mathrm{g}}_i = b_0 - b_1 \ln y_{it_0} + \varepsilon_i, \tag{2.10}$$

where the explained variable $\bar{\mathrm{g}}_i$ denotes the average growth rate over the sample period of real per capita GDP in country i , the parameter $0 < b_1 < 1$ denotes the speed of convergence, the explanatory variable y_{it_0} denotes the initial real per capita GDP, an ε_i is the normally distributed error term with a zero mean and a constant variance. The average growth rate $\bar{g}_i$ is computed as $(1/T)\sum_{t=t_0}^{t_0+T-1}(\ln y_{it+1} - \ln y_{it})$, which reduces to $(\ln y_{it_0+T} - \ln y_{it_0})/T$. Instead of just using the sample endpoints to compute the average growth rate, one can drop the logarithm and rather average year-to-year growth rates as follows $(1/T)\sum_{t=t_0}^{t_0+T-1}(y_{it+1} - y_{it})/y_{it}$.

I have used model (2.10) to test for convergence across the ten provinces of Canada and the eight countries that makes up the customs and currency union called UEMOA. [17] UEMOA is a French acronym standing for Union Économique et Monétaire Ouest-Africain, which means in English the West African Economic and Monetary Union. These countries share a common currency called the CFA Franc. The UEMOA was created in January 1994. Its last member, which is also the only non French speaking country of the organization, joined in May 1997. The data for Canada come from Statistics Canada and those of the UEMOA from the Central Bank of West African States.

The sample period used in testing for absolute convergence across Canada ranges from 1981 to 2009. The ordinary least squares (OLS) estimates are reported below. The t-ratios,*i.e.*, the ratios of the estimated parameters to their respective standard deviations, are put in brackets.

$$\begin{aligned} \hat{\bar{\mathrm{g}}}_i &= .22 - .02 \ln y_{1981,i} \\ &\quad (5.79) \;\; (-5.36) \\ \bar{R}^2 &= .75 \quad t_{5\%}(8) = 1.86 \\ \sigma_{\ln y_{1981}} &= .27 \quad \sigma_{\ln y_{2009}} = .14. \end{aligned} \tag{2.11}$$

The two estimated parameters are statistically significant, *i.e.*, the absolute value of their t-ratios, 5.79 and 5.36, are greater than the 5 percent critical value, 1.86. The slope parameter points to convergence. The standard deviation of the logarithm of real per capita GDP decreased in 2009 compared to 1981. This means a reduction over time of disparities between the ten provinces. Figure 2.2 plots the actual and fitted values. Note that a similar exercise by Barro and Sala-i Martin (2004, p 46-7) using personal income data over the period 1880-2000 also shows absolute convergence across the US adjoining states. [18]

For the UEMOA, the sample period, which ranges from 1971 to 2010, has been subdivided into two: (1) 1971-1996 to test for absolute convergence across the eight countries before the union was created in 1994 and before it was joined by its last member country in 1997, and (2) 1997-2010 to test for convergence since the last member country joined the union. The OLS estimates for the two subsample

[17] Canada is made up of ten provinces and three territories.

[18] US adjoining states are the forty eight US states that are located in the middle of North America. This excludes, from the US, Alaska, Hawaii, and all the off-shore territories.

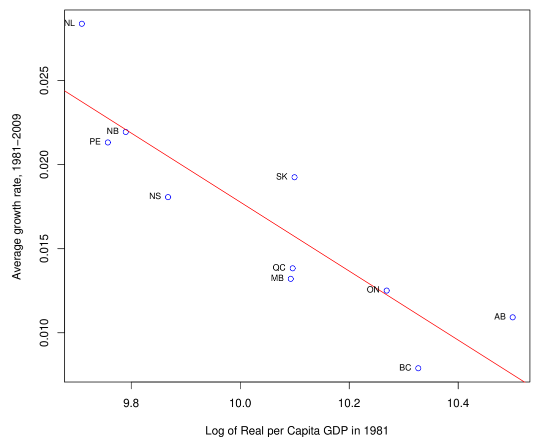

Figure 2.2: Convergence of Real GDP across Canada's Ten Provinces — Alberta (AB), British Columbia (BC), Manitoba (MB), New Brunswick (NB), Newfoundland and Labrador (NL), Nova Scotia (NS), Ontario (ON), Prince Edward Island (PE), Quebec (QC), and Saskatchewan (SK).

periods are displayed below.

$$
\begin{aligned}
\hat{\bar{g}}_i &= .1 - .006 \ln y_{1971,i} \\
&\quad (1.522) \quad (-1.07) \\
\bar{R}^2 &= .021 \quad t_{5\%}(6) = 1.94 \\
\sigma_{\ln y_{1971}} &= .64 \quad \sigma_{\ln y_{1996}} = .59
\end{aligned} \tag{2.12}
$$

$$
\begin{aligned}
\hat{\bar{g}}_i &= .34 - .023 \ln y_{1997,i} \\
&\quad (2.23) \quad (-2.02) \\
\bar{R}^2 &= .31 \quad t_{5\%}(6) = 1.94 \\
\sigma_{\ln y_{1997}} &= .59 \quad \sigma_{\ln y_{2010}} = .47
\end{aligned} \tag{2.13}
$$

Before 1997, even though the sign and the magnitude of the estimated speed of convergence are correct, there is no statistically significant evidence of convergence and the model only explains 2. 1% of the observed variability in the data. On the other hand, for the subsample period 1997-2010, there are evidence supporting the absolute convergence hypothesis. The estimated speed of convergence is statistically significant and higher. Besides, the explanatory power of the model improved. One also observed a decrease in the dispersion of real per capita GDP across theses countries. This means the creation of the customs and currency union indeed contributed towards economic integration in the eight UEMOA's members countries. Figure 2.3 plots the actual and fitted data for the two subsample periods.

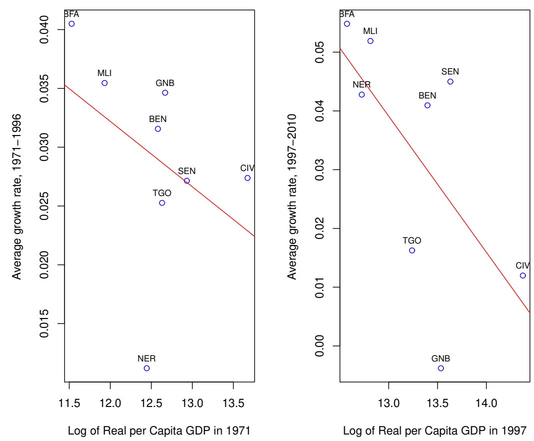

Figure 2.3: Convergence of Real GDP across the Eight UEMOA's Member Countries — Benin (BEN), Burkina Faso (BFA), Côte d'Ivoire (CIV), Guinea-Bissau (GNB), Mali (MLI), Niger (NER), Senegal (SEN), and Togo (TGO).

3 Some Endogenous Growth Models

Three popular endogenous growth theories are presented in this section. These theories are the Uzawa-Lucas Jr, the Romer, and the Aghion and Howitt models. In the Lucas Jr model, growth is driven by human capital accumulation whereas in the other two models, it is driven by product innovations. These three models are sometimes referred to as the first generation of endogenous growth models. Some criticisms and contributions to the model of Romer by Jones (1995), Benassy (1998), and Alvarez-Pelaez and Groth (2005) are also presented. Thereafter, I have carried out some empirical investigations. The first set of investigations shows the importance of distinguishing, in a growth model, between different types of labor according to households' skills. As a matter of fact, in Canada, unskilled labor and its share in aggregate income are decreasing whereas the proportion in the labor force of workers with a university degree is increasing. The second set of investigations show the positive contribution of labor broken down by educational attainment to growth in Canada. The last investigations show the positive relationship between GDP and the R&D expenditure across Canada.

3.1 The Uzawa-Lucas Model

Lucas Jr (1988), due to the inability of the Solow-Swan model to account for the diversity observed between rich and poor countries, added to that model human capital accumulation as an engine of growth that complements technological progress. Population grows at the exogenous rate n. Households are endowed with the CIES utility function specified in (2.3) on page 9. Each household allocates his time endowment normalized to unity between human capital accumulation through education $e(t)$ and labor $1-e(t)$. Lucas Jr endogenized human capital in the same way as Uzawa (1965) modeled technological change. The evolution of human capital depends on $e(t)$ as follows

$$\dot{h}(t) = \psi e(t)h(t),$$

where the variable $h(t)$ denotes human capital and $\psi > 0$ its productivity parameter. The labor force is defined as

$$L = \int_0^\infty L(h)dh,$$

where $L(h)$ denotes the number of households with the skill level h. The effective labor force is defined as $L^e = (1-e)hL$, where it is assumed that households are identical and have the same level of human capital and supply the same hour. The aggregate resource constraint is:

$$L(t)c(t) + \dot{K}(t) = AK(t)^\alpha \left[(1-e(t))h(t)L(t)\right]^{1-\alpha} h_a(t)^\beta,$$

where the right-hand side element is the aggregate production function. The variable $h_a(t)$, in the aggregate production function, is the average level of human capital and equals $h(t)$, and the term $h_a(t)^\beta$ captures the external effects of human capital. The aggregate production function exhibits increasing returns due to the presence of this term.

The Euler equations from maximizing households' utility subject to the aggregate resource constraint and the human capital production function are [19]

$$\theta\frac{\dot{c}}{c} = \alpha\frac{Y(t)}{K(t)} - \rho \tag{3.1a}$$

$$\frac{\dot{Y}(t)}{Y(t)} + \frac{\dot{e}(t)}{1-e(t)} + \frac{1-\alpha+\beta}{1-\alpha}\psi = \alpha\frac{Y(t)}{K(t)} + \frac{1-\alpha+\beta}{1-\alpha}\psi e(t), \tag{3.1b}$$

where $Y(t) = AK(t)^\alpha \left[(1-e(t))h(t)L(t)\right]^{1-\alpha} h_a(t)^\beta$.

Relation (3.1a) governs the inter-temporal substitution of consumption while (3.1b) governs the intra-temporal trade-off between education and labor.

Let now the constants ν and κ denote respectively the rate of growth of per capita human capital and consumption along the balanced growth path (BGP). One has $\kappa = \nu(1-\alpha+\beta)/(1-\alpha)$, which means consumption grows faster than human capital. Both variables would grow at the same rate in the absence of the external effect of human capital, *viz*, when $\beta = 0$.

The balanced growth share of time allocated to human capital accumulation is constant. Both the aggregate stock of physical capital and output grow at the rate $\kappa + n$. Evaluating the two Euler equations along the BGP gives the optimal rate of growth of human capital

$$\nu^* = \frac{\psi}{\theta} - \frac{1-\alpha}{1-\alpha+\beta}\frac{\rho-n}{\theta}. \tag{3.2}$$

This growth rate positively depends on the human capital productivity parameter ψ, the parameter of its external effect β, and on the share of physical capital income in the aggregate output α. On the other hand, it negatively depends on the time preference rate ρ.

[19] See details in the appendix.

3.2 The Romer Model

Romer's 1990 model is based on three premises. First, technological progress is instrumental in economic growth. Second, it results from the design of new and distinct durables by private profit-maximizing R&D firms, *viz.* innovations are horizontal. Third, it is a *nonrival* and *partially excludable* input. An input is nonrival if it can be used simultaneously by several firms. An input is said to be excludable if other firms can be prevented from using it. Technological knowledge is nonrival because the knowledge underlying previous innovations is freely used by all R&D firms in designing new durables. As examples, consider the use of the microprocessor in the design of the first cellphone by Motorola, Inc or the use of the charge-coupled device in the design of the first digital still camera by Kodak Company. Technological progress is partially excludable because each R&D firm obtains a patent for its design. This patent is then sold to a selected intermediate firm who will manufacture the durable and be its exclusive supplier.

The economy consists of three production sectors: the R&D, the intermediate, and the final sectors.

The R&D sector – It combines human capital H_A with the existing stock of technological knowledge A to design $\dot{A}$ new durables. A design is associated with one unit of technological knowledge. Thus, A, in addition to being the level of technological knowledge, also denotes the number of durables designed up to time t.

$$\dot{A}(t) = BH_A(t)A(t), \tag{3.3}$$

where B denotes the productivity parameter. They face the following optimization problem

$$\begin{aligned} &\max_{H_A(t)} P_A(t)BH_A(t)A(t) - w(t)H_A(t) \\ &H_A(t) : P_A(t)BA(t) = w(t), \end{aligned} \tag{3.4}$$

where P_A is the competitive price the patent of a design is sold to an intermediate firm.

The intermediate sector – It produces durables out of the final good and the designs purchased from the R&D firms. Let the continuous variable $i \in [0, A]$ index the durables. The production of one unit of each durable i requires η units of the final good rented from households at the price r and purchasing beforehand from the R&D firm i its license at the price P_A

$$P_A(t) = \int_t^\infty \pi(\tau, i) \exp\left\{-\int_t^\tau r(s)ds\right\} d\tau, \tag{3.5}$$

where r denotes the real interest rate and π the monopoly profit made by intermediate firm i

$$\begin{aligned} \pi(t,i) = &\max_{x(t,i)} p(t,i)x(t,i) - \eta r(t)x(t,i) \\ x(t,i) : &\frac{p(t,i) - \eta r(t)}{p(t,i)} = -\frac{\partial p(t,i)}{\partial x(t,i)}\frac{x(t,i)}{p(t,i)} \end{aligned} \tag{3.6}$$

where p denotes the monopoly price at which the durable i is rented to the final sector and x the quantity supplied. According to (3.6), the intermediate firm's profit is maximized when its markup rate equals the inverse of the price elasticity of demand.

The final sector – The final output Y is produced using as inputs unskilled labor L, human capital H_Y, and the available set of durables rented from the intermediate firms.

$$Y(t) = \left[\int_0^{A(t)} x(t,i)^\alpha di\right] H_Y(t)^\beta L(t)^{1-\alpha-\beta} \tag{3.7}$$

In the production function (3.7), the elasticity of substitution between any pair of durables, say durables 1 and 2, is $1/(1-\alpha) > 1$. [20] This means no durable is a close substitute (a case where the elasticity of substitution is infinite) or a perfect complement (a case where the elasticity of substitution is zero) to any other. It turns out from the final sector profit maximization problem that

$$x(t,i): \ \alpha x(t,i)^{\alpha-1} H_Y(t)^\beta L(t)^{1-\alpha-\beta} = p(t,i) \tag{3.8a}$$

$$H_Y(t): \ \beta \frac{Y(t)}{H_Y(t)} = w(t) \tag{3.8b}$$

Substituting (3.8a) into (3.6), one has

$$p(t,i) = p(t) = \frac{\eta r(t)}{\alpha} \tag{3.9a}$$

$$\pi(t,i) = \pi(t) = (1-\alpha)p(t)x(t), \quad i \in [0, A(t)], \tag{3.9b}$$

meaning all durables, at equilibrium, are supplied in the same quantity and at the same price. It emerges from the above relations that the markup rate in the intermediate sector equals $1-\alpha$

Durables do not depreciate and one has the following aggregate resource constraint $\dot{K} = Y - C$, where C denotes aggregate consumption and K, the capital stock, is defined as follows

$$K(t) = \eta \int_0^{A(t)} x(t) di = \eta A(t) x(t). \tag{3.10}$$

Population, unskilled labor, and human capital are exogenous. Households are endowed with a CIES utility function defined over consumption just as in (2.3) on page 9.

Along the BGP, Y, K, C, and A all grow at the constant rate BH_A.

$$\mathrm{g} = BH_A = \frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{C}}{C} = \frac{r-\rho}{\theta} \tag{3.11}$$

All the other variables are constant. Evaluating the equations along the BGP and after some substitution and rearrangement, the growth rate can be expressed in terms of the fundamentals as follows [21]

$$\mathrm{g} = \frac{BH - \Lambda\rho}{1 + \theta\Lambda}, \tag{3.12}$$

[20] Let F designate the production function specified in (3.7), the elasticity of substitution between any two durables, say durables 1 and 2, is defined as $d\ln(x_2/x_1)/d\ln(F_{x_1}/F_{x_2})$, where F_{x_1} and F_{x_2} are respectively the marginal products of durables 1 and 2.

[21] See the details in Appendix D.

where $\Lambda = \beta / [(1-\alpha)\alpha]$ and $H = H_A + H_Y$.

According to (3.12), growth rate positively depends on the aggregate stcok of human capital. This means an economy with a larger stock of human capital will experience a faster growth than underdeveloped economies that have a low level of human capital. It also appears that unskilled labor has no growth effect.

According to relation (3.11), the per capita output growth rate is proportional to the number of households engaged in R&D. This implies that if the latter variable, say, doubled, so should the former. This is termed the *scale effect* prediction. Jones (1995) showed that between 1950 and 1087, the size of the labor force engaged in R&D in the US quintupled whereas the per capita output growth rate remained stationary. The same observation was made in such other advanced economies as France, Germany, and Japan. This led Jones to develop what he called a *semi-endogenous* growth model. His amendment to the model of Romer is to replace (3.3), the R&D sector's design production function, with the following relation

$$\dot{A}(t) = BH_A(t)^{\chi} A(t)^{\xi},\ 0 < \chi \leq 1.$$

It follows that the growth rate becomes

$$\mathrm{g} = \frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K} = \frac{\dot{C}}{C} = BH_A(t)^{\chi} A(t)^{\xi - 1}. \tag{3.13}$$

Since g, the growth rate, is constant along the BGP, log-differentiating (3.13) yields

$$\chi \frac{\dot{H}_A}{H_A} + (\xi - 1)\frac{\dot{A}}{A} = 0 \Leftrightarrow \chi n_A + (\xi - 1)\mathrm{g} = 0.$$

It follows from the above relation that

$$\mathrm{g} = \frac{\chi n_A}{1 - \xi}.$$

The contribution of Jones is qualified as semi-endogenous because it says the growth rate depends not on the level of the human capital used in the R&D sector but rather on its exogenous growth rate.

There are several other contributions to the model of Romer. Some of these contributions relate to the magnitude of the human capital allocated to R&D. The model of Romer predicts that too little human capital is allocated to R&D. This is due to the fact that the knowledge underlying previous innovations are freely used by the current R&D firms. Consequently, P_A, the market price of a design, is lower than its social value and human capital is under-compensated. Benassy (1998) and Alvarez-Pelaez and Groth (2005) showed that the model of Romer can generate excess R&D when one relaxes some strong assumptions on the parameters of (3.7), the final output technology.

Benassy suggested replacing (3.7) with

$$Y(t) = A^{\alpha + \upsilon - 1} \left[\int_0^{A(t)} x(t,i)^{\alpha} di \right] H_Y(t)^{\beta} L(t)^{1-\alpha-\beta}, \quad \upsilon > 0 \tag{3.14}$$

The purpose of doing so is to dissociate, at equilibrium, the elasticity of the final output with respect to the existing number of durables (also known as degree of returns to specialization) from $1-\alpha$, the markup rate in the intermediate sector. In the model of Romer, at equilibrium, all durables are supplied in the same quantity as it appears in (3.10) and consequently (3.7), the final output technology, becomes

$$Y(t) = A(t)^{1-\alpha}\left[\frac{K(t)}{\eta}\right]^{\alpha} H_Y(t)^{\beta} L(t)^{1-\alpha-\beta},$$

with the degree of returns to specialization being equal to $1-\alpha$. With (3.14), the production function suggested by Benassy (1998), one instead has at equilibrium

$$\begin{aligned} Y(t) &= A(t)^{\upsilon}\left[\frac{K(t)}{\eta}\right]^{\alpha} H_Y(t)^{\beta} L(t)^{1-\alpha-\beta}, \\ &= \left[\frac{K(t)}{\eta}\right]^{\alpha}\left[A(t)^{\frac{\upsilon}{1-\alpha}} H_Y(t)\right]^{\beta}\left[A(t)^{\frac{\upsilon}{1-\alpha}} L(t)\right]^{1-\alpha-\beta}, \end{aligned}$$

and the degree of returns to specialization becomes υ. He then showed that with a sufficiently low υ, too much R&D can occur in the model of Romer. The above specification of the production function shows that, in the model of Benassy, output and physical capital growth rate differs from the growth rate of technological knowledge. Both growth rates are related as follows

$$\mathrm{g}_A = \frac{1-\alpha}{\upsilon}\mathrm{g}, \tag{3.15}$$

where g designates the growth rate of final output and physical capital and g_A designates the growth rate of technological knowledge. One can see from (3.15) that, for υ sufficiently low, precisely for $\upsilon < 1-\alpha$, $\mathrm{g}_A = BH_A > \mathrm{g}$.

As for Alvarez-Pelaez and Groth (2005), they furthered the contribution of Benassy by dissociating the capital share, α, from the markup rate. Recall from (3.9b) that, in the model of Romer, the markup rate is $1-\alpha$. To have a markup of $1-\epsilon$ in the intermediate sector, they then suggested the following final output production function

$$Y(t) = A(t)^{\upsilon}\left\{A(t)\left[\frac{1}{A(t)}\int_0^{A(t)} x(t,i)^{\epsilon} di\right]^{\frac{1}{\epsilon}}\right\}^{\alpha} H_Y(t)^{\beta} L(t)^{1-\alpha-\beta}, 0<\epsilon<1,$$

where ϵ is the substitution parameter. Technological knowledge growth rate expressed in terms of the fundamentals then becomes [22]

$$\begin{aligned} \mathrm{g} &= \upsilon\frac{BH - \Lambda'\rho}{1-\alpha+\Lambda'\left[(\theta-1)\upsilon+1-\alpha\right]} \\ \text{where } \Lambda' &= \frac{\beta}{(1-\epsilon)\alpha}. \end{aligned} \tag{3.16}$$

Observe that for $\upsilon = 1-\alpha$ and $\epsilon = \alpha$, (3.16) equals (3.12), the growth rate in the model of Romer. For just $\epsilon = \alpha$, it is equal to the growth rate in the model of Benassy. The lower are υ, the return to specialization, and ϵ, the substitution parameter, the higher the growth rate of technological knowledge and accordingly the human capital used in R&D are.

[22] See details in Appendix E.

3.3 The Aghion and Howitt Model

Schumpeter (1942) presented the concept of *creative destruction* as the essence of capitalism. By the oxymoron creative destruction, he was referring to the continuing invention or discovery of new goods, methods of production, and markets, which keeps revolutionizing fundamentally the economic structure, destroying the old one after creating a new one. Aghion and Howitt (1992), following Schumpeter, built a model in which growth is driven by the innovation of improved quality durables that render existing ones out-of-date. Their economy consists of infinitely lived households and three sectors, which are the R&D, intermediate, and final sectors.

The households – They are endowed with preferences that are linear in consumption. They are also endowed with one unit of labor that they supply inelastically with no disutility. Moreover, they are heterogeneous in their abilities. They are either skilled or unskilled. The L^s skilled households are hired by the R&D and intermediate firms whereas the L^u unskilled households work in the final sector. Because preferences are linear in consumption, (2.2) implies that, at any time $t \geq 0$, the interest rate equates the time preference rate.

The R&D sector –R&D firms compete to come up with the design of the next generation of durable, which will be the $i+1$st innovation. In addition to indexing innovations, $i = 0, 1 \ldots$ also denotes the time interval $[t_i, t_{i+1}[$ over which the i-th innovation is the most up-to-date. Over the interval i, L^s_{Ai} skilled households are assigned each the design of a new durable. Each of these equally qualified households engaged in R&D has the same and constant probability $0 < \lambda < 1$ to innovate. The *expected* number of innovations per unit of time is therefore λL^s_{Ai}. This number is also referred to as the aggregate arrival rate of innovations. As for the *actual* number of innovations that occurs per unit of time, it follows a Poisson process with arrival rate λL^s_{Ai}. [23] The number of innovations being a Poisson random variable, it follows that the length of the interval i is also random and exponentially distributed with parameter λL^s_{Ai}. The successful innovator is granted a patent, which he sold at the competitive price P_{Ai+1} to an intermediate monopolist who will produce the durable. R&D firms therefore choose L^s_{Ai} so as to maximize their *expected* profits defined as

$$\max_{L^s_{Ai}} \lambda L^s_{Ai} \mathrm{E}\left(P_{Ai+1}\right) - w_i L^s_{Ai}$$

with as FOC

$$\mathrm{E}\left(P_{Ai+1}\right) = \frac{w_i}{\lambda}, \tag{3.17}$$

where E denotes the expectation operator and comes from the fact the length of the interval i is uncertain.

The intermediate sector – The intermediate firm manufactures the durable and rent it to the final good firms at the monopoly price p_i. The production of one unit of durable requires the labor of one skilled household. The quantity x_i of the latest durable supplied results from the intermediate firm's profit maximization problem

$$\pi_i = \max_{x_i} p_i x_i - w_i x_i, \tag{3.18}$$

[23] The Poisson distribution is a discrete probability distribution used to model the number of events occurring in a fixed time interval or in specified intervals such as distance, area, or volume. For more details, see among others, Casella and Berger (2001, pp 92-4).

where π_i denotes the profit made. The FOC of the above problem is

$$\frac{\partial p_i}{\partial x_i} x_i + p_i = w_i. \tag{3.19}$$

The price $\mathrm{E}(P_{Ai})$ the intermediate monopolist bids for the patent is the expected present value of the profits he will make over the random time interval i.

$$\begin{aligned} P_{Ai} &= \int_{t_i}^{t_{i+1}} \pi_i \exp\left[(\tau - t_i) r\right] d\tau \\ &= \frac{\pi_i}{r}\left[1 - \exp\left(-r\Delta t_i\right)\right] \end{aligned}$$

where $\Delta t_i = t_{i+1} - t_i \sim \text{exponential}(\lambda L^s_{Ai})$ because t_{i+1}, the time his monopoly position will be destroyed by the next innovation, is unknown and

$$\begin{aligned} \mathrm{E}(P_{Ai}) &= \lambda L^s_{Ai} \frac{\pi}{r} \int_0^\infty \left[1 - \exp(-r\Delta t_i)\right] \exp(-\lambda L^s_{Ai} \Delta t_i) d\Delta t_i \\ &= \frac{\pi_i}{r + \lambda L^s_{Ai}}. \end{aligned} \tag{3.20}$$

It emerges from (3.20) that more on-going R&D, *i.e.*, a higher L^s_{Ai}, shortens the duration of the current intermediate firm's monopoly position and as a result the present value of its expected flow of profits. This is what is called the effect of creative destruction.

The final sector – The final good firms production technology is Hicks neutral

$$y_i = A_i F(x_i), \; F' > 0, \; F'' < 0 \tag{3.21}$$

where y_i and A_i respectively denote the final output and the productivity parameter. The function F exhibits constant returns to scale. The adoption of the latest innovation raises the productivity by the factor $\mu > 1$,

$$A_i = \mu^i A_0. \tag{3.22}$$

The parameter μ is also referred to as the size of innovation. Since L^u the quantity of unskilled labor is fixed, one can ignore the wage bill in the final good firms' profit maximization problem

$$\begin{aligned} &\max_{x_i} A_i F(x_i) - p_i x_i \\ &x_i : A_i F'(x_i) = p_i. \end{aligned} \tag{3.23}$$

Now evaluating both (3.18) and (3.19), the intermediate firm's profit and FOC, using the above result, one gets

$$\begin{aligned} \omega_i = \frac{w_i}{A_i} &= F'(x_i) + F''(x_i) x_i \\ \pi_i &= -x_i^2 A_i F''(x_i) = A_i \tilde{\pi}(\omega_i). \end{aligned} \tag{3.24}$$

Equating (3.17) to the first lead of (3.20), substituting for (3.24), then normalizing both sides with A_i and rearranging, yields the arbitrage equation

$$w_i = \lambda\mu \frac{\tilde{\pi}(\omega_{i+1})}{r + \lambda L^s_{Ai+1}}. \tag{3.25}$$

Skilled labor market clears whenever

$$L^s = L^s_{Ai} + \tilde{x}(\omega_i), \tag{3.26}$$

where $x_i = \tilde{x}(\omega_i)$, with $\tilde{x}'(\omega_i) < 0$. The above two equations determine the equilibrium level of wage and labor used in R&D. The balanced growth equilibrium level of labor used in research L^s_A appears to increase with a decrease in the interest rate r, or an increase in the other parameters, which are the arrival rate λ, the size of innovations μ, and the available quantity of skilled labor L^s .

Besides, along the BGP, the production function can be expressed as

$$y_i = A_i F(L^s - L^s_{Ai}),$$

implying

$$\begin{aligned} y_{i+1} &= \mu y_i \\ \ln y(t_{i+1}) - \ln y(t_i) &= \ln \mu. \end{aligned}$$

If one sets $t_{i+1} = t_i + 1$, the growth rate can be written as

$$\mathrm{g}_i = \ln y(t_i + 1) - \ln y(t_i) = \ln \mu \times \ \# \text{ of innovations},$$

Knowing that # of innovations $\sim$ Poisson(λL^s_A),

$$\begin{aligned} \mathrm{E}(\mathrm{g}_i) &= \ln \mu \mathrm{E}(\# \text{ of innovations}) = \lambda L^s_A \ln \mu \\ \mathrm{Var}(\mathrm{g}_i) &= (\ln \mu)^2 \mathrm{Var}(\# \text{ of innovations}) = \lambda L^s_A (\ln \mu)^2. \end{aligned} \tag{3.27}$$

It thus appears that both the average growth rate, $\mathrm{E}(\mathrm{g}_i)$, and its variance , $\mathrm{Var}(\mathrm{g}_i)$, positively depend on L^s_A the balanced growth level of labor used in R&D. The signs of the *ceteris paribus* effects of r, λ, μ and L^s_A on $\mathrm{E}(\mathrm{g}_i)$ and $\mathrm{Var}(\mathrm{g}_i)$ are therefore the same as the signs of their effects on L^s_A.

3.4 Empirical Investigations

The Aghion and Howitt and the Romer models predict a positive relation between growth rate and skilled labor. In this subsection, I investigate empirically this prediction. Beforehand, I have distinguished between three types of labor: unskilled, semi-skilled, and skilled labor. By unskilled labor, I refer to the households with primary and secondary education. Semi-skilled households are those with some post-secondary diploma and skilled households are those with a university degree. I have presented the evolution of their respective proportions in the labor force and shares of labor income in GDP. The data used in the first two sets of investigations relate to Canada's business sector, are from Statistics Canada and taken over the sample period 1961-2006 (46 years). The last investigations in this subsection study the relationship between GDP and R&D in Canada.

3.4.1 The Importance of Distinguishing Labor by Type of Skill

Figure 3.1 plots four labor shares: the total labor share and those of unskilled, semi-skilled, and skilled labor. The total labor share is stable over the whole sample period with an average of .6. The share of the unskilled households' labor income in

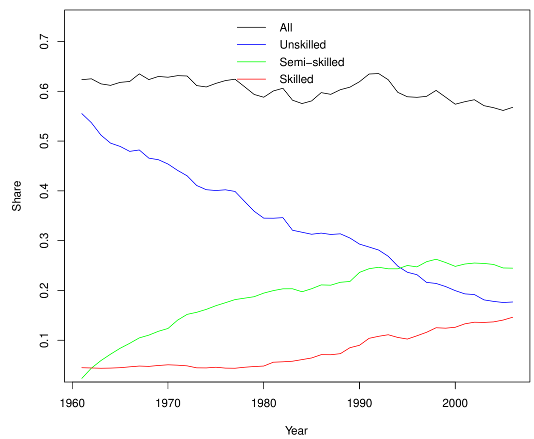

Figure 3.1: Labor Share Composition in Canada, 1961-2006 .

Table 3.1: Labor Share Composition, Canada, 1961-2006

Components	Mean	Standard Deviation
All	.6	.02
Unskilled	.34	.11
Semi-skilled	.19	.07
Skilled	.08	.04

the aggregate output is negatively trended whereas those of both semi-skilled and skilled households are trended upwards. At the beginning of the sample period, 89.1% of the labor income were remunerating unskilled households. It is only 31.1% of that income that goes to them at the end of the sample period. In 1961, semi-skilled and skilled labor accounted respectively for 3.7% and 7.2% of the overall labor income. In 2006, it is rather semi-skilled labor and no longer unskilled labor which became the most important component of labor income. As a matter of fact, in 2006, semi-skilled and skilled households' labor incomes accounted respectively for 43.1% and 25.8% of the total labor income. Table 3.1 displays the mean and standard deviation of the labor shares. The standard deviations indeed point to a high variability in the labor shares of unskilled and semi-skilled households. Below, I have reported the OLS estimates of the linear relationship between the three components of labor share.

$$\begin{aligned} \hat{\beta}_t^s &= .38 - .597\beta_t^u - 537\beta_t^{ss} \\ &\quad (10.7) \;\; (-10.9) \;\; (-5.8) \\ \bar{R}^2 &= .89 \;\; t_{2.5\%}(43) = 2.02, \end{aligned} \tag{3.28}$$

where β_t^s, β_t^u, and β_t^{ss} respectively denote the skilled, unskilled,semi-skilled households' labor shares at time t. The estimated parameters are all statistically significant. The negative sign of the slope parameters indicates that the labor shares of

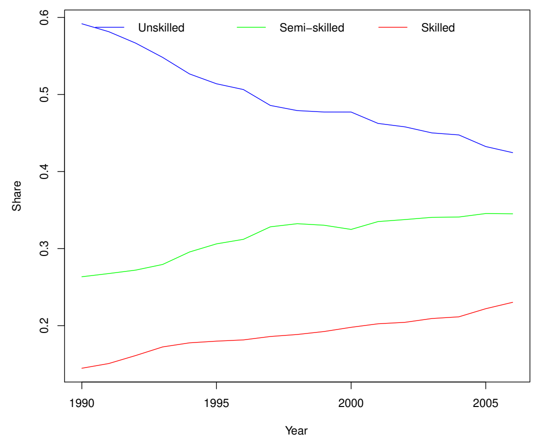

Figure 3.2: The Share of Unskilled, Semi-Skilled, and Skilled Households in Employment in Canada, Labor Force Survey Statistics, Statistics Canada, 1990-2006 .

both unskilled and semi-skilled households decrease over time while that of skilled households is increasing. The factor influencing this dynamics is the improvement in households' level of education in Canada. As one could see from Figure 3.2, the share of households with a university degree (the skilled households) in employment keeps rising over time while the proportion of unskilled households is decreasing.

These studies show that the three types of labor do not follow the same dynamics, hence the importance of distinguishing between them in modeling the aggregate production function as Romer (1990), Aghion and Howitt (1992), and Mankiw, Romer, and Weil (1992) did. A growth model should be able to account for changes in the level and the shares of the different types of labor.

3.4.2 Economic Growth and Skilled Labor

Human capital or skilled labor appears in the endogenous growth literature as the main source of growth. The purpose of the current investigation is to check this empirically. The econometric model to be estimated is

$$\mathrm{g}_t = b_0 + b_1 L_t^s + \varepsilon_t \tag{3.29}$$

where g_t, L_t^s and ε_t respectively denote the real GDP growth rate, skilled labor, and the error term. The data used for L_t^s is the chained Fisher aggregation of the hours worked by households with a university degree. L_t^s turns out to be a trended variable whereas g_t is stationary. I therefore replace the explanatory variable by $\Delta \ln L_t^s$, the first-difference of its logarithm. The OLS estimates are

$$\begin{aligned} \hat{\mathrm{g}}_t &= .015 + .415 \Delta \ln L_t^s \\ &\quad (1.83) \quad (3.05) \\ \bar{R}^2 &= .16 \quad t_{2.5\%}(43) = 2.02. \end{aligned} \tag{3.30}$$

This show a positive relationship between skilled labor and economic growth as some theories predicted. The slope parameter is statistically significant. The intercept term is not significantly different from zero. The explanatory power of the model is not that high since the adjusted determination coefficient $\bar{R}^2$ is only .16.

To improve the explanatory power of the model, I have included the two other types of labor, *i.e.*, semi-skilled and unskilled labor. Here are the estimates

$$\begin{aligned} \hat{g}_t &= .023 + .55\Delta \ln L_t^u + .107\Delta \ln L_t^{ss} + .105\Delta \ln L_t^s \\ &\quad (3.08) \quad (3.9) \qquad\quad (3.91) \qquad\quad (.86) \\ \bar{R}^2 &= .56 \qquad t_{2.5\%}(43) = 2.02. \end{aligned} \tag{3.31}$$

All the coefficients in (3.31) are correctly signed. The $\bar{R}^2$ is much higher but this time skilled labor no longer contributes significantly to economic growth. Dropping this variable yields

$$\begin{aligned} \hat{g}_t &= .029 + .621\Delta \ln L_t^u + .105\Delta \ln L_t^{ss} \\ &\quad (8.75) \quad (5.39) \qquad\quad (3.86) \\ \bar{R}^2 &= .56 \qquad t_{2.5\%}(43) = 2.02. \end{aligned} \tag{3.32}$$

The fact that the skilled labor coefficient in (3.31) is not significant arises from the collinearity between this variable and unskilled labor. The correlation coefficient between $\Delta \ln L_t^s$ and $\Delta \ln L_t^u$ is .57. This brings to mind a theoretical finding one can derive from the model of Romer (1990). When human capital is the only factor accounting for the difference between the skilled and unskilled households' labor income, the quantity of unskilled labor and that of the skilled labor used in the final sector are directly proportional.

3.4.3 Output and R&D

In this subsection, I empirically investigate the relationship between output and the expenditure on R&D using an log-linear econometric model

$$\ln Y_t = b_0 + b_1 \ln I_{RD,t} + \varepsilon_t, \tag{3.33}$$

where the explanatory variable $I_{RD,t}$ denotes the annual real expenditure on R&D and the slope parameter $b_1 > 0$ can be interpreted as the *elasticity of output with respect to the knowledge R&D expenditure*. This latter parameter indicates the percentage change in output induced by a one-percent increase in the real expenditure on R&D. I carried out the investigations with data of Canada using three techniques: (1) the *cross-section* estimation, *i.e.*, I estimated model (3.33) separately for each of the ten provinces of Canada, (2) the *pooled* estimation, *i.e.*, I piled up one after another the data of the ten provinces to run a single regression instead of ten as in the previous case, and (3) the group estimation, *i.e.* I simply used the data of Canada, which are the sum of data of the ten provinces plus the the three other territories of Canada. The data used to measure ΔA_t and Y_t are Statistics Canada's 1981-2006 annual series on (1) the real gross domestic expenditure on R&D funded and performed by all sectors in the fields of Natural Science and Engineering and (2) real GDP. The scatter plot of the two time series are displayed in Figure 3.3 for each of the ten provinces. On the figure, it clearly appears that

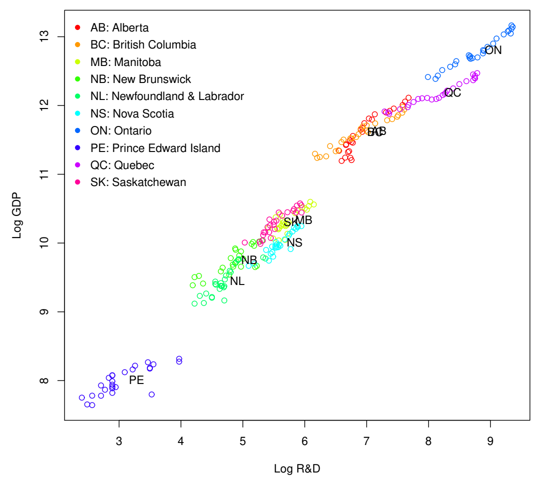

Figure 3.3: Scatter Plot of Log Gross Domestic Expenditure on R&D and Log GDP across the Ten Provinces of Canada, Statistics Canada, 1981-2006 .

the relationship between the two variables is linear and that the two variables are positively related. The provinces experiencing the highest R&D activities (Ontario, Quebec, British Columbia, and Alberta) have the highest GDPs and those such as The Prince Edward Island accumulating less knowledge through R&D have the lowest GDPs.

The results of the regressions are displayed in Table 3.2. Comparing the t-ratios, which are in parentheses, to the corresponding critical values, which are on the bottom of the table, it appears that all the coefficients are highly statistically significant. They are all correctly signed, *i.e.* positive. The adjusted determination coefficients, $\bar{R}^2$ are also quite high.

4 Some Directions for Future Research

On March 17, 2011, Quebec's former Liberal Finance Minister, Mr Raymond Bachand, announced in his 2011-2012 budget speech an increase in university tuition. From Fall 2012 till 2017, tuition would increase each year by $ 325 to reach $ 3 793 in 2017. To protest against this decision, students started on February 13, 2012 what became the longest student strike in Quebec's history. At the beginning of March, the number of students on strike was estimated at 120 000. [24] On September 5, the unpopular Liberal government was defeated at the general elections. The Parti Québécois who won the elections abolished the decision to increase tuition and recommended its indexation to the rate of growth of households' disposable income, which roughly was 3%.

[24]For some details on this, see the article *Grève étudiante québécoise de 2012* (in English, *2012 Quebec student strike*) on Wikipedia.

Table 3.2: Regressing Real GDP on Real Gross Domestic Expenditure on R&D, Canada, 1981-2006

	Intercept	Slope	$\bar{R}^2$
Albertta	6.113	.789	.869
	(14.32)	(12.89)	
British Columbia	8.266	.482	.958
	(59.37)	(23.95)	
Manitoba	6.655	.639	.623
	(11.804)	(6.502)	
New Brunswick	7.415	.478	.628
	(20.94)	(6.57)	
Newfoundland & Labrador	6.239	.681	.749
	(16.97)	(8.69)	
Nova Scotia	5.851	.739	.854
	(17.17)	(12.15)	
Ontario	7.964	.548	.966
	(44.22)	(26.82)	
Pince Edward Island	6.836	.379	.645
	(39.87)	(6.81)	
Quebec	9.156	.37	.955
	(69.92)	(23.06)	
Saskatchewan	6.473	.687	.837
	(19.31)	(11.38)	
Pooled data	5.628	.823	.975
	(109.15)	(99.86)	
Canada	8.18	.58	.986
	(62.2)	(42.03)	

$t_{5\%}(24)=1.71$, $t_{5\%}(258)=1.65$

Students who protested against Liberals' decision claimed that a rise in tuition would make higher education less affordable and reduce university attainment. Are they right? What are actually the impacts of a rise in tuition on university attainment, human capital accumulation, R&D, and economic growth? Several articles in the literature including Glomm and Ravikumar (1992), Tran-Nam, Truong, and Van Tu (1995), and Shimomura and Tran-Nam (1997) dealt with education financing. But few studies delved into the specific issue of the impacts of a rise in the tuition rate on the time allocated to education and consequently human capital accumulation and growth.

Besides, further contributions can be made to some existing growth theories such as the model of Romer and that of Aghion and Howitt. In these models, growth is explained by the amount of human capital used in R&D. But this latter variable is kept constant for convenience reason. Instead of merely having technological progress, the exogenous determinant of growth in neoclassical models, replaced by a human capital that is held constant, one can endogenize this latter variable assuming households accumulate it through education. Furthermore, the definition of physical capital in the model of Romer is limited to the stock of producer durables. Augmenting this model with the stock of structures as Greenwood, Hercowitz, and Krusell (1997) did to study the impacts of investment-specific technological change could also be interesting. These are issues I am tackling nn my PhD thesis (Accolley, 2015a,b).

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Appendices

A The Fundamental Equation of the Neoclassical Growth Theory

The law of motion of (physical) capital, K, is

$$\dot{K} = I - \delta K,$$

where I is firms' gross investment. At equilibrium, gross investment equals households' saving. And saving is a constant fraction of households' income, which itself equals aggregate output. One can therefore rewrite the above relation as follows

$$\dot{K} = sF(K, L) - \delta K.$$

Divide both sides of the above relation by the population, L, to express the aggregate output in intensive form

$$\frac{\dot{K}}{L} = sf(k) - \delta k, \tag{A.1}$$

where $k \equiv K/L$ and $f(k) \equiv F(K, L)/L$. Differentiating the ratio k with respect to the time variable t, one has

$$\dot{k} = \frac{\dot{K}}{L} - nk, \text{ where } n = \frac{\dot{L}}{L}$$

Substituting the expression of $\dot{K}/L$ from the above relation into (A.1) gives

$$\dot{k} = sf(k) - (n + \delta)k$$

B Solving a Dynamic Optimization Problem in Continuous Time

Consider some identical and infinitely lived households seeking to maximize their lifetime utility given their budget constraint. Their optimization problem in per capita terms is

$$\begin{aligned} \max_{a(t),\ c(t)} \quad & \int_0^{\infty} u\left[c(t)\right] \exp\left[(n - \rho)t\right] dt \\ \text{subject to:} \quad & \dot{a}(t) = w(t) + \left[r(t) - n\right] a(t) - c(t). \end{aligned}$$

The function u is the instantaneous utility function. The endogenous variables $a(t)$ and $c(t)$ are the per capita financial assets and consumption. Households take as given real interest rate $r(t)$ and real wage $w(t)$. The parameters n and ρ respectively denote the population growth and the time preference rates. Households begin and end their lives without any financial asset.

Their optimization problem can equivalently be written as

$$\begin{aligned} \max_{a(t),\ c(t)} \quad & \int_0^{\infty} u\left[c(t)\right] \exp\left[(n - \rho)t\right] dt \\ & + \int_0^{\infty} \mu(t) \left\{w(t) + \left[r(t) - n\right] a(t) - c(t) - \dot{a}(t)\right\} dt, \end{aligned}$$

where the variable $\mu(t)$ is refereed to as Lagrange multiplier or the shadow price of capital. Let's define now the function $\mathcal{H}$ called present-value Hamiltonian function.

$$\begin{aligned}\mathcal{H}\left[c(t), a(t), \mu(t), t\right] = u\left[c(t)\right]\exp\left[(n-\rho)t\right] \\ + \mu(t)\left\{w(t) + \left[r(t)-n\right]a(t) - c(t)\right\},\end{aligned} \tag{B.1}$$

which enables us to rewrite the optimization problem as

$$\max_{a(t),\ c(t)} \quad \int_0^\infty \mathcal{H}\left[c(t), a(t), \mu(t), t\right]dt - \int_0^\infty \mu(t)\dot{a}(t)dt.$$

Integrating by parts the last element of the above problem results in

$$\max_{a(t),\ c(t)} \quad \int_0^\infty \mathcal{H}\left[c(t), a(t), \mu(t), t\right]dt + \int_0^\infty \dot{\mu}(t)a(t)dt \tag{B.2}$$

with as first-order conditions (FOCs)

$$c(t): \quad \frac{du\left[c(t)\right]}{dc(t)}\exp\left[(n-\rho)t\right] = \mu(t) \tag{B.3a}$$

$$a(t): \quad \mu(t)[r(t)-n] = -\dot{\mu}(t). \tag{B.3b}$$

Solving the above system of two equations gives

$$-\frac{\frac{d^2u[c(t)]}{dc(t)^2}}{\frac{du[c(t)]}{dc(t)}}c(t)\frac{\dot{c}(t)}{c(t)} = r(t) - \rho, \tag{B.4}$$

C The Optimization Problem in the Uzawa-Lucas Model

Infinitely lived and identical households seek to

$$\max \quad \int_0^\infty L(t)\frac{c(t)^{1-\theta}-1}{1-\theta}\exp(-\rho t)dt$$

subject to

$$\begin{aligned}&\text{(a)} \quad \dot{K}(t) = AK(t)^\alpha\left[(1-e(t))h(t)L(t)\right]^{1-\alpha}h(t)^\beta - L(t)c(t) \\ &\text{(b)} \quad \dot{h}(t) = \psi e(t)h(t)\end{aligned}$$

This problem is equivalent to

$$\begin{aligned}\max \int_0^\infty \mathcal{H}(c, e, h, K, \mu_1, \mu_2, t)dt + \int_0^\infty \dot{\mu}_1(t)K(t)dt + \int_0^\infty \dot{\mu}_2(t)h(t) \\ - \left[\mu_1(t)K(t)\right]_0^\infty - \left[\mu_2(t)h(t)\right]_0^\infty\end{aligned}$$

where

$$\begin{aligned}\mathcal{H}(c, e, h, K, \mu_1, \mu_2, t) = L(t)\frac{c(t)^{1-\theta}-1}{1-\theta}\exp(-\rho t) \\ +\mu_1(t)\left\{AK(t)^\alpha\left[(1-e(t))h(t)L(t)\right]^{1-\alpha}h(t)^\beta - L(t)c(t)\right\} \\ +\mu_2(t)\psi e(t)h(t)\end{aligned}$$

The FOCs

$$c(t): \quad c(t)^{-\theta}\exp(-\rho t) = \mu_1(t) \tag{C.1a}$$

$$e(t): \quad \mu_1(t)(1-\alpha)\frac{Y(t)}{1-e(t)} = \mu_2(t)\psi h(t) \tag{C.1b}$$

$$h(t): \quad \mu_1(t)(1-\alpha+\beta)\frac{Y(t)}{h(t)} + \mu_2(t)\psi e(t) = -\dot{\mu}_2(t) \tag{C.1c}$$

$$K(t): \quad \mu_1(t)\alpha\frac{Y(t)}{K(t)} = -\dot{\mu}_1(t) \tag{C.1d}$$

with $Y(t) = AK(t)^{\alpha}\left[(1-e(t))h(t)L(t)\right]^{1-\alpha} h(t)^{\beta}$,

The Euler equations are

$$\theta\frac{\dot{c}(t)}{c(t)} = \alpha\frac{Y(t)}{K(t)} - \rho \tag{C.2a}$$

$$\frac{\dot{\mu}_2(t)}{\mu_2(t)} = -\frac{1-\alpha+\beta}{1-\alpha}\psi + \frac{\beta}{1-\alpha}\psi e(t). \tag{C.2b}$$

One gets the Euler equation (C.2a) after solving (C.1a) and (C.1d) getting rid of $\mu_1(t)$. To have (C.2b), one first expresses $\mu_1(t)$ as a function of $\mu_2(t)$ using (C.1b) and then plugs it into (C.1c).

Along the BGP, $\frac{\dot{c}}{c} = \kappa$ and $\frac{\dot{h}}{h} = \nu$, it follows from (C.2a) that

$$\begin{aligned}\frac{Y}{K} &= \frac{\theta\kappa+\rho}{\alpha}\\ &= AK^{\alpha-1}\left[(1-e)hL\right]^{1-\alpha} h^{\beta}.\end{aligned} \tag{C.3}$$

Dividing the aggregate resource constraint by K, one has

$$\frac{Lc}{K} + + \frac{\dot{K}}{K} = \frac{Y}{K}. \tag{C.4}$$

The second element on the left-hand side of (C.4), $\dot{K}/K$, is constant. Now, differentiating (C.4) with respect to time recalling from (C.3) that its right-hand side element equals the constant $(\theta\kappa+\rho)/\alpha$, gives the balanced growth rate of capital

$$\begin{aligned}\frac{\dot{K}}{K} &= \frac{\dot{L}}{L} + \frac{\dot{c}}{c}\\ &= n + \kappa.\end{aligned} \tag{C.5}$$

Differencing the logarithm of the second line of (C.3) with respect to time and rearranging helps establish a relation between the balanced growth rates of per capita consumption and human capital

$$\kappa = \frac{1-\alpha+\beta}{1-\alpha}\nu. \tag{C.6}$$

Let's go back to the FOCs, From (C.1b), it turns out that the rate of growth of shadow price $\mu_2(t)$ is

$$\frac{\dot{\mu}_2(t)}{\mu_2(t)} = \frac{\dot{\mu}_1(t)}{\mu_1(t)} + \frac{\dot{Y}(t)}{Y(t)} + \frac{\dot{e}(t)}{1-e(t)} - \frac{\dot{h}(t)}{h(t)},$$

which gives after substituting for (C.1d)

$$\frac{\dot{\mu}_2(t)}{\mu_2(t)} = -\alpha\frac{Y(t)}{K(t)} + \frac{\dot{Y}(t)}{Y(t)} + \frac{\dot{e}(t)}{1-e(t)} - \frac{\dot{h}(t)}{h(t)}. \tag{C.7}$$

We end up with two expressions of the rate of growth of μ_2, relations (C.2b) and (C.7). Equating the two relations and evaluating them along the BGP, gives

$$(\alpha-\theta)\kappa + (1-\alpha+\beta)\nu + n - \rho = -\frac{1-\alpha+\beta}{1-\alpha}\psi + \frac{\beta}{1-\alpha}\nu.$$

Let's substitute for (C.6) into the above relation and solve for ν to get

$$\nu^* = \frac{\psi}{\theta} - \frac{1-\alpha}{1-\alpha+\beta}\frac{\rho-n}{\theta}, \tag{C.8}$$

which is the optimal balanced growth rate of human capital.

D The Growth Rate in the Model of Romer

According to relation (3.5), the exclusive right to manufacture and supply a durable is sold to intermediate firms at the price

$$P_A(t) = \int_t^{\infty} \pi(\tau)\exp\left\{-\int_t^{\tau} r(s)ds\right\} d\tau.$$

Deriving the above expression with respect to t, applying Leibniz rule gives

$$\dot{P}_A(t) = r(t)P_A(t) - \pi(t). \tag{D.1}$$

Since $P_A(t)$ is stationary, along the BGP, the above relation becomes,

$$P_A = \frac{\pi}{r}. \tag{D.2}$$

It emerged from (3.9b) that the intermediate firm's profit along the BGP is:

$$\pi = (1-\alpha)px$$

According to (3.10), along the BGP, x equals $\hat{K}/\eta$ with $\hat{K} \equiv K/A$. Plugging this into the above relation gives,

$$\pi = \frac{1-\alpha}{\eta}p\hat{K}. \tag{D.3}$$

Plugging (D.3) into (D.2) gives

$$P_A = \frac{1-\alpha}{\eta}\frac{p}{r}\hat{K}. \tag{D.4}$$

From (3.4), the R&D firms profit maximization problem, one has

$$\begin{aligned}\hat{w} &= P_A B \\ &= B\frac{1-\alpha}{\eta}\frac{p}{r}\hat{K},\end{aligned}$$

with $\hat{w} \equiv w/A$. Equating the above expression to (3.8b) yields

$$\beta \frac{\hat{Y}}{H_Y} = \frac{1-\alpha}{\eta} \frac{p}{r} \hat{K}, \text{ with } \hat{Y} \equiv \frac{Y}{A}, \Rightarrow$$
$$H_Y = \frac{\eta}{B} \frac{\beta}{1-\alpha} r \frac{\hat{Y}}{p\hat{K}}.$$

Plugging (3.10) into (3.7) and (3.8a), one finds out that the term $\hat{Y}/p\hat{K}$ in the above expression equals $1/(\alpha\eta)$. H_Y therefore becomes

$$H_Y = \frac{\Lambda}{B} r, \text{ where } \Lambda = \frac{\beta}{(1-\alpha)\alpha}. \tag{D.5}$$

Relation (3.11) states the growth rate along the BGP is,

$$\mathrm{g} = BH_A = B\left(H - H_Y\right) \tag{D.6a}$$
$$= \frac{r-\rho}{\theta} \Rightarrow r = \theta \mathrm{g} + \rho. \tag{D.6b}$$

Plugging (D.5) into (D.6a) gives

$$\begin{aligned} \mathrm{g} &= BH - \Lambda r = BH - \Lambda(\theta g + \rho) \\ &= \frac{BH - \Lambda\rho}{1 + \Lambda\theta}. \end{aligned} \tag{D.7}$$

E The Growth Rate in the Model of Alvarez-Pelaez and Groth

The final output production function in the model of Romer is replaced with

$$Y(t) = X(t)^{\alpha} \left[A(t)^{\frac{\upsilon}{1-\alpha}} H_Y(t)\right]^{\beta} \left[A(t)^{\frac{\upsilon}{1-\alpha}} L(t)\right]^{1-\alpha-\beta},$$
$$\text{with } X(t) = A(t) \left[\frac{1}{A(t)} \int_0^{A(t)} x(t,i)^{\epsilon} di\right]^{\frac{1}{\epsilon}}, \ 0 < \epsilon < 1.$$

The above function exhibits human capital and labor augmenting technological progress. This implies that along the BGP, final output and physical capital grows at the constant rate

$$\mathrm{g} = \frac{\upsilon}{1-\alpha} \mathrm{g}_A,$$

where g_A designates the growth rate of technological knowledge. Since $K(t) = \eta A(t) x(t)$, it follows that, along the BGP, $x(t)$, the quantity supplied of a durable, will grow at the rate $\left[\upsilon/(1-\alpha) - 1\right] \mathrm{g}_A$. So will $\pi(t)$, the profit made by an intermediate firm, and $P_A(t)$, the price a design is sold.

In the final sector, the FOC with respect to $x(i,t)$, which also represents the demand for durable $x(i,t)$, is

$$x(t,i): \ \alpha X(t)^{\alpha} \left[A(t)^{\frac{\upsilon}{1-\alpha}} H_Y(t)\right]^{\beta} \left[A(t)^{\frac{\upsilon}{1-\alpha}} L(t)\right]^{1-\alpha-\beta} \times$$
$$\left[\frac{1}{A(t)} \int_0^{A(t)} x(t,i)^{\epsilon} di\right]^{\frac{1}{\epsilon}-1} x(t,i)^{\epsilon-1} = p(t,i)$$

Given the demand function for durables, evaluating along the BGP (3.6), the intermediate firm's optimization problem, using the above FOC, one has

$$p = \frac{\eta}{\epsilon} r \tag{E.1a}$$
$$\tilde{\pi} = \frac{1-\epsilon}{\eta} p\hat{K}, \tag{E.1b}$$

with $\tilde{\pi} \equiv \pi / A^{\frac{\upsilon}{1-\alpha}-1}$ and $\hat{K} \equiv K/A^{\frac{\upsilon}{1-\alpha}}$

Let us now evaluate (D.1), the law of motion of the price of a design, along the BGP. We get

$$\tilde{P}_A = \frac{\tilde{\pi}}{r - \left(\frac{\upsilon}{1-\alpha} - 1\right) BH_A},$$

$\tilde{P}_A \equiv P_A / A^{\frac{\upsilon}{1-\alpha}-1}$. Plugging this latter result into (E.1b) yields

$$\tilde{P}_A = \frac{1-\epsilon}{\eta\left[r - \left(\frac{\upsilon}{1-\alpha} - 1\right) BH_A\right]} p\hat{K}.$$

Given $\hat{w} = B\tilde{P}_A$ in the R&D sector, we have

$$\hat{w} = B\frac{1-\epsilon}{\eta\left[r - \left(\frac{\upsilon}{1-\alpha} - 1\right) BH_A\right]} p\hat{K}.$$

In the final sector, since wage equals the marginal product of human capital, it follows that

$$\begin{aligned} H_Y &= \beta \hat{Y}/\hat{w} \\ &= \frac{\beta}{B(1-\epsilon)}\left[r - \left(\frac{\upsilon}{1-\alpha} - 1\right) BH_A\right] \eta \frac{\hat{Y}}{p\hat{K}} \end{aligned}$$

Recall from the previous appendix that the term $\eta\hat{Y}/p\hat{K}$ in the above relation equals $1/\alpha$, the inverse of the capital share. Plugging this into the above relation yields

$$H_Y = \frac{\Lambda'}{B}\left[r - \left(\frac{\upsilon}{1-\alpha} - 1\right) BH_A\right],$$
$$\text{where } \Lambda' = \frac{\beta}{(1-\epsilon)\alpha}.$$

From (2.4), we know that the rate of growth of consumption is related to interest rate as follows $r = \theta\dot{C}/C + \rho$. Along the the BGP, $\dot{C}/C = [\upsilon/(1-\alpha)]BH_A$. Plugging this information into the above relation and replacing H_Y with the expression $H - H_A$ yields ater rearrangement

$$\mathrm{g} = \upsilon \frac{BH - \Lambda'\rho}{1 - \alpha + \Lambda'\left[(\theta-1)\upsilon + 1 - \alpha\right]} \tag{E.2}$$