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# Dynamic Panics: Theory and Application to the Eurozone<sup>\*†</sup>

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## Abstract

It is shown in a standard quantitative model of sovereign debt and default that sentiment shocks can alter the distribution of fundamental defaults. The channel through which this occurs is a new type of dynamic lender coordination problem in sovereign debt markets that I call a *dynamic panic*. During a dynamic panic of this kind, expectations of future negative investor sentiments dilute the price of long-term debt; the sovereign's optimal response to such a panic can be to borrow aggressively, which justifies investors' fears of dilution. Such panics generate naturally many of the unique features the recent Eurozone crisis, and so I explore policy implications in this environment. I find that rate ceilings are an ineffective policy tool but that liquidity provision, appropriately implemented, can eliminate such panics.

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**Keywords:** Sovereign Debt Crises, Confidence-Driven Crises, Long-Term Debt

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# 1. Introduction

Since the seminal contribution of [Eaton and Gersovitz \(1981\)](#), there has been a folk intuition in the sovereign debt and default literature that market sentiments can have real effects. The intuition is as follows: When investors expect the sovereign to default they demand a high spread, which makes repayment costly and raises the default frequency; on the other hand, when lenders do not expect a default they demand a low spread, which reduces the burden of repayment and with it the default frequency.<sup>1</sup>

Casting the impact of sentiments in this light alone, however, can be overly restrictive. There are other channels through which sentiment dynamics can impact sovereign debt markets, even while retaining the tractable Markov-Perfect structure preferred by the quantitative literature. In particular, this paper shows that sentiment shocks can alter the distribution of *fundamental* defaults. In other words, negative sentiment shocks can induce the sovereign to venture into regions of the state space where fundamental default is far more likely. The channel through which this occurs is a dynamic lender coordination failure that I call a *dynamic panic*.

During a dynamic panic of this kind, which I will call a *non-default-relevant* dynamic panic, a negative shock to sentiment today sends a signal that future sentiment will likely be low, which dilutes the price of long-term debt. In anticipation of this dilution, investors demand compensation in the form of high spreads. In the face of these non-fundamental high spreads, the sovereign can find it optimal to borrow excessively rather than default or delever because the typical default-risk channel<sup>2</sup> that induces saving in the face of adverse shocks is no longer present since sentiment shocks themselves never cause default. This excessive borrowing dilutes the value of long-term debt and justifies investors' initial fears.

These non-default-relevant dynamic panics are not a mere theoretical curiosity. I will show that, with only minor modification<sup>3</sup>, they are already present in standard quantitative models in the literature. I will provide as an example the calibrated model of [Chatterjee and Eyigungor \(2012\)](#), which was not initially designed to allow for such non-fundamental borrowing crises.

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<sup>1</sup>This intuition was formalized in some form in the models of [Calvo \(1988\)](#) and [Cole and Kehoe \(1996\)](#).

<sup>2</sup>For an in-depth discussion of this mechanism, see [Arellano \(2008\)](#). Essentially, during an adverse fundamental shock, there are two competing forces: A borrowing motive driven by a high marginal utility, and a saving motive driven by an increase in the country's interest rate, which jumped because of increased default risk. In quantitative applications, the latter nearly always dominates, though there can be exceptions e.g. [Conesa and Kehoe \(2012\)](#).

<sup>3</sup>The only modifications that need to be made are an augmentation of the state space to include a non-fundamental and a slight increase in the degree of absolute risk-aversion.

This unusual feature of borrowing into high spreads, which comes quite naturally out of the crises in my model, is a hallmark feature of the Eurozone crisis and the one that has attracted the most attention from the recent literature (see [Lorenzoni and Werning \(2013\)](#), [Corsetti and Dedola \(2013\)](#), or [Conesa and Kehoe \(2012\)](#)). In light of this similarity, I explore the policy implications of dynamic panics. I find that in this environment an interest rate ceiling, which has been proposed as a potential policy, would be ineffective.

To see why an interest rate ceiling is ineffective during a dynamic panic, it is helpful to understand why it would normally work. The justification for such a policy is grounded in [Calvo's \(1988\)](#) framework in which there are two ways a sovereign can generate the same of revenue: Issue a small amount of debt at low spreads, which are low because the probability of default is low, or a large amount of debt at high spreads, which are high because the probability of default is high. Through the lens of this model, distressed Eurozone countries were 'stuck' in the latter situation and therefore a simple, credible cap on the market rate would be enough to rule out this sub-optimal equilibrium. However, during a dynamic panic, the sovereign is always borrowing on the left side of this 'Laffer curve', even during a crisis, since it can freely choose its debt level and would never place itself on the Pareto-dominated right side.<sup>4</sup> Thus, an interest rate ceiling in this environment is isomorphic to a revenue cap on debt issuance and will only reduce government consumption and increase the likelihood of default.

Even though rate ceilings are ineffective, I do find that the provision of liquidity, which I interpret to be a policy that the ECB undertook in its OMT program, was effective. In providing liquidity, the central bank credibly pledges to purchase sovereign debt at potentially sub-market rates. The model suggests that such a policy is effective at removing sentiment fluctuations, but that its welfare consequences are ambiguous. This is because dynamic panics will not be a randomization over fundamental multiplicity, and thus there may not be a 'good' regime on which to coordinate. As a consequence, the regimes induced by a dynamic panic may look radically different in terms of their debt levels, default frequencies, and average spreads, than an alternative fundamental equilibrium that does not act on sentiment dynamics.

This welfare trade-off can be understood in the context of the debate between the core and the periphery: The periphery wants the central bank to provide liquidity to protect them from malignant market sentiments, while the core fears that with such a backstop the periphery will rack up unsustainable debt

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<sup>4</sup>For a graphical illustration of this, refer to [Figure 5](#).

levels and bring about more frequent fundamental crises. Both channels are active in the model. Under some non-trivial regularity assumptions, it can be demonstrated that the key to the welfare consequences of such a policy are the correlation of confidence with the re-entry shock following a default-induced period of exclusion: If market sentiment is high upon re-entry, then the low-spreads equilibrium corresponding to the mid-2000s is sustainable and the policy is likely welfare-improving; however, if market sentiment is low upon re-entry, then the low spreads of the mid-2000s did not constitute a sustainable equilibrium. Given

In summary, this paper makes the following points: First, it outlines a new dynamic lender coordination problem, which is the susceptibility of the standard sovereign debt environment to dynamic panics; second, it generates borrowing into high spreads endogenously in the context of an already canonical model; third, it demonstrates that interest rate ceilings would be an ineffective policy in combating such crises; and fourth, it highlights the trade-off faced in the provision of liquidity and argues that, conditional on observed spreads, this policy was likely welfare-improving.

The rest of the paper is organized as follows. In Section 2, I review the relevant literature. In Section 3, I construct a simple model in which dynamic panics are completely characterizable and driven by default alone and explore their basic properties. In Section 4, I augment the simple model until it resembles a canonical model of endogenous sovereign default, which is augmented to allow for sentiment dynamics. I contrast the sort of panics that can arise in this model with those in the simple model, focusing on the possibility for panics to be driven by excessive borrowing by virtue of the long-term nature of the debt. I then explore several necessary features of dynamic panics in this environment and their relationship to the Eurozone crisis as well as impact of several plausible policies designed to counter such panics. Section 5 concludes.

## 2. Literature Review

This paper contributes to several different strands of the literature. First, it contributes to the quantitative literature outlining the dynamics of sovereign debt and default episodes. This literature takes the seminal framework of [Eaton and Gersovitz \(1981\)](#) and applies it quantitatively to primarily Latin American economies to match business cycle statistics and the empirical regularities of developing nations. Noteworthy papers in this vein include [Aguiar and Gopinath \(2006\)](#) and [Arellano \(2008\)](#). There is a nice

summary of this tradition in [Aguiar and Amador \(2014\)](#).

This literature has also developed a branch that explicitly considers debt of longer maturities, of which prominent examples include [Hatchondo and Martinez \(2009\)](#), [Chatterjee and Eyigungor \(2012\)](#), and [Arelano and Ramanarayanan \(2012\)](#). The lesson from this branch is that, apart from expected default, the movements in the expected future price of the debt can have a significant impact on spreads today. This effect has been called ‘dilution risk’, and [Chatterjee and Eyigungor \(2012\)](#) show that it accounts for a substantial fraction of long-term spreads. Dilution risk will feature prominently in my exploration of non-default-relevant dynamic panics.

The framework developed by this literature has also been the benchmark for a string of recent applied work studying default episodes. This is in large part because of the tractability of the assumption of Markov-Perfection. Some prominent examples include [Mendoza and Yue \(2012\)](#), [Gornemann \(2014\)](#), [Salomao \(2014\)](#), and [Na et al. \(2014\)](#), who study respectively the impact of default on international private lines of credit, long-term growth, credit default swaps, and optimal devaluation policy.

A common thread in this tradition is the lack of multiplicity or self-fulfilling dynamics. In this class of models, default is always driven by an unfortunate sequence of fundamental shocks and often the underlying equilibrium is unique. This uniqueness result was recently formalized by [Auclert and Rognlie \(2014\)](#). A recent exception is [Passadore and Xandri \(2014\)](#), who find sufficient conditions under which multiplicity of equilibria can exist for the case of short-term debt and use these conditions to bound the impact of sunspot activity. This paper will have a similar goal, but will focus primarily on the recursive dynamics of sunspot activity itself rather than its bounds. It also explores the interaction of sunspot activity with the unique features of long-term debt.

This paper also contributes to the recent literature on the Eurozone. As of yet, the academic literature has had little time to keep pace with developments that took place in the Eurozone over the past 6 years or so. However, several noteworthy pieces have emerged that have tried to deal seriously with the peculiar circumstances surrounding the sovereign debt crisis in the Eurozone. These papers have been both empirical and structural. On the empirical side, recent work has taken aim at demonstrating the confidence-driven nature of this crises by documenting an unusually weak correlation between economic fundamentals and CDS spreads. Some prominent examples include [De Grauwe and Ji \(2013\)](#) and [Aizenman](#)

[et al. \(2013\)](#). This work will rely in some sense on these empirical findings in its placement of malignant market sentiments at the heart of the story in its theory of the crisis.

On the structural side, much emphasis has been placed on the unusual phenomenon of borrowing into high spreads and its concomitant drastic effect on debt-to-GDP ratios. [Conesa and Kehoe \(2012\)](#) have termed this phenomenon ‘gambling for redemption,’ and have argued that being mired in a deep recession is a necessary condition for such behavior. [Broner et al. \(2014\)](#) and [Corsetti and Dedola \(2013\)](#) have also built models featuring borrowing into high spreads. The former emphasizes the crowding out effect of sovereign debt issuance when there is domestic preference for debt and the latter argues that the access to liquidity that the central bank provides is more important for preventing such crises than the printing press.

To the author’s knowledge, the only others that have highlighted the role of dynamic coordination problems in the recent Eurozone crisis are [Lorenzoni and Werning \(2013\)](#). These authors also argue for a Laffer-curve type multiplicity in the spirit of [Calvo \(1988\)](#), but with the explicit inclusion of long-term debt. In their environment, as in mine, a dynamic lender coordination failure can place the economy on a malignant trajectory of high spreads and debt ratios. They term such a crisis a ‘slow-moving’ crisis. The key difference between their paper and mine is that I place more structure on the coordination failure itself and give it the form of a persistent sunspot. This allows me to explore the possibility of such crises in the standard Eaton-Gersovitz framework in which the government can commit to a level of debt as well as to a level of revenue. This small difference has important policy implications. For instance, in their environment, an interest rate ceiling would be an effective tool at alleviating slow-moving crises, while in mine such a policy will tend to induce more default.

## 3. Simple Model

### 3.1. Environment

In this section I construct a simple sovereign debt environment in the tradition of [Eaton and Gersovitz \(1981\)](#) and [Arellano \(2008\)](#) but with no intrinsic uncertainty and then characterize completely how the model reacts to non-fundamental confidence. In the subsequent section I will relax many of the restrictive assumptions and explore quantitatively how a richer, more standard model reacts to confidence shocks.

Specifically, I will explore how sentiment fluctuations can generate borrowing panics and the key model features required to do so.

For now, consider an infinitely-lived sovereign borrower that receives a constant endowment,  $y$ , in each period. This sovereign has an increasing flow utility,  $u(\cdot)$ , over consumption in each period and discounts the future at a rate  $\beta < 1$ . The only uncertainty in this model is extrinsic confidence,  $\xi$ , which can take one of two values,  $\{\xi_L, \xi_H\}$ . I assume that  $\xi$  follows a symmetric Markov process with transition probability  $\eta$ .<sup>5</sup>

He has a constant stock of debt,  $b$ , and he can either choose to roll over that debt at an exogenously given price,  $q$ , which may depend on the level of confidence. If he does not roll over this debt, then he defaults on it. He makes this default decision as soon as the extrinsic uncertainty is realized i.e. before he goes to the auction to roll over his debt. When he defaults, he is excluded from credit markets forever and pays a constant additive cost  $\phi(y)$  in every subsequent period. I will restrict attention to equilibria that are Markov-Perfect in confidence, and so we can write his Bellman equation, conditional on repayment, as follows:

$$V(\xi) = u(y - b + q(\xi)b) + \beta E_{\tilde{\xi}|\xi}[\max\{V(\tilde{\xi}), X\}] \quad (1)$$

where  $X$ , which is the value of default, can be computed as follows:

$$X = u(y - \phi(y)) + \beta X$$

Notice that because there is no re-entry the value of  $X$  is independent of any particular equilibrium. Hence, it is not an equilibrium object. The sovereign will borrow from a unit mass of risk-neutral, deep-pocketed lenders with an outside option with return  $R$ . These lenders price default risk according to a no-arbitrage condition, so in equilibrium the price must be given by:

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<sup>5</sup>The assumption of symmetry can be easily relaxed at the cost of a lengthier exposition.

$$q(\xi) = \frac{1}{R} E_{\tilde{\xi}|\xi}[\mathbf{1}\{V(\tilde{\xi}) \geq X\}] \quad (2)$$

We are now ready to define an equilibrium in our simple environment. In particular, a **Markov-Perfect Equilibrium** will be a pair of functions  $\{V(\xi), q(\xi)\}$  such that

1. Given  $q(\xi)$ , the value function  $V(\xi)$  solves Recursion 1
2. Given  $V(\xi)$ , the pricing function  $q(\xi)$  solves Recursion 2

I will call a Markov-Perfect Equilibrium a **Fundamental Markov-Perfect Equilibrium** if in it confidence has no real effects. If confidence does have real effects, I will call the Markov-Perfect Equilibrium a **Confidence-Waves Equilibrium**. In a Confidence-Waves Equilibrium, a shift from  $\xi_H$  to  $\xi_L$  will be a **Dynamic Panic**.

### 3.2. Characterizing Confidence-Waves Equilibria

In this simple environment, it is possible to make analytic statements regarding the entire set of Confidence-Waves Equilibria. In particular, the following theorem can be established:

**Theorem 3.1.** *The set of parameters over which a Confidence-Waves Equilibrium exists is completely characterized by the following two conditions:*

1.  $\frac{R-1+\eta}{R}b \leq \phi(y)$
2.  $\frac{\beta\eta}{1-\beta(1-\eta)} \left[ u\left(y - \frac{R-1+\eta}{R}b\right) - u(y - \phi(y)) \right] < u(y - \phi(y)) - u\left(y - \frac{R-\eta}{R}b\right)$

**Proof** See [Appendix A](#)

Theorem 3.1 provides a set of both necessary and sufficient conditions for the potential for sentiment dynamics in this simple model of sovereign default.<sup>6</sup> The first condition will ensure that repayment is optimal for some value of  $\xi = \xi_H$  without loss of generality. The second condition ensures that, in addition, default is optimal in  $\xi_L$ . Since there is no other source of uncertainty, this is the only way in

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<sup>6</sup>Note that we cannot say that such conditions are absolutely necessary for sunspot activity since, as I will show, non-sunspot equilibria will also exist for these same parameterizations.

which confidence can have real effects in a Markov-Perfect and thus these conditions completely characterize the conditions necessary for sunspot activity.

From these conditions, we can derive several useful corollaries that highlight key model elements required for confidence to have a real impact. The first is that confidence must be persistent.

**Corollary 3.2.** *In any Confidence-Waves Equilibrium,  $\eta < 1/2$ .*

**Proof** See [Appendix A](#)

Why must confidence be persistent? When default is driven by market sentiment, it must be that high spreads themselves cause the default and low spreads themselves cause repayment. However, spreads reflect anticipated default *in the future*, not contemporaneous default.

Suppose that we were trying to establish an equilibrium in which default only occurred in the face of low confidence. If confidence was transient, then high confidence today would imply that quite likely default would occur tomorrow. But this would drive down the price of debt today relative to the low confidence state. Since the price discrepancy can be the sole driver of default discrepancies, this would induce default in the high-confidence state instead of the low confidence state, which is a contradiction. Thus, if confidence has real effects, it must be persistent.

So confidence must be persistent. But there is another important characteristic about the sovereign debt environment that is not immediately obvious. It is summarized in the following corollary:

**Corollary 3.3.** *Let  $\Theta(u)$  be the set of parameters for which a Confidence-Waves Equilibrium exists given a utility function,  $u$ . If  $\hat{u}$  is more concave than  $u$ , then  $\Theta(u) \subseteq \Theta(\hat{u})$ .*

**Proof** See [Appendix A](#)

In words, any parameterization for which a Confidence-Waves Equilibrium exists will continue to do so as you increase the degree of risk-aversion of the sovereign. Further, the set of parameters over which confidence fluctuations can occur expands with the degree of risk-aversion. Why is this? It is because a more concave utility functions will punish a repaying sovereign in utility terms more severely when debt service costs are expensive and thus when consumption is low. This will occur when confidence is low.

This punishment must be severe enough such that default is optimal even though there is some probability of re-entering the high-confidence, no-default regime.

### 3.2.1. Relationship to Rollover Crises and Multiplicity

An easy misinterpretation of Theorem 3.1 is that I am simply finding conditions for a ‘rollover crisis’ as in Cole and Kehoe (1996). This is not at all what I am describing here. During a rollover crisis, an individual investor fears that other investors will not show up to the auction to roll over the sovereign’s short-term debt. If the sovereign defaults in response, then the investors’ fears are justified and they do not show up.

This is not what is going on with my dynamic panics. First, this is because the timing is different: The default decision occurs *prior* to the sovereign’s debt auction. Thus, lenders cannot experience such a contemporaneous coordination failure in my set-up. Second, lenders in my set-up are not panicking about the behavior of other lenders today; rather, they are panicking about the behavior of lenders *tomorrow*. Because the sentiment shock is persistent, when it is shocked today, lenders anticipate that lenders tomorrow will offer a low price, which will induce default tomorrow. This fear induces them to offer a low price today, which in turn induces default today. It is for this reason that I call my confidence crises dynamic panics, since the intertemporal dimension is crucial.

This is formalized in the following corollary. During a rollover crisis, the equilibrium price of debt,  $q$ , equals zero i.e. the sovereign cannot raise any revenue at the auction. However, during a dynamic panic the price of debt falls, but it is not zero.

**Corollary 3.4.** *During a dynamic panic, debt can be auctioned at  $q(\xi_L) > 0$ .*

Thus, unlike a rollover crisis, it is *possible* to raise revenues at auctions; it is simply sub-optimal to do so.

The last necessary condition I will discuss is the relationship of Confidence-Waves Equilibria to multiplicity. In particular, we can claim the following:

**Corollary 3.5.** *If a Confidence-Waves Equilibrium exist, then the full-default and full-repayment equilibrium both exist as well.*

**Proof** See [Appendix A](#)

This finding accords with the recent findings of [Passadore and Xandri \(2014\)](#), who find that multiplicity is a necessary pre-condition to sunspot activity. In other words, there must be some room for strategic complementarities of the sort that would induce multiple equilibria in order for a Confidence-Waves Equilibrium to exist.

This is not to say that Confidence-Waves Equilibria are simply randomizing over existing multiplicity. While this is one way that they can be generated, it is also possible that the sunspot is randomizing over pricing schedules *that are not themselves equilibria*. In [Appendix A](#) I provide an analytic example of such a case when there is some intrinsic uncertainty as well.

This result is similar in kind to that of [Gottardi and Kajii \(1999\)](#), who argue that sunspots need not randomize over existing multiplicity, but only over ‘potential multiplicity’. They define to potential multiplicity to be the existence of multiple equilibria for a reallocation of endowments. In my set-up, such ‘potential multiplicity’ arises if there exist pricing schedules and default strategies that are close to satisfying the equilibrium conditions. Confidence fluctuations can randomize over these schedules even though they themselves are not equilibria.

## 4. Full Model

In this section I will describe a model of sovereign debt and default that subsumes several of the standard quantitative sovereign default models in the literature. I then explore the potential for dynamic panics in these already standard models and characterize several of their necessary features, honing in on the capacity of sentiment shocks to generate borrowing panics.

This model will be a superset of [Chatterjee and Eyigungor \(2012\)](#), which in turn subsumes the influential model of [Arellano \(2008\)](#). In particular, there will be intrinsic as well as extrinsic uncertainty. There will also be an endogenous borrowing choice and debt of longer maturities.

### 4.1. Environment

The theoretical model will allow for three stochastic processes: A fundamental endowment shock,  $y \in \mathcal{Y}$ , a continuous, fundamental subsistence consumption level shock,  $\tilde{m} \in [\underline{m}, \bar{m}]$ , and a non-fundamental

confidence shock,  $\xi \in \Xi$ . Both the endowment and the confidence shocks are assumed to be persistent and  $\mathcal{Y}$  and  $\Xi$  are assumed to be discrete sets. In particular, I assume that confidence is binary i.e.  $\xi \in \{\xi_L, \xi_H\}$  and that there is a symmetric transition probability,  $\eta < 1/2$ . The preference shock,  $\tilde{m}$ , is assumed to be iid over time and will therefore not impact the price in equilibrium. Its continuous nature, however, will smooth over the discrete nature of sovereign's decisions in expectation, which helps both with existence and computation. Its range is assumed to be fairly small.

The sovereign borrower chooses a level of consumption,  $c$ , how much to borrow from abroad,  $b' \in \mathcal{B}$ , and whether or not to default. It is assumed that every element in  $\mathcal{B}$  is non-negative and that  $0 \in \mathcal{B}$ . He receives a flow utility,  $u(\cdot)$  from consumption, which I assume is increasing and strictly concave. Debt is long-term and matures stochastically at a rate  $\lambda$ ; in each period that it does not mature it pays a coupon  $\kappa$ .

I will focus on Markov-Perfect Equilibria and so I can write the sovereign's problem recursively. Taking as given the demand schedule for its debt from foreign investors,  $q(y, \xi, b')$ , the government solves the following Bellman, which is conditional on repayment this period:

$$\begin{aligned}
 V(y, \xi, m, b) &= \max_{c \geq 0, b' \in \mathcal{B}} u(c - m) + \beta \mathcal{V}(y, \xi, b') & (3) \\
 \text{s.t. } c &\leq y - [\lambda + (1 - \lambda)\kappa]b + q(y, \xi, b')[b' - (1 - \lambda)b]
 \end{aligned}$$

I assume that if the sovereign faces an empty budget set, he must default. The continuation value allows for default is ex-post optimal, and is thus given by

$$\mathcal{V}(y, \xi, b') = E_{(\tilde{y}, \tilde{\xi}, \tilde{m})|(y, \xi)}[\max\{V(\tilde{y}, \tilde{\xi}, \tilde{m}, b'), X(\tilde{y})\}]$$

I will denote the sovereign's borrowing policy function to be  $a(y, \xi, m, b)$ .

I assume that when the country defaults, it is excluded from credit markets temporarily. It will suffer some additive output loss,  $\phi(y)$ , in each period of this exclusion. It re-enters stochastically at a rate  $\pi_{RE}$  and it does so with a deterministic level of confidence  $\bar{\xi}$ , after which confidence follows its typical Markov-process. This assumption helps solidify the representation of  $\xi$  as confidence, since presumably confidence is highly correlated with the re-entry shock, though it is not entirely clear in which direction that correlation goes. It may be that creditors are leery of a defaulter's return to credit markets and

confidence is low; this would accord with the findings of Ozler (1993), who finds that borrowing costs tend to be higher for defaulters upon re-entry. Alternatively, it may be the case that the simple fact that lenders are now willing to lend to the sovereign again reflects a positive shift in sentiment. Which story is true will have important theoretical and policy consequences, as I will outline later.

Notice that this assumption will imply that the default value is independent of  $\xi$  even though  $\xi$  remains non-fundamental. This feature will be important both computationally and theoretically. Also upon re-entry, it is assumed to have no debt obligations.<sup>7</sup>

Under these assumptions, we can express the value of default as

$$X(y) = u(y - \phi(y) - \bar{m}) + \beta E_{\tilde{y}, \tilde{m}|y}[(1 - \pi_{RE})X(\tilde{y} - [m - \tilde{m}]) + \pi_{RE}V(\tilde{y}, \bar{\xi}, \tilde{m}, 0)] \quad (4)$$

Notice that, as in Chatterjee and Eyigungor (2012), during the first period of default the sovereign faces the worst  $\tilde{m}$  shock but experiences it as its normal stochastic process thereafter.

I assume that the sovereign borrows from a unit mass of short-lived, risk-neutral, deep-pocketed lenders that price default and dilution risk against some asset with a risk-free return,  $R$ . These lenders care about the endowment and confidence level today, since these objects provide information regarding their distribution tomorrow, and the level of borrowing today,  $b'$ , since indebtedness will also provide information regarding the default frequency tomorrow. Since the debt is long-term, they also care about the expected future price of their debt and thus about the degree to which the sovereign borrows tomorrow. The pricing recursion thus becomes

$$q(y, \xi, b') = \frac{1}{R} E_{(\tilde{y}, \tilde{\xi}, \tilde{m})|(y, \xi)} \left[ 1\{V(\tilde{y}, \tilde{\xi}, \tilde{m}, b') \geq X(\tilde{y})\} \times [\lambda + (1 - \lambda)(\kappa + q(\tilde{y}, \tilde{\xi}, a(\tilde{y}, \tilde{\xi}, \tilde{m}, b')))] \right] \quad (5)$$

#### 4.1.1. Equilibrium Definition

A **Markov-Perfect Equilibrium** is a set of functions  $V(y, \xi, m, b)$ ,  $a(y, \xi, m, b)$ ,  $X(y)$ , and  $q(y, \xi, b')$  such that

1.  $V(y, \xi, m, b)$  satisfies Recursion 3 when given  $X(y)$  and  $q(y, \xi, b')$  and implies the borrowing policy function  $a(y, \xi, m, b)$

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<sup>7</sup>This last assumption can be relaxed to allow for haircuts provided we restrict attention to the set of equilibria for which default incentives are increasing in the level of debt, which is a benign and plausible restriction.

2.  $X(y)$  satisfies Recursion 4 when given  $V(y, \xi, m, b)$
3.  $q(y, \xi, b')$  solves Recursion 5 given  $V(y, \xi, m, b)$ ,  $X(y)$ , and  $a(y, \xi, m, b)$

A **Confidence-Waves Equilibrium** is a Markov-Perfect Equilibrium in which either  $V(y, \xi_L, m, b) \neq V(y, \xi_H, m, b)$ ,  $a(y, \xi_L, m, b) \neq a(y, \xi_H, m, b)$ , or  $q(y, \xi_L, b) \neq q(y, \xi_H, b)$  for some  $(y, m, b)$  in the fundamental state space. In other words, a Confidence-Waves Equilibrium is a Markov-Perfect Equilibrium in which sentiment shocks matter. A **Dynamic Panic** is defined to be a transition from  $\xi_H$  to  $\xi_L$  in the context of a Confidence-Waves Equilibrium, since such a panic will represent a shift in lender sentiment regarding the behavior of future lenders.

## 4.2. Theoretical Results

In what follows I characterize the theoretical properties of Confidence-Waves Equilibria. In particular, I will show that dynamic panics at longer maturities exhibit several features peculiar to the Eurozone crisis, including excessive persistence and borrowing into high spreads. Because of this, I explore several policy implications of long-term dynamic panics and discuss briefly their applicability to the Eurozone crisis.

The first result tells us that confidence shocks are indeed an equilibrium phenomenon: Both lenders and the sovereign must actively respond to the shock in order for them to have any real effects:

**Proposition 4.1.** *Let  $d(y, \xi_L, m, b)$  denote the default policy of the sovereign. In any CW Equilibrium, **both** of the following must hold*

1.  $\exists(y, m, b)$  such that either  $d(y, \xi_L, m, b) \neq d(y, \xi_H, m, b)$  or  $a(y, \xi_L, m, b) \neq a(y, \xi_H, m, b)$
2.  $\exists(y, b')$  such that  $q(y, \xi_L, b') \neq q(y, \xi_H, b')$

**Proof** See [Appendix A](#)

Having established the equilibrium nature of these sentiment shocks, I now begin to characterize them by defining a new term:

**Definition** A Confidence-Waves Equilibrium is **Default-Relevant** if the realization of  $\xi$  matters for the default decision of the sovereign at some point in the fundamental state space.

In other words, if the equilibrium is default-relevant then there is a fundamental state for which the sovereign defaults when confidence is low and repays when confidence is high. It is possible to have an equilibrium that is non-default-relevant even though  $\tilde{m}$  is continuously distributed provided that the range of  $\tilde{m}$  is fairly small. I will provide an example momentarily.

With this definition in hand, we can distinguish how maturity will influence the underlying impact of confidence.

**Proposition 4.2.** *If the debt is short-term i.e.  $\lambda = 1, \kappa = 0$ , then any Confidence-Waves Equilibrium must be default-relevant.*

**Proof** See [Appendix A](#).

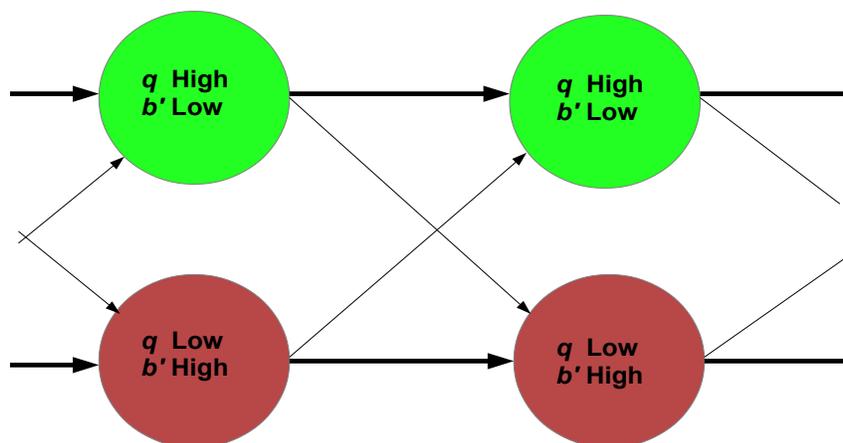
Proposition 4.2 tells us that if the sunspot has any real effects, it must at some point make the difference between the sovereign defaulting and repaying. This proposition disappears when we extend the maturity of the debt. It will still be the case that default-relevance is sufficient to have an active sunspot, but it will no longer be necessary, since the sunspot can affect the future price of the debt if it affects borrowing behavior.

So how can we have a non-default-relevant dynamic panic? There are essentially two components: Investors' pricing of dilution and sovereign excessive borrowing. For intuition, suppose that the path of primary deficits is taken as given. During a non-default-relevant dynamic panic, investors today anticipate aggressive borrowing *tomorrow*, which dilutes the price of long-term debt today. Since debt prices are lower today, additional borrowing is required to fill the same primary deficit. But since the sovereign has borrowed more today, he will have more to roll over tomorrow and thus borrow more tomorrow, justifying lenders' fears of dilution. This intuition holds up even when the sovereign is allowed to optimally choose the path of primary deficits i.e. it is more costly for him to change alter this path than it is to reduce the elevated risk of default.

The sovereign cannot escape the impact of these sentiment shocks because investors are afraid not of contemporaneous borrowing behavior, but future borrowing behavior, and he lacks the ability to commit to borrowing in the future, just as he lacks the ability to commit to repayment.

Figure 1 demonstrates the intuition. With longer term debt lenders care not only about whether the

sovereign defaults tomorrow, but about the future price of the debt. Thus, if lenders in period  $t$  anticipate lenders in  $t+1$  to panic, it need not be the case that default probabilities actually rise in  $t+1$ , since lenders in  $t$  already care about the price of debt in  $t+1$ . But Proposition 4.1 tells us that the sovereign must respond somehow to this shock, even though he does not change his default behavior. Thus, he changes his borrowing behavior, and in particular he must borrow in the face of a panic to justify the panic occurring in the first place. He borrows more today to fill the same primary deficit at higher spreads and is expected to borrow more tomorrow to roll over this larger debt burden.



**Figure 1:** Non-Default-Relevant Dynamic Panics: An Illustration

Thus, non-default-relevant dynamic panics are a way for sentiments to have real effects without ever impacting directly the default decision of the sovereign. Rather, they alter the distribution over future fundamental defaults by increasing sovereign indebtedness through excessive borrowing. This additional indebtedness justifies lenders fears, which are of dilution risk.

In a sense, a non-default-relevant dynamic panic is the opposite of an investor ‘run’ as in [Diamond and Dybvig \(1983\)](#) or [Rodrik and Velasco \(1999\)](#). Investors in such a panic do not fear that investors tomorrow will withdraw lending from the sovereign. They are afraid of quite the reverse i.e. that lenders tomorrow will lend excessively to the sovereign, which will drive down the expected future price of the debt. Since

long-term bondholders care about this future price, they too will panic and demand higher spreads.

Although the causal chain of a long-term dynamic panic is driven by borrowing, in equilibrium we will simply have two different regimes: One in which spreads, default probabilities, and borrowing are all low and another in which all of these objects are high. The only time this will not be true is in the period of the initial panic itself, since here default probabilities will not increase, though spreads and borrowing will. This can be seen in the numerical example provided in Figure 2.<sup>8</sup> This example shows how a non-default-relevant dynamic panic can affect borrowing and pricing behavior. In particular, we get two distinct pricing and borrowing regimes: One in which low borrowing occurs at low spreads and one in which high borrowing occurs at high spreads.

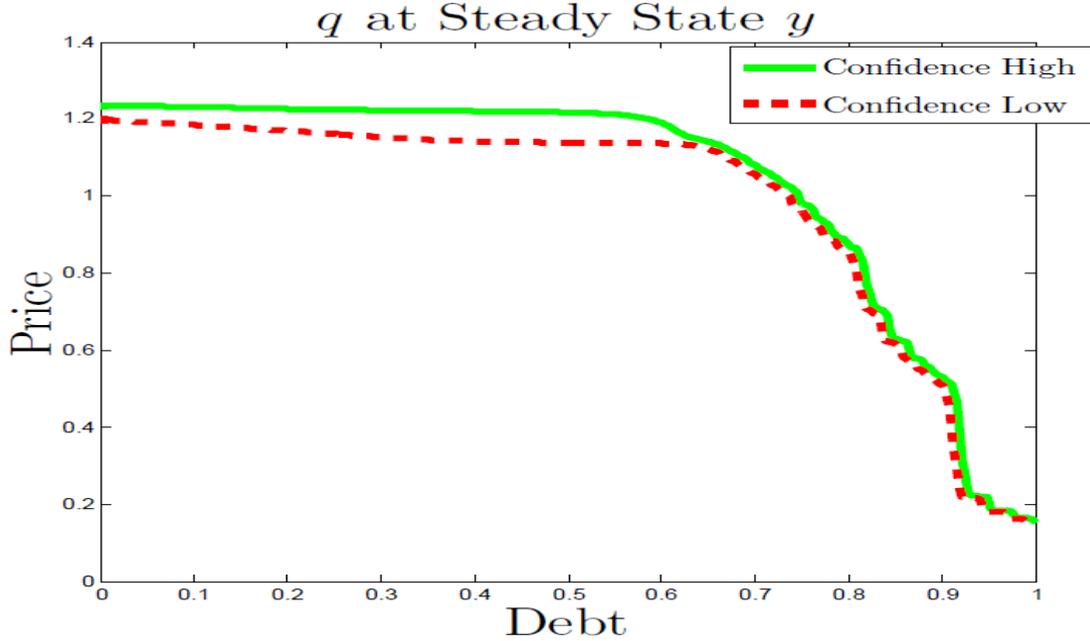
Notice that in Figure 2 that the demand schedules across panic and non-panic states are very similar for high levels of debt. This is because these far right portions of the demand curve reflect immediate default risk, which does not change across confidence states when the equilibrium is non-default-relevant. However, there is a substantial difference in the pricing schedules with regards to low debt levels. This is because debt prices at low levels don't reflect immediate default risk but rather dilution risk: Lenders are concerned about the future trajectory of sovereign borrowing, which will be necessarily be higher during a panic than not.

Further, we can see from a simulation path in Figure 3 that a regime change is associated with higher levels of borrowing.<sup>9</sup> Figure 3 compares the spreads and consumption paths of two sample economies which face the same endowment shocks. The first economy does not respond to non-fundamental activity and the second economy is in a confidence-waves equilibrium, experiencing a dynamic panic. One can see that when the confidence falls, it immediately and substantially increases spreads; by a factor of about two in this example. However, consumption changes very little; in other words, the sovereign undergoes little to no fiscal consolidation in response to this shock. Rather, he borrows more from abroad to fill the budgetary gap. This is seen in Figure 4, which plots the response of the debt level to the same sequence

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<sup>8</sup>To calibrate these examples, I simply use the parameterization of Chatterjee and Eyigungor (2012) and add a large but non-binding subsistence level of consumption to induce greater absolute risk-aversion in the flow utility function, which is assumed to be CRRA. Increased risk-aversion increases the utility cost of high sentiment-driven spreads and thus increases responsiveness of the sovereign in terms of borrowing and default behavior to such spread shocks.

<sup>9</sup>Notice that both spreads and debt ratios are contemporaneous. High confidence-driven spreads induce excessive borrowing in the next period while high fundamental-driven spreads induce deleveraging in the following period.



**Figure 2:** Demand Functions in a Long-Term Dynamic Panic (Non-Default-Relevant)

of shocks as Figure 4.

I will address the optimality of excessive sovereign borrowing in a moment, but before I do I will define one more term to describe the sort of panic outlined in this quantitative model.

**Definition** A dynamic panic is **monotone** at  $y$  if for every  $b' \in \mathcal{B}$  we have that  $q(y, \xi_H, b') \geq q(y, \xi_L, b')$ .

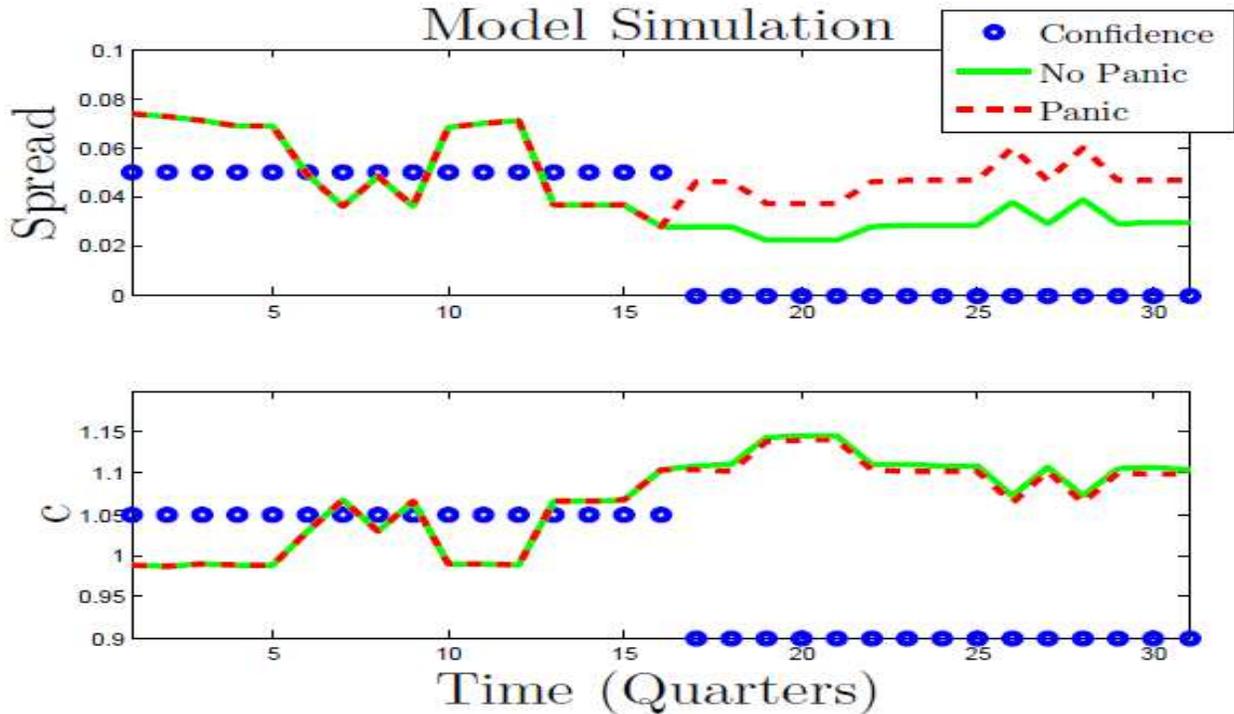
There are numerous necessary features of monotone, non-default-relevant dynamics panics that formalize the intuition just outlined. I will now outline each of them in turn.

**Proposition 4.3.** *If, in a non-default-relevant Confidence-Waves Equilibrium, the dynamic panic is monotone at some  $\hat{y} \in \mathcal{Y}$ , then there is a subset of fundamental states,  $S \subset \mathcal{Y} \times \mathcal{M} \times \mathcal{B}$  such that if  $(y, m, b) \in S$ , we must have that*

$$a(y, \xi_H, m, b) < a(y, \xi_L, m, b)$$

*In these states, future default frequencies must rise.*

**Proof** See [Appendix A](#)

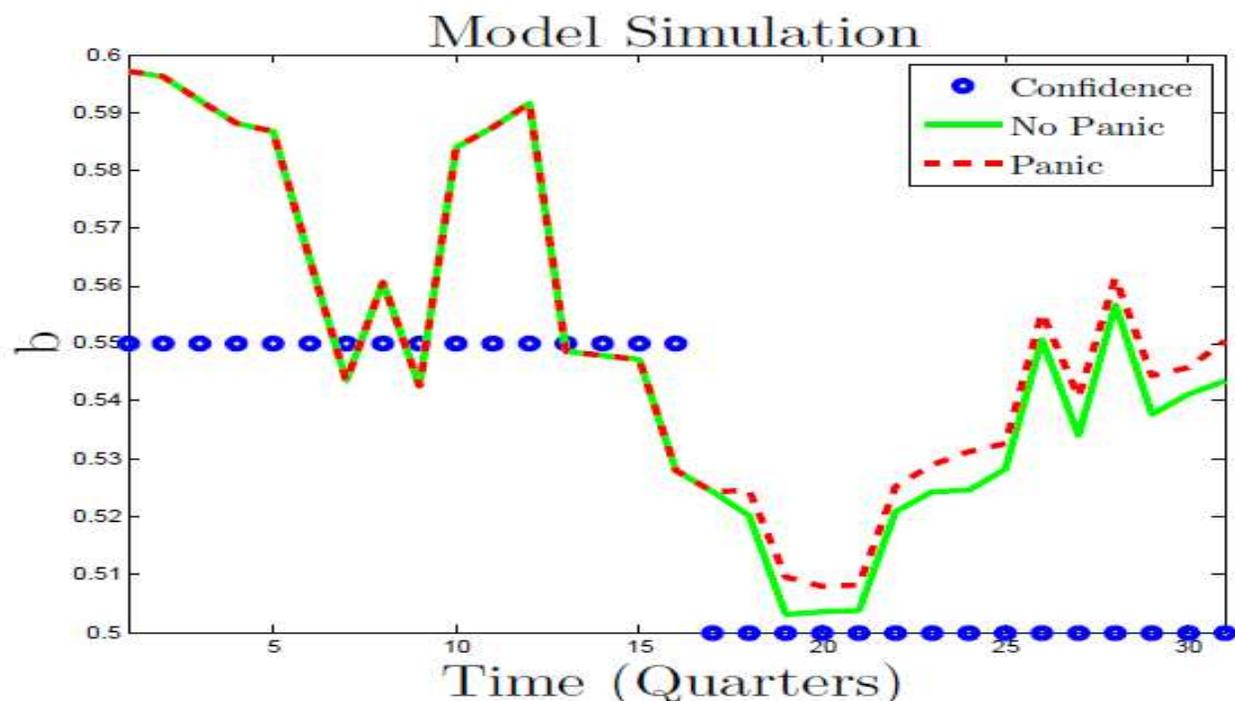


**Figure 3:** Simulation of a Long-Term Dynamic Panic (Non-Default-Relevant)

This behavior of borrowing into high spreads is a *necessary* condition of non-default-relevant long-term dynamic panics. The sovereign has three options in the face of an adverse shock: Default, delever, or borrow into the spreads. This third option, in which little to no domestic fiscal adjustment takes place, *is a requirement* to generate non-default-relevant dynamic panics, since this is the only response that justifies the lender panic. This is one of the key features of such panics that I will utilize in my structural estimation.

But why would such excessive borrowing behavior be the optimal response of the sovereign? The reason is twofold. First, at longer maturities, the amount of debt that needs to be rolled over at panic spreads will be lower than it will be for shorter maturities; thus, such panic borrowing is on average less costly.

Second, the typical default-risk channel that generates overly volatile consumption is absent without default-relevance. To understand why, consider what usually happens in response to an adverse fundamental shock in period  $t$  in the style of [Arellano \(2008\)](#). Such a shock has two consequences: First, it reduces income in period  $t$ , generating a borrowing motive; and second, it increases default risk in period  $t+1$ , which in equilibrium raises interest rates and generates a saving or delevering motive. In quantitative



**Figure 4:** Simulation of a Long-Term Dynamic Panic (Non-Default-Relevant)

applications, this latter effect dominates and saving occurs in bad times, generating consumption volatility that is greater than output volatility.

However, during a monotone, non-default-relevant confidence shock, default risk remains unchanged in period  $t + 1$ ; instead, it increases in periods  $t + 2, t + 3, \dots$  as a result of higher future borrowing. Thus, the magnitude of the equilibrium rate effect is necessarily smaller for a confidence shock of this kind, and so too is the saving motive. In equilibrium, it is necessarily dominated and higher borrowing occurs.

In practice, this result is far stronger than the fairly weak claim of this proposition: Such borrowing into high spreads tends to happen in nearly every state of the world that is realized on the equilibrium path. In fact, in the face of a panic the sovereign tends to not change his consumption or default behavior at all and instead borrows additionally to fill the same primary deficit.

This result is quite striking upon reflection. It tells us that during a dynamic panic, the sovereign will *willfully increase his debt position and default probability*. This stands in stark contrast to the behavior sovereigns in such models as [Cole and Kehoe \(1996\)](#), in which case the sovereign either delevers or defaults in response to the negative shock. The intuition behind this result is that the debt here is of longer maturity

and therefore the average cost of new debt issuance, which is what is directly affected by the price, is not nearly as high as it is for short-term debt, the stock of which the sovereign must roll over every period. It is therefore much more willing to increase its debt position in the hopes of recovery.

#### 4.2.1. Policy Implications

I now explore two new policy implications in this environment. The first is the efficacy of a rate ceiling. Many authors, including [Corsetti and Dedola \(2013\)](#) and [Lorenzoni and Werning \(2013\)](#) have argued that an interest rate ceiling could have been an effective tool in combating malignant market sentiments. The reason is the following: A graph of revenue versus debt at any debt-auction ought to be parabolic since low levels of debt will have high prices and thus raise revenue but high levels of debt will have lower prices due to increased default probabilities and thus actually lower revenue. Some examples of such Laffer-curves can be seen in [Figure 5](#).

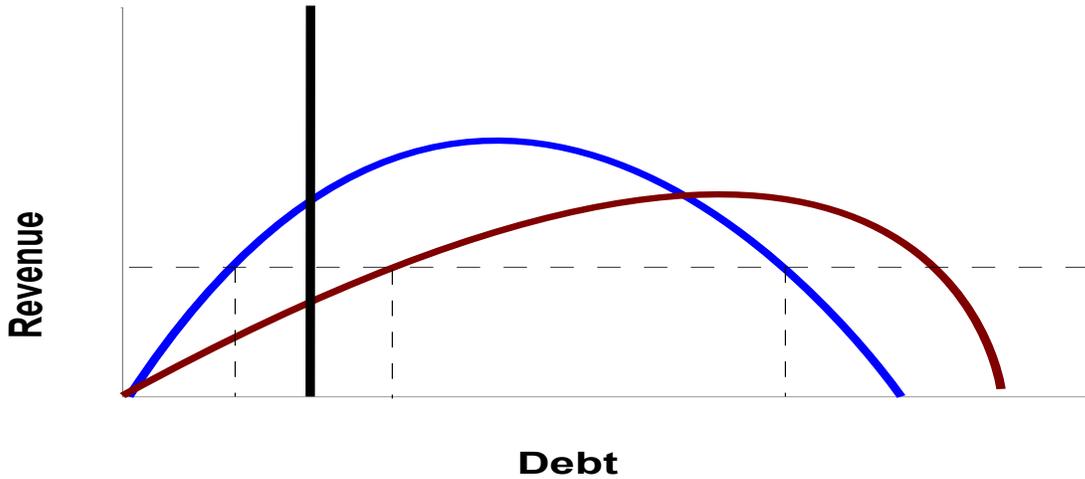
These authors follow [Calvo \(1988\)](#) in asserting that the confidence crisis experienced by the Eurozone was a result of the sovereign winding up on the right-hand side of this Laffer-curve i.e. raising the same amount of revenue but with higher debt and worse prices. In the presence of such a crisis, an interest rate ceiling can be an effective tool since it forces the investors to coordinate on the good equilibrium on the left-hand side of the Laffer curve.

This policy implication is lost when the crisis at hand is a dynamic panic and not a Calvo-style crisis, as is made clear by the following proposition.

**Proposition 4.4.** *During a dynamic panic, a binding interest rate ceiling is equivalent to a revenue cap on debt issuance. Thus a binding, temporary interest rate ceiling will increase the probability of default.*

**Proof** See [Appendix A](#)

There is an intuitive graphical exposition of [Proposition 4.4](#) in [Figure 5](#). The black line represents the debt cap imposed by the rate ceiling. Consider the level of revenue raised by the horizontal dashed line. The typical Calvo-style multiplicity dictates that the sovereign is on the far-right intersection with the blue curve, and thus a rate ceiling such as this forces investors to coordinate back on the good equilibrium on the left side of the curve.



**Figure 5:** Laffer-Curves: Non-crisis (Blue) and Crisis (Red)

However, during a dynamic panic, we are not on the right side of the blue curve; we are in fact on the left side of a new red curve that implies less revenue raised for any given level of debt. The debt cap imposed by the black line then simply implies a revenue cap. Given this revenue cap, consumption must drop in the case of repayment and so repayment becomes less attractive and default frequencies rise.

It is important to note that an interest rate ceiling is different than the arguably successful measures that the ECB took to avert the crisis such as the Outright Monetary Transactions (OMT) bond-buying program. As noted by [Corsetti and Dedola \(2013\)](#), these programs were not rate ceilings but guarantees that the ECB would purchase government debt at sub-market interest rates. I follow [De Grauwe \(2011\)](#) in calling such a policy *liquidity provision*. The next proposition demonstrates that in fact such a policy may be effective.

**Proposition 4.5.** *Liquidity provision can eliminate the impact of confidence fluctuations without the need to actually purchase any assets. The resulting economy will still suffer from a weakly positive probability of default driven by the problem of limited commitment.*

**Proof** See [Appendix A](#).

Proposition 4.5 tells us that the ECB can in fact judiciously provide liquidity to eliminate the impact

of malignant market sentiments, as they effectively did with the OMT Program. Such a policy does not actually require a purchasing of the debt, so long as the implied demand schedule is consistent with an equilibrium not subject to confidence fluctuations.

The proposition also tells us that under the resulting economy will continue to suffer from a weakly positive probability of default. This is actually a fairly accurate description of the OMT program, which did not provide unconditional liquidity, but required that certain sustainability measures be met. [Wolf \(2014\)](#) highlights that this aspect of the program drew criticism from its opponents, since it would presumably not be there to provide liquidity precisely when member countries needed it most: During a crisis. My modelling choice for liquidity provision allows for precisely such a crisis to occur while simultaneously eliminating the direct impact of sentiment fluctuations.

It is clear from the above proposition that the central bank can eliminate confidence fluctuations, but it is less clear if it would want to. Provision of liquidity in this sense can essentially costlessly shift the economy from an equilibrium with sunspot activity to one without. However, it is not clear a priori that the equilibrium without sunspots is better than its sunspots counterpart. To make a welfare statement one way or the other will require some knowledge of the relationship of Confidence-Waves Equilibria to underlying multiplicity. Unfortunately for the policy-maker, the regimes over which the sunspot randomizes are not both themselves equilibria. It turns out that at most one of them is. This result is formalized in the following proposition:

**Proposition 4.6.** *Either  $\lim_{\eta \rightarrow 0} q(y, \xi_L, b')$  is an equilibrium pricing schedule or  $\lim_{\eta \rightarrow 0} q(y, \xi_H, b')$  is an equilibrium pricing schedule, but not both.*

**Proof** See [Appendix A](#).

This result tells us that either the panic regime or the high confidence regime is close to an equilibrium, but not both. The result follows from the assumption that the confidence shock is perfectly correlated with the re-entry shock. The two pricing regimes will converge to what may at first appear to be distinct equilibria, but they must share a common value of default. This default value can be associated with at most one equilibrium, and so the other pricing schedule must not be an equilibrium.

This result has a very useful corollary that helps clarify the consequences of liquidity provision:

**Corollary 4.7.** *Suppose that we have a set of non-default-relevant Confidence-Waves Equilibria that are monotone for all  $y \in \mathcal{Y}$  as well as continuous in  $\eta$ . If  $\bar{\xi} = \xi_H$ , then the provision of liquidity can return the economy to an equilibrium with lower spreads than the high-confidence regime.*

This result tells us that for a certain class of equilibria, those that are continuous in  $\eta$  and monotone for all  $y$ , the consequences of liquidity provision depend critically on expectations regarding investor confidence in the re-entry following a default. In particular, if high investor confidence is associated with re-entry, then it will be the case that the central bank can essentially shift the economy back to a regime that is even better than the high-confidence state; however, if low investor confidence is associated with re-entry, then providing liquidity in this way will only worsen the panic, since the only sustainable regime is the panic.

Corollary 4.7 does not necessarily imply that the provision of liquidity is welfare-improving: This is because in states of the world where the sovereign wishes to delever and repurchase debt, it is more costly for him to do since debt prices are high. In practice however, since these sovereign governments are myopic, this tends to imply that the provision of liquidity is welfare-improving. Since we did see spreads fall on impact in the European case, it is quite likely that in this case,  $\bar{\xi} = \xi_H$ ,<sup>10</sup> and thus the provision of liquidity was likely welfare-improving in this context.

## 5. Conclusion

In this paper, I characterized a new type of dynamic lender coordination problem, which I call dynamic panics. I demonstrated their existence in the standard quantitative sovereign debt model as well as characterized their basic properties. In particular, I showed that they appear as true panics in the sense of monotone price shifts, and that their existence implies that the uniqueness result associated with fundamental Markov-perfection is a fragile one. I also show that such panics can affect both long-term and short-term debt, but if they affect short-term debt it must at some point act through the default channel.

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<sup>10</sup>If  $\bar{\xi} = \xi_L$ , we would have seen spreads rise mildly at the announcement in 2012 rather than fall drastically.

However, with long-term debt we can have crises driven solely by borrowing behavior, which I argue occurred in the Eurozone periphery. I further demonstrated that in this environment interest rate ceilings are ineffective but that liquidity provision can eliminate the malignant impact of market sentiments.

This paper lays the groundwork for much potential future research. First and foremost, there is a clear need to take this mechanism to Eurozone data in a serious way. In [Stangebye \(2015\)](#) I attempt this with an auxiliary DSGE model with a unique equilibrium, but an exploration of the quantitative capacity of true sentiment shocks to generate such crises is called for.

Further, I have only just begun to outline the theoretical properties of these confidence-waves and have only been able to prove their existence for short-term debt, though computational examples with long-term debt can be found. An existence theorem for the case of long-term debt would likely be quite enlightening.

In addition, the dynamic panics may have significantly broader implications than simply in sovereign debt markets. There is no reason why we would not expect such panics in markets for, say, municipal debt or commercial paper. A more in-depth exploration of the potential for dynamic panics in these markets would also prove illuminating.

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# Appendix A. Theoretical Proofs

## Appendix A.1. Proof of Theorem 3.1

To characterize the set of Confidence-Waves Equilibria, note first that there is only one way that  $\xi$  can have real effects: It must induce the sovereign to default in one state and repay in the other. If the sovereign defaults in both states, then confidence has no effects; the same is true if the sovereign repays in both states. Without loss of generality, let us search for an equilibrium in which the sovereign repays in  $\xi_H$  and defaults in  $\xi_L$ .

If this is the case, then the equilibrium pricing function must be given as follows:

$$\begin{aligned} q(\xi_L) &= \frac{\eta}{R} \\ q(\xi_H) &= \frac{1-\eta}{R} \end{aligned}$$

If we impose the default strategy in the continuation value of the sovereign, then we can write the Bellman of the sovereign conditional on repayment in  $\xi_H$  as follows:

$$V(\xi_H) = u\left(y - b + \frac{1-\eta}{R}b\right) + \beta[\eta X + (1-\eta)V(\xi_H)]$$

Notice that if we difference this expression with the value of default and call this object  $M(\xi_H) = V(\xi_H) - X$ , then we have

$$\begin{aligned} M(\xi_H) &= u\left(y - \frac{R-1+\eta}{R}b\right) - u(y - \phi(y)) + \beta(1-\eta)M(\xi_H) \\ \rightarrow M(\xi_H) &= \frac{u\left(y - \frac{R-1+\eta}{R}b\right) - u(y - \phi(y))}{1 - \beta(1-\eta)} \end{aligned}$$

In order for default to be the optimal response, it will be both necessary and sufficient that  $M(\xi_H) \geq 0$ . This condition is precisely the first assumption under the assumption of an increasing utility function.

We will also require that  $V(\xi_L) - X = M(\xi_L) < 0$  i.e. default is optimal in the low-confidence state. This Bellman can be written as

$$V(\xi_L) = u\left(y - b + \frac{\eta}{R}b\right) + \beta[(1-\eta)X + \eta V(\xi_H)]$$

We can again take the difference with  $X$  to define  $M(\xi_L)$ :

$$M(\xi_L) = u\left(y - \frac{R - \eta}{R}b\right) - u(y - \phi(y)) + \beta\eta M(\xi_H)$$

$$\rightarrow M(\xi_L) = u\left(y - \frac{R - \eta}{R}b\right) - u(y - \phi(y)) + \frac{\beta\eta}{1 - \beta(1 - \eta)} \left[ u\left(y - \frac{R - 1 + \eta}{R}b\right) - u(y - \phi(y)) \right]$$

This last expression will be strictly less than zero if and only if the second assumption holds. In other words, the flow difference must be largely negative; enough so to compensate for the smaller but positive continuation value difference.

The intuition here is that the cost of debt service is greater than the default costs when confidence is low since debt prices are also very low. As such, it is no longer worthwhile to service the debt and default becomes optimal. However, when confidence is high so are debt prices and so the cost of debt service is now lower than default costs.

Thus, the two conditions in the theorem are both necessary and sufficient for the existence of Confidence-Waves Equilibrium. ■

### Appendix A.1.1. Proof of Corollary 3.2

To see why this persistence holds, note that the second assumption requires the following to be true

$$u(y - \phi(y)) - u\left(y - \frac{R - \eta}{R}b\right) > 0$$

$$\rightarrow \frac{R - \eta}{R}b - \phi(y) > 0$$

i.e. the cost of debt service in the low confidence state is strictly greater than the default costs. If we take the difference of this expression with the first assumption, we arrive at the result:

$$\frac{R - 1 + \eta}{R}b - \frac{R - \eta}{R}b < 0$$

$$R - 1 + \eta - R + \eta < 0$$

$$\eta < 1/2$$

■

### Appendix A.1.2. Proof of Corollary 3.3

Notice first that in any Confidence-Waves Equilibrium, it must be that  $\frac{R-1+\eta}{R}b \leq \phi(y) < \frac{R-\eta}{R}b$ . If we increase the concavity, then the utility difference between the first and second of these terms will increase more than the utility difference between the second and third terms. But this will imply that the second condition of Theorem 3.1 will continue to hold.

Notice, however, that this result does not hold if we make  $u$  more convex. In fact, some Confidence-Waves Equilibria can disappear when this happens. ■

### Appendix A.1.3. Proof of Corollary 3.5

First, let us find the conditions that govern the two non-sunspot equilibria: Full default and full repayment. Under the assumption of full-default, the price must be 0 in equilibrium. If we insert default as the optimal strategy in the continuation value, we derive the following Bellman:

$$V = u(y - b) + \beta X$$

If we take the difference of this value with  $X$  we arrive at

$$M = u(y - b) - u(y - \phi(y))$$

We require that  $M < 0$  in order for this to be an equilibrium, which will be true provided  $b > \phi(y)$ . Notice that this is implied by our second assumption, which requires that

$$\begin{aligned} \frac{R-\eta}{R}b - \phi(y) &> 0 \\ \rightarrow b - \phi(y) &> 0 \end{aligned}$$

Thus, if a Confidence-Waves Equilibrium exists, so too does the full-default equilibrium.

To verify the full-repayment equilibrium, the procedure is the same. Notice that the price here is  $\frac{1}{R}$ :

$$\begin{aligned} V &= u\left(y - b + \frac{1}{R}b\right) + \beta V \\ \rightarrow M &= u\left(y - \frac{R-1}{R}b\right) - u(y - \phi(y)) + \beta M \\ \rightarrow M &= \frac{u\left(y - \frac{R-1}{R}b\right) - u(y - \phi(y))}{1 - \beta} \end{aligned}$$

We require that  $M \geq 0$  in order for this to be an equilibrium, which is true provided  $\frac{R-1}{R}b \leq \phi(y)$ . But this follows direction from the first assumption which states that  $\frac{R-1+\eta}{R}b \leq \phi(y)$ . Thus, if a Confidence-Waves Equilibrium exists, so too does the full-repayment equilibrium. ■

#### Appendix A.1.4. Sunspots Randomizing over Non-Equilibrium Pricing Schedules: An Example

I provide a simple example here of a case in which sunspot activity can randomize over non-equilibrium pricing schedules. This example demonstrates that the regimes to which sunspots transition may not themselves be sustainable, which has important policy implications.

Consider again the simple model, but now suppose that  $u(c) = c$  and that there is some simple intrinsic uncertainty as well. In particular,  $y \in \{y_1, y_2\}$  and changes regimes with probability  $p$ . Continue to suppose that default costs can depend on  $y$ .

I now outline the conditions that define several non-sunspots equilibria. Using the techniques outlined in the proof of Theorem 3.1, it can be shown that a full repayment equilibrium exists if and only if the following condition holds:

**Assumption FR:**  $(\phi(y_i) - \frac{r}{1+r}b) + \frac{\beta p}{1-\beta(1-p)} (\phi(y_{-i}) - \frac{r}{1+r}b) \geq 0$  for both states,  $i$ .

We can also find conditions under which an alternative equilibrium exists: One in which repayment occurs in  $y_2$  but default occurs in  $y_1$ . This equilibrium can exist if and only if the following conditions hold:

**Assumption INT1:**  $\phi(y_2) \geq \frac{p+r}{1+r}b$

**Assumption INT2:**  $\left(\frac{1+r-p}{1+r} + \frac{\beta p}{1-\beta(1-p)} \frac{p+r}{1+r}\right) b > \phi(y_1) + \frac{\beta p}{1-\beta(1-p)} \phi(y_2)$

Now that we have outlined tightly how two non-sunspots equilibria can arise(or not arise), I will show how a sunspots equilibrium can randomize over these pricing schedules even if one of them is not an equilibrium. In particular, we will search for a Confidence-Waves Equilibrium in which the following holds:

$$V(\xi_1, y_1) < X(y_1) \leq V(\xi_2, y_1)$$

$$X(y_2) \leq V(\xi_1, y_2) \leq V(\xi_2, y_2)$$

Thus, we are searching for a sunspots equilibrium in which the sunspot randomizes over the RF default strategy and the INT default strategy. This pattern of default and repayment will suggest the following pricing schedule:

$$q(\xi_1, y_1) = \frac{p + \eta - \eta p}{1 + r}$$

$$q(\xi_2, y_1) = \frac{1 - \eta + \eta p}{1 + r}$$

$$q(\xi_1, y_2) = \frac{1 - p + \eta p}{1 + r}$$

$$q(\xi_2, y_2) = \frac{1 - \eta p}{1 + r}$$

Notice that, under sunspot persistence, the following is true:

$$q(y_1) < q(\xi_1, y_1) < q(\xi_2, y_1) < \frac{1}{1 + r}$$

$$q(y_2) < q(\xi_1, y_2) < q(\xi_2, y_2) < \frac{1}{1 + r}$$

where  $q(y_i)$  is the price of an INT equilibrium (if it exists, which it may not). Thus the sunspot is randomizing over these two regimes in which the sovereign follows either FR or INT.

In words, if this pricing schedule is an equilibrium, then it implies a welfare improvement over the interior solution without confidence waves. This is because the overall probability of default has fallen, since the sovereign does not always default in fundamental state 1. Even though it appears as if this might be a convexification over the risk-free equilibrium and the interior one, we will show that it is not in a moment.

To determine whether an sunspots equilibrium exists, define  $M_k^{ij} = V(\xi_i, y_j) - X(y_k)$ . It is sufficient for the above equilibrium to exist provided that  $M_1^{11} < 0$  and  $M_1^{21}, M_2^{12}, M_2^{22} \geq 0$ . Assuming the appropriate default/repayment scheme in the continuation value, we can difference the Bellmans of the value and

default functions to derive these objects as functions of themselves as follows:

$$\begin{aligned}
M_1^{11} &= \phi(y_1) - \frac{1+r-p-\eta+\eta p}{1+r}b + \beta\eta(1-p)M_1^{21} + \beta p[(1-\eta)M_2^{12} + \eta M_2^{22}] \\
M_1^{21} &= \phi(y_1) - \frac{r+\eta-\eta p}{1+r}b + \beta(1-\eta)(1-p)M_1^{21} + \beta p[\eta M_2^{12} + (1-\eta)M_2^{22}] \\
M_2^{12} &= \phi(y_2) - \frac{r+p-\eta p}{1+r}b + \beta\eta p M_1^{21} + \beta(1-p)[(1-\eta)M_2^{12} + \eta M_2^{22}] \\
M_2^{22} &= \phi(y_2) - \frac{r+\eta p}{1+r}b + \beta(1-\eta)p M_2^{21} + \beta(1-p)[\eta M_2^{12} + (1-\eta)M_2^{22}]
\end{aligned}$$

The above is a linear system of four equations in four unknowns. The analytic solution to this system is quite complicated and difficult to characterize (though feasible to find), but it is quite easy to determine computationally the solution for simple parameter values.

Consider the following parameterization, but which satisfies Assumptions FR and INT2, but which violates INT1. Thus, for these parameters, the full-repayment scheme is an equilibrium but the interior one is not.

$$\begin{aligned}
\beta &= 0.98 \\
p &= 0.01 \\
r &= 0.02 \\
b &= 1.0 \\
\phi(y_1) &= 0.020 \\
\phi(y_2) &= 0.029
\end{aligned}$$

For varying values of  $\eta$  (non-fundamental persistence), we get the following differences. Recall that an equilibrium exists if  $M_1^{11} < 0$  and the rest are positive.

Value of $\eta$	0.0	0.002	0.05
$M_1^{11}$	-0.9703	-0.9681	-0.9507
$M_1^{21}$	0.1310	0.0468	-0.6068
$M_2^{12}$	-0.0138	0.0066	0.0372
$M_2^{22}$	0.3582	0.3101	0.0646

We can see that, as assumed, when  $\eta = 0$  and there is no switching, there is no equilibrium. Also, if  $\eta$  is too large, we lose the equilibrium as well. However, for some persistent process e.g.  $\eta = 0.002$ , a CW equilibrium does exist. Thus, in a sunspots equilibrium we can sustain a temporary pattern of default in state 1 and repayment in state 2 even when this pattern of default is not itself an equilibrium.

This is possible because INT is close enough to satisfying the equilibrium conditions i.e. it is a ‘potential’ equilibrium. In particular, the value of repayment in the high state is just a bit too small to justify repayment on its own. However, once we insert a sunspot that gives us the opportunity to jump back into a more beneficial regime, the price of repayment in  $y_2$  increases enough to suddenly make repayment worthwhile. Thus the non-equilibrium INT regime is a potential equilibrium and thus can be randomized over in a sunspots equilibrium.

## Appendix A.2. Proof of Proposition 4.1

This can be shown by contradiction. Suppose that we had a Confidence-Waves Equilibrium with a pricing schedule that never depended on  $\xi$ . Once given the pricing schedule, the sovereign’s Bellman equation becomes a contraction on  $V$  and  $X$ . Since the only channel through which  $\xi$  can affect the sovereign’s payoff is through the price, the resulting, unique fixed point will not depend on fluctuations in  $\xi$ . Thus,  $\xi$  would have no effect in equilibrium, which contradicts the fact that the equilibrium is a Confidence-Waves Equilibrium.

The same argument holds if we suppose that sovereign behavior never depended on  $\xi$ , since the lenders’ pricing recursion is also a conditional contraction on  $q$ . Thus, any Confidence-Waves Equilibrium must feature an active change of behavior on both sides of the market in response to sentiment shocks. ■

### Appendix A.3. Proof of Proposition 4.2

When the equilibrium is not default relevant, then for any  $(y, m, b)$ , we will have either  $V(y, \xi, m, b) \geq X(y)$  or  $X(y) > V(y, \xi, m, b)$  for all  $\xi \in \Xi$ . Further, if the debt is short-term, then we will have

$$\begin{aligned} q(y, \xi_1, b') &= \frac{1}{R} E_{(\tilde{y}, \tilde{\xi}, \tilde{m})|(y, \xi_1)} \left[ 1\{V(\tilde{y}, \tilde{\xi}, \tilde{m}, b') \geq X(\tilde{y})\} \right] = \frac{1}{R} E_{(\tilde{y}, \tilde{\xi}, \tilde{m})|(y, \xi_0)} \left[ 1\{V(\tilde{y}, \tilde{\xi}, \tilde{m}, b') \geq X(\tilde{y})\} \right] \\ &\rightarrow q(y, \xi_1, b') = q(y, \xi_0, b') = q(y, b') \end{aligned}$$

and so the sunspot does not affect the price. ■

### Appendix A.4. Proof of Proposition 4.3

First, note that under the restriction of Markov-Perfection, the default policy of the sovereign is increasing in  $b$ .<sup>11</sup> We know that, in equilibrium, the price schedule  $q$  must reflect default risk in *all* future periods, not just the subsequent period. Further, since the equilibrium is non-default-relevant, we know that a confidence-driven price discrepancy in period  $t$  does not reflect a difference in the default probabilities in period  $t + 1$ . Thus, the confidence-driven price discrepancy must reflect increased default risk in periods  $t + 2, t + 3, \dots$ . Since confidence never drives default, it must be the case that the distribution over fundamental states induced by  $\xi_L$  implies a higher default frequency than that induced by  $\xi_H$ . However, since the distribution of future endowment realizations is identical across the two regimes, it must be the case that debt levels are higher in *some* states of the world under  $\xi_L$  than  $\xi_H$ . Since confidence is persistent, this must have been generated by strictly higher borrowing in  $\xi_L$  than  $\xi_H$  for some fundamental states in the future, in particular, those states which are most likely. ■

### Appendix A.5. Proof of Proposition 4.4

First, note that whether the economy is in a crisis or not the sovereign optimally borrows on the left-hand side of the ‘Laffer curve’, which plots revenue against debt issuance. This is because the sovereign can commit to not only to revenue raised at debt auctions, but also to the amount of debt issued. Thus, given to issuance options yielding the same revenue, the sovereign will always choose the one with less debt, since the value function is decreasing the level of debt.

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<sup>11</sup>See Chatterjee and Eyigungor (2012) for a proof of this claim. Their results generalize to my environment.

During a dynamic panic, the entire Laffer curve shifts but the sovereign continues to remain on the left-hand side of it. Therefore, if we implement a binding interest-rate ceiling, it will necessarily lower the quantity of debt that can be issued. This is because the demand curve for debt is downsloping<sup>12</sup>, so a price floor (rate ceiling) translates directly to a ceiling on debt issuance. A ceiling on debt issuance also places a ceiling on the revenue that can be raised, since the sovereign is located on an upsloping portion of the Laffer curve. Since the ceiling is temporary, tomorrow the sovereign can expect to resume with the equilibrium dynamics.

Denote the value of the sovereign who faces the original equilibrium demand functions with an interest rate ceiling as  $\hat{V}(y, \xi, m, b')$ . Note that since  $\hat{V}(y, \xi, m, b')$  is the objective function of the same maximization as  $V(y, \xi, m, b')$  but with an additional constraint, specifically one on revenue, we will necessarily have  $\hat{V}(y, \xi, m, b') \leq V(y, \xi, m, b')$ . Therefore, the probability of default has risen. ■

## Appendix A.6. Proof of Proposition 4.5

This result follows because, ignoring the non-fundamental  $\xi$ , the model becomes isomorphic to the model of [Chatterjee and Eyigungor \(2012\)](#). Thus, the existence result they provide for long-term debt *without* confidence fluctuations still holds. Denote this equilibrium price of debt to be  $q(y, b')$ .

The European Central Bank can pledge liquidity by guaranteeing to purchase debt at a schedule  $q(y, b')$  for the foreseeable future. If it does so, it will induce the sovereign to adopt the policy rules from the equilibrium free of confidence shifts. When this happens, investors will lend to the sovereign at the price  $q(y, b')$ , since it is in fact an equilibrium price, and the ECB never actually has to purchase the debt. ■

## Appendix A.7. Proof of Proposition 4.6

To see this result, first notice that since the equilibria are continuous in  $\eta$ , we will have two limiting pricing schedules:  $q_L(y, b') = \lim_{\eta \rightarrow 0} q(y, \xi_L, b')$  and  $q_H(y, b') = \lim_{\eta \rightarrow 0} q(y, \xi_H, b')$ . Now, if  $\bar{\xi} = \xi_L$ , then by continuity in  $\eta$  it must be the case that the equilibrium conditions are satisfied at  $q_L(y, b')$ ,  $V_L(y, m, b)$ , and  $X_L(y)$ , the latter two being the implied sovereign value functions (since, conditional on the pricing schedule, the sovereign's problem is a contraction).

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<sup>12</sup>See [Chatterjee and Eyigungor \(2012\)](#) for a proof of this.

However, it cannot be the case that  $q_H$  is part of a distinct equilibrium. This is because a distinct equilibrium would entail that  $X_L(y) \neq X_H(y)$ , but we since the equilibrium valuation of default does not depend on  $\xi$ , we will have  $X_L(y) = X_H(y) \forall y$ . In other words, these two pricing schedules cannot be two distinct equilibria since they share a common value of default. A symmetric argument can be made when  $\bar{\xi} = \xi_H$ , and thus the confidence sunspot can only ever be randomizing over *one* equilibrium pricing schedule. The other schedule is not an equilibrium. ■