From endogenous growth to stationary state: The world economy in the mathematical formulation of the Ricardian system

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From endogenous growth to stationary state: The world economy in the mathematical formulation of the Ricardian system

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Abstract: We analyze international trade in a Pasinetti-Ricardo growth model in the world economy scenario in which several small trading countries coexist and international commodity prices are determined by the interplay of supply and demand amongst them. We demonstrate that all the trading countries eventually reach the stationary state, though this process is not monotonic and the dynamics of capital and population may actually push some countries toward the stationary state and others away from it. We also use our model to assess an argument which Malthus employed in the second edition (1803) of *An Essay on the Principle of Population* to support a policy of agricultural protectionism.

Keywords: Ricardo, Pasinetti, international trade, endogenous growth, world economy, stationary state.

***JEL* Classification:** B12, B16, B31

1. Introduction

Ever since Whewell's exposition (1831), the temptation to dress Ricardo's *Principles* in a mathematical garb has proved irresistible. More recently, mathematical renditions of Ricardo's growth theory were stimulated both by Piero Sraffa's edition of the *Works and Correspondence of David Ricardo* and the renewal of interest in the theory of economic growth ignited by Nicholas Kaldor (1955-56) and Robert Solow (1956). Similarly, the blossoming of the 'new' growth theory, starting in the late 1980s-early 1990s with the contributions by Romer (1986), Lucas (1988) and Rebelo (1991), prompted an innovative assessment of Classical economists as forerunners of contemporary endogenous growth theorists (Kurz and Salvadori 1988 and 2003). This stream of literature has shown that Pasinetti's 1960 formulation of Ricardo's economics turns out to share the same analytical structure as that of an AK endogenous growth model, if a mechanism is at work to prevent diminishing returns in those sectors that produce wage-goods. In particular, Fiaschi and Signorino (2003 and 2006) and D'Alessandro and Salvadori (2008) obtain endogenous growth in

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Pasinetti (1960) through a change in the assumptions concerning the consumption patterns of the various social classes; while in Salvadori and Signorino (2015b) the same analytical result is achieved through the transformation of Pasinetti's closed economy into an open one.[2] The small open economy we studied in Salvadori and Signorino (2015b) is characterized by endogenous growth since the growth rate is bounded from below, although this occurs since international commodity prices are assumed to be given and constant over time. Accordingly, in order to tackle the issue of the dynamics of international commodity prices, a different set-up is required. This is the main reason why in this paper we analyze the relationship between growth and international trade in the world economy scenario, that is, a scenario in which several small trading countries coexist and international commodity prices are determined by the interplay of supply and demand amongst them. Unsurprisingly, the main result we obtain is that, while in the scenario of the small open economy trading countries may grow forever, in the world economy scenario the stationary state makes its comeback. In particular, we demonstrate that all the trading countries under scrutiny eventually reach the stationary state. Since the Earth is a closed economy and the overall amount of land cannot be increased, the world economy cannot grow forever in the absence of technological progress. Yet this process is not monotonic, and the dynamics of capital and population may actually push some countries toward the stationary state and others away from it. Only when all countries share the same rate of profits does the capital and population dynamics push all countries toward the stationary state.

[2] It hardly needs stressing that Ricardo himself envisaged free corn-trade between a net corn-exporting agricultural country and a net corn-importing manufacturing one as an effective mechanism to halt diminishing returns in the agricultural sector of the manufacturing country and thus prevent the fall of the rate of profits and the rate of capital accumulation in the latter. As is well-known, in Chapter 7, 'On Foreign Trade', of his *Principles* Ricardo wrote: "It has been my endeavour to shew throughout this work, that the rate of profits can never be increased but by a fall in wages, and that there can be no permanent fall of wages but in consequence of a fall of the necessaries on which wages are expended. If, therefore, by the extension of foreign trade, or by improvements in machinery, the food and necessaries of the labourer can be brought to market at a reduced price, profits will rise. *If, instead of growing our own corn, or manufacturing the clothing and other necessaries of the labourer, we discover a new market from which we can supply ourselves with these commodities at a cheaper price, wages will fall and profits rise*; but if the commodities obtained at a cheaper rate, by the extension of foreign commerce, or by the improvement of machinery, be exclusively the commodities consumed by the rich, no alteration will take place in the rate of profits" (*Works* I, p. 132, emphasis added).

Besides the finalization of the formal analysis of growth and international trade developed in an earlier paper, in this paper we pursue a second, more historiographical-oriented goal. We aim to provide an analytical framework to assess some significant issues of the corn-trade policy debates that took place in Great Britain in the early decades of the 19th century. Our focus is on the controversy between Malthus, the agricultural protectionist, and Ricardo, the champion of free corn-trade. In Salvadori and Signorino (2015a), we analyzed the issue of defense versus (trade-induced) opulence that characterized the Malthus-Ricardo exchange preceding the enactment of the Corn Laws in March 1815. Malthus (1815) envisaged defense and (trade-induced) opulence as two mutually exclusive options and he had no hesitation in choosing the former. By contrast, Ricardo (1815) excluded any such trade-off, arguing that, even in the case of war or poor domestic harvest, foreign agricultural countries would bear the brunt of serious economic losses if they opted for restrictions on their corn exports to Great Britain. Actually, in his 1815 protectionist pamphlet, *Grounds of an Opinion on the Policy of Restricting the Importation of Foreign Corn*, Malthus built his case for food autarky on a few theoretical arguments he had already presented twelve years before, in Chapters VII to X of Book III of the second edition of *An Essay on the Principle of Population* (1803). Yet Malthus (1803) outlined a further argument to support agricultural protectionism that he did not re-state in his *Observations on the Corn Laws* (1815). It concerned the dynamics of international trade between agricultural and manufacturing countries in a world economy scenario. Accordingly, we employ the model developed in this paper to evaluate Malthus's 1803 world economy argument for agricultural protectionism.

The paper is structured as follows. In section 2, we develop the model of a Pasinetti-Ricardo small open economy. In sections 3 and 4, we investigate the world economy scenario and its dynamic properties. In section 5, we examine an argument employed by Malthus (1803) to support his proposal of agricultural protectionism. Finally, section 6 concludes.

2. A Pasinetti-Ricardo small open economy

We assume that there exist n small open countries: *A, B, ... , N*. With regard to production technology and income expenditure patterns, we make for each of these countries exactly the same assumptions as in Pasinetti (1960): in each country, there are only two single-product sectors, agriculture and industry, which produce corn and gold, respectively. Corn is produced by means of labor and a natural resource (land) that can be neither produced nor destroyed; while gold is produced by labor alone, without the use of land. In each country, there are three social classes:

workers, capitalists and landlords. Workers provide labor, live on their wages since they have no other source of income, and spend the whole of their income on corn. Capitalists do not work nor own land, make investments by advancing corn-wages to workers and gain profits on their investments.[3] Landlords own land, do not work nor invest capital, and live by renting their land to capitalists. For the sake of simplicity, we assume that capitalists spend their income on corn alone, while landlords consume only gold. Say's law of markets and universal free competition hold.

Pasinetti (1960) formalized technology in the two sectors of his economy by means of two simple production functions. As regards the agricultural sector, the physical quantity of corn produced, X_1, is a non-decreasing concave function of the number of workers employed in corn production, N_1:

$$X_1^{(J)} = f_J\left(N_1^{(J)}\right), \tag{1.1}$$

$$f_J(0) \geq 0, \tag{1.2}$$

$$f_J'(0) > \overline{x}, \tag{1.3}$$

$$f_J''\left(N_1^{(J)}\right) < 0, \tag{1.4}$$

$$\lim_{N_1^{(J)} \to \infty} f_J'\left(N_1^{(J)}\right) < \overline{x} \tag{1.5}$$

where $\overline{x}$ is the natural wage-rate in terms of corn. We added a reference to country *J* in order to avoid repetitions. For the sake of simplicity, $\overline{x}$ is given and uniform in all *n* countries.

Inequality (1.2) has an obvious meaning: even without using labor, land alone either produces a (small) amount of corn or produces nothing, but, taking account of inequality (1.3), any (small) amount of labor is undoubtedly enough to produce some corn. Inequality (1.3) has a strong meaning: marginal productivity of labor is larger than the natural wage-rate in terms of corn $\overline{x}$. This means that when a small amount of labor is employed in the production of corn, this labor is able to produce at least the amount of corn needed to pay labour at the wage rate $\overline{x}$ and, accordingly, the economy is viable. Without this assumption, the economy cannot reproduce itself,

[3] In Pasinetti's model, there is no capital good proper and, accordingly, capitalists buy corn both for personal consumption and investment purposes. Capitalists gain their profits since production requires time and corn-wages need to be advanced to workers.

also because of inequality (1.4). The meaning of inequality (1.4) is crystal-clear: marginal productivity of labor in agriculture is decreasing. If inequality (1.5) does not hold, the stationary state cannot exist (see Kurz and Salvadori 1998: 72-3). Pasinetti (1960) introduced inequality (1.5) just to have the stationary state (see Pasinetti 1960: 87, eq. 20). We assume that all countries have the same technology, but not the same qualities of land nor the same area for each land quality. Consequently, if, say, countries *C* and *D* have the same best quality of land, then $f_C'(0) = f_D'(0)$, but they may differ at any other level of agricultural population.

With regard to the industrial sector, Pasinetti (1960) assumed that the physical quantity of gold produced, X_2, is proportional to the number of workers employed in gold production, N_2:

$$X_2^{(J)} = aN_2^{(J)}. \tag{2}$$

Note that once again we have assumed that all *n* countries share the same technology.

Countries *A, B, ... , N* are small open economies and, therefore, following Salvadori and Signorino (2015b), corn (gold) may be imported into one country from another by exporting gold (corn). From the perspective of a given country, international trade may be considered a kind of alternative technology to produce commodities: while in a closed economy commodities may only be produced domestically, in an open set-up commodities may also be acquired through trade with foreign countries. Therefore, for each international price of gold in terms of corn, p_2, we also have the following 'production functions':

$$I_1^{(J)} = p_2 a N_{I1}^{(J)}, \tag{3}$$

$$I_2^{(J)} = \frac{f\left(N_{I2}^{(J)} + N_1^{(J)}\right) - f\left(N_1^{(J)}\right)}{p_2}, \tag{4}$$

where $I_1^{(J)}$ is the quantity of corn imported into country *J*, $N_{I1}^{(J)}$ is the quantity of labor employed in the production of gold that is exported in order to import foreign corn into country *J*, $I_2^{(J)}$ is the quantity of gold imported into country *J* and $N_{I2}^{(J)}$ is the quantity of labor employed in the production of corn that is exported in order to import foreign gold into country *J*.

Equations (3) and (4) imply that the import-export activity does not require time nor does it incur transport costs: the gold produced for export ($aN_{I1}^{(J)}$) is instantaneously transformed into imported

corn ($I_1^{(J)}$) and the corn produced for export ($f_J\left(N_{I2}^{(J)}+N_1^{(J)}\right)-f_J\left(N_1^{(J)}\right)$) is instantaneously transformed into imported gold ($I_2^{(J)}$). Obviously,

$$N_J = N_1^{(J)} + N_2^{(J)} + N_{I1}^{(J)} + N_{I2}^{(J)} \tag{5}$$

$$K_J = W_J = \bar{x}N_J\,, \tag{6}$$

where N_J is the total number of workers in country *J*, W_J is the total wage bill in country *J*, and K_J is the physical stock of capital in country *J* (both measured in terms of corn).

In Salvadori and Signorino (2015b), we determined two thresholds in the course of the process of capital accumulation, both depending on the international relative price of the two traded commodities. The lower threshold, $\bar{N}_J(p_2)$, is the unique solution to the equation

$$ap_2 = f_J'\left(\bar{N}\right)$$

provided that

$$p_2 < \frac{f_J'(0)}{a} \tag{7}$$

otherwise $\bar{N}_J(p_2)=0$. If $N_J \le \bar{N}_J(p_2)$, then the whole working population in country *J* is employed in the production of corn, part of which, $R_J = f\left(N_J\right)-N_J f'\left(N_J\right)$, is exported to import foreign gold. The rate of profits is determined by equation

$$r_J = \frac{f_J'\left(N_{I2}^{(J)}+N_1^{(J)}\right)}{\bar{x}}-1\,. \tag{8}$$

An increment of N_J implies a reduction in the rate of profits. On the contrary, if $N_J > \bar{N}_J(p_2) > 0$, then $\bar{N}_J(p_2)$ workers are employed in the domestic production of corn whereas $N_J-\bar{N}_J(p_2)$ workers are employed in the domestic production of gold. The rate of profits is defined by equation (8) with $N_{I2}^{(J)}+N_1^{(J)}=\bar{N}_J\left(p_2\right)$ or

$$r_J = \frac{ap_2}{\bar{x}}-1 \tag{9}$$

which results in the same r_J. An increment in N_J has no impact on the rate of profits. Note that $f_J\left(\bar{N}_J(p_2)\right)$ is the maximum amount of corn that can profitably be produced at home at the given

international relative price p_2. Furthermore, if $N_J > \bar{N}_J(p_2) = 0$, all workers are employed in the domestic production of gold, that is exported in order to obtain the corn consumed by workers and capitalists (landlords have no income and no consumption). The rate of profits is defined by equation (9), which does *not* obtain the same value determined by equation (8).

The higher threshold is

$$\bar{\bar{N}}_J(p_2) = \bar{N}_J(p_2) + \frac{f_J\left(\bar{N}_J(p_2)\right) - \bar{N}_J(p_2) f_J'\left(\bar{N}_J(p_2)\right)}{ap_2}.$$

Note that $\bar{\bar{N}}_J(p_2) = \bar{N}_J(p_2) = 0$ if inequality (7) does not hold. If $\bar{N}_J(p_2) < N_J < \bar{\bar{N}}_J(p_2)$, the value of rent exceeds the value of domestic gold production in country *J*. Consequently, a share of the domestic corn production is exported abroad to pay for foreign gold imports. If, instead, $N_J > \bar{\bar{N}}_J(p_2)$, the value of rent is lower than that of domestic gold production. As a consequence, a share (or the whole if $\bar{\bar{N}}_J(p_2) = \bar{N}_J(p_2) = 0$) of domestic gold production is exported to pay for the import of foreign corn.

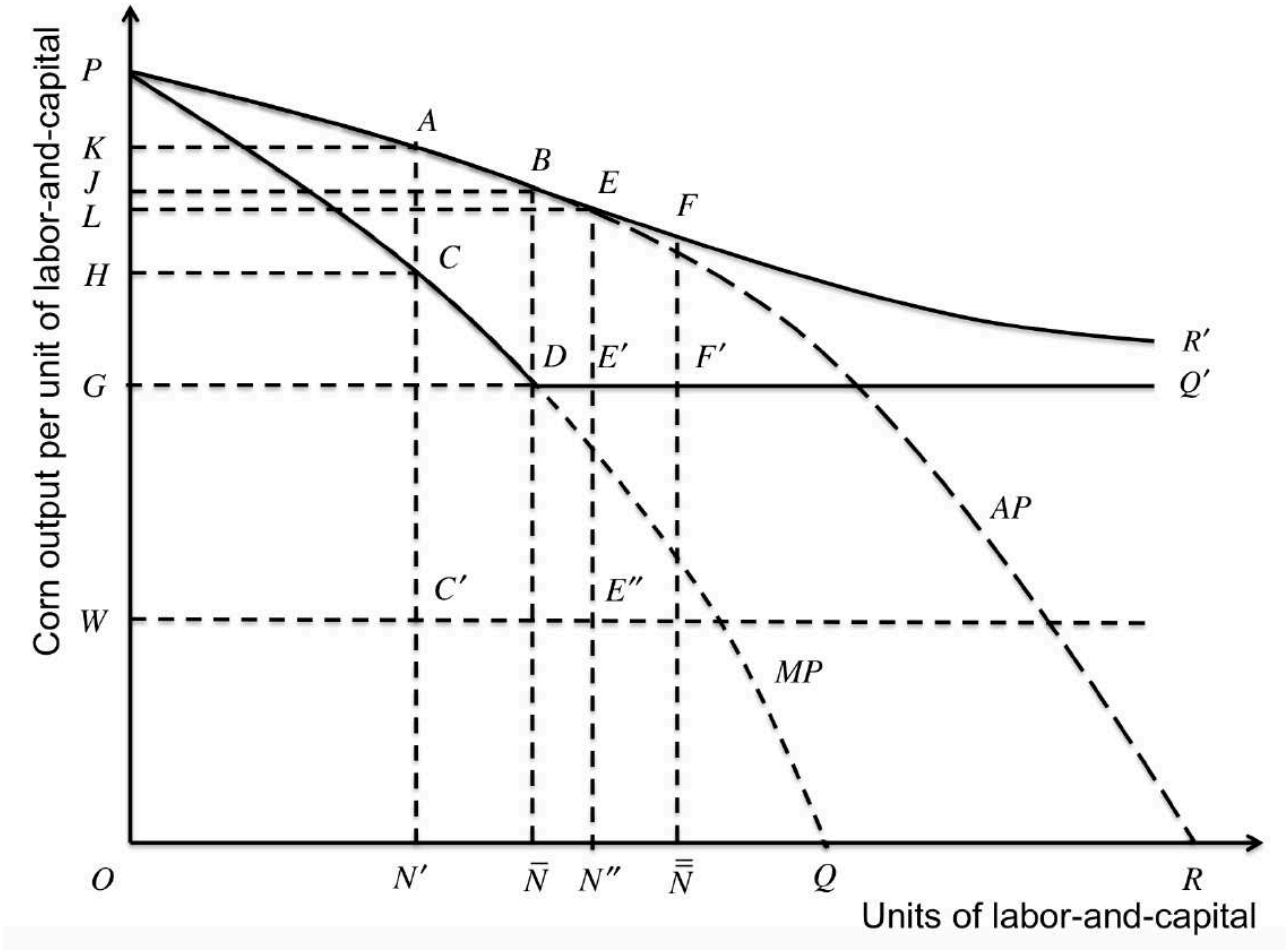

Figure 1

In Salvadori and Signorino (2015), we illustrated our analysis by means of a diagram similar to that used by Kaldor (1955-56): see Figure 1. The curve *PDQ* is the marginal productivity of labor-and-capital in the closed economy; similarly, the curve *PBR* is the average productivity of labor-and-capital in the closed economy. However, in the open economy they become *PDQ'* and *PBR'*, respectively. The length of segment *OW* equals $\bar{x}$ and the length of segment *OG* equals $a\bar{p}_2 / \bar{p}_1$. If $N \le \bar{N}$, as N', the rent equals the area of rectangle *HKAC*, the wage bill, and therefore the capital is the area of rectangle *OWC'N'*, and the profit is the area of rectangle *WHCC'*. Hence, the rate of profits, that is the ratio between profit and capital, is the ratio between the lengths of segments *WH* and *OW* since the two rectangles *OWC'N'* and *WHCC'* have the same base. An increase in *N* increases the rent and the wage bill; it may increase or decrease profits, but undoubtedly decreases the rate of profits. If $N > \bar{N}$, as in N'', the rent is equal to the area of rectangle *GLEE'*, which in turn is equal to the area of rectangle *GJBD* since curve *BR'* is built in such a way as to impose this equality, the wage bill equals the rectangle *OWE''N''*, and the profit equals the rectangle *WGE'E''*.

An increase in N leaves both rent and profit rate unchanged, but increases, proportionally, both profits and capital. Finally, $\bar{\bar{N}}$ is determined in such a way that rectangles $GJBD$ and $\bar{N}DF'\bar{\bar{N}}$ have the same area.

3. The Ricardian world economy scenario

The international (and domestic) price of gold in terms of corn, p_2, is taken as given by any and each trading country. In the world economy scenario, p_2 is determined by the condition that the total amount of rents paid worldwide is able to buy the whole of the gold globally produced:

$$\sum_J \left[f_J(N_{1J}) - f_J'(N_{1J}) N_{1J} \right] = a p_2 \sum_J N_{2J} \qquad (10)$$

where $N_{1J} = N_1^{(J)} + N_{I2}^{(J)}$ and $N_{2J} = N_2^{(J)} + N_{I1}^{(J)} = N_J - N_{1J}$.

In this section, we determine the equilibrium of the world economy when the population in the country J is N_J^* (all J). All changes in quantity variables are comparative static changes and have no dynamic meaning. We find the equilibrium of the world economy by means of a mental experiment. We start with a fictitious state in which $N_J = 0$ for each J; then we increase N_A from 0 to N_A^* keeping $N_J = 0$ for each $J \neq A$; then we increase N_B from 0 to N_B^* keeping $N_A = N_A^*$ and $N_J = 0$ for each $J \neq A, B$; and so on and so forth.

The first stage concerns a closed economy and we obviously obtain:

$$r_A = \frac{f_A'(N_{1A})}{\bar{x}} - 1 = \frac{a p_2}{\bar{x}} - 1$$

$$N_{2A} = \frac{f_A(N_{1A}) - N_{1A} f_A'(N_{1A})}{a p_2}$$

$$N_A^* = N_{1A} + N_{2A} \qquad (11)$$

The first three equations uniquely determine r_A, p_2, N_{2A} as functions of N_{1A}; then the fourth equation determines N_{1A}.

The relative price p_2 thus determined is also the international price of gold in terms of corn at the very beginning of the second stage. When N_B is small, country B produces *only* corn, that is,

country B is an agricultural country with no industrial sector. Therefore, the industrial country A must produce gold for both countries.[4] Hence, the increase in N_B implies an increase in N_{2A} and hence a decrease in N_{1A}, which implies an increase in r_A and p_2. Obviously, Ricardo's results hold in this case. When there are no legal restrictions to international trade between the industrial country A and the agricultural country B, the rate of profits in the industrial net corn-importing country A and the price of the industrial commodity go up and the industrial country A moves away from the stationary state. Things are different for the agricultural net corn-exporting country B where

$$r_B = \frac{f_B'(N_B)}{\overline{x}} - 1 > r_A = \frac{f_A'(N_{1A})}{\overline{x}} - 1 = \frac{ap_2}{\overline{x}} - 1 \qquad (12)$$

and an increase in N_B implies a reduction in r_B. More precisely, we have to add to equalities (11) and (12) the equality

$$N_{2A} = \frac{f_A(N_{1A}) - N_{1A}f_A'(N_{1A}) + f_B(N_B) - N_B f_B'(N_B)}{ap_2} \qquad (13)$$

Consequently,

$$\frac{dN_{1A}}{dN_B} = \frac{f_B''(N_B)N_B}{f_A'(N_{1A}) - N_A f_A''(N_{1A})} < 0$$

The process continues in this way until either the second stage is completed ($N_B = N_B^*$) or the two countries attain the same profit rate ($r_B = r_A$).[5] In the second case, the formerly agricultural country B develops a domestic industrial sector, that is, it starts producing gold at home and the increase in N_B implies a decrease in $r_B = r_A$ and in p_2. Both countries move toward the stationary state. More precisely, equations (12) and (13) are replaced by equations

$$r_B = \frac{f_B'(N_B)}{\overline{x}} - 1 = r_A = \frac{f_A'(N_{1A})}{\overline{x}} - 1 = \frac{ap_2}{\overline{x}} - 1 \qquad (14)$$

$$N_{2A} + N_{2B} = \frac{f_A(N_{1A}) - N_{1A}f_A'(N_{1A}) + f_B(N_{1B}) - N_{1B}f_B'(N_{1B})}{ap_2} \qquad (15)$$

[4] This means that country A is above the higher threshold identified in section 2.

[5] This means that country B is above the lower threshold identified in section 2.

and we have to add to Equation (11) also the equation

$$N_B = N_{1B} + N_{2B} \tag{16}$$

Consequently:

$$\frac{dN_{1A}}{dN_B} = \frac{ap_2}{f_A'(N_{1A}) + f_B'(N_{1B})\dfrac{f_A''(N_{1A})}{f_B''(N_{1B})} - f_A''(N_{1A})(N_A + N_B)} > 0$$

$$\frac{dN_{1B}}{dN_B} = \frac{f_A''(N_{1A})}{f_B''(N_{1B})}\frac{dN_{1A}}{dN_B} > 0$$

Note that, while the formerly agricultural country *B* starts industrializing, the formerly industrial country *A* increases its domestic corn production and reduces its domestic gold production. Consequently, country *A* tends to become an agricultural country once again.[6] This process goes on until either the second stage is completed ($N_B = N_B^*$) or $N_{1A} = N_A^*$. In the second case, the formerly industrial country *A* has ceased not only to export gold, the industrial commodity, but even to produce it and now produces *only* corn, the agricultural commodity. Put differently, the industrial status of a trading country is not carved in stone and therefore is not irreversible. Subsequently, country *A*'s rate of profits is constant and the increase in N_B implies a decrease in r_B and p_2. More precisely, equations (14) and (15) are replaced by equations

$$r_A = \frac{f'(N_A^*)}{\overline{x}} - 1 > r_B = \frac{f_B'(N_{1B})}{\overline{x}} - 1 = \frac{ap_2}{\overline{x}} - 1$$

$$N_{2B} = \frac{f_A(N_A^*) - N_A^* f_A'(N_A^*) + f_B(N_{1B}) - N_1^{(B)} f_B'(N_{1B})}{ap_2}$$

where equation (16) still holds. Consequently:

$$\frac{dN_{1B}}{dN_B} = \frac{f_A'(N_{1B})}{f_A'(N_{1B}) - N_{1B} f_B''(N_{1B})} > 0$$

The process goes on in this way until $N_B = N_B^*$.

[6] This means that country *A* progressively goes backwards and reduces its distance from the higher threshold identified in section 2. Country *A* can even go below the higher threshold and reach the lower threshold.

At the beginning of the third stage, we may have either that $r_B \neq r_A$ or that $r_B = r_A$. In the former case, the increase in N_C implies an increase in the lowest rate of profits (say r_A) and in p_2, whereas r_C is decreasing and the other rate of profits (say r_B) is constant until $r_B = r_A$ (unless we are stopped by the condition $N_C = N_C^*$). Subsequently (but we start from here if at the beginning of the third stage we have $r_B = r_A$), the increase in N_C implies an increase in $r_B = r_A$ and in p_2 until $r_C = r_B = r_A$ (unless we are stopped by the condition $N_C = N_C^*$).[7] The increase in N_C then implies a decrease in $r_C = r_B = r_A$ and in p_2 until either $N_{1A} = N_A^*$ or $N_{1B} = N_B^*$ (unless we are stopped by the condition $N_C = N_C^*$); say that $N_{1B} = N_B^*$.[8] Subsequently, the increase in N_C implies a decrease in $r_C = r_A$ and in p_2 whereas r_B is constant until $N_{1A} = N_A^*$ (unless we are stopped by the condition $N_C = N_C^*$).[9] Afterwards, the increase in N_C implies a decrease in r_C and p_2 whereas r_A and r_B are constant until $N_C = N_C^*$.

And so on and so forth.

In the above exposition, we implicitly assumed that $f_J'(0)$ is the same for each *J*, that is, that each country has at least one plot of best-quality land. Otherwise, with no loss of generality, we may assume that

$$f_A'(0) \leq f_B'(0) \leq \ ... \ \leq f_S'(0)$$

This done, the analysis unfolds exactly as before. The only difference is that now at some stage (but not in the first) country *A* (and possibly *B*, *C* …) may be in the condition that it does not produce corn domestically and produces only gold. In such a case, $N_{1A} = 0$, $N_{2A} = N_2^*$, and

$$r_A = \frac{ap_2}{\bar{x}} - 1$$

We are confident that interested readers would be able to complete the analysis by themselves.

[7] This means that country *C* is above the lower threshold; it is not possible to detect whether or not country *B* is above the higher threshold.

[8] This means that country *B* is again below the lower threshold.

[9] This means that country *A* is again below the lower threshold.

4. Dynamic properties

All the formal relations we found among variables are static in the sense that they do not refer to changes over time of the magnitudes under scrutiny. Pasinetti (1960) analyzed the dynamic path of two magnitudes, population and capital. The former affects the real wage rate: we ignored it since in this paper we assume a given and constant real wage rate. As to the latter, Pasinetti assumed that the derivative of capital with respect to time is a known increasing function of profits that can be expressed as follows:[10]

$$g = \frac{\dot{K}}{K} = \varphi(r), \quad \varphi'(r) > 0 . \tag{17}$$

In the world economy scenario, there are as many rates of profits as countries. If the rate of profits is uniform among countries,[11] then any increase in any country's working population results in a fall in the common rate of profits and in the price of the industrial commodity. That is, all countries move toward the stationary state. Things are different in the case in which the rates of profits differ among countries.[12] In this case, an increase in any working population of a country with the lowest rate of profits results in a fall in this rate of profits and in the price of the industrial commodity, but leaves the other countries' rates of profits above this level unchanged. By contrast, an increase in any other working population leads to a fall in the rate of profits of its own country, but an increase in the lowest rate of profits and in the price of the industrial commodity, leaving the other countries' rates of profits at different levels unchanged.

In this paper we prefer not to formalize the equivalent of function (17) for the world economy. Two polar cases are possible. In the first capitalists invest only in countries with the highest rate of profits. In this case the international price of gold and the rate of profits in the countries with the lowest rate of profits go up. Hence, the countries with the lowest rate of profits move away from the stationary state. Contextually, the rate of profits in the countries with the highest rate of profits goes

[10] See the argument developed by D'Alessandro and Salvadori (2008) and restated by Salvadori and Signorino (2015b) in this regard.

[11] This means that all countries are above the lower threshold and at least one is above the higher threshold.

[12] This means that not all countries are above the lower threshold, though at least one country is above the higher threshold.

down and the countries with the highest rate of profits move toward the stationary state. In all the other countries the rates of profits are unchanged and so are rents, even if landlords may buy less gold with their rents. In the second case capitalists invest only in their own country and function (17) applies. In this case in all countries capital and working populations increase, but they increase more in countries with the highest rate of profits. One way or other, the rate of profits tends to level out among trading countries. As soon as this happens, any increase in capital and working population in any country pushes all countries toward the stationary state.

Between these two polar cases, we may assume that the growth rate of capital and working population in a country is a function of the rate of profits in all countries. However, whatever assumption we make, the stationary state is the unavoidable conclusion of the story. Yet though the stationary state is the unavoidable destiny of the world economy, the path each country follows toward the stationary state is monotonic *if and only if* all countries have exactly the same rate of profits. In particular, the population increase in countries that have a rate of profits higher than the lowest rate pushes the countries with the lowest rate of profits in the opposite direction.

5. Malthus's 1803 world economy argument for agricultural protectionism

In a companion paper (Salvadori and Signorino 2015c) we analyzed the main arguments employed by Malthus in Chapters VII to X of Book III of the second edition of *An Essay on the Principle of Population* (1803) to endorse legal protection for British agriculture. The issue was a hot one at the time: the Anglo-French war had just started again and, in 1804, the British Parliament enacted new protectionist Corn Laws. To put it in a nutshell, Malthus (1803) championed the (re)introduction of bounties to promote the exports of British corn abroad and duties to discourage the imports of cheap foreign corn into the British market on the basis of three arguments:

1) a policy of free foreign corn imports is highly unsuitable for a large landed country such as Great Britain,

2) a policy of free foreign corn imports delivers a fatal blow to the British agricultural sector and entails a growing threat to British national security, particularly in wartime, and

3) a policy of free foreign corn imports is abortive in the long run since agricultural countries are going to develop their domestic industrial sectors and cut their corn exports to Great Britain.

In what follows, we focus on Malthus's third argument, the most interesting one for the purposes of this paper. Malthus adopted a world economy perspective in which the dynamics of international commodity prices is determined by the interplay of demand and supply among several trading countries. According to Malthus, the division and specialization of international labor and the ensuing pattern of international trade between manufacturing and agricultural countries is, at best, a short-run phenomenon. As noted by Hollander (1997), Malthus envisaged international corn trade "as no more than a means of balancing temporary requirements, not as a trade upon which could be based the international division of labor" (Hollander 1997: 837; see also *idem*: 818-20). The foundation for Malthus's third argument is that international corn trade stimulates the economic growth of trading countries and the latter phenomenon displays feedback effects on the international corn market. In the long run, such feedback effects drastically change the economic conditions that make it worthwhile for the various countries to participate in the international corn trade.

Malthus's chain of reasoning is the following. In a given moment of time, a few foreign agricultural countries enter the international corn market as large net corn-exporting countries: the high agricultural rate of profits, due to their corn exports, induces capital accumulation in their internal agricultural sector. As time passes, the agricultural rate of profits falls and foreign farmers progressively shift their own capital from agriculture into their own manufactures. As foreign domestic manufactures start developing, formerly agricultural countries experience a structural change from a pattern of deeply unbalanced, agriculture-based growth to a pattern of roughly balanced growth. Therefore, within these formerly agricultural countries, an increasingly growing share of their domestic corn production is going to be devoted to match their growing domestic demand for corn. On such a premise, Malthus drew the conclusion that these countries will find it worth exporting steadily decreasing amounts of corn to the countries that have entered into international trade as predominantly manufacturing countries. When this scenario materializes, manufacturing countries that have let their agricultural sector decline, stubbornly relying on constant yearly foreign corn imports, are going to face soaring difficulties in procuring food. The rate of growth of such manufacturing countries will inevitably slow down:

> Nothing perhaps will shew more clearly the absurdity of that artificial system, which prompts a country, with a large territory of its own, to depend upon others for its food, than the supposition of the same system being pursued by many other states. If France, Germany, and Prussia, were to become manufacturing nations, and to consider agriculture as a secondary concern, how would their wants, in the indispensable article of food, be supplied. The increasing demand for corn, would tend certainly to encourage the growth of it in Russia and America; but we know that in these countries, at present, particularly in America, the natural progress of population is not very greatly

> checked; and that, as their towns and manufactories increase, the demand for their own corn will of course increase with them. […] But allowing that at first, and for some time, the increasing demands of these manufacturing countries might be adequately supplied; yet this could not in the nature of things last long. *The manufacturers, from the decay of agriculture in their own countries, would annually want more; and Russia and America, from their rapidly increasing population, and the gradual establishment of manufactures at home, would annually be able to spare less*. (446-7, emphasis added)

It is worth stressing that in the 1817 fifth edition of his *Essay* Malthus reiterated the same argument and better clarified the underlying chain of reasoning. The relevant passage is worth quoting at length:

> A nation which is obliged to purchase from others nearly the whole of its raw materials, and the means of its subsistence, is not only dependent entirely upon the demands of its customers, as they may be variously affected by indolence, industry or caprice, but it is subjected to a necessary and unavoidable diminution of demand in the natural progress of these countries towards that proportion of skill and capital which they may reasonably be expected after a certain time to possess. *It is generally an accidental and temporary, not a natural and permanent division of labour, which constitutes one state the manufacturer and the carrier of others*. While, in these landed nations, agricultural profits continue very high, it may fully answer to them to pay others as their manufacturers and carriers; but *when the profits on land fall, or the tenures on which it can be held are not such as to encourage the investment of an accumulating capital, the owner of this capital will naturally look towards commerce and manufactures for its employment*; and, according to the just reasoning of Adam Smith and the Economists, finding at home both the materials of manufactures, the means of subsistence, and the power of carrying on their own trade with foreign countries, they will probably be able to conduct the business of manufacturing and carrying for themselves at a cheaper rate than if they allowed it to continue in the hands of others. *As long as the agricultural nations continued to apply their increasing capital principally to the land, this increase of capital would be of the greatest possible advantage to the manufacturing and commercial nation*. It would be indeed the main cause and great regulator of its progress in wealth and population. *But after they had turned their attention to manufactures and commerce, their further increase of capital would be the signal of decay and destruction to the manufactures and commerce, which they had before supported*. And thus, in the natural progress of national improvement, and without the competition of superior skill and capital, a purely commercial state must be undersold and driven out of the markets by those who possess the advantage of land. (Malthus 1817, III.ix.13)

In the light of the model developed in this paper, we may reconstruct Malthus's analysis of international trade as follows. For Malthus, a monotonic process of structural change characterizes the economy of any agricultural trading country: the development of domestic manufactures is an irreversible phenomenon and, most of all, is a phenomenon fully independent of what happens abroad. As we showed above, none of these three propositions is true: the dynamics of a foreign working population deeply influences the growth pattern of a given trading country, the industrial status of a country is a reversible one and worldwide dynamics may be non-monotonic. Moreover,

our analysis makes it crystal clear that Malthus based his analysis on a crucial but unstated assumption, that is, the assumption of an asymmetry between capitalists living in agricultural countries and those living in industrial countries. While the former quickly react to changes in relative prices and rates of profits and progressively shift their own capital from their internal agricultural sector toward their internal manufacturing sectors; the latter do not react the same way and do not progressively move their own capital into their internal agricultural sectors as the international price of corn rises and, as a consequence, the profitability of agricultural investment increases.

6. Concluding remarks

In a previous contribution (Salvadori and Signorino 2015a) we analyzed the issue of international trade in a Pasinetti-Ricardo growth model. We employed a simplifying assumption: the economy under scrutiny was a small open one and, accordingly, international commodity prices were taken as given and constant. Conversely, in this paper we analyzed the issue of international trade in a Pasinetti-Ricardo growth model in the world economy scenario, that is, a scenario in which several small trading countries coexist and international commodity prices are determined by the interplay of supply and demand amongst them. The main finding attained in this paper is as follows: while in our previous paper, with international commodity prices taken as given, trading countries may grow forever, in the world economy scenario we herein analyze the stationary state makes its comeback. In particular, we demonstrate that all the trading countries under scrutiny eventually reach the stationary state. Yet this process is not a monotonic one and the dynamics of capital and population may actually push some countries toward the stationary state and others away from it. Only when all countries share the same rate of profits, does the capital and population dynamics push all countries toward the stationary state.

Moreover, we showed that the model developed in this paper may be used to appraise an argument Malthus employed in the second edition of his *An Essay on the Principle of Population* (1803) to support a policy of agricultural protectionism. Malthus argued that a policy of free foreign corn imports was abortive in the long run, since agricultural countries were going to develop their domestic industrial sectors and cut their corn exports toward Great Britain. Malthus described a simple and linear process of structural change within the economy of any agricultural trading country. For Malthus, the development of domestic manufactures in a formerly agricultural country was an irreversible phenomenon and fully independent of what happened abroad. In our model, the

domestic dynamics of a trading country may be much more complicated than the simple and linear one imagined by Malthus since the growth of the foreign working population deeply influences the growth pattern of other trading countries and the agricultural/industrial status of a country is fully reversible. Moreover, we showed that Malthus based his analysis of international trade dynamics on an *ad hoc* asymmetry assumption as regards the behavior of capitalists living in agricultural countries and those living in industrial countries.

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