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On The Transmission of Continuous Cultural Traits*

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Abstract

This paper generalizes the discrete cultural transmission model proposed by Bisin and Verdier (2001) to continuous trait space. The resulting cultural evolutionary dynamic can be characterized by a continuous imitative dynamic in a population game in which a player's payoff is equal to the aggregate cultural intolerance he has towards other agents. We show that cultural heterogeneity is always preserved. In addition, we model each agent's cultural intolerance towards another agent as an increasing function of *cultural distance* — the distance between that other agent's trait and his own trait in the trait space. This captures people's general tendencies of evaluating culturally more distant people with stronger biases, and it is most easily modeled on a continuous trait space. We find that the curvature of the cultural intolerance function plays an important role in determining the long-run cultural phenomena. In particular, when cultural intolerance is a convex function of cultural distance, only the most extremely polarized state is a stable limit point.

Keywords: Cultural transmission, Continuous trait space, Cultural evolution, Imitative Dynamic, Polarization.

JEL Classifications: A14, C72, C73, D10, Z13

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1 Introduction

During the past decade, inter-generational cultural transmission has become an important subject in economics (see Bisin and Verdier (2011) for an extensive survey). Economists realize that to have a better understanding of economic behavior and outcomes, it is essential to understand how economically relevant cultural traits such as preferences, attitudes, opinions and beliefs are formed and transmitted across generations over time. Bisin and Verdier (2001) propose the seminal utility-maximizing framework for studying inter-generational transmission of discrete cultural traits, which is later generalized by Bisin et al (2009) and Montgomery (2010) to multiple-trait settings. While Bisin and Verdier’s (2001) model is widely used to study socioeconomic phenomena such as corruption (Hauk and Saez-Marti (2002)), religious intermarriage (Bisin et al (2004)) and cooperation (Tabellini (2008)), the discrete nature of the model may limit its applicability to the study of traits such as risk preferences, patience and interethnic attitudes (Panebianco (2014) and Buechel et al (2014)), which are better modeled as continuous.

In this paper, we generalize Bisin and Verdier (2001)’s model to continuous trait space. The cultural transmission process we consider is of probabilistic nature as in Bisin and Verdier (2001). Parents can costly exert effort to increase the probability of successful inculcation of their own traits to their children. If inculcation fails, a child adopts the trait from a randomly chosen role model in the society. We also maintain the important assumption called “imperfect empathy” in Bisin and Verdier (2001). That is, parents care about their children’s adopted cultural traits but they are biased in that they are intolerant towards other traits (*cultural intolerance*).¹

We find that the resulting cultural evolutionary dynamic can be viewed as a continuous imitative dynamic (cf. Cheung (2015))² in a population game in which a player’s payoff is equal to the aggregate cultural intolerance he has towards other agents. Montgomery (2010) shows that in the finite trait setting, the resulting cultural evolutionary dynamic can be viewed as the discrete replicator dynamic given specific functional forms. Our result generalizes Montgomery (2010)’s result in two directions: first, we extend his result from the finite trait setting to the continuous trait setting; second, we extend his result from the replicator dynamic to the more general imitative dynamics. More importantly, we show that imperfect empathy is sufficient to generate the commonly observed phenomenon of cultural heterogeneity as in Bisin and Verdier (2001).

Note that our model contrasts to several recent works including Bisin and Topa (2003), Doepke and Zilibotti (2008), Pichler (2010), Vaughan (2010), Panebianco (2014) and Buechel et al (2014),

¹Formally, imperfect empathy means that parents evaluate their children’s actions through their own (the parental) utility. This indicates that parents desire their children to adopt their (the parents’) traits.

²The replicator dynamic is a special case of imitative dynamics.

who consider continuous trait cultural transmission models based on Cavalli-Sforza and Feldman’s (1973) non-probabilistic framework: a child’s trait is a weighted sum of his parent’s trait and the average trait of the whole society. Both Panebianco (2014) and Buechel et al (2014) show that when the weights are determined by parents’s effort, imperfect empathy alone cannot lead to cultural heterogeneity. Instead, the cultural transmission process converges people to the social average trait. Compared to these works, our model preserves Bisin and Verdier’s (2001) insights on the connection between imperfect empathy and cultural heterogeneity and at the same time accommodates the needs for a model on continuous trait space.

Next, we equip the model with a novel feature: cultural intolerance as an increasing function of *cultural distance*, i.e., the distance between two traits in the trait space.³ This is consistent with the recent empirical literature on cultural economics (e.g., Guiso, Sapienza and Zingales (2006, 2009), Spolaore and Waacziarg (2009, 2013)) who find that people are less comfortable with culturally more distant people.⁴

We find that the curvature of the cultural intolerance function is crucial for determining the prediction of the cultural evolutionary dynamic. In particular, when the cultural intolerance function is a convex function of cultural distance, only those states that consist of no more than two traits are limit points, which are also rest points (or stationary points), of the cultural evolutionary dynamic. And among those states, only the most extremely polarized one is Lyapunov stable. The intuition is as follows. Parents make inculcation efforts that increase with aggregate cultural intolerance. Thus traits experiencing the highest aggregate cultural intolerance grow most quickly, which alters the profile of aggregate cultural intolerance. Under convexity of the cultural intolerance function, the highest aggregate cultural intolerance is experienced by the two most extreme traits. Therefore, the dynamic always has the tendency to move away from the current state as long as there are some positive masses on both of the most extreme traits, until the population eventually splits into two opposing groups with these two most extreme traits. This result shows that convexity of the cultural intolerance function is a sufficient condition for polarization.

When the cultural intolerance function is a strictly concave function of cultural distance, polarization is no longer Lyapunov stable under the cultural evolutionary dynamic. Moreover, the set of limit points (also rest points) under a strictly concave cultural intolerance function is a strict

³we consider the continuous trait space as described by a closed interval on the real line.

⁴As argued in Spolaore and Wacziarg (2009, 2013), differences in cultural traits can hinder development through a barrier effect. Less closely related societies are less likely to learn from each other, and so the diffusion of complex technological and institutional innovations is limited. Guiso, Sapienza and Zingales (2009) find that in the context of economic exchange, people from culturally more similar countries tend to trust each other more and breach contracts less frequently. See also Felbermayr and Toubal (2010) and Hahn (2013).

superset of the set of limit points under a convex cultural intolerance function. That is, the cultural evolutionary dynamic under a strictly concave cultural intolerance function can lead to more diversified population states. Intuitively, since the increment in cultural intolerance decreases as cultural distance increases, one can always find population states in which some traits different from the two most extreme traits is associated with aggregate cultural intolerance higher than those of the two most extreme traits. This result implies that non-concavity of the cultural intolerance function serves as a necessary condition for polarization.

In sum, our model provides an cultural transmission explanation for why a society may be divided into two opposing groups which are defined by their conflicting preferences or beliefs.⁵

The rest of this paper is organized as follows. Section 2 describes the inter-generational cultural transmission mechanism for continuous trait space, and derives the cultural evolutionary dynamic. Section 3 reviews the stability and convergence results on imitative dynamics for later analysis. Section 4 first shows that cultural heterogeneity is always preserved. Then we investigate the curvature of the cultural intolerance function and shows how it affects the long-run behavior of the cultural evolutionary dynamic. Section 5 concludes. Some proofs and definitions omitted from the text are presented in the Appendix.

2 Inter-generational Cultural Transmission Mechanism

2.1 Background

Inter-generational cultural transmission mechanisms are first formally introduced by Cavalli-Sforza and Feldman (1981) and Boyd and Richerson (1985). Building on those works, Bisin and Verdier (2001) introduce a model in which the probability that a child adopts a certain trait is endogenously determined by the parent’s effort. One crucial assumption made by Bisin and Verdier (2001) is that the parents have “imperfect empathy”. That is, a parent’s effort choice decision is motivated by his evaluation of his child’s payoff, and the evaluation is based on the parent’s own utility function (paternalistic altruism). Hence, a parent is always bias against other traits (*cultural intolerance*)

⁵Polarization is an important empirical phenomenon. For example, “cultural war” in the United States which describes the tug-of-war between the liberal left and the conservative right, has been a heated political topic for decades (see Hunter (1992)). Polarization also has significant impacts on economics and politics. As argued by Sartori (1966, 1976), public ideological movement towards extremism serves as the basis for the creation of highly polarized multiparty systems. Polarization in people’s political beliefs, also known as political polarization, has been investigated in the literature of political economics. For example, Dixit and Weibull (2007) propose an elegant model to illustrate that political polarization can arise from heterogeneous beliefs about the nature of the world even people’s are homogeneous. See also McMurray (2015). In addition, confirmation bias in psychology (see Nickerson (1998), Rabin and Schrag (1999)) can also give rise to polarization in beliefs. Yet to our limited knowledge, no work has been done on explaining polarization in persistent traits such as preferences.

and want to inculcate his own trait to his child. It turns out that parents with stronger cultural intolerance exert higher effort on inculcation. Nevertheless, the parent to child transmission is probabilistic. Therefore, there is still some chance that the child adopts a trait that is different from his parent's. In this section, we follow the same spirit of Bisin and Verdier (2001) and build up a cultural transmission mechanism, but on continuous trait space.

Consider a population of unit mass of agents. Each agent in the population has a trait from set $T = [0, 1]$.⁶ Let \mathcal{B} be the Borel σ -algebra on T . Note that if the trait space T were finite, then the population state at a particular time could be described by a vector in $\mathbb{R}^{|T|}$,⁷ as in Bisin and Verdier (2001), Bisin et al (2009), and Montgomery (2010). On the other hand, for the case of continuous trait space we consider, the population state is instead described by a probability measure over T . Denote by $\mathcal{M}_1^+(T)$ the space of probability measures on (T, \mathcal{B}) . Then a *population state* is a distribution of traits over T and is described by a probability measure $\mu \in \mathcal{M}_1^+(T)$.

At each time t , agents in the population are selected uniformly at random, and the selected agent becomes a parent and bears a child. Assume that the child does not have a defined trait, and the parent has to decide how much effort he would exert to inculcate his own trait into his child. This is called “direct vertical” socialization in Bisin and Verdier (2001). If the parent fails to inculcate his own trait to his child, the child will randomly search for a role model in the population and adopt the trait of the role model. This is called “oblique” socialization in Bisin and Verdier (2001). After the child's trait is formed, the parent will be replaced by the child in the current population.

2.2 Parents' Decisions

Consider the decision of a parent with trait $z \in T$ on how much effort to put into inculcation. Let V_{zy} denote the parent's belief of his child's payoff if his child finally adopts trait $y \in T$. Assume that $V_{zz} \geq V_{zy}$ for any $y \in T$. This reflects the “imperfect empathy” assumption in Bisin and Verdier (2001): a parent always believes that his child would be better off to adopt his own trait. Without loss of generality, we assume that $V_{zy} \in [0, 1]$ for all $z, y \in T$.

Denote by $\Delta_{zy} := V_{zz} - V_{zy}$ the *cultural intolerance* a z -parent has towards trait $y \in T$. We assume that V_{zy} is continuous in z and y , and hence Δ_{zy} is continuous in z and y . This assumption captures the following two important features of our cultural transmission model with continuous trait space, which are absent when the trait space is finite:

⁶Here we follow the classic spatial models by Hotelling (1929) and Downs (1957). In fact, T can be any closed interval on \mathbb{R} . We just normalize it to $[0, 1]$.

⁷ $|A|$ denotes the number of elements in set A .

- i) The continuity of V_{zy} in y means that the evaluation of utility for the other trait y by a z -parent is continuous in y .
- ii) The continuity of V_{zy} in z means that the intolerance structure/pattern of a z -parent is continuous in the trait z of the parent.

One can immediately see that $\Delta_{zz} = 0$ for any $z \in T$, which means that the cultural intolerance a z -parent has towards his own trait is always zero. Also, we have $\Delta_{zy} \in [0, 1]$ for any $z, y \in T$.

Let $e \in \mathbb{R}_+$ denote the effort exerted by a parent. Let $d : \mathbb{R}_+ \rightarrow [0, 1]$ be the probability function on effort. That is, $d(e)$ is the probability that a parent successfully inculcates his own trait to his child when he exerts effort e . We assume that $d : \mathbb{R}_+ \rightarrow [0, 1]$ is a twice-differentiable function that satisfies $d(0) = 0$, $d' > 0$ and $d'' \leq 0$. Let $c(e)$ be the cost associated with effort $e \in \mathbb{R}_+$, where $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is the cost function that is twice-differentiable and satisfies $c(0) = 0$, $c'(0) = 0$, $c' \geq 0$ and $c'' > 0$.

At state $\mu \in \mathcal{M}_1^+(T)$, a z -parent solves the following maximization problem:

$$\max_{e_z} d(e_z)V_{zz} + (1 - d(e_z)) \int_{y \in T} V_{zy} \mu(dy) - c(e_z). \quad (\text{M})$$

The z -parent maximizes the expected payoff of his child from his own point of view (i.e., based on his own), taking into account both the possibility that he successfully inculcates his own trait into his child, as well as the possibility that he fails to do so and his child adopts the trait of a randomly selected role model in the population. Since

$$\begin{aligned} & d(e_z)V_{zz} + (1 - d(e_z)) \int_{y \in T} V_{zy} \mu(dy) - c(e_z) \\ = & \int_{y \in T} V_{zy} \mu(dy) + d(e_z) \int_{y \in T} (V_{zz} - V_{zy}) \mu(dy) - c(e_z) \\ = & \int_{y \in T} V_{zy} \mu(dy) + d(e_z) \int_{y \in T} \Delta_{zy} \mu(dy) - c(e_z), \end{aligned}$$

the maximization problem (M) is equivalent to

$$\max_{e_z} d(e_z) \int_{y \in T} \Delta_{zy} \mu(dy) - c(e_z). \quad (\text{M}')$$

Taking the first order condition, we have

$$d'(e_z) \int_{y \in T} \Delta_{zy} \mu(dy) = c'(e_z),$$

which, writing $(\frac{c'}{d'})^{-1}$ for $\frac{(c')^{-1}}{(d')^{-1}}$, gives

$$e_z^* = \left(\frac{c'}{d'}\right)^{-1} \left(\int_{y \in T} \Delta_{zy} \mu(dy) \right). \quad (1)$$

Note that since c' is strictly increasing and d' is weakly decreasing, the function $\frac{c'}{d'}$ is strictly increasing, and hence its inverse $(\frac{c'}{d'})^{-1}$ exists and is strictly increasing. From the assumptions on $c(\cdot)$ and $d(\cdot)$, we have

$$d''(e_z) \int_{y \in T} \Delta_{zy} \mu(dy) - c''(e_z) < 0.$$

So the objective function in (M') is strictly concave in e_z , and hence e_z^* is the unique maximizer. Since $c'(0) = 0$, we have

$$\frac{c'(0)}{d'(0)} = 0 \leq \int_{y \in T} \Delta_{zy} \mu(dy).$$

Taking $(\frac{c'}{d'})^{-1}$ yields

$$0 \leq \left(\frac{c'}{d'}\right)^{-1} \left(\int_{y \in T} \Delta_{zy} \mu(dy) \right) = e_z^*.$$

Since $\int_{y \in T} \Delta_{zy} \mu(dy) \in [0, 1]$ and $(\frac{c'}{d'})^{-1}$ is continuous, $e_z^* \geq 0$ is bounded.

Note that $(\frac{c'}{d'})^{-1}$ is a strictly increasing function, and so e_z^* strictly increases with $\int_{y \in T} \Delta_{zy} \mu(dy)$, a z -parent's aggregate cultural intolerance. We summarize the result and this observation in the following proposition.

Proposition 1 *At state $\mu \in \mathcal{M}_1^+(T)$, the optimal effort exerted by a z -parent is⁸*

$$e_z^*(\mu) = \left(\frac{c'}{d'}\right)^{-1} \left(\int_{y \in T} \Delta_{zy} \mu(dy) \right) \geq 0,$$

which is bounded. In particular, the z -parent's optimal effort $e_z^(\mu)$ strictly increases with his aggregate cultural intolerance $\int_{y \in T} \Delta_{zy} \mu(dy)$.*

Intuitively, the greater a z -parent's aggregate cultural intolerance is, the better off the z -parent believes his child would be if his child adopts trait z rather than a randomly chosen one, and hence the more effort the z -parent exerts.

⁸Note that the optimal effort e_z^* in (1) depends on the population state μ .

2.3 Cultural Evolutionary Dynamic

As in Montgomery (2010), given the inter-generational cultural transmission mechanism, we derive the cultural evolutionary dynamic as follows.

Given the optimal effort and hence the probability of successful transmission derived for each parent in the population, the cultural evolutionary dynamic is characterized by the following differential equation on $\mathcal{M}_1^+(T)$:

$$\dot{\mu}(A) = \int_{y \in A} \int_{z \in T \setminus A} (1 - d(e_z^*)) \mu(dz) \mu(dy) - \int_{y \in A} \int_{z \in T \setminus A} (1 - d(e_y^*)) \mu(dz) \mu(dy), \quad (\text{D})$$

for all $A \in \mathcal{B}$. The term $\dot{\mu}(A)$ is the rate of change in the mass of agents with traits in set A . The first term on the RHS of (D) is the ‘‘inflow’’ of children whose parents’ traits are not in set A but who themselves adopt traits in set A , and the second term is the ‘‘outflow’’ of children whose parents’ traits are in set A but who themselves adopt traits not in set A . We can then simplify (D) to

$$\begin{aligned} \dot{\mu}(A) &= \int_{y \in A} \int_{z \in T \setminus A} (d(e_y^*) - d(e_z^*)) \mu(dz) \mu(dy) \\ &= \int_{y \in A} \int_{z \in T} (d(e_y^*) - d(e_z^*)) \mu(dz) \mu(dy) \\ &= \int_{y \in A} d(e_y^*) \mu(dy) - \mu(A) \int_{z \in T} d(e_z^*) \mu(dz). \end{aligned}$$

To further understand the cultural evolutionary dynamic, consider a random matching environment in which a unit mass of agents are randomly matched in pairs to play a two-player symmetric game with strategy set T and payoff function Δ_{xy} .⁹ Define

$$F_x(\mu) := \int_{y \in T} \Delta_{xy} \mu(dy), \quad \text{for } \mu \in \mathcal{M}_1^+(T) \text{ and } x \in T. \quad (2)$$

Then $F_x(\mu)$ is the expected payoff of an agent playing pure strategy $x \in T$ at population state $\mu \in \mathcal{M}_1^+(T)$. Note that $F(\mu)$ is the payoff profile over T at state μ , and we simply denote it as $F(\mu)$. From Example 1 of Cheung (2014), F , as a map from $\mathcal{M}_1^+(T)$ to $\mathcal{C}_b(T)$ where $\mathcal{C}_b(T)$ is the space of bounded continuous functions on T with the supremum norm, defines a population game.¹⁰

⁹That is, Δ_{xy} is the single match payoff of an agent playing strategy x against an opponent playing strategy y .

¹⁰A population game is identified with a map from $\mathcal{M}_1^+(T)$ to $\mathcal{C}_b(T)$ that is continuous with respect to the weak topology (cf. Cheung, 2014, Section 2.1). A sequence of measures $\mu_n \in \mathcal{M}_1^+(T)$ converges weakly to $\mu \in \mathcal{M}_1^+(T)$, written $\mu_n \xrightarrow{w} \mu$, if $\int_S f d\mu_n \rightarrow \int_S f d\mu$ for all $f \in \mathcal{C}_b(T)$. A map $F : \mathcal{M}_1^+(T) \rightarrow \mathcal{C}_b(T)$ is continuous with respect to

Now we have

$$d(e_z^*(\mu)) = d\left(\left(\frac{c'}{d'}\right)^{-1}\left(\int_{y \in T} \Delta_{zy} \mu(dy)\right)\right) = d\left(\left(\frac{c'}{d'}\right)^{-1}(F_z(\mu))\right).$$

Dynamic (D) becomes

$$\dot{\mu}(A) = \int_{y \in A} d\left(\left(\frac{c'}{d'}\right)^{-1}(F_y(\mu))\right) \mu(dy) - \mu(A) \int_{z \in T} d\left(\left(\frac{c'}{d'}\right)^{-1}(F_z(\mu))\right) \mu(dz). \quad (D')$$

Cheung (2015) shows that monotonicity conditions for imitative dynamics are most naturally expressed in terms of Radon-Nikodym derivatives. From (D'), we have $\dot{\mu} \ll \mu$ and so the Radon-Nikodym derivative $\frac{d\dot{\mu}}{d\mu}$ exists.¹¹ In particular,

$$\frac{d\dot{\mu}}{d\mu}(y) = d\left(\left(\frac{c'}{d'}\right)^{-1}(F_y(\mu))\right) - \int_{z \in T} d\left(\left(\frac{c'}{d'}\right)^{-1}(F_z(\mu))\right) \mu(dz). \quad (3)$$

Since $d\left(\left(\frac{c'}{d'}\right)^{-1}(\cdot)\right)$ is a strictly increasing function and the second term on the RHS of (3) is independent of y , we have for any $x, y \in T$,

$$\frac{d\dot{\mu}}{d\mu}(y) > \frac{d\dot{\mu}}{d\mu}(x) \iff F_y(\mu) > F_x(\mu).$$

Hence, the cultural evolutionary dynamic satisfies the payoff monotonicity condition (PM) in Cheung (2015), and hence is equivalent to an imitative dynamic for the population game F defined by (2).¹²

From Theorem 2 of Cheung (2015) and the discussion after that theorem, dynamic (D') is well-defined if $d\left(\left(\frac{c'}{d'}\right)^{-1}(\cdot)\right)$ is Lipschitz continuous, which is guaranteed by the assumptions on $c(\cdot)$ and $d(\cdot)$. We summarize the result in the following theorem.

Theorem 1 (The Cultural Evolutionary Dynamic is Well-defined)¹³ *The cultural evolutionary dynamic is equivalent to an imitative dynamic for the population game F defined by (2), and is well-defined (i.e., for any initial distribution of traits, solutions for the dynamic exist and are unique). Furthermore, solutions to the dynamic are continuous in the initial distribution of*

the weak topology if $F(\mu_n) \rightarrow F(\mu)$ (in the supremum norm) for any sequence $\{\mu_n\} \subseteq \mathcal{M}_1^+(T)$ such that $\mu_n \xrightarrow{w} \mu$.

¹¹Note that $\dot{\mu}$ is a finite signed measure on (T, \mathcal{B}) . For any signed measure φ and any positive measure ν , we say φ is absolutely continuous with respect to ν , written $\varphi \ll \nu$, if for any $A \in \mathcal{B}$, $\nu(A) = 0$ implies $\varphi(A) = 0$.

¹²See Section 3.1 of Cheung (2015) for the definition of imitative dynamics for games with continuous strategy space.

¹³Cf. Cheung, 2015, Theorem 2. By putting $\rho_{zy}(\pi) = 1 - d\left(\left(\frac{c'}{d'}\right)^{-1}(\pi(z))\right)$ as the conditional switch rate from z to y under payoff profile π into Theorem 2 of Cheung (2015), we obtain the result.

traits.

In particular, when $c(e) \equiv \frac{1}{2}e^2$ and $d(e) \equiv e$, we have $d((\frac{c'}{d'})^{-1}(e)) \equiv e$. Denote $\bar{F}(\mu) := \int_{x \in T} F_x(\mu) \mu(dx)$, which is the *population-weighted average payoff* obtained by the unit mass of agents at state μ . Then (D') becomes

$$\begin{aligned} \dot{\mu}(A) &= \int_{y \in A} F_y(\mu) \mu(dy) - \mu(A) \int_{z \in T} F_z(\mu) \mu(dz) \\ &= \int_{y \in A} (F_y(\mu) - \bar{F}(\mu)) \mu(dy), \end{aligned}$$

which is the replicator dynamic for the population game F defined by (2).

As in Montgomery (2010), we can reinterpret our cultural evolutionary dynamic as an imitative dynamic for the population game F defined by (2), and can use the game-theoretic approach to study the dynamic. Under the reinterpretation, “traits” become “pure strategies” (elements in T), the “cultural intolerance that a z -parent has towards trait y ” becomes the “single match payoff of an agent playing pure strategy z against an opponent playing pure strategy y ” (Δ_{zy}), and the “aggregate cultural intolerance of a z -parent” becomes the “expected payoff of an agent playing pure strategy z in the random matching game” ($\int_{y \in T} \Delta_{zy} \mu(dy)$).

3 Stability and Convergence for Imitative Dynamics

To study stability under the cultural evolutionary dynamic, we need to consider “closeness” and “neighborhoods” of population states, which depend on the choice of topology for the space of measures. We find it most appropriate to use the weak topology on $\mathcal{M}_1^+(T)$ to study dynamic stability for the following reason.¹⁴

Suppose that two traits $x, y \in [0, 1]$ are very close to each other in the trait space. If we adopt strong topology, the monomorphic state corresponding to x and that corresponding to y are considered to have the maximum distance in the state space. On the other hand, the two monomorphic states described above are regarded to be close to each other in the weak topology.¹⁵ Therefore, weak topology allows us to have a more natural definition of “closeness” of population states.

Some terms standard in dynamical systems theory and in evolutionary game theory (e.g., Lyapunov stability, ω -limit points) are used. Their formal definitions can be found in Appendix A.1.

¹⁴See also Oechssler and Riedel (2002) for an extensive discussion on the issues of choosing between the strong topology and the weak topology.

¹⁵See Appendix A.1.

Let $F : \mathcal{M}_1^+(T) \rightarrow \mathcal{C}_b(T)$ be a population game. A population state $\mu^* \in \mathcal{M}_1^+(T)$ is a *Nash equilibrium* (NE) of F if

$$F_y(\mu^*) \leq F_z(\mu^*), \quad \forall z \in \text{supp}(\mu^*), \quad \forall y \in T. \quad (\text{NE})$$

We denote by $NE(F)$ the set of Nash equilibria of F , which can be described as

$$NE(F) := \left\{ \mu \in \mathcal{M}_1^+(T) : x \in \text{supp}(\mu) \Rightarrow F_x(\mu) = \max_{y \in T} F_y(\mu) \right\}.$$

Define the set of *restricted equilibria*¹⁶ of F by

$$RE(F) := \left\{ \mu \in \mathcal{M}_1^+(T) : x \in \text{supp}(\mu) \Rightarrow F_x(\mu) = \max_{y \in \text{supp}(\mu)} F_y(\mu) \right\}.$$

In words, μ is a restricted equilibrium of F if it is a Nash equilibrium of a restricted version of F in which only strategies in the support of μ can be played. In particular, all Nash equilibria are restricted equilibria. An imitative dynamic in F has the following properties:¹⁷

- a) the set of rest points of the dynamic coincides with $RE(F)$;
- b) any non-Nash rest points are not Lyapunov stable under the dynamic.

In the remainder of the paper, we focus on cultural intolerance structures that satisfy $\Delta_{xy} = \Delta_{yx}$ for any $x, y \in T$. This means that parents with different traits have symmetric cultural intolerance towards each other. Such intolerance structures resemble the payoff structures for potential games (in particular, doubly symmetric games) with continuous strategy space (cf. Oechssler and Riedel (2002), Cheung (2014)).¹⁸ When $\Delta_{xy} = \Delta_{yx}$ for any $x, y \in T$, the population game F defined by (2) is a potential game.¹⁹

Below are some useful existing stability and convergence results on imitative dynamics in potential games. Consider an imitative dynamic in potential game F . Suppose that the initial condition is $\xi \in \mathcal{M}_1^+(T)$.²⁰ The ω -limit set $\omega(\xi)$ is the set of all points that the solution trajectory from ξ approaches arbitrarily closely infinitely often in the weak topology.²¹ The elements in $\omega(\xi)$ are called

¹⁶Cf. Sandholm, 2010, Section 5.4.6.

¹⁷Cf. Cheung, 2015, Propositions 2 and 3.

¹⁸See Montgomery (2010) for a discussion on cultural intolerance structures that resemble the payoff structures for potential games with finite number of strategies when the trait space is finite.

¹⁹See Section 5 of Cheung (2014) for the formal definition of potential games in continuous strategy settings, and Example 2 therein.

²⁰Initial condition means initial distribution of strategies (or traits).

²¹See the formal definition of $\omega(\xi)$ in Appendix A.1.

ω -limit points. The set $\Omega := \bigcup_{\xi \in \mathcal{M}_1^+(S)} \omega(\xi)$ denotes the set of all ω -limit points of all solution trajectories. We have the following:²²

- i) $\Omega = \text{set of rest points} = RE(F)$;
- ii) if F has a unique Nash equilibrium μ^* , then μ^* is Lyapunov stable.

4 Cultural Heterogeneity and Polarization

4.1 Cultural Heterogeneity

One powerful conclusion drawn from Bisin and Verdier (2001) is that imperfect empathy is sufficient to generate cultural heterogeneity, a phenomenon commonly observed in the real world. Recently, Panebianco (2014) and Buechel et al (2014) claim that imperfect empathy alone can no longer lead to cultural heterogeneity when the trait space is continuous. However, the model they consider are based on the non-probabilistic cultural transmission model by Cavalli-Sforza and Feldman (1973). In this section, we show that cultural heterogeneity is preserved in probabilistic cultural transmission model on continuous trait space:

Proposition 2 *Any monomorphic state is not a Nash equilibrium of the population game F defined by (2), and thus is not Lyapunov stable under the cultural evolutionary dynamic.*

Proof. See Appendix A.2.

Proposition 2 shows that cultural homogeneity is unstable under the cultural evolutionary dynamic. The reason is as follows. Consider the induced population game F defined by (2). In a monomorphic state, every agent is using the same strategy, which gives them zero payoff. Meanwhile, using any other strategy gives them a positive payoff. Hence, agents have incentive to deviate. This implies that any monomorphic state cannot be a Nash equilibrium.

Proposition 2 demonstrates that the key observation of cultural heterogeneity in Bisin and Verdier (2001) in the finite trait setting is naturally generalized to the continuous trait setting.

4.2 Polarization

In this section, we model cultural intolerance as a strictly increasing function of *cultural distance*, that is, the distance between two traits in the trait space. This is consistent with the recent

²²Point (i) is the same as Part (a) of Theorem 3 in Cheung (2015). Point (ii) follows from Lemma 3, Theorem 3 and Remark 5 in Cheung (2014), and Proposition 1 in Cheung (2015).

contributions from the literature on empirical cultural economics, e.g., Guiso, Sapienza and Zingales (2006, 2009), Felbermayr and Toubal (2010), Hahn (2013), Spolaore and Waacziarg (2009, 2013), who suggest that people have greater intolerance for culturally more distant people. Also, as argued by Akerlof and Kranton (2000), people tend to identify with culturally similar people but to distinguish themselves from culturally distant people.

Let $h : [0, 1] \rightarrow [0, 1]$ be a continuous and strictly increasing function that satisfies $h(0) = 0$, and let $\Delta_{xy} = h(|x - y|)$. We call h the *cultural intolerance function*.

In what follows, we investigate how the curvature of the cultural intolerance function h shape the predictions of the cultural evolutionary dynamic. First, we explore the case in which the cultural intolerance function is convex and we have the following main result of the paper:

Theorem 2 *Suppose that the cultural intolerance function h is convex. Then the population state $\mu^* = \frac{1}{2}\delta_0 + \frac{1}{2}\delta_1$ is the unique Nash equilibrium of the population game F defined by (2). Thus since F is a potential game, μ^* is Lyapunov stable under the cultural evolutionary dynamic.*

Proof. See Appendix A.2.

Theorem 2 shows that convexity is the key leading to a polarized state in which only the two most extreme traits coexist. The intuition of the proof is as follows. Since the cultural intolerance function is convex, cultural intolerance is intensified as cultural distance increases. Therefore, given any population state, either one (or both) of the two most extreme traits 0 and 1 is associated with the highest aggregate cultural intolerance because these two traits are furthest away from other traits on average. Hence, in the induced population game F defined by (2) where the intolerance function is viewed as the single match payoff function, either one (or both) of 0 and 1 is a best response. As long as there are some positive masses on both 0 and 1, the dynamic always has the tendency to move away from the current population state and increases the mass on either one (or both) of 0 and 1.

Nevertheless, since the cultural evolutionary dynamic is equivalent to an imitative dynamic, if initially the mass on either 0 or 1 is zero, then the dynamic can never lead to a population state with positive masses on both 0 and 1. The intuition behind is that, when the children search for role models, they will never meet a role model that does not exist at the first place. Therefore, we need to consider some more general long-run predictions of the dynamic that are not necessarily stable.

Note that Ω depends on the dynamic studied, which in turn depends on the cultural intolerance function h . From now on, we denote by $\Omega(h)$ the set of all ω -limit points of all solution trajectories

for the cultural evolutionary dynamic under cultural intolerance function h .

The following proposition shows that when the cultural intolerance function is convex, the ω -limit points (also rest points) are precisely those population states that consist of no more than two traits.

Proposition 3 *Suppose that the cultural intolerance function h is convex and that F is the population game defined by (2). Then*

$$\Omega(h) = \text{set of rest points} = RE(F) = \left\{ \frac{1}{2}\delta_a + \frac{1}{2}\delta_b : 0 \leq a \leq b \leq 1 \right\}.$$

Proof. See Appendix A.2.

Note that under imitative dynamics, any non-Nash rest points are not Lyapunov stable. Hence, when the cultural intolerance function is convex, the most extremely polarized state $\frac{1}{2}\delta_0 + \frac{1}{2}\delta_1$ is the only ω -limit point (also rest point) that is Lyapunov stable.

Next, we consider the case in which the cultural intolerance function is strictly concave. Intuitively, any strictly concave cultural intolerance function can be viewed as less extreme than any convex cultural intolerance function. One may conjecture that polarization is more difficult to sustain and more diversified population states tend to appear as ω -limit points (also rest points). The following theorem confirms this conjecture.

Theorem 3 *Suppose that the cultural intolerance function h is strictly concave. We have the following:*

- i) *Under h , the population state $\frac{1}{2}\delta_0 + \frac{1}{2}\delta_1$ is not a Nash equilibrium of the population game F defined by (2), and thus is not Lyapunov stable under the cultural evolutionary dynamic.*
- ii) *Let \tilde{h} be a convex cultural intolerance function. Then $\Omega(h) \supsetneq \Omega(\tilde{h})$. In particular, $\Omega(h)$ contains some more diversified population states (i.e., those states with supports having more than two different traits).*

Proof. See Appendix A.2.

Part (i) of Theorem 3 shows that polarization is no longer a stable phenomenon when the cultural intolerance function is strictly concave. Part (ii) shows that when the cultural intolerance function is strictly concave, the set of ω -limit points (also rest points) for the cultural evolutionary dynamic is richer and contains some more diversified population states. The intuition behind Theorem 3

is that when the increment in cultural intolerance decreases as cultural distance increases, it is no longer the case that either one (or both) of the two most extreme traits 0 and 1 is associated with the highest aggregate cultural intolerance for any population state. Instead, at states where the distribution of traits places most of the weight on 0 and 1, some trait different from the two most extreme traits 0 and 1 is associated with even higher aggregate cultural intolerance compared to 0 and 1. Therefore, the dynamic always has the tendency to move away from these states.

From the above results, we see that whether the cultural intolerance function h is convex or concave is critical for polarization. That h is convex is a sufficient condition for the population state $\frac{1}{2}\delta_0 + \frac{1}{2}\delta_1$ to be the only ω -limit point (also rest point) that is Lyapunov stable. Also, that h is not strictly concave is a necessary condition for $\frac{1}{2}\delta_0 + \frac{1}{2}\delta_1$ to be the only ω -limit point (also rest point) that is Lyapunov stable.

5 Conclusion

This paper generalizes the discrete cultural transmission model proposed by Bisin and Verdier (2001) to continuous trait space. We first find that the resulting cultural evolutionary dynamic can be characterized by a continuous imitative dynamic in a population game in which a player's payoff is equal to the aggregate cultural intolerance he has towards other agents. We show that cultural heterogeneity is always preserved given the assumption of imperfect empathy.

Second, we model an agent's cultural intolerance towards another agent in the population as a strictly increasing function of cultural distance. This is most easily modeled on a continuous trait space. We find that whether the cultural intolerance function is convex or concave is crucial and leads to distinct cultural phenomena. When the cultural intolerance function is convex, as time passes the population will be divided into two groups with the two most extreme traits in the trait space. Moreover, such population state is the only ω -limit point (also rest point) that is Lyapunov stable under the cultural evolutionary dynamic. On the other hand, when the cultural intolerance function is strictly concave, polarization is no longer stable and more diversified population states consisting of more traits appear as ω -limit points (also rest points).

There are other possible directions for future study. First, in our model agents do not interact with each other within their own generations. Extending the current model to allow social interactions would be an important extension and would help to connect the literature of cultural transmission tighter to the literature of evolution.²³ Second, the trait space we consider in this paper is a closed interval on the real line. It is of interest to investigate whether polarization could

²³See Heifetz, Shannon and Spiegel (2007a,b) for their study of evolution on continuous trait spaces.

arise when the trait space is more complicated (e.g., the trait space is a unit circle, which can be used to describe people's preferences for geographic locations).

A Appendix

A.1 Weak Topology, and Definitions for the Study of Stability and Convergence

The weak topology is related to weak convergence of measures. The weak topology on $\mathcal{M}_1^+(T)$ is metrized by the Prohorov metric κ , which is defined by

$$\kappa(\mu, \nu) := \inf\{\varepsilon > 0 : \mu(A) \leq \nu(A^\varepsilon) + \varepsilon \text{ and } \nu(A) \leq \mu(A^\varepsilon) + \varepsilon, \forall A \in \mathcal{B}\},$$

where $A^\varepsilon := \{x \in T : |x - y| < \varepsilon \text{ for some } y \in A\}$.²⁴ Specifically, $\mu_n \in \mathcal{M}_1^+(T)$ converges weakly to $\mu \in \mathcal{M}_1^+(T)$ if and only if $\kappa(\mu_n, \mu) \rightarrow 0$. In other words, weak convergence and convergence in the Prohorov metric are equivalent. Also, if $\mu = (1 - \varepsilon)\delta_x + \varepsilon\delta_u$ with $0 \leq \varepsilon \leq 1$, where δ is the Dirac delta measure,²⁵ then $\kappa(\mu, \delta_x) = \min\{\varepsilon, |x - u|\}$. In particular, the distance between two monomorphic states in the weak topology agrees with the underlying metric in the continuous trait space T , i.e., $\kappa(\delta_x, \delta_u) = |x - u|$, when x and u are close to each other in T . We use the Prohorov metric κ to measure the distances between population states.

Let $\mu \in \mathcal{M}_1^+(T)$ and $Y \subseteq \mathcal{M}_1^+(T)$. The distance between μ and the set Y in the weak topology is $\kappa(\mu, Y) := \inf\{\kappa(\mu, \nu) : \nu \in Y\}$. The ε -neighborhood of Y (in the weak topology) is $Y^\varepsilon := \{\mu \in \mathcal{M}_1^+(T) : \kappa(\mu, Y) < \varepsilon\}$. Consider an evolutionary dynamic on $\mathcal{M}_1^+(T)$ (like our cultural evolutionary dynamic) that admits a unique forward solution from each initial condition, and suppose that solutions to the dynamic are continuous in their initial conditions. Let $Z \subseteq \mathcal{M}_1^+(T)$ be a closed set. We say Z is *Lyapunov stable* under the dynamic if for every $\varepsilon > 0$ there exists $\delta > 0$ such that every solution of the dynamic that starts in Z^δ is contained in Z^ε .

Let $\xi \in \mathcal{M}_1^+(T)$, and let $\{\mu_t\}_{t \in [0, \infty)}$ be the solution trajectory to the dynamic with $\mu_0 = \xi$. The ω -limit $\omega(\xi)$ is the set of all points that the solution trajectory from ξ approaches arbitrarily closely infinitely often in the weak topology:

$$\omega(\xi) := \{\psi \in \mathcal{M}_1^+(T) : \exists \{t_k\}_{k=1}^\infty \text{ with } \lim_{k \rightarrow \infty} t_k = \infty \text{ such that } \mu_{t_k} \xrightarrow{w} \psi \text{ as } k \rightarrow \infty\}.$$

Since $\mathcal{M}_1^+(T)$ is compact in the weak topology, $\omega(\xi)$ is nonempty. The set $\Omega := \bigcup_{\xi \in \mathcal{M}_1^+(T)} \omega(\xi)$

²⁴Cf. Billingsley, 1999, p. 72–73.

²⁵ δ_x is the monomorphic state in which all agents have trait $x \in T$.

denotes the set of all ω -limit points of all solution trajectories, which provides a basic notion of recurrence for deterministic dynamics.

A.2 Some proofs from Section 4

Proof of Proposition 2. Let $z \in T$ and consider monomorphic state δ_z . Recall that $F_x(\mu) := \int_{y \in T} \Delta_{xy} \mu(dy)$. So $F_x(\delta_z) = \Delta_{xz}$. Then $F_z(\delta_z) = 0 < F_y(\delta_z)$ for any $y \in T \setminus \{z\}$. Hence, δ_z is not a Nash equilibrium. *Q.E.D.*

Proof of Theorem 2. First, we prove that μ^* is a Nash equilibrium. For any $x \in (0, 1)$,

$$\begin{aligned} F_x(\mu^*) &= \frac{1}{2}(h(x) + h(1-x)) \\ &= \frac{1}{2}(h(x \cdot 1 + (1-x) \cdot 0) + h((1-x) \cdot 1 + x \cdot 0)) \\ &\leq \frac{1}{2}(xh(1) + (1-x)h(0) + (1-x)h(1) + xh(0)) \text{ (by convexity of } h(\cdot)\text{)} \\ &= \frac{1}{2}h(1) = F_0(\mu^*) = F_1(\mu^*). \end{aligned}$$

Therefore, μ^* is a Nash equilibrium.

Next we prove that the Nash equilibrium is unique. To do so, it suffices to prove that for any $\mu \neq \mu^*$ and any $x \in (0, 1)$, x is never a best response. Let us write down the payoffs corresponding to all the strategies for any $\mu \in \mathcal{M}_1^+(T)$:

$$\begin{aligned} F_0(\mu) &= \int_{[0,1]} h(y) \mu(dy); \\ F_1(\mu) &= \int_{[0,1]} h(1-y) \mu(dy); \end{aligned}$$

and for any $x \in (0, 1)$,

$$F_x(\mu) = \int_{[0,x]} h(x-y) \mu(dy) + \int_{[x,1]} h(y-x) \mu(dy).$$

Suppose for a $x \in (0, 1)$ such that $F_x(\mu) \geq F_0(\mu)$, which implies

$$\begin{aligned} F_x(\mu) - F_0(\mu) &= \int_{[0,x]} h(x-y) \mu(dy) + \int_{[x,1]} h(y-x) \mu(dy) - \int_{[0,1]} h(y) \mu(dy) \\ &= \int_{[0,x]} (h(x-y) - h(y)) \mu(dy) + \int_{[x,1]} (h(y-x) - h(y)) \mu(dy) \\ &= \int_{[0,x]} (h(x) + h(x-y) - h(y) - h(x)) \mu(dy) \end{aligned}$$

$$\begin{aligned}
& + \int_{[x,1]} (-h(x) + h(y-x) - h(y) + h(x)) \mu(dy) \\
= & h(x)(\mu([0,x]) - \mu([x,1])) + \int_{[0,x]} (h(x-y) - h(y) - h(x)) \mu(dy) \\
& + \int_{[x,1]} (h(y-x) - h(y) + h(x)) \mu(dy) \geq 0.
\end{aligned}$$

Since, $h(x-y) - h(y) - h(x) \leq 0$ for any $y \in [0, x]$, we have $\int_{[0,x]} (h(x-y) - h(y) - h(x)) \mu(dy) \leq 0$. Moreover, by convexity of $h(\cdot)$, $h(y-x) + h(x) \leq h(y)$ for any $y \in [x, 1]$. Hence, $\int_{[x,1]} (h(y-x) - h(y) + h(x)) \mu(dy) \leq 0$. This implies that $\mu([0, x]) - \mu([x, 1]) \geq 0$. Let $\mu([0, x]) - \mu([x, 1]) = \sigma$, where $\sigma \geq 0$. Since $\mu([0, x]) + \mu([x, 1]) = 1$, we can rewrite $\mu([0, x]) = \frac{1+\sigma}{2}$ and $\mu([x, 1]) = \frac{1-\sigma}{2}$. On the other hand, we have

$$\begin{aligned}
F_x(\mu) - F_1(\mu) & = \int_{[0,x]} h(x-y) \mu(dy) + \int_{[x,1]} h(y-x) \mu(dy) - \int_{[0,1]} h(1-y) \mu(dy) \\
& = \int_{[0,x]} (h(x-y) - h(1-y)) \mu(dy) + \int_{[x,1]} (h(y-x) - h(1-y)) \mu(dy) \\
& = \int_{[0,x]} (h(x) + h(x-y) - h(1-y) - h(x)) \mu(dy) \\
& \quad + \int_{[x,1]} (-h(x) + h(y-x) - h(1-y) + h(x)) \mu(dy) \\
& = h(x)(\mu([0, x]) - \mu([x, 1])) + \int_{[0,x]} (h(x-y) - h(1-y) - h(x)) \mu(dy) \\
& \quad + \int_{[x,1]} (h(y-x) - h(1-y) + h(x)) \mu(dy).
\end{aligned}$$

Given that $\mu([x, 1]) = \frac{1-\sigma}{2}$, then

$$\int_{[x,1]} (h(y-x) - h(1-y) + h(x)) \mu(dy) \leq \frac{1-\sigma}{2} (h(1-x) + h(x)),$$

and equality holds if and only if $\mu(\{1\}) = \frac{1-\sigma}{2}$ since $h(\cdot)$ is strictly increasing. Moreover, by convexity of $h(\cdot)$, $h(x-y) + h(1-x) \leq h(1-y)$ for any $y \in [0, x]$. So,

$$\begin{aligned}
\int_{[0,x]} (h(x-y) - h(1-y) - h(x)) \mu(dy) & \leq \int_{[0,x]} (-h(1-x) - h(x)) \mu(dy) \\
& = -\mu([0, x]) (h(1-x) + h(x)) \\
& = -\frac{1+\sigma}{2} (h(1-x) + h(x)). \tag{4}
\end{aligned}$$

Hence, we have

$$\begin{aligned}
F_x(\mu) - F_1(\mu) &\leq h(x)\sigma - \frac{1+\sigma}{2}(h(1-x) + h(x)) + \frac{1-\sigma}{2}(h(1-x) + h(x)) \\
&= -h(1-x)\sigma \\
&\leq 0,
\end{aligned}$$

where both equalities hold if and only if $\sigma = 0$ and $\mu([x, 1]) = \mu(\{1\}) = \frac{1}{2}$ and equality holds in (4). Note that when $\sigma = 0$ and $\mu([x, 1]) = \mu(\{1\}) = \frac{1}{2}$,

$$F_x(\mu) - F_0(\mu) = \int_{[0,x]} (h(x-y) - h(y) - h(x)) \mu(dy) + \frac{1}{2}(h(1-x) - h(1) + h(x)).$$

Since we assume that $F_x(\mu) - F_0(\mu) \geq 0$ and by convexity of $h(\cdot)$, $\frac{1}{2}(h(1-x) - h(1) + h(x)) \leq 0$, we need $\int_{[0,x]} (h(x-y) - h(y) - h(x)) \mu(dy) \geq 0$. However, this can only happen when $\mu([0, x]) = \mu(\{0\}) = \frac{1}{2}$.

Therefore, for any $\mu \neq \mu^*$, if a $x \in (0, 1)$ yields a weakly higher payoff than 0 does, i.e., $F_x(\mu) \geq F_0(\mu)$, then it must be the case that 1 yields a strictly higher payoff than x does, i.e., $F_1(\mu) > F_x(\mu)$. Similarly, we can prove that if a $x \in (0, 1)$ yields a weakly higher payoff than 1 does, i.e., $F_x(\mu) \geq F_1(\mu)$, then it must be the case that 0 yields a strictly higher payoff than x does, i.e., $F_0(\mu) > F_x(\mu)$. This proves that $x \in (0, 1)$ is never a best response for any $\mu \neq \mu^*$. In other words, in any Nash equilibrium μ of the population game, $\mu((0, 1)) = 0$.

It is straightforward to show that for any $\mu \in \mathcal{M}_1^+(T)$ that satisfies $\mu((0, 1)) = 0$, if $\mu(\{0\}) > \mu(\{1\})$, then $F_1(\mu) > F_0(\mu)$, and if $\mu(\{1\}) > \mu(\{0\})$, then $F_0(\mu) > F_1(\mu)$. Therefore, μ^* is the unique Nash equilibrium. *Q.E.D.*

Proof of Proposition 3. It is trivial that monomorphic states are restricted equilibria. Any dimorphic state having the form $\mu = \frac{1}{2}\delta_x + \frac{1}{2}\delta_y$ ($x \neq y$) satisfies $F_x(\mu) = \frac{1}{2}h(|x-y|) = F_y(\mu)$, and so is a restricted equilibrium. In addition, any dimorphic state $\mu = \alpha\delta_x + (1-\alpha)\delta_y$ with $\alpha \neq \frac{1}{2}$ is not a restricted equilibrium since $F_x(\mu) \neq F_y(\mu)$.

Next we prove that any population state μ having more than 2 points in its support cannot be a restricted equilibrium. Let x be the minimum of $\text{supp}(\mu)$ and y be the maximum of $\text{supp}(\mu)$.²⁶ Suppose there exists $z \in \text{supp}(\mu)$ such that $z \neq x$ and $z \neq y$. Then by the same logic as in the proof of Theorem 2, we have either $F_x(\mu) > F_z(\mu)$ or $F_y(\mu) > F_z(\mu)$. Hence, μ is not a restricted equilibrium. *Q.E.D.*

²⁶Note that $\text{supp}(\mu)$ is a closed set.

Proof of Theorem 3.

Part (i): Let $\tilde{\mu} = \frac{1}{2}\delta_0 + \frac{1}{2}\delta_1$. Then $F_0(\tilde{\mu}) = \frac{1}{2}h(1) = F_1(\tilde{\mu})$, but

$$F_{\frac{1}{2}}(\tilde{\mu}) = \frac{1}{2}h\left(\frac{1}{2}\right) + \frac{1}{2}h\left(\frac{1}{2}\right) = h\left(\frac{1}{2}\right) = h\left(\frac{1}{2} \times 1 + \frac{1}{2} \times 0\right) > \frac{1}{2}h(1) + \frac{1}{2}h(0) = \frac{1}{2}h(1).$$

Hence, $\tilde{\mu}$ is not a Nash equilibrium.

Part (ii): Any $\mu = \frac{1}{2}\delta_a + \frac{1}{2}\delta_b$ with $0 \leq a \leq b \leq 1$ is a restricted equilibrium when $h(\cdot)$ is strictly concave. Moreover, it is easy to check that $\frac{\frac{1}{2}h(\frac{1}{2})}{2h(\frac{1}{2}) - \frac{1}{2}h(1)}\delta_0 + \frac{h(\frac{1}{2}) - \frac{1}{2}h(1)}{2h(\frac{1}{2}) - \frac{1}{2}h(1)}\delta_{\frac{1}{2}} + \frac{\frac{1}{2}h(\frac{1}{2})}{2h(\frac{1}{2}) - \frac{1}{2}h(1)}\delta_1$ is a restricted equilibrium when $h(\cdot)$ is strictly concave. *Q.E.D.*

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