Teaching Practices and the Management of Student Motivation, Effort and Achievement

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2016

Teaching Practices and the Management of Student Motivation, Effort and Achievement[*]

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Abstract

Student motivation is primordial for educational success. We develop a theoretical model in which a teacher manages student motivation through the choice of teaching practices. We show that only high-ability students can be motivated by extrinsically-oriented teaching practices. For low-ability or myopic students, intrinsically-oriented teaching practices are more effective in fostering student achievement. Furthermore, the choice of teaching practices depends on their relative costs, the teacher's objective function (utilitarian or Rawlsian), and the teacher's time preferences. We draw important policy implications regarding teacher effectiveness, the harmfulness of not tailoring teaching practices to student types, and how to limit student dropouts.

Keywords: Teaching practices; cognitive and non-cognitive skills; student achievement; utilitarian and Rawlsian maximizers; achievement goal theory

JEL-classification: A12; C70; D03; D04; I24

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[*] We thank Hans Bonesrønning, John Dagsvik, Torberg Falch, Manon Garrouste, Robert Gary-Bobo, Torbjørn Hægeland, Alessandro Tampieri, and Ragnar Torvik, as well as participants at the educational governance and finance workshop in Oslo 2014 and the LAGV conference in public economics in Aix-en-Provence 2015 for useful comments and suggestions. The usual disclaimer applies.

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1 Introduction

Schooling is vital to the formation of human capital and hence to economic growth. Most schools, however, experience gaps between their mandate to promote high academic achievements by their students and their actual performances. These gaps can to a large extent be explained by the lack of student effort. Insufficient effort can have severe private, social and economic consequences as it may damage students' learning curves, decrease their performances, and trigger dropouts (Belfield and Levin, 2007). To increase student effort, policy makers and economists have so far mainly focused on the creation of competitive learning environments (e.g., introducing educational standards, school accountability, and quasi-markets) and the use of extrinsic incentives (e.g., grades, rankings, monetary and non-monetary rewards). Little attention has been paid to within-classroom education policies, and how teachers through the choice of teaching practices can also foster intrinsic motivation in students to increase effort, limit student dropouts, and improve life chances.[1]

According to educational psychologists, the main determinant of student effort is their motivation (e.g., Wigfield et al., 2009). Student motivation, defined as the internal force that moves students to engage in a learning task, builds on four main factors: Their extrinsic valuation for the task, their intrinsic interest, their self-concept of ability, and their perception of control over the task. These four motivational factors differ among students, and evolve through their stages of development. They are also affected by students' ups and downs in achievement. In fact, students with different sources of motivation react differently to failure. For example, students who are intrinsically motivated tend to develop a stronger failure tolerance and are more willing to work harder after a failure than students who are extrinsically motivated (e.g., Anderman and Wolters, 2006). Furthermore, teachers can alter students' motivational pattern and thereby increase student effort and achievement through the choice of teaching practices and the design of the classroom environment (Ames, 1992; Wolters, 2004).

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[1] See Koch et al.(2015) for an overview of the effectiveness and pitfalls of providing extrinsic incentives in the context of education, and how they seem to function for some but not all students depending among others on ability.

In this article, we build on insights of educational psychology and develop a theoretical model to study how a teacher manages student motivation, effort, and achievement through the choice of teaching practices. We emphasize that since schooling is compulsory and a long-term contract[2], the teachers' role is to motivate students, enable them to learn and do as well as they can academically, and prevent them from dropping out.[3] We first study the management of motivation over the short run: Faced with homogeneous students, how does the teacher tailor teaching practices to student types to foster achievement? Faced with heterogeneous students, how does the teacher prioritize? We next study the management of motivation over the long run: How can the teacher design a classroom environment to maintain students' motivation over time, particularly after failure? We also characterize various constraints that may prevent the teacher from choosing what best suits students' needs. These constraints are either related to differences among teachers in their innate talent to teach, to the institutional setting in schools, or both. Our framework enables us to draw important policy implications regarding teacher effectiveness, the harmfulness of not tailoring teaching practices to student types, the design of teacher contracts and educational reforms, and how to prevent student dropout. Teachers are the most important school-based factor in determining student achievement, and teaching practices constitute their main instrument to foster performance. This article is to the best of our knowledge the first theoretical economic analysis of the interplay between teachers' choice of teaching practices and their effects on student motivation, effort and achievement.

To open the black box of the classroom and specify the set of teaching practices we rely on the achievement goal theory (Nicholls, 1984; Dweck, 1986; Ames, 1992; Elliot, 2005). This theory

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[2] Compulsory education, which often includes elementary and middle school, lasts up to around ten years.

[3] In contrast to firms where workers have outside options and employers can fire inefficient employees, students in compulsory education have no outside option before a certain age, and the length is determined ex-ante. Teachers should therefore do their best to keep students on track. In fact, learning basic skills (compulsory education) requires direct intervention by teachers (Rosen, 1987). In contrast, as students move upward in the educational hierarchy and education becomes optional (high school, tertiary education), students become more independent and can better indulge in motivational self-regulation. For an analysis of motivation and goal setting in firms and how motivation in this context is considered as self-regulation, see Koch and Nafziger (2011).

is one of the main motivational theories in educational psychology and emphasizes the dual role of (teachers' design of) classroom structures and (students') achievement goals. Achievement goals refer to a student's subjective representation of the purposes of a learning task, the way success is defined, and the role of effort and ability in achievement. The achievement goal literature considers two main goals (Nicholls, 1984; Dweck, 1986). Students with a *mastery goal (or task goal)* focus on learning, developing new skills, and improving their competence. They use self-referenced (or intrapersonal) standards and view success as evidence of effort. Students with a *performance goal (or ability goal)* focus on proving their competence to themselves and others; they want to obtain high grades or outperform other students. They believe in normative (or interpersonal) standards and that achievement strongly depends on and therefore signals their ability. A classroom structure, on the other hand, is defined by the way a teacher designs learning tasks, shares authority and motivates students. The fundamental idea is that the choice of the classroom structure conveys a message to students about learning goals and alters their initial goal orientation (Ames, 1992). A mastery goal structure refers to teaching practices that emphasize understanding and personal improvement. A performance goal structure refers to teaching practices that emphasize grades and rankings. When students perceive a classroom structure as mastery (performance)-oriented, they become more mastery (performance)-goal-oriented themselves (Wolters, 2004).[4,5]

Extensive empirical research in educational psychology shows that holding a mastery goal fosters adaptive study behaviors such as effort, deep processing of the learning material, task enjoyment, low levels of test anxiety, and persistence in the face of difficulties or failure (Anderman and Wolters,

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[4] Mastery and performance goals echo intrinsic and extrinsic motivation. Extrinsic motivation is based on incentives coming from outside such as student grades. Intrinsic motivation comes from inside the student and gives no reward except for the task itself (e.g., Frey and Jegen, 2001). In the economic literature, the focus is on how the use of extrinsic incentives may crowd out intrinsic motivation, e.g., Benabou and Tirole (2003). They use an informed principal-agent framework where the teacher has better information regarding students' ability than the students themselves. They build on self determination theory. This theory focuses on to which degree an individual's behavior is self-motivated and self-determined. We, on the other hand, build on achievement goal theory and focus on how the teacher can foster both intrinsic and extrinsic motivation in students. Hence, the "principal" in our model has a more active role.

[5] There exists a 2x2 achievement goal framework where goals exist both as approach (acquiring positive possibilities) and avoidance (avoiding negative possibilities) (Elliot and McGregor, 2001). In this article, we focus on approach goals.

2006). There is a noteworthy omission from the list of positive outcomes, however: Empirical studies have not established a link between mastery goals and academic achievement. This puzzle has been explained by exams consisting of multiple choice questionnaires, which may favor surface learning over deep learning. Another explanation is that mastery-oriented students seem to spend time on material that is personally interesting to them, but not relevant to the test (Senko and Miles, 2008). In contrast, empirical research shows that a performance goal fosters effort and academic achievement (Anderman and Wolters, 2006). The positive relationship between performance goals and grades has been explained by the fact that performance-oriented students seek to align their learning agenda with that of the teacher by carefully trying to identify the assessment criteria (Senko and Miles, 2008). However, there are also negative consequences: Performance-oriented students tend to have higher levels of test anxiety. It could also be more difficult for these students to preserve their level of engagement in the long run, notably after a failure (Covington and Omelich, 1979).

Although achievement goal theory is rich in empirical evidence, it lacks a parsimonious framework. In this article, we develop a theoretical model where a teacher manages student motivation, effort, and achievement through the choice of a classroom structure. We consider a situation with a teacher and a class of students with a learning task to achieve. There is a test at the end of the period to verify whether students have acquired some knowledge or not. The teacher's choice of classroom structure alters both the students' initial goal orientation and the efficiency of effort. The teacher's objective is to maximize student grades. The model helps to understand the complex interactions at play between the classroom environment, students' motivational patterns, and achievements.

We first consider a static framework where students are homogeneous and the classroom structures (performance or mastery) have the same implementation cost for the teacher. We show that the optimal choice of the teacher depends on a unique index - the PAM index - which summarizes the students' PAttern of Motivation. This index depends positively on student ability and initial performance goal orientation, but negatively on their mastery goal orientation. When this index is high, the teacher promotes academic achievement by designing a performance goal structure. In fact, the effort exerted by students under this structure reflects their ability level. Furthermore, students are more focused on the teacher's demands, so that each unit of effort is more efficient. When this index is low, the teacher cannot solely rely on extrinsic incentives to foster student motivation and promotes higher achievement by choosing a more mastery-oriented classroom structure. By doing so, the teacher creates a learning environment in which students' effort is also fueled by their interest in the task, and where they get some satisfaction independently of the test result. The consequence is that students display an effort level that is independent of their ability level (Proposition 1).

Faced with homogeneous students and equal structure costs, the teacher can design the classroom environment that best matches student needs ("tailoring"). Nevertheless, this situation is somewhat utopic. We therefore study how the teacher adapts the choice of the classroom structure when (i) a mastery goal structure is more costly to implement than a performance goal structure, and (ii) students are heterogeneous. Faced with these two constraints, it is more difficult for the teacher to tailor the classroom structure to student types, and hence to keep all students motivated. When there is a cost difference between structures, the students with a low PAM index experience the biggest decline in achievement. The reason is that the teacher chooses a structure that is too performance-oriented for them. When students are heterogeneous, the teacher has to choose which student(s) to prioritize. The objective set by the teacher affects the choice of the classroom structure. We show that when student ability is more dispersed, a utilitarian teacher chooses a more performance-oriented classroom structure to maximize the average achievement, and to do so, targets a student whose PAM index is *above* that of the average student. In contrast, a Rawlsian teacher, in the same situation, considers the PAM index of the most at-risk student and may choose a structure more oriented toward mastery to keep this student motivated (Proposition 2).

The results of Propositions 1 and 2 are obtained in a static framework. To take into account that schooling is a long-term contract and building motivation is a long-term process, we consider a dynamic version of the model. We show that the teacher, if sufficiently forward-looking, chooses a first-period classroom structure that is more mastery-oriented than in the static case. Even though this is at the expense of a (slight) short-run decrease in achievement, the teacher is able to develop the student's failure tolerance which leads the student to persist in exerting effort after a failure (Proposition 3). The time horizon of the teacher plays the role of a third constraint that can hinder the teacher from tailoring teaching practices to student types.

This article is related to the new microeconomics of education. This literature (e.g., Correa and Gruver, 1987; Costrell, 1994; Betts, 1998; Akerlof and Kranton, 2002; Bonesrønning, 2004; De Fraja and Landras, 2006; De Fraja et al., 2010; Fryer, 2011; Bettinger, 2012) considers student effort to be the most important input to education production. The focus of attention is on the use of extrinsic incentives (e.g., educational standards, grading practices, monetary incentives), students' social position and identity in schools, and teacher and student efforts in a prisoner's dilemma situation. However, students' intrinsic motivation, the efficiency and dynamics of student effort, and the teacher's role in avoiding a low effort-low effort equilibrium characterizing in a prisoner's dilemma situation are often disregarded. In this article, we aim at integrating these elements.

This article is also related to the vast empirical literature on teacher quality.[6] Chetty et al. (2014) use student achievement gains to estimate teacher value-added, i.e., a measure of unobserved teacher quality, and find that teachers matter, vary in effectiveness, and have long-lasting impacts on student achievement and later outcomes. As teaching practices are one of the most important devices for teachers to increase student achievement, some of the heterogeneity in teachers' value-added is likely to be explained by the choice of teaching practices. Only a few recent articles try

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[6] See Hanushek and Rivkin (2006) for a review.

to identify both effective teachers and teaching practices. Rouse et al. (2013) show that teaching practices changed in meaningful ways due to accountability pressure and led to an increase in student achievement. Lavy (2015) finds that "instillment of knowledge and enhancement of comprehensions" (i.e., traditional teaching practices) has a very strong positive effect on test scores of students from low socioeconomic backgrounds, whereas classroom techniques that endow students with "analytical and critical skills" (i.e., modern teaching practices) have a very strong effect among students from educated families. Bietenbeck (2014) finds that modern and traditional teaching practices promote different cognitive skills. Whereas traditional teaching practices increase students competency in solving routine problems, modern teaching practices foster students reasoning skills. Bietenbeck also provides evidence that standardized tests do not measure reasoning skills well.[7]

Although none of the aforementioned articles on teaching practices study mastery and performance, as modern and traditional teaching practices relate to which problems to solve (i.e., cognitive skills) and not patterns of motivation (i.e., non-cognitive skills and effort enhancement), their results echo ours: First, teaching practices seem to matter for student achievement. Second, they need to be tailored to student types. In contrast to these papers, however, we study not only how teaching practices relate to student achievement, but also how they relate to student motivation and effort. By studying the underlying mechanism of effort, we can explain why extrinsic motivation is harmful for certain types of students and why providing some intrinsic motivation can be beneficial. In addition, we consider three constraints (cost difference between classroom structures, student heterogeneity, and the time horizon of the teacher) that can hinder the teacher from applying the optimal classroom

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[7] This article is also related to the growing literature on non-cognitive skills and how such assets relate to economic payoffs. Algan, Cahuc and Shleifer (2013) analyze the relationship between teaching practices and social capital. They study horizontal teaching practices (working in groups) versus vertical teaching practices (teachers lecturing). They find that the former generates social capital and economic growth. Heckman (2013) focuses on early interventions and how non-cognitive qualities such as motivation, self-confidence, and other social-emotional qualities later can influence student test scores. Heckman et Kautz (2012) study how student motivation/soft skills impact student achievement. Jackson (2014) zooms in on how teachers also affect non-cognitive abilities that are non test-score outcomes. We focus, however, on how teachers can augment students' non-cognitive abilities through their choice of classroom structure and how these non-cognitive abilities affect students' study effort and achievement.

structure of the unconstrained setting. The lack of tailoring, either due to the teacher's insufficient talent to teach or organizational constraints, is likely to explain why some students do not succeed in school and why some drop out. We suggest some solutions, in terms of selection of teachers and design of educational reforms, to relax these constraints and hence to decrease the private and social costs of inadequate education, notably in the context of middle school (see Belfield and Levin (2007) for the estimation of such costs).

The article proceeds as follows: Section 2 presents the static model. Section 3 presents the dynamic framework. Section 4 derives some policy implications of our results in the context of middle school. Section 5 concludes.

2 Static Management of Student Motivation

In this section, we consider a model with a teacher and a class of students who interact during one period. In Section 2.1, we consider homogeneous students and study how the teacher in this setting is able to tailor teaching practices to student types. Thereafter, we consider a first constraint, a cost difference between structures, that makes the tailoring of teaching practices more difficult: We calculate the loss in achievement of providing extrinsic incentives for all student types, regardless of their motivational pattern (a "one-size-fits-all" classroom policy). In Section 2.2, we consider heterogeneous students and study how the choice of the optimal classroom structure depends on whether the teacher is a utilitarian or a Rawlsian maximizer. Student heterogeneity constitutes a second constraint that can prevent the teacher from tailoring teaching practices to student types.

2.1 The case of homogeneous students

2.1.1 The Model

Students are identical and risk-neutral. We focus on the problem of one (representative) student.

The student ("he"). He has a learning task to perform, that is, a knowledge to acquire. There is a test at the end of the period to verify whether this knowledge has been acquired or not. We assume that the test result measures the students' knowledge acquisition without any noise and that test topics are stipulated in the school- or national curriculum. Furthermore, the teacher's grading practice is independent of teaching practices. The grade is solely based on the test result. The student is endowed with an initial goal orientation, which can be regarded as his non-cognitive skills, where $v \in [0,1]$ represents his performance orientation and $\gamma \in [0,2]$ represents his mastery orientation. The student has a cognitive ability $\theta \in [0,1]$ and exerts effort $e \in [0,1]$.[8,9]

The teacher ("she"). We assume that she has complete information about student characteristics. This is a fair assumption faced with a representative student. Later, we will consider heterogeneous students and the teacher will only know the distribution of student characteristics. The teacher chooses a classroom structure, s, in a continuum of differentiated structures, $[0,1]$. The structure describes the way the teacher designs tasks, shares authority and motivates students. The structure $s = 1$ ($s = 0$) corresponds to the situation in which the teacher chooses a pure performance (mastery) goal structure.[10] An intermediate structure, $s \in (0,1)$, is referred to as a multiple goal structure, and corresponds to a mixture of the two classroom structures. In this case, the teacher spends a proportion s of the time emphasizing the importance of performance as a learning goal, and a proportion $1-s$ of the time emphasizing the importance of mastery as a learning goal.[11]

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[8] The learning task can consist of acquiring a particular competence (say, learning to add numbers), understanding the content of a course (say, basic algebra), or in a broader perspective, acquiring the skills and knowledge associated with a specific education level (say, elementary school). Note that ability and the goal orientation are task specific.

[9] We will see that in some cases, a student with $\gamma = 2$ exerts the maximum effort, even when his ability is nil.

[10] Under a pure performance classroom structure, the teacher exerts total control over the classroom activities and provides whole class instruction. Learning tasks are repetitive and the time available to perform a particular task is fixed. The recognition of achievement is public and based on normative standards. The teacher emphasizes that the primary objective of students is to obtain a good grade and demonstrate ability (Ames, 1992). Under a pure mastery goal structure, the teacher uses a variety of learning tasks to challenge students and disregard whole class instructions. Students are given opportunities to participate during the class and the time available to perform a task is flexible. The recognition of achievement is private and based on self-referenced standards. Effort is valued. The teacher emphasizes that the primary objective of students is to understand the learning material (Ames, 1992).

[11] The dimensions defining the structures are referred to by the acronym TARGET: Task, Autonomy, Recognition, Grouping, Evaluation and Time (Ames, 1992). We model these dimensions through the variable s. Note, however, that in our case the way to evaluate students does not vary across structures.

Test result. We assume that it is equal to

$$x = \begin{cases} 1 & \text{with probability} \quad \theta s e \\ 0 & \text{with probability} \quad 1 - \theta s e \end{cases} \tag{1}$$

where 1 means success and 0 means failure. The probability of passing the test is increasing in student effort, e. It is also increasing in θs, which we interpret as the efficiency of effort. The efficiency of effort is increasing in student ability. It also increases as the classroom structure becomes more performance-oriented. The reason for this assumption is twofold. First, when the teacher stresses performance by choosing a higher s, the student becomes more attentive to her demands, so that the efficiency of effort and the probability of passing the test increase. Conversely, when the teacher stresses mastery, the student spends more time on material that he personally finds interesting, but less time on material relevant to the test (Senko and Miles, 2008). Second, under a mastery-oriented structure, the teacher may partly deviate from test-related subjects to foster the student's intrinsic task value. We assume that the pure mastery goal classroom structure $s = 0$ nullifies the probability of success: It corresponds to the extreme situation where the teacher underemphasizes the importance of the test result, and/or deviates too much from test-related subjects.

Payoffs. The student maximizes the following expected utility function:

$$(\theta s e) \times \left[v + \gamma(1-s)e - 0.5e^2\right] + (1 - \theta s e) \times \left[0 + \gamma(1-s)e - 0.5e^2\right] \tag{2}$$

The student has two sources of utility, extrinsic and intrinsic, which correspond to the two achievement goals. The first term in the square brackets represents the payoff of pursuing a performance goal. When the student succeeds in the test, he is able to demonstrate his ability and gets a payoff equal to v; when he fails the test, he gets 0.[12] The term $\gamma(1-s)e$ represents the payoff of pursuing

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[12] We take the test result as a dichotomous variable. Nevertheless, under risk neutrality the model is equivalent to a

a mastery goal and γ describes the intensity of this goal in the student's initial goal orientation. This payoff appears regardless of the test result and is increasing in student effort. The reason is that a mastery-oriented student derives satisfaction from developing new skills and achieving a sense of mastery based on self-referenced standards. In contrast to a performance-oriented student who views success as evidence of ability, a mastery-oriented student views success as evidence of effort (Ames, 1992). The student's intrinsic satisfaction is higher as the teacher increases the mastery goal structure because this structure fosters interest in learning. We assume that the student's mastery goal orientation totally vanishes when the teacher chooses a pure performance goal structure, $s = 1$. Finally, the cost of effort takes the quadratic shape, $0.5e^2$. For the moment, we assume that v and γ do not depend on θ, which means that the student's goal orientation is not affected by his ability level. However, we later extend the analysis to include heterogeneous students which enables us to consider positive or negative correlations between θ, v and γ.

The teacher is risk neutral. Her primary concern is student achievement. She achieves a gross utility equal to w when the student succeeds in the test. A rationale for this assumption is that the teacher operates in a school accountability context: She is accountable for student achievement. The teacher internalizes this environment by maximizing the student's grade. Whereas maximizing student grades is standard in the economics of education literature[13], it is a novelty to take the classroom structure as the teacher's decision variable. We write the teacher's expected utility function as $(\theta se)w - c_p s - c_m(1-s)$ where $c_m \geq c_p$. The term c_m (c_p) is the constant unitary cost of applying a mastery (performance) goal structure. Without loss of generality, we normalize w to 1.

Timing of the game. First, the teacher chooses a classroom structure, $s \in [0,1]$. Second, the

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model where (i) the grade is continuous and equal to θes (with possibly an extra noise term) and (ii) the (expected) payoff of the student is $v\theta es + \gamma(1-s)e - 0.5e^2$.

[13] Tests are necessary in schools to evaluate students based on an objective measure (i.e., test results measure the knowledge that has been acquired). Teachers thus maximize student test scores. Teachers cannot base their evaluations on a subjective measure, such as student effort, as students later in life compete for seats in non-compulsory education and in the labor market.

student observes s and exerts effort $e \in [0,1]$. Third, the student takes the test and obtains a result $x \in \{0,1\}$.

2.1.2 The equilibrium

The strategies are s for the teacher and $e(s)$ for the student. We characterize the subgame perfect equilibrium. Maximizing expression (2) with respect to e yields

$$e^*(s) = \min\left\{\gamma\big(1 + (\frac{\theta v}{\gamma} - 1)s\big), 1\right\} \tag{3}$$

The way student effort changes with respect to the structure s depends on the value of $\frac{\theta v}{\gamma}$. This index takes values in the interval $[0, +\infty[$ and summarizes the initial motivational pattern (initial goal orientation and cognitive ability) of the student in a single value. A low (high) $\frac{\theta v}{\gamma}$ means that the student has a low (high) ability level and/or a low initial performance (mastery) goal orientation. When $\frac{\theta v}{\gamma} > 1$ ($\frac{\theta v}{\gamma} < 1$), effort increases as the teacher chooses a more performance-oriented (mastery-oriented) classroom structure. A student with $\frac{\theta v}{\gamma} = 1$ provides effort independently of the structure designed by the teacher. We will refer to $\frac{\theta v}{\gamma}$ as the PAM index, where the acronym PAM stands for Performance, Ability and Mastery, or PAttern of Motivation.

We now consider the maximization problem of the teacher. We first assume that the two structures are equally costly for the teacher: $c_p = c_m$. Considering homogeneous students and no cost difference between classroom structures constitutes our benchmark case, a "first best" situation. Later, we will add constraints on the teacher, by considering situations where (i) the teacher faces a higher cost of providing a mastery structure relatively to a performance structure, and (ii) the teacher faces heterogeneous students and needs to prioritize. We will analyze how the teacher adjusts her behavior to these constraints and how they affect student motivation and achievement.

The case of equal costs between classroom structures

Under equal costs, the teacher solves $s^* = \arg\max_{s\in[0,1]} \theta s e^*(s) - c_m$. The solution is

$$s^* = \begin{cases} \frac{1}{2}\left(\frac{1}{1-\frac{\theta v}{\gamma}}\right) & \text{if } \frac{\theta v}{\gamma} \leq \frac{1}{2} \\ 1 & \text{if } \frac{\theta v}{\gamma} \geq \frac{1}{2} \end{cases} \tag{4}$$

The equilibrium is fully described by expressions (3) and (4). At equilibrium, the student's effort $e^*(s^*)$ is equal to $\frac{\gamma}{2}$ when $\frac{\theta v}{\gamma} \leq \frac{1}{2}$ and θv when $\frac{\theta v}{\gamma} \geq \frac{1}{2}$. We sum up the results in Proposition 1:

Proposition 1 *At equilibrium, the teacher chooses a pure performance goal structure when* $\frac{\theta v}{\gamma} \geq \frac{1}{2}$. *The student then exerts an effort proportional to his ability level. When* $\frac{\theta v}{\gamma} < \frac{1}{2}$, *the teacher chooses a multiple goal structure. The student then exerts an effort independent of his ability level.*

When $\frac{\theta v}{\gamma} > 1$, both effort $e^*(s)$ and its efficiency θs increase as the teacher chooses a more performance-oriented classroom structure. The teacher therefore chooses a pure performance goal structure, $s^* = 1$. When $\frac{\theta v}{\gamma} \leq 1$, the teacher faces a trade-off between the level of effort and the efficiency of effort. There are two cases. When $\frac{\theta v}{\gamma}$ belongs to $\left[\frac{1}{2}, 1\right]$ the teacher still chooses $s^* = 1$. The student's effort would be higher if the classroom structure were more oriented toward mastery goals. However, this mastery-induced effort would be less efficient and the probability of passing the test would decrease. When $\frac{\theta v}{\gamma} < \frac{1}{2}$, the teacher chooses a multiple goal structure that conveys both performance and mastery goals: $s^* = \frac{1}{2}\left(\frac{1}{1-\theta v/\gamma}\right)$.[14] In doing so, she induces an effort level $e^*(s^*) = \frac{\gamma}{2}$, which is independent of the student's ability θ. In this case, the benefit of breaking the ability-effort connection is higher than the loss resulting from the reduced efficiency of effort. The equilibrium path and payoffs are described in Table 1 (U^t denotes the teacher's payoff and U^p denotes the student's payoff).

To clarify our findings, we consider four polar student types. Consider first the two representative students ($\theta = 1, v = 0, \gamma = 2$) and ($\theta = 0, v = 1, \gamma = 2$). Despite being different in characteristics, these students share the same PAM index: $\frac{\theta v}{\gamma} = 0$. The teacher therefore chooses the same classroom

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[14] The optimal structure s^* is always larger than 1/2: a pure mastery goal structure nullifies the efficiency of effort.

Table 1: Equilibrium Payoffs

	s^*	$e^*(s^*)$	U^{t^*}	U^{p^*}
If $\frac{\theta v}{\gamma} \geq \frac{1}{2}$	1	θv	$\theta^2 v$	$\frac{(\theta v)^2}{2}$
If $\frac{\theta v}{\gamma} \leq \frac{1}{2}$	$\frac{1}{2}\frac{\gamma}{\gamma-\theta v}$	$\frac{\gamma}{2}$	$\frac{1}{4}\frac{\gamma^2\theta}{\gamma-\theta v}$	$\frac{\gamma^2}{8}$

structure for both of them. A performance goal structure would demotivate these students, so the teacher designs the multiple goal structure $s^* = \frac{1}{2}$. Faced with this structure, these students exert the maximum level of effort: $e^*(s^*) = \frac{\gamma}{2} = 1$. The student $(\theta = 1, v = 0, \gamma = 2)$ succeeds with probability 0.5. This student is a high-ability student, but his motivation for effort is not to get an A, but rather to study what he personally finds interesting.[15] The teacher's role in this regard is to let this student unfold his interest and at the same time make sure that he stays somewhat focused on the test assignment. The student $(\theta = 0, v = 1, \gamma = 2)$ succeeds in the test with probability 0. In our framework, effort and ability are strategic complements in achievement: Being motivated is a necessary, but not a sufficient condition to succeed in the test. This means that even if the teacher is able to foster interest in the task, it is difficult for low-ability students to succeed. Now consider the two representative students $(\theta = 1, v = 1, \gamma = 0)$ and $(\theta = 0.1, v = 1, \gamma = 0)$. Their PAM indexes are infinite. The teacher chooses a pure performance structure, $s^* = 1$, for both of them because they are completely performance-oriented. The student $(\theta = 1, v = 1, \gamma = 0)$ exerts the maximum level of effort $e^*(s^*) = 1$ and succeeds the test with probability 1 due to his high ability and efficiency of effort. The student $(\theta = 0.1, v = 1, \gamma = 0)$ exerts effort $e^*(s^*) = 0.1$ and succeeds the test with probability 0.01. Again, students with the same PAM index can attain very different levels of achievement, because of the role of student ability in the probability of success.

Several educational psychologists have long advocated the development of a multiple goal structure to foster student achievement. According to them, the multiple goal structure enables students to combine the best features from mastery and performance goals: Empirical studies show that whereas

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[15] The student is not motivated to get an A, say on a Spanish vocabulary test, but rather to speak Spanish fluently.

mastery fosters interest and joy of learning, performance fosters students to become more attentive to the teacher's demands (Senko and Miles, 2008; Harackiewicz et al., 2008). Our results show a more nuanced picture as our PAM index includes student ability in addition to students' initial goal orientation. When ability is low or the student has a high ability but is not very performance-oriented, the level of effort and its efficiency vary in opposite directions as the classroom structure changes. The multiple goal structure then corresponds to the best mix of incentives. On the other hand, when ability is high and the student is extrinsically motivated, a performance structure causes both high and efficient effort, thereby promoting high academic achievement.

Cost difference between classroom structures and loss in achievement

We now consider a cost difference between structures: $\triangle c \equiv c_m - c_p > 0$. Indeed, it is reasonable to think that mastery-oriented classroom structures require more attention, involvement, and understanding of student needs, and therefore more effort from the teacher. In fact, under mastery-oriented structures the teaching is more personalized and there is no whole class instruction as with performance structures. To determine the optimal structure under cost difference, we first consider $\triangle c \leq \gamma^2/8v$. Solving $\max_{s\in[0,1]}(\theta s e^*(s)) - c_p s - c_m(1-s)$, where $e^*(s)$ is given by (3), yields

$$s^*_{\triangle c} = \begin{cases} 1 & \text{if} \quad \frac{\theta v}{\gamma} \leq \frac{1-\sqrt{1-\frac{8v}{\gamma^2}\triangle c}}{4} \\ \frac{1}{2}\left(\frac{1+\frac{\triangle c}{\theta\gamma}}{1-\frac{\theta v}{\gamma}}\right) & \text{if} \quad \frac{1-\sqrt{1-\frac{8v}{\gamma^2}\triangle c}}{4} \leq \frac{\theta v}{\gamma} \leq \frac{1+\sqrt{1-\frac{8v}{\gamma^2}\triangle c}}{4} \\ 1 & \text{if} \quad \frac{\theta v}{\gamma} \geq \frac{1+\sqrt{1-\frac{8v}{\gamma^2}\triangle c}}{4} \end{cases}$$

When $\triangle c \geq \gamma^2/8v$ the optimal classroom structure $s^*_{\triangle c}$ is uniformly equal to one. Not surprisingly, in both cases, the teacher chooses a more performance-oriented structure when there is a cost difference between structures: $s^*_{\triangle c} \geq s^*$. More interestingly, the teacher designs a pure performance goal structure for students with a low PAM index. Designing a multiple goal structure would induce higher effort and achievement from these students. Yet from the teacher's point of view, the increase in student achievement would not compensate for the higher cost of designing a multiple goal structure. In fact, the cost difference acts as a constraint: The teacher is no longer able to tailor teaching practices to students with a low PAM index, inducing a loss in achievement for these students.

The organizational setting in schools may accentuate the cost difference between classroom structures: By definition, larger classes increase the cost of applying (personalized) mastery-oriented structures.[16] In addition, recent educational policies, like school accountability, may have increased the cost difference and induced teachers to become more performance-oriented.[17] Furthermore, there is presumably heterogeneity in cost differences among teachers. Indeed, it is reasonable to think that (i) designing mastery-oriented structures requires more talent and pedagogical skills than applying performance goal structures, and (ii) teachers vary in their talent to design mastery goal structures as these structures are less standardized. Heterogeneity in cost differences is likely to explain variation in teacher value-added (see, Chetty et al., 2014), that is varying teacher quality, as these differences impact to what extent teachers are able to tailor teaching practices to student types.

To highlight the importance of teacher quality on student achievement, we calculate the loss in achievement of having a high-cost teacher ($\triangle c \geq \gamma^2/8v$) instead of a low-cost teacher ($\triangle c = 0$). The loss is an upper bound: A high-cost type designs a pure performance goal structure, $s = 1$, for all student types, regardless of their motivational pattern, whereas a low cost type designs the "first-best" structure s^*, defined in (4). The loss (in %) equals $100 * \left|\frac{\theta^2 v - \theta s^* e^*(s^*)}{\theta s^* e^*(s^*)}\right|$ (see Table 1). We have,

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[16] It is reasonable to think that a higher cost of applying a mastery structure is not only a question about class size per se, but also student heterogeneity. We leave the relation between optimal classroom structure, student heterogeneity, and class size to future research.

[17] Test-based school accountability, defined as the ranking of schools on the basis of students' test score, puts teachers under a lot of pressure to produce satisfactory student test scores. Testing students can easily become their main priority. Hence, time is missing to focus on student learning and to implement more time-consuming mastery-based structures. Teachers therefore transmit the extrinsic incentives imposed at the school level to the classrooms and neglect the use of intrinsic motivation. We comment on school accountability also in Section 2.2 and Section 3.

$$loss = \begin{cases} 100\left(1-4(1-\frac{\theta v}{\gamma})\frac{\theta v}{\gamma}\right) & if\ \frac{\theta v}{\gamma} \leq \frac{1}{2} \\ 0 & if\ \frac{\theta v}{\gamma} \geq \frac{1}{2} \end{cases}$$

The achievement loss is decreasing in the PAM index on $\left[0, \frac{1}{2}\right]$. When $\frac{\theta v}{\gamma}$ is close to zero, the loss approaches 100%. When $\frac{\theta v}{\gamma} = \frac{1}{8}$ it is equal to 56.25% and when $\frac{\theta v}{\gamma} = \frac{1}{4}$ it is equal to 25%. In other words, a low PAM-index student is very ill-adapted to the "one size fits all" performance goal structure designed by a high-cost teacher and would benefit a lot from a tailored multiple goal structure designed by a low-cost teacher. Loss in achievement is likely to have adverse effects on the long run motivation of these students and may trigger them to drop out. We will come back to this point in Section 3 and Section 4. In the rest of the article, we consider cases where there is no cost difference, $\triangle c = 0$. The teacher then simply maximizes the expected grade of the student.

2.2 The case of heterogeneous students

We now extend the analysis beyond the case of a representative student and consider a group of heterogeneous students. That is, we move from the simplest teaching situation in which the teacher is teaching one (representative) student to a more complex and realistic situation where the teacher faces a dilemma to ensure adequate education simultaneously to several types of students. This extension is interesting for two reasons: First, how does the teacher adapt her choice when students have different abilities and different goal orientations? In other words, how can the teacher achieve her desired distribution of learning gains and elicit her desirable student behavior from both those who are weak and lagging behind and those who can much and are far ahead? Second, how does the teacher behave when ability, the mastery-, and the performance-goal orientations are correlated?

To answer these questions, we consider a class with a population of students of size one. Each student is characterized by his ability θ, performance goal orientation v, and mastery goal orientation,

γ. The parameters θ, v and γ are distributed according to the density $f(\theta, v, \gamma)$ defined on $[\underline{\theta}, 1] \times [\underline{v}, 1] \times \left[\underline{\gamma}, 2\right]$ with $\underline{\theta} > 0$, $\underline{v} > 0$ and $\underline{\gamma} > 0$. We assume that $f(\underline{\theta}, \underline{v}, \underline{\gamma}) > 0$. Let $\rho_{\theta\gamma}$ denote the coefficient of correlation between θ and γ and $\rho_{\theta^2 v}$ the coefficient of correlation between θ^2 and v.

We consider two types of teachers who differ in the objective they pursue: The utilitarian teacher maximizes the average expected grade, $\int_0^1 \int_0^1 \int_0^2 \theta s e^*(\theta, v, \gamma, s) f(\theta, v, \gamma) d\gamma dv d\theta$ (where $e^*(\theta, v, \gamma, s) = max\left\{\theta v s + \gamma(1-s), 1\right\}$). The optimal classroom structure is[18]

$$s^*_{ut} = \begin{cases} \frac{1}{2(1-E[\theta^2 v]/E[\theta\gamma])} & \text{if } \frac{E\left[\theta^2 v\right]}{E[\theta\gamma]} \leq \frac{1}{2} \\ 1 & \text{if } \frac{E\left[\theta^2 v\right]}{E[\theta\gamma]} \geq \frac{1}{2} \end{cases} \tag{5}$$

The Rawlsian teacher maximizes the expected grade of the most at-risk student $(\underline{\theta}, \underline{v}, \underline{\gamma})$, i.e., the student with the lowest ability and the lowest performance- and mastery orientations. In this case, we can use the results from Section 2.1.2. The Rawlsian teacher chooses the pure performance goal structure $s^*_{ra} = 1$ if $\frac{\underline{\theta} \times \underline{v}}{\underline{\gamma}} \geq \frac{1}{2}$, but the multiple goal structure $s^*_{ra} = 1/(2 - 2\frac{\underline{\theta} \times \underline{v}}{\underline{\gamma}})$ if $\frac{\underline{\theta} \times \underline{v}}{\underline{\gamma}} < \frac{1}{2}$.

To analyze how the two types of teachers adapt their classroom structure when faced with heterogeneous students and to compare their behavior, it is convenient to define the concept of mean-preserving class diversification. Suppose that the class was homogeneous before becoming heterogeneous. We say that the class has undergone a mean-preserving diversification if the average ability, the average mastery goal orientation and the average performance goal orientation are equal in the initial state and the final state. More precisely,

Definition. Consider a class which is initially composed of homogeneous students of type $(\widehat{\theta}, \widehat{v}, \widehat{\gamma})$. We say that the class undergoes a mean-preserving diversification if the distribution of student types becomes $f(\theta, v, \gamma)$ with $E\left[\theta\right] = \widehat{\theta}$, $E\left[v\right] = \widehat{v}$, $E\left[\gamma\right] = \widehat{\gamma}$, and $\sigma(\theta) \geq 0$, $\sigma(v) \geq 0$ and $\sigma(\gamma) \geq 0$ with at least one strict inequality.

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[18] We use the fact that$\int_0^1 \int_0^1 \int_0^2 \theta s e^*(\theta, v, \gamma, s) f(\theta, v, \gamma) d\gamma dv d\theta = s^2 E\left[\theta^2 v\right] + (1-s) s E\left[\theta\gamma\right]$.

Let s^* be the optimal classroom structure before the diversification. Note that the utilitarian and Rawlsian teachers choose the same structure in the initial state because the class is homogeneous. For the sake of simplicity, we focus on the case where the initial structure, s^*, and the final structures, s^*_{ut} and s^*_{ra}, are interior solutions.[19] This yields the following result:

Proposition 2 *Consider a class that is subject to a mean-preserving diversification. Then the utilitarian teacher (the Rawlsian teacher) increases the performance goal structure after the diversification,* $s^*_{ut} > s^*$ *(* $s^*_{ra} > s^*$*), if and only if* $\rho_{\theta\gamma} < \frac{\sigma(\theta)E[\gamma]}{\sigma(\gamma)E[\theta]} + \frac{\sigma(\theta^2)\sigma(v)E[\gamma]}{\sigma(\theta)\sigma(\gamma)E[\theta]E[v]}\rho_{\theta^2 v}$ *(* $\frac{E[\theta]}{\underline{\theta}} < \frac{E[\gamma]/\underline{\gamma}}{E[v]/\underline{v}}$*).*

Proposition 2 shows that faced with a mean-preserving classroom diversification, the behavior of the utilitarian teacher and the Rawlsian teacher contrast sharply. Consider the utilitarian teacher. Suppose first that $\rho_{\theta\gamma} = 0$ and $\rho_{\theta^2 v} = 0$. In this case, the utilitarian teacher increases the performance goal structure after the diversification. In fact, the probability to succeed in the test is θse. That is, there is a double advantage of being a high-ability type due to the strategic complementarity between effort and ability in student achievement. By the same reasoning, being a low-ability student entails a double handicap because both effort, e, and the efficiency of effort, θs, are negatively affected. When student ability is dispersed, the double handicap is exacerbated: It becomes more difficult to motivate low-ability students, but easier to motivate high-ability students. The utilitarian teacher therefore maximizes the average achievement by fostering the performance of *above*-average students and designs a structure that is more performance-oriented.

When correlations are nil, the optimal structure s^*_{ut} can also be easily written in terms of the PAM index. We have $s^*_{ut} = \frac{1}{2\left(1-\left(1+CV^2(\theta)\right)\frac{E[\theta]E[v]}{E[\gamma]}\right)}$ when $\left(1+CV^2(\theta)\right)\frac{E[\theta]E[v]}{E[\gamma]} \leq \frac{1}{2}$, where $CV(\theta) \equiv \frac{\sigma(\theta)}{E[\theta]}$ is the coefficient of variation of ability. To maximize the average performance, the teacher designs a classroom structure which is optimal for the above-average student whose PAM index is

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[19] That is, we have $\frac{\widehat{\theta}\times\widehat{v}}{\widehat{\gamma}} < \frac{1}{2}$, $E\left[\theta^2 v\right] \leq \frac{1}{2}E\left[\theta\gamma\right]$ and $\frac{\underline{\theta}\times\underline{v}}{\underline{\gamma}} < \frac{1}{2}$. Note that $s^* = \frac{1}{2(1-\widehat{\theta}\widehat{v}/\widehat{\gamma})}$.

$\left(1 + CV^2(\theta)\right)\frac{E[\theta]E[v]}{E[\gamma]}$. The more heterogeneous in ability the class is (*i.e.* the higher $CV(\theta)$), the more distant this above-average student is from the average student $\Big(E[\theta], E[v], E[\gamma]\Big)$.

Suppose now that $\rho_{\theta\gamma} > 0$ and $\rho_{\theta^2 v} \geq 0$. When the dispersion of abilities, $\sigma(\theta)$, is low compared to the dispersion of mastery goals, $\sigma(\gamma)$, the double handicap effect is weaker. If the correlation between student ability and the mastery goal orientation is sufficiently high, the teacher chooses to induce effort based on students' intrinsic motivation in order to exploit the strategic complementarity between the mastery-induced effort and ability in the student achievement function, θse. To do so, she increases the mastery goal structure. In both cases (when correlations are nil or positive), the performance of the average student decreases.[20]

Faced with the same mean-preserving class diversification, the Rawlsian teacher compares $\frac{E[\theta]E[v]}{E[\gamma]}$, the PAM index of the average student, to $\frac{\underline{\theta}\times\underline{v}}{\underline{\gamma}}$, the PAM index of the most at-risk student $(\underline{\theta}, \underline{v}, \underline{\gamma})$. When $\frac{E[\theta]E[v]}{E[\gamma]} > \frac{\underline{\theta}\times\underline{v}}{\underline{\gamma}}$, the teacher chooses a structure more mastery-oriented than in the homogeneous case, in order to foster the intrinsic motivation of student $(\underline{\theta}, \underline{v}, \underline{\gamma})$. Note that the inequality can be written as $\frac{E[\theta]}{\underline{\theta}} > \frac{E[\gamma]/\underline{\gamma}}{E[v]/\underline{v}}$: Contrary to the utilitarian teacher, a "high" dispersion of abilities makes the Rawlsian teacher increase the mastery goal structure, after the diversification. Note that the performance of the average student also decreases with the Rawlsian teacher.

Neal and Schanzenbach (2010) study the effect of the "no child left behind"-reform that among others aimed at improving the educational lot of disadvantaged students and holding schools more accountable for student progress. They find that test-based school accountability systems induce teachers to distribute their effort differently: Teachers only allocate effort to students in the middle of the achievement distribution following such reforms. As a consequence, test scores among students in the middle of the achievement distribution increases, whereas students in the bottom of the distri-

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[20] In a meta-analysis, Senko, Hulleman, and Harackiewicz (2011) find that ability correlates only weakly with performance goals (mean = .10) and mastery goals (mean = .08). This suggests that utilitarian teachers should choose a more performance-oriented structure following a mean-preserving classroom diversification.

bution score lower or the same. In terms of our model, high-stake school accountability may encourage teachers to adopt more utilitarian preferences, and hence become more performance-oriented. Even if average achievement increases with a utilitarian teacher, the motivation and the achievement of the at-risk student $(\underline{\theta}, \underline{v}, \underline{\gamma})$ may be negatively affected, notably if his PAM index, $\frac{\underline{\theta}\times\underline{v}}{\underline{\gamma}}$, differs a lot from $\left(1+CV^2(\theta)\right)\frac{E[\theta]E[v]}{E[\gamma]}$. Policymakers must therefore in order to reduce unintended responses to accountability schemes, keep in mind how teachers' objectives can be affected when designing school accountability systems. They must also take into account student heterogeneity and carefully consider which students are to be prioritized as teachers cannot tailor teaching practices to all student types when students are too heterogeneous.

3 Dynamic Management of Student Motivation

Up until now, we have focused on how a teacher can facilitate success in one test. In other words, we have dealt with the management of student motivation in the short run. The classroom structure, however, also affects the way students react to test results, most notably after a failure. To study the management of student motivation in the long run, we introduce a twice repeated version of the static model presented in Section 2.1. We assume that failure in the first period affects the student's attitude towards schooling in two ways. First, the probability to pass the test in the second period decreases. This assumption echoes the cumulative nature of knowledge.[21] Second, we assume that a failure negatively affects the student's mastery goal orientation in the second period *unless* the teacher chooses a classroom structure that is sufficiently mastery-oriented in the first period. In fact, designing a mastery-oriented classroom structure in the first period can be understood as a long term investment in student failure tolerance, and as a way of maintaining student motivation over time, especially after a failure. Within this dynamic framework, we will see that the teacher faces a trade-off

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[21] Think about two successive courses where understanding the material taught in course one (say, math I) is a prerequisite for understanding the material taught in course two (math II).

between promoting better achievement in the short run through a performance structure, or allowing the student to overcome a potential failure and stay motivated in the long run by implementing a multiple goal structure. The dynamic framework also highlights the role of teacher and student time preferences.

3.1 The Dynamic Model

There are two periods denoted by $t = 1, 2$.

The student. In period t, the student exerts effort e_t. We denote by x_t the random variable equal to 1 if the test in period t is successful and 0 otherwise.

The teacher. She chooses a classroom structure $s_t \in [0, 1]$. As before, a higher (lower) s_t means that the structure is more performance (mastery)-oriented. An intermediate level of s_t corresponds to a multiple goal structure. The different classroom structures impose the same cost on the teacher. The teacher's payoff function for period t depends on the expected test result in this period, $E(x_t|h_t)$, where h_t is the history of the game at the beginning of period t.

Test result. For period one, we take

$$x_1 = \begin{cases} 1 & \text{with probability} \quad \theta s_1 e_1 \\ 0 & \text{with probability} \quad 1 - \theta s_1 e_1 \end{cases} \tag{6}$$

We assume that the probability of success in period two is unchanged after a success in period one, but negatively affected after a failure as knowledge is thought to be cumulative. That is, the student needs to acquire some knowledge in period one in order to better succeed in period two. If the realized value of x_1 is equal to 1, then:

$$x_2 = \begin{cases} 1 & \text{with probability} \quad \theta s_2 e_2 \\ 0 & \text{with probability} \quad 1 - \theta s_2 e_2 \end{cases} \tag{7}$$

However, if the realized value of x_1 is equal to 0, then:

$$x_2 = \begin{cases} 1 & \text{with probability} \quad \theta s_2 e_2/2 \\ 0 & \text{with probability} \quad 1 - \theta s_2 e_2/2 \end{cases} \tag{8}$$

Goal orientation. To concentrate on dynamic issues, we consider a representative student with a constant performance goal orientation, $v = 1$, and an initial mastery goal orientation $\gamma_1 = 1$. We consider that $\theta \geq \frac{1}{2}$. That is, a pure performance structure would be optimal in the static framework. We assume that the student's mastery goal orientation is unaffected after a success: $\gamma_2(1) = 1$. Nonetheless, after a failure in period one, the mastery goal orientation is affected by the teacher's choice of classroom structure in the first period. We take:

$$\gamma_2(0) = \begin{cases} 1 & \text{if} \quad s_1 \leq \widehat{s} \\ 0 & \text{if} \quad s_1 > \widehat{s} \end{cases}$$

for a given $\widehat{s} < 1$. In other words, the student keeps his initial mastery goal orientation after a failure if the teacher chooses a classroom structure in the first period that is sufficiently mastery-oriented. Otherwise, the mastery goal orientation vanishes and the student becomes fully performance-oriented in the second period. The rationale behind this assumption comes from empirical evidence. After a failure, performance-goal oriented students tend to develop more negative self-related thoughts and less interest in learning than mastery-goal oriented students (e.g., Ames 1992; Anderman and Wolters, 2006). The reason is that performance-oriented students attribute failure to a lack of ability, whereas mastery-oriented students believe that lack of effort is to blame (Ames, 1992). In consequence, performance-oriented students feel powerless to change when schooling is not longer easy. In contrast, mastery-oriented students keep their interest in learning and think they can engender success by working harder.

Payoffs. The student's expected payoff in period t after history h_t can be written $E(x_t|h_t)+\gamma_t(h_t)(1-s_t)e_t-0.5e_t^2$. We have $h_1=\emptyset$, $h_2\in\{0,1\}$, and by assumption $\gamma_1(h_1)=1$. The student's total payoff is the discounted sum of his per-period payoffs. Let δ^p denote his discount factor.

The teacher's expected payoff in period t is $E(x_t|h_t)$. We assume that the total payoff for the teacher is the discounted sum of her per-period payoffs. Let δ^t denote her discount factor.

Timing of the game and strategies. In each period $t=1,2$:

- The teacher chooses a classroom structure $s_t\in[0,1]$.
- The student observes s_t and exerts an effort level $e_t\in[0,1]$.
- There is a test in the end of period t with result x_t.
- The teacher and the student observe the realized value of x_t.

Strategies are $s_1, s_2(1)$ $s_2(0)$ for the teacher and $e_1(s_1)$, $e_2(1,s_2)$ and $e_2(0,s_2)$ for the student.

3.2 The Subgame Perfect Equilibrium

We solve the second period subgame given a classroom structure s_1 and effort e_1. We determine s_2 and e_2 conditional on the test result in period one. If the student is successful in period one ($x_1=1$), then we have $e_2^*(1,s_2)=(\theta-1)s_2+1$ and $s_2^*(1)=1$. These results follow from expressions (3) and (4), and the assumption $\theta\geq 1/2$. After a success in period one, the probability of passing the second test is sufficiently high to justify the teacher choosing a performance goal structure in period two. From Table 1, we know that at equilibrium the (expected) payoff for the student in period two is $U_2^{p^*}(1)=\theta^2/2$ and the (expected) payoff for the teacher is $U_2^{t^*}(1)=\theta^2$. If the student fails the test in period one ($x_1=0$), then his probability of success in period two decreases. We consider two cases:

(i) $s_1 > \widehat{s}$: The teacher designed a performance goal structure in period one. In this case, the mastery goal of the student totally vanishes after the failure, $\gamma_2(0) = 0$. In period two, the student chooses an effort level $e_2^*(0, s_2) = \theta s_2/2$ and the teacher chooses the classroom structure $s_2^*(0) = 1$: A performance structure is the only way to motivate a student who has lost his intrinsic interest in learning. At equilibrium, the (expected) payoff for the student in period two is $U_2^{p^*}(0) = \theta^2/8$ and the (expected) payoff for the teacher is $U_2^{t^*}(0) = \theta^2/4$.

(ii) $s_1 \leq \widehat{s}$: The teacher designed a multiple goal structure in period one to preserve the student's mastery goal after a failure, $\gamma_2(0) = 1$. In period two, the student chooses an effort level $e_2^*(0, s_2) = (\theta/2 - 1)s_2 + 1$ and the teacher chooses the classroom structure $s_2^*(0) = \frac{1}{2}\frac{1}{1-\theta/2}$. By establishing a multiple goal structure in period two, the teacher can build on the preserved intrinsic motivation of the student to induce effort. At equilibrium, the (expected) payoff for the student in period two is $U_2^{p^*}(0) = 1/8$. The (expected) payoff for the teacher is $U_2^{t^*}(0) = \frac{1}{4}\frac{\theta/2}{1-\theta/2}$.

We solve period one knowing $e_2^*(.)$ and $s_2^*(.)$. The student, for a given classroom structure s_1, maximizes $\theta s_1 e_1 + (1 - s_1)e_1 - \frac{1}{2}e_1^2 + (\theta s_1 e_1)\delta^p U_2^{p*}(1) + (1 - \theta s_1 e_1)\delta^p U_2^{p*}(0)$. We obtain:

$$e_1^*(s_1) = \begin{cases} min\left\{1 + (\theta - 1)s_1 + (\theta s_1)\delta^p(\frac{\theta^2}{2} - \frac{\theta^2}{8}), 1\right\} & \text{if} \quad s_1 > \widehat{s} \\ min\left\{1 + (\theta - 1)s_1 + (\theta s_1)\delta^p(\frac{\theta^2}{2} - \frac{1}{8}), 1\right\} & \text{if} \quad s_1 \leq \widehat{s} \end{cases} \tag{9}$$

In period one, the student makes more effort in the dynamic model than in the static one (where effort would be equal to $1 + (\theta - 1)s_1$). The existence of a second period extends the benefits of being successful in the first period, as the student's capacity to succeed in the second test depends on his initial achievement. The supplementary effort is higher, the more patient the student is.

The teacher chooses s_1^* to maximize the discounted sum of her per-period payoffs: $\theta s_1 e_1^*(s_1) + (\theta s_1 e_1^*(s_1)) \times \delta^t U_2^{t^*}(1) + (1 - \theta s_1 e_1^*(s_1)) \times \delta^t U_2^{t^*}$ where $e_1^*(s_1)$ is given by (9). Two structures are potentially optimal: $s_1^* = 1$ and $s_1^* = \widehat{s}$. The total expected payoff when she chooses $s_1^* = 1$ is:

$$\theta(\theta + \frac{3}{8}\delta^p\theta^3)(1 + \frac{3}{4}\delta^t\theta^2) + \delta^t\frac{\theta^2}{4} \tag{10}$$

The total expected payoff when the teacher chooses $s_1^* = \widehat{s}$ is:

$$\left(\theta(\theta\widehat{s} + 1 - \widehat{s} + \theta\widehat{s}\delta^p\frac{4\theta^2 - 1}{8})\widehat{s}\right)\left(1 + \delta^t(\theta^2 - \frac{1}{4}\frac{\theta}{2-\theta})\right) + \frac{1}{4}\delta^t\frac{\theta}{2-\theta} \tag{11}$$

We denote by $\widetilde{s}(\theta)$ the value of $\widehat{s}$ that equalizes (10) and (11). The determinant of the corresponding second degree equation is $\Delta = 1 - 4(1 - \theta - \theta\frac{4\theta^2-1}{8}\delta^p)(\theta + \eta)$, with $\eta = -\frac{8\delta^t(1-\theta^2)(1-\theta)^2 - 3\delta^p(4+3\delta^t\theta^2)(2-\theta)\theta^3}{32(2-\theta)+\delta^t(32\theta^2(2-\theta)-8\theta)}$.

We first consider the case of a myopic student: His discount factor is nil, $\delta^p = 0$. Here, the determinant Δ is positive.[22] We find $\widetilde{s}(\theta) = \frac{1-\sqrt{\Delta}}{2(1-\theta)}$. When $\widetilde{s}(\theta)$ is below (above) $\widehat{s}$, the structure $\widehat{s}$ yields a higher (lower) payoff for the teacher than $\widetilde{s}(\theta)$. As a result, the teacher prefers the structure $s_1 = \widehat{s}$ ($s_1 = 1$) to the structure $s_1 = 1$ ($s_1 = \widehat{s}$). Consequently, the optimal classroom structure in period one for a given ability θ and structure $\widehat{s}$ is:

$$s_1^* = \begin{cases} 1 & \text{if} \quad \widehat{s} < \widetilde{s}(\theta) \\ \widehat{s} & \text{if} \quad \widehat{s} \geq \widetilde{s}(\theta) \end{cases}$$

The function $\widetilde{s}(\theta)$ is represented in Figure 1 for $\delta^t = 1$. The left part of the figure shows the optimal classroom structure for a myopic student of different ability levels. The lower the student's ability, θ, the larger the area in which the teacher chooses the multiple goal structure, $\widehat{s}$, in the first period. By promoting a mix of both mastery and performance goals, the teacher accepts that the student performs less well in the first period in order for him to be able to overcome a possible failure. The choice of the multiple goal structure, $\widehat{s}$, is less appropriate for a high-ability student for two reasons. First, it induces a decrease in the expected grade in the first period compared to the situation with a

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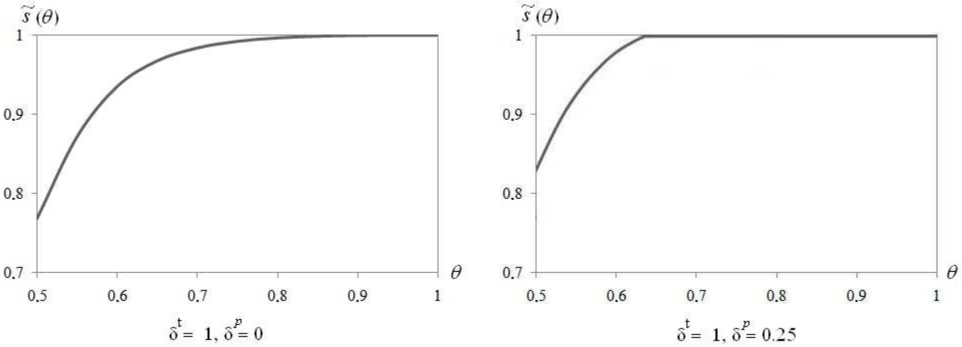
[22] This is because $\eta < 0$ implies $4(1 - \theta - \theta\frac{4\theta^2-1}{8}\delta^p)(\theta + \eta) < 4(1 - \theta)(\theta) \leq 1$.

performance structure. Second, the probability of passing the test is greater for a high-ability student, so the benefit of developing the student's failure tolerance is reduced.

Note that $\widetilde{s}(\theta)$ increases as δ^t decreases and that $\widetilde{s}(\theta) = 1$ for any θ when $\delta^t = 0$.[23] That is, as the teacher becomes less forward-looking, she is less willing to develop the student's failure tolerance at the expense of sacrificing his performance in the first period. Therefore, a short time horizon, either innate or caused by the organizational setting, acts as a third constraint that prevents the teacher from tailoring teaching practices to student types.

Figure 1: The threshold values



$\delta^t = 1$, $\delta^p = 0$

$\delta^t = 1$, $\delta^p = 0.25$

We now study the effect of increasing δ^p starting from zero for a given positive value of δ^t. We have $\widetilde{s}(\theta) = \frac{1-\sqrt{\Delta}}{2(1-\theta-\theta\delta^p\frac{4\theta^2-1}{8})}$. It can be verified that $\widetilde{s}(\theta)$ is increasing in δ^p (see the right part of Figure 1 drawn for $\delta^p = 0.25$). A more patient student exerts a higher level of effort in period one to successfully enter period two. The extra effort is nevertheless smaller when the structure is more mastery-oriented in period one, because the student is then in a sense more "insured" against failure. As a consequence, the teacher finds it less interesting to design a multiple goal structure in the first period when the student is forward looking. It can be verified that $\widetilde{s}(\theta)$ does not exist when δ^p is above 0.56. In this case, the teacher chooses a pure performance goal structure, which brings us back to the static case. We sum up the results in the following proposition.

Proposition 3 *In a dynamic context, the teacher, if sufficiently patient, chooses a first-period goal*

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[23] When δ^t increases, η decreases. In turn, Δ increases and $\tilde{s}(\theta)$ decreases.

structure that is more mastery-oriented than in the static case, if the student is not too patient. This choice of structure enables the teacher to develop the failure tolerance of the student at the expense of a short-term decrease in achievement.

The results in proposition 3 correspond to the idea developed in the achievement goal literature, which states that, by choosing a multiple goal structure, the teacher uses performance to spur efficient effort in the short run and uses mastery to increase student's failure tolerance in the long run (Ames, 1992; Barron and Harackiewicz, 2001). In addition, the proposition underlines the role played by student and teacher time preferences: The multiple goal structure is more effective when the teacher is sufficiently patient and the student is not too patient.

Results on time preferences enable us to draw additional policy implications. As only a sufficiently patient teacher can succeed in motivating students over time, it is important to design teacher contracts that select and retain teachers that embody this quality. In addition, educational policies should be designed to induce teachers to keep focusing on students' long term achievements. The design of some recent school accountability reforms, however, seem to push teachers in the opposite direction, which may negatively affect students' motivation in the long run. Evidence shows that teachers preemptively hold students back from taking the test (Hanushek and Raymond, 2002), increase the use of special education placements (Jacob, 2005), substitute away from low-stakes subjects (Figlio, 2006), teach for the test (Jacob, 2005), and cheat (Jacob and Levitt, 2003).

Our results also indicate that developing a multiple goal structure is the best practice when the student is not too patient. Several studies suggest that young students (i.e., in elementary school) are myopic, notably due to the development of the brain (e.g., Lavecchia et al. 2014). Young students therefore do not properly trade off the immediate costs and the long term benefits from education. Hence, mastery-oriented classroom structures are even more suited to young students, and seem therefore particularly adapted to the context of elementary school.

4 How to manage the transition from elementary to middle school?

It is well established that in many countries, schooling becomes more competitive on the transition from elementary school to middle school (e.g. Eccles et al. 1993). In middle school, students typically face whole class instruction and have fewer opportunities to participate in class. Teachers emphasize grades and the demonstration of ability relative to others, and attach more importance to the final achievement than to effort and progress made by the students. Evidence describes a decline in students' motivation and achievement in middle school (e.g., Eccles et al., 1993). The reason could be that classrooms are too performance-oriented. While adolescents in middle school progressively strengthen their time preference for the future (i.e., δ^p increases) and develop a normatively based conception of ability (i.e., v increases and γ decreases) (Harackiewicz et al., 1998), a more competitive classroom environment may still have negative consequences for many students. In fact, the changes in psychological and cognitive development do not occur at the same time, the same rate, or in the same amount for all adolescents. For many of them, there is a mismatch between their stage of development and the learning environment (Eccles et al. 1993; Midgley, 1993). Our model helps to understand the reasons and the consequences of this mismatch: First, middle-school teachers are confronted with a more demanding curriculum, tighter time constraints, and larger classes than elementary school teachers, and hence a higher cost of applying a mastery-oriented structure. We saw in Section 2.1 that a higher cost difference makes it more difficult for the teacher to tailor teaching practices to student types. Classroom structures become too performance oriented for students with a low PAM index. Second, middle school teachers may have a more utilitarian point of view regarding teaching practices in contrast to elementary school teachers who may have a more Rawlsian point of view. We saw in Section 2.2 that with heterogeneous students, a utilitarian teacher prioritizes above-average students, and neglects at risk- and the average students. Third, middle school teachers might be less forward looking than elementary school teachers as they often follow students in only one subject (and not in almost all subjects as in elementary school) and for a shorter time period (middle school lasts around 3 years). We saw in Section 3 that a less patient teacher is less able to develop the students' failure tolerance. To facilitate the transition from elementary to middle school, it could therefore be important to design teacher contracts that select and retain teachers that have a low innate cost of applying a mastery structure, and also design educational reforms that decrease the cost of adopting such a structure. Moreover, middle school policies should incite teachers to adopt more Rawlsian preferences, and aim to make teachers more forward looking. A smoother transition with more mastery-oriented structures in middle school could keep low-ability, intrinsically motivated, and myopic students more motivated, increase their success probability, and prevent them from later dropping out of high school.[24]

5 Conclusion

This article opens the black box of the classroom and studies the microeconomic foundations of teaching practices and student motivation in schools. We construct a theoretical model based on achievement goal theory that clarifies how teaching practices affect student motivation, effort, and achievement in different classroom settings. We highlight an important trade-off between level and efficiency of student effort. We also show that the optimal teaching practice depends on an unique index embodying student ability (cognitive skills) and their goal orientation (non-cognitive skills).

When students are homogeneous and implementation costs of classroom structures are equal, the teacher is able to tailor teaching practices to students types. For high-ability and extrinsically motivated students, the teacher designs a performance goal structure to promote achievement, whereas

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[24] See Belfield and Levin (2007) for the tremendous private and social costs related to inadequate education and high school dropout in the USA. Applying a mastery structure for many of these students could probably have been highly beneficial relative to the cost of not doing so. In fact, students that begin high school are unlikely to begin with the same set of preferences, abilities, or motivation, given their diversity of experiences and background. A better experience in middle school could help to improve their performances and thereby reduce high school dropout rates.

for low-ability students and high-ability students that are intrinsically motivated, the teacher designs a multiple goal structure. Although this structure comes at the cost of a lower efficiency of effort, it triggers and/or maintains these students' interest in the task, even after a failure. In fact, a multiple goal structure is the optimal mix between the strong, but potentially unstable incentives corresponding to performance goals, and the more stable, but potentially less efficient incentives corresponding to mastery goals.

Our framework also highlights three constraints - cost difference between classroom structures, student heterogeneity, and teachers' myopic behavior - that can prevent teachers from tailoring teaching practices to student types. The existence of these constraints, which can explain why some students do not succeed in school, brings forward several recommendations. First, teachers with a low innate cost of applying mastery-oriented structures should be selected and retained as these teachers are likely to engender teacher value added. In addition, the organizational setting (class size, lack of teacher training, performance-oriented policies) should not prevent teachers from choosing the appropriate classroom structure. Second, the objectives as regards to which students are to be prioritized - Utilitarian, Rawlsian, or mixed - should be explicitly and carefully chosen as teachers cannot tailor teaching practices to all student types when students are too heterogeneous. Third, teachers should be willing to develop students' failure tolerance to safeguard their long run motivation. Notably, teacher payment schemes and school accountability systems should be designed to select forward looking teachers as well as induce teachers to stay non-myopic.

This article is one first attempt to understand how teachers can manipulate student motivation, effort and achievement through the choice of teaching practices. In future research, we would like to study the optimal classroom design and inquire how teachers can tailor teaching practices to student types by taking into account simultaneously student heterogeneity and class size.

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