"Why Can't the Long-Term Unemployed Find Jobs? A Possible Explanation and Dynamic Implications", MSc thesis, London School of Economics, 1986 (MSc Econometrics and Mathematical Economics).

Kollmann, Robert

1986

LONDON SCHOOL OF ECONOMICS

WHY CAN'T THE LONG TERM UNEMPLOYED FIND JOBS ?

A POSSIBLE EXPLANATION AND DYNAMIC IMPLICATIONS

June 1986

Submitted by

Robert Kollmann in
partial fulfilment of the
degree requirements of the
M.Sc. in Econometrics and
Mathematical Economics

The author wishes to thank Dr.J.Moore
for his guidance in writing this essay.

1. INTRODUCTION

It is a feature of most labour markets that firms are unable to determine the precise characteristics (skills) of unemployed workers without incurring considerable costs.

Firms which want to hire workers are thus induced to "exploit" any signal which can inform them about the likelihood that particular unemployed workers meet their skill requirements before incurring these costs.

Workers who meet the requirements of a high proportion of firms are likely to be the first to leave unemployment: the duration of unemployment spells is thus an "informative signal" which firms can use when evaluating job applicants.

In section 2 of this essay, we show that this may lead firms to disregard job applications made by workers whose unemployment spell exceeds a "cut-off" duration. In section 3 we show that the existence of such a "cut-off" duration affects the dynamic characteristics of the economy: it leads to an asymmetry in its adjustment to positive as opposed to negative shocks; furthermore increases in the amplitude of these shocks (holding constant their mean and their frequency) influence the average employment level(1), while increases in their frequency (holding constant their amplitude and their mean) reduce the amplitude of the fluctuation of employment ("inertia").

Employment exhibits serial correlation: a one-time shock to the economy leads to an adjustment path which extends over several periods. The

(1) We identify a condition under which an increase in the amplitude of shocks leads to an increase in average employment.

model developed in this essay is an equilibrium model (the wage rate is fully flexible). The serial correlation result is obtained even in the absence of misperceptions , i.e. even for perfectly anticipated shocks and although there are no costs of adjustment.

2. THE MODEL

2.1 THE "INGREDIENTS"

The model consists of a fixed number N of workers[(A)] and a non-predetermined number of firms [free entry (exit) of firms into (out of) the economy].

Workers and firms are heterogeneous: there are two types of workers (denoted by A and B (the proportion of type A (type B) workers will be denoted by a (b))) whose characteristics (skills) imply that they differ in the following way: for each firm, the probability that a randomly chosen worker meets its skill requirement is greater if that worker is a type-A than if he is a type-B worker (the former probability will be denoted by p_A and the latter by p_B).

We give here an example of an economy with the features described in the text; assume that there exists a continuum of skills which can be represented by a circle.

Each worker has a set of skills which corresponds to an interval on the circle.

What distinguishes type-A from type-B workers is the length of their "skill intervals" (i.e. the breadth of their skills); denote this length by J_A and J_B respectively. The "skill intervals" of type-A and type-B workers are evenly distributed over the circle. Each firm requires its workers to have one skill. When entering into the economy (we assume free entry), firms can "locate" their "skill requirement" anywhere on the circle.
As the "skill intervals" of type-A and type-B workers are evenly distributed over the circle, firms will choose the locations of their "skill requirements" in such a way that they too are evenly distributed over the circle. If $J_A > J_B$, this implies that $p_A > p_B$.

In the model time is continuous.

(A) To avoid problems of indivisibilities it is best to interpret N as the interval [0,N].

All workers are "infinitely" lived, with an instantaneous death probability of z. Workers who die are immediately "replaced" by newborn workers in such a way that the composition of the workforce remains unchanged; all newborn workers enter unemployment.

Each firm can employ at most one worker. Gross per period productivity of that worker equals a positive constant if he meets his firm's skill requirement.

Firms are unable to determine what the skills of unemployed workers are, without incurring a cost (denoted by C)(1).

Under the assumption that gross productivity produced by an "unsuitable" worker is negative i.e. that a damage results from employing such a worker and that this damage is sufficiently high, firms accept to bear the cost of finding out what the skills of unemployed workers are.

The firm's acceptance to incur this cost can be explained in an alternative way (thereby avoiding the assumption - which is perhaps unrealistic - that there is a substantial damage from employing "unsuitable" workers) if we suppose that before being able to work for a particular firm, each worker has to go through an apprenticeship which aims at teaching him essential firm-specific skills which only "suitable" workers are able to learn (C can then be interpreted as the cost of this apprenticeship).

Firms can conduct "interviews" (i.e. determine the characteristics of unemployed workers) once every period of unitary length, and each period they can interview one worker.

(1) Moral hazard (on the part of the firm) deters workers from accepting to pay this cost.

We assume that a similar "capacity constraint" exists for workers too: "on average" each worker can be interviewed once every period of unitary length (see appendix pp.1/2). We assume that firms can costlessly determine the duration of the unemployment spells of any unemployed worker.

Workers who meet the skill requirements of the largest number of firms have the highest probability of leaving unemployment; this implies that the probability that unemployed workers meet a firm's skill requirement decreases as their unemployment spells get longer.

Consider the group of workers who - at a certain point in time - have been unemployed for t periods ("cohort t" for short).

The criterion used by a firm when deciding whether it is worthwhile to interview a "cohort t" worker (and not a worker from another "cohort") is the maximization of the expected present discounted value of its profit ("asset value" for short).

Given the possibility of free entry of firms into the economy and under the assumption that the wage rate is fully flexible, in equilibrium the wage offered to cohort t workers is such that:

(1) in the market for cohort t unemployed, the number of interviewees equals the number of firms; furthermore the asset value of all these firms is zero (if all markets are in equilibrium, firms are thus indifferent between entering any labour market and staying outside);

(2) all cohort t workers accept to be interviewed and hired (i.e. the wage is not smaller than the reservation wage).

The wage rate which meets condition (1) can be smaller than the reservation wage. If this is the case, the market for cohort t workers "closes", i.e. no cohort t workers are interviewed.

2.2 FORMAL EXPOSITION

In equilibrium the asset value of all firms which have a vacancy equals zero. This implies that the asset value of a firm which employs a worker equals the expected present discounted value of the net profit stream yielded by that worker.

Let $A_t^E(T)$ denote the asset value of a firm which at T hires a worker in market t.

(1) We have: $$A_t^E(T) = \int_{s=0}^{\infty} [y^e(T+s/T) - w_t(T)].e^{-(z+i).s}ds$$

where $y^e(T+s/T)$ is what firms at T expect "instantaneous" output at T+s to be and where $W_t(T)$ is the one period wage prevailing in market t at T [(A)], while z and i denote a worker's instantaneous death probability and the discount rate used by firms.

The asset value of a firm which at T has a vacancy and which conducts an interview in market t shall be denoted by $A_t^V(T)$.

(2) We have: $$A_t^V = \pi_t(T).A_t^E(T) + (1-\pi_t(T)).e^{-i}.A_{t'}^V.(T+1) - C$$

where $\pi_t(T)$ denotes the probability for a firm of conducting a successful interview in market t at T, and C stands for the cost of conducting an interview.

(A) We assume that unless an unforseen fall in gross productivity takes place (see the discussion of this case on p.16) this wage is paid to the worker during his entire employment period.

Formula (2) can be interpreted in the following way: with probability $\pi_t(T)$ the interview conducted by the firm is successful; this implies that the firm's asset value becomes $A_t^E(T)$. If the interview is unsuccessful, by assumption, the firm has to wait one period of unitary length before it can again conduct an interview. Its asset value (discounted back to T) in the case of an unsuccessful interview becomes $e^{-i}.A_{t'}^V(T+1)$ (if it plans to enter market t' at T + 1). $A_t^V(T)$ is thus equal to the expected value of what the firm's asset value becomes when the firm has conducted its interview minus the cost of conducting that interview.

In equilibrium $A_t^V(T) = A_{t'}^V(T') = 0 \quad \forall\ t,t',T,T'$ and hence

$$(2') \quad 0 = \pi_t(T)\ .\ A_t^E(T) - C.$$

(1) implies that $A_t^E(T) = \int_{s=o}^{\infty} y^e(T+s/T).e^{-(z+i).s}ds - \dfrac{W_t(T)}{z+i}$

and hence (using (2'))

$$(3) \quad W_t(T) = (z+i).\int_{s=o}^{\infty} y^e(T+s/T).e^{-(z+i).s}ds - \frac{z+i}{\pi_t(T)}.C\ .$$

To give content to equation (3), $\pi_t(T)$ has to be determined. If in each labour market, interviewees are randomly allocated to firms,

(4) we have that: $\pi_t(T) = p_A.a(T,t) + p_B.b(T,t)$,

where a(T,t) and b(T,t) denote the proportion of type-A and type-B workers in market t at T.

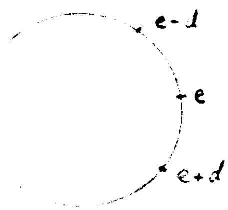
In real economies, unemployed receive some information about the skill requirements of particular firms (e.g. through job advertisements).

This information allows them to evaluate in which "sectors" of the economy their skills are sought after.

In the model, the assumption of a random allocation of interviewees to firms on an economy-wide level can easily be replaced by one in which unemployed workers correctly identify sets of firms which have skill requirements which are "close" to their own skills: assume that a firm with a skill requirement corresponding to e.g. point e in the continuum of skills described above (see p. 3) receives job applications from all workers the

midpoints of whose skill intervals lie in the interval $[e-d,e+d]$ (see Fig. 1).

Fig. 1



By assumption the midpoints of the skill intervals ot type-A and type-B workers are evenly distributed over the "skill circle".

Hence amongst the job applications received by the firm "located" at e, a fraction $a(T,t)$ comes from type-A and a fraction $b(T,t)$ come from type-B workers. Assume that $J_A < 2d$ (J_A (J_B) denotes the length of the skill intervals of type-A (type-B) workers).

Only type-A (type-B) workers the midpoint of whose skill interval lies in the interval $[e - J_A/2, e + J_A/2]$ ($[e - J_B/2, e + J_B/2]$) meet the firm's skill requirement (see Fig. 2).

Fig. 2

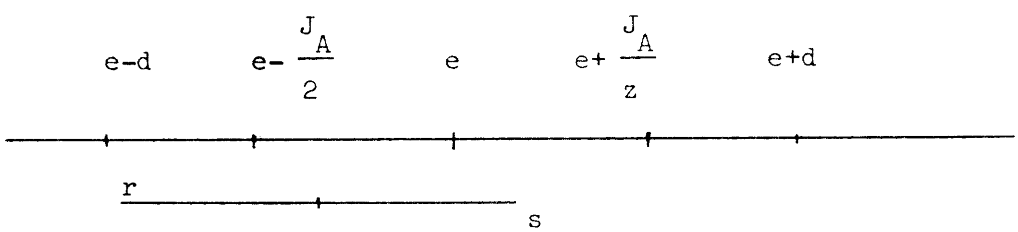


Fig. 2 shows the skill intervals of two of the type-A job applicants (denoted by 'r' and 's' respectively). Worker 'r' meets the firm's skill requirement (the midpoint of his skill interval lies in the interval $[e - \frac{J_A}{2}, e + \frac{J_A}{2}]$), while worker 's' doesn't.

If the firm randomly selects an interviewee amongst its job applicants, the probability of a successful interview is thus equal to

$$\pi_t(T) = a_t(T,t).\frac{J_A}{2d} + b_t(T,t).\frac{J_A}{2d}.$$

Set $J_A/2d = p_A$ and $J_B/2d = p_B$ and we obtain equation (4).

It is shown in the appendix that

$$(5)\quad a(T,t) = \frac{1}{1+(b/a).e^{(p_A-p_B).t'}} \quad \text{and hence}$$

$$(5')\quad \pi_t(T) = (p_A-p_B).\frac{1}{1+(b/a).e^{(p_A-p_B).t'}} + p_B,$$

(where a (b) denotes the proportion of type-A (type-B) workers in the total workforce) if during t' periods interviews have before T been conducted in the cohort of unemployed aged t at T.

It follows from (5') that for $p_A \neq p_B$ the probability of conducting a successful interview is a decreasing function of t'. Furthermore, assuming $p_A > p_B$, $\lim_{t'\to\infty} \pi_{t'} = p_B$: as t' tends to infinity, only type-B workers subsist in the group of unemployed.

The wage rate defined in (3) guarantees that the first condition of the equilibrium concept formulated on p. 5 is met.

The second condition defining an equilibrium requires that w_t is not smaller than the reservation wage (which will be denoted by $\bar{w}$):

$$(6)\quad w_t \geq \bar{w}$$

Inequality (6) allows to determine the minimal probability of conducting a successful interview compatible with equilibrium in the market for cohort-t workers at T (we denote this probability by $\pi_t^{Min}(T)$): it implies that

$$(7)\quad \pi_t^{Min}(T) = \frac{(z+i)C}{(z+i).\int_0^\infty y^e(T+s/T)e^{-(i+z).s}ds-\bar{w}}.$$

The markets for which $\pi_t(T) < \pi_t^{Min}(T)$ "close", i.e. for these markets the wage rate compatible with (2') (i.e. which allows firms which have a vacancy to make net profits (when they hire a worker), the expected present discounted value of which compensates them for the cost of conducting interviews) is smaller than the reservation wage.

We can deduce from $\lim_{t'\to\infty} \pi_{t'} = p_B$, that if $p_B < \pi^{Min}$ a "cut-off" duration of unemployment can exist i.e. a finite number $\tilde{t}$ which is such that if there is a market $\tilde{t}$ which is such that $\pi_{\tilde{t}} = \pi^{Min}$ then all markets t for which $t > \tilde{t}$ close.

It follows from equation (7) that an increase in anticipated gross productivity reduces π^{Min} (when y^e increases, firms can afford to interview unemployed with a smaller probability of being successfully interviewed).

3. DYNAMIC PROPERTIES OF THE MODEL

In what follows, we will submit an economy in which a cut-off duration of unemployment exists to gross productivity shocks.

These shocks can be technological (variations in physical productivity which follow from the introduction of new techniques), reflect variations in the firms' output in terms of e.g. a consumption good (if y is expressed in units of that good), or in the cost of materials. They affect employment through their impact on the cut-off duration; (shocks to other exogenous variables - e.g. the cost of conducting interviews - influence employment through the same channel).

We will consider 3 types of shocks:

(1) a permanent reduction in gross productivity (section 3.1),

(2) a permanent increase in gross productivity (section 3.2),

and (3) a gross productivity cycle, i.e. a sequence of phases of high and low gross productivity (section 3.3).

As the analysis of the impact of these types of shocks on total (un) employment and on the structure of (un) employment is rather technical, we shall now give a verbal exposition of t..e principal results which we obtain: A permanent reduction (increase) in gross productivity leads to a sluggish adjustment path of total employment which extends over several periods; employment is thus characterised by "persistence" (serial correlation). There is furthermore an asymmetry in the adjustment of employment to "positive" as opposed to "negative" shocks:

an unanticipated permanent fall in gross productivity leads to an immediate fall in the cut-off duration of unemployment and hence to the immediate closure of the markets for workers whose unemployment spell exceeds the new cut-off duration. If workers who (before the fall in gross productivity) were hired in the markets which close under the impact of the fall in gross productivity continue to be employed after the fall, total employment decreases until all these workers have died away and - as by assumption workers are "infinitely" lived - the adjustment of employment to its new steady state level is infinitely long.

We turn now to the case of a permanent increase in gross productivity. As the composition of a cohort of unemployed does not change if its members are not interviewed, the probability of conducting a successful interview is constant in all those markets which before an unanticipated increase in gross productivity are closed. This implies that all these markets are opened up by an unanticipated increase in gross productivity. This situation however only subsists until the probability of conducting a successful interview in those cohorts in which interviews start to be conducted under the impact of the increase in gross productivity has been driven down to its new minimal value. If this process is of finite length (i.e. if the cut-off duration of unemployment which corresponds to the increased level of gross productivity is finite) the adjustment of total employment which follows an increase in gross productivity too is of finite length.

The effects of an anticipated permanent fall (increase) in gross productivity are similar to those of an unanticipated fall (increase), except for the fact that the adjustment of employment begins <u>before</u> the actual variation in gross productivity takes place.

If the economy is hit by a gross productivity cycle, employment exhibits "inertia", i.e. increases in the frequency of the gross productivity cycle (holding constant its mean and its amplitude) reduce the amplitude of the fluctuation of employment.

Intuitively this can be explained in the following way: employment falls during phases of low productivity and rises during phases of high productivity. As employment exhibits "persistence", the amount by which it falls (rises) during a phase of high (low) gross productivity - and hence the amplitude of employment fluctuations - decreases as the length of the high (low) productivity phases is reduced (i.e. as the frequency of the gross productivity cycle increases).

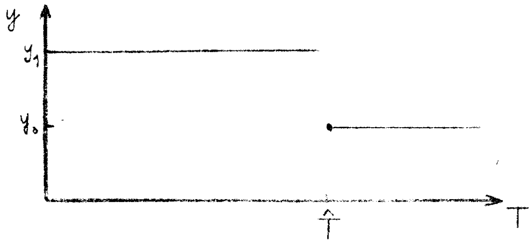
We furthermore show that - if workers' death probability is sufficiently low - an increase of the amplitude of the gross productivity cycle (holding constant its mean and its frequency) can be "desirable" as it increases total employment.

The intuition behind this result is that an increase in the amplitude of the gross productivity cycle (holding constant its mean) reduces π^{Min} during the high productivity phases of the cycle. It therefore increases the total length of the time span during which interviews are conducted in each cohort and thereby reduces the total number of unemployed in the cohorts when these reach the increased cut-off duration of unemployment which obtains during the high productivity phase of the cycle.

If workers' death probability (z) is small, this leads to a large reduction in the total number of permanent unemployed.

3.1 EFFECTS OF A PERMANENT REDUCTION IN GROSS PRODUCTIVITY

Fig. I



We first analyse the effect of an unforseen permanent reduction in gross productivity: at $T < \hat{T}$, gross productivity is equal to y_1 (see Fig. I) and at $\hat{T}$ it falls to y_0.

Denote the number of type-A (type-B) workers whose unemployment spell at T is t periods long by A(T,t) [B(T,t)].

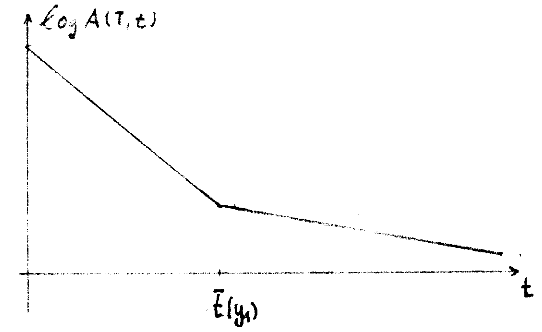
<u>Before $\hat{T}$</u> $\quad A(T,t) = A(T,0).e^{-(p_A+z).t}$ for $0 \leqslant t \leqslant \bar{t}(y_1)$ and

$$A(T,t) = A(T,0).e^{-(p_A+z).\bar{t}(y_1)}.e^{-z(t-\bar{t}(y_1))} \quad \text{for } \bar{t}(y_1) \leqslant t \qquad \text{(A)}$$

where $\bar{t}(y_1)$ is the cut-off duration of unemployment corresponding to the level of gross productivity y_1.

One thus obtains the following "age-profile" of the "population" of unemployed type-A workers at $T < \hat{T}$:

Fig. IIa



$\pi_t(T)$ and $w_t(T)$ are decreasing functions of t (for $t \leqslant \bar{t}(y_1)$):

(A) See Appendix p. 4.

Fig. IIb

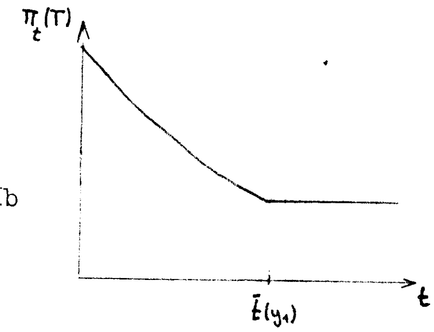
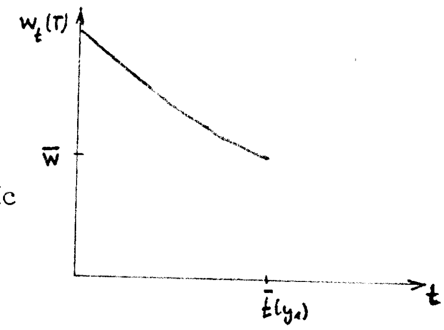


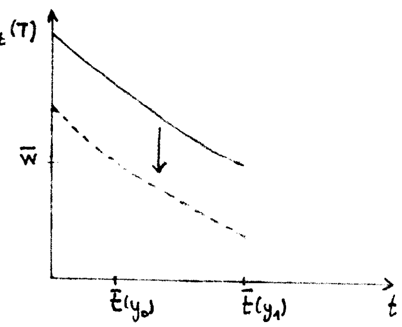
Fig. IIc



Note that $\pi_t(T)$ is constant for $t > \bar{t}(y_1)$ as for these markets the only cause of departure out of unemployment is death and by assumption the instantaneous death probability is identical for type-A and type-B workers; hence the composition of a group of unemployed workers does not change once the duration of their unemployment spell exceeds the cut-off duration.

<u>At $\hat{T}$</u> gross productivity is reduced from y_1 to y_0 (see Fig. Ia). This leads to an instantaneous increase in π^{Min} and to a downward shift of the wage schedule in Fig. IIc by the same number of units as the reduction in gross productivity (see Fig. III).

Fig. III



The fall in the wage rate (A) is necessary to "compensate" firms for the sudden fall in gross productivity.

(A) See equation (3): $w_t(T) = (z+i) \int_{s=0}^{\infty} y_1 . e^{-(z+i).s} ds - \frac{z+i}{\pi_t(T)} .C$ if $y^e(T+s/T) = y_1$ for all $s \geq 0$ (this is the case for $T < \hat{T}$).

Hence $w_t(T) = y_1 - \frac{z+i}{\pi_t(T)} .C$ for $T < \hat{T}$. Similarly,

$$w_t(T) = y_0 - \frac{z+i}{\pi_t(T)} .C \text{ for } T \geq \hat{T}.$$

Fig. III shows that the fall in gross productivity at $\hat{T}$ leads to an immediate fall in the cut-off duration of unemployment from $\bar{t}(y_1)$ to $\bar{t}(y_0)$ and thus to the immediate closure of the markets for unemployed workers whose age lies in the interval $[\bar{t}(y_0),\bar{t}(y_1)]$.

Workers hired in those markets before $\hat{T}$, continue to be employed after the reduction in gross productivity (A).

Now we analyse the adjustment path which follows the fall in gross productivity.

Consider the "age-profile" of the population of unemployed type-A workers at $\hat{T}+\Delta$ $(\Delta > 0)$, i.e. Δ periods after the fall in gross productivity (the "profile" for type-B workers is similar to the one for type-A workers).

Using the same notation as previously we have:

(i) $A(\hat{T}+\Delta,t) = A(\hat{T},0).e^{-(z+p_A).t}$ for $0 < t < \bar{t}(y_0)$,

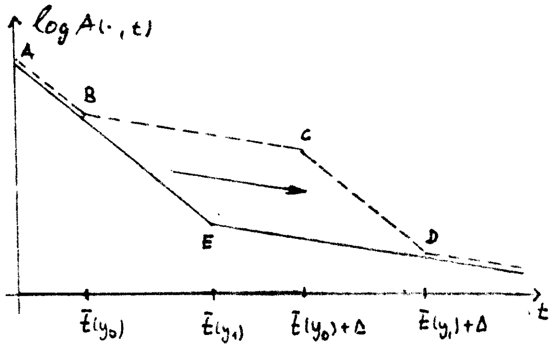
(ii) $A(\hat{T}+\Delta,t) = A(\hat{T}+\Delta,\bar{t}(y_0)).e^{-z(t-\bar{t}(y_0))}$ for $\bar{t}(y_0) < t < \bar{t}(y_0)+\Delta$,

(iii) $A(\hat{T}+\Delta,t) = A(\hat{T}+\Delta,\bar{t}(y_0)+\Delta).e^{-(z+p_A).(t-[\bar{t}(y_0)+\Delta])}$ for $\bar{t}(y_0)+\Delta <$ $t < \bar{t}(y_1)+\Delta$,

(iv) $A(\hat{T}+\Delta,t) = A(\hat{T}+\Delta,\bar{t}(y_1)+\Delta).e^{-z(t-[\bar{t}(y_1)+\Delta])}$ for $\bar{t}(y_1)+\Delta < t$.

(A) We assume that this is so even when the new level of gross productivity is smaller than the wage at which they were hired; in this case it is in the mutual interest of the concerned workers and their employers to bargain out a new wage which is smaller than the new gross-productivity level: if no such agreement is reached (i.e. if firms are forced to stop production), the firms' asset value falls to zero; denote the expected wage which the workers can earn when no agreement is reached by $\hat{w}$ (necessarily $\hat{w}<y_0$). Hence an agreement which leads to a wage $\tilde{w}$ with $\hat{w}<\tilde{w}<y_0$ is beneficial to workers and their employers.

Fig. IV



The continuous line and the broken line represent the age profile at $\hat{T}$ and $\hat{T}+\Delta$ respectively.

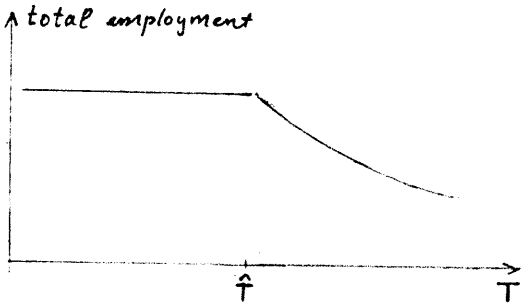
After the fall in gross productivity, interviews continues to be conducted in the markets of the interval $[0,\bar{t}(y_0)]$. Hence the corresponding part of the "age-profile" does not change between $\hat{T}$ and $\hat{T}+\Delta$ (see Fig. IV). Interviews cease however to be conducted in markets $t > \bar{t}(y_0)$. This explains the segment BC of the profile at $\hat{T}+\Delta$ the slope of which is $-z$: consider the group of unemployed type-A workers aged $\bar{t}(y_0)$ at $\hat{T}$; it consists of $A(\hat{T},\bar{t}(y_0))$ individuals and Δ periods later of $A(\hat{T}+\Delta,\bar{t}(y_0)+\Delta) = A(\hat{T},\bar{t}(y_0)).\ e^{-z.\Delta}$ individuals.

Unemployed workers hired in the markets which close at $\hat{T}$ continue to be employed at $\hat{T}+\Delta$: the group of unemployed type-A workers aged $\bar{t}(y_0)+e$ at $\hat{T}$ (with $0 \leq e \leq \bar{t}(y_1)-\bar{t}(y_0)$) consists of $A(\hat{T},\bar{t}(y_0)+e)$ individuals at $\hat{T}$ and of $A(\hat{T}+\Delta,\bar{t}(y_0)+e+\Delta) = A(\hat{T},\bar{t}(y_0)+e).e^{-z\Delta}$ individuals Δ periods later, the only reason of "departure" from the group being death (this explains the segment CD).

Graphically one sees that, starting from the profile at $\hat{T}$, the profile at $\hat{T}+\Delta$ is obtained by sliding to the South-East the segment BE (along the arrow in Fig. IV). This is accompanied by an increase in total unemployment: after the fall in gross productivity total employment falls until all workers who were hired in markets which close under the impact of the reduction in gross productivity, have died away.

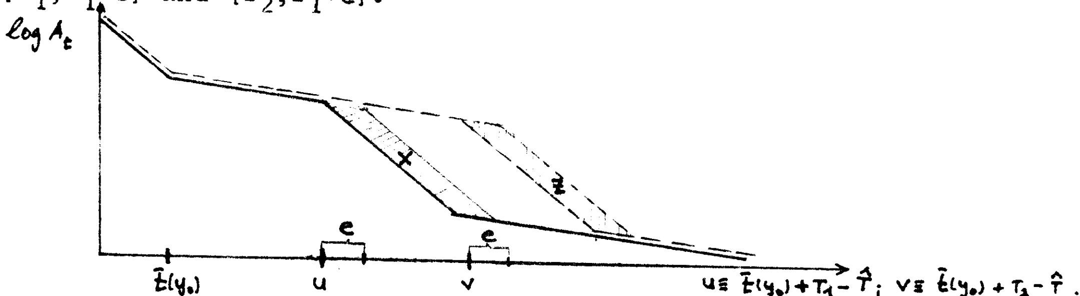
Given the assumption that workers are "infinitely" lived, the adjustment which follows a fall in gross productivity is thus infinitely long(A).

FIG. V



We now assume that from $T = \tilde{T} < \hat{T}$ on, firms anticipate the fall in gross productivity which takes place at $\hat{T}$. Thus:

(A) It is easy to see that the adjustment path depicted in Fig. V is convex: consider two points in time after the fall in gross productivity: $\hat{T} < T_1 < T_2$.
Consider the profile at T_1 (continuous line) and at T_2 (broken line); consider the fall in employment which takes place in the intervals $[T_1, T_1+e]$ and $[T_2, T_1+e]$.



One can get an idea about the amount of these reductions in employment by considering the superficies of X and Z. These superficies are identical.
Note however that what is measured on the vertical axis is the logarithm of the number of type-A workers. Hence the reduction in employment corresponding to X is <u>greater</u> than the amount of the reduction corresponding to Z. This implies that the 2nd derivative of the adjustment path of employment w.r.t. time is positive (the "rapidity" of the adjustment decreases over time).

$y^e(T+s/T) = y_1 \ \forall s \geq 0$ at all $T < \tilde{T}$

$$\left.\begin{array}{lll} y^e(T+s/T) & = y_1 & \text{for } s \in [0, \hat{T}-T] \\ y^e(T+s/T) & = y_0 & \text{for } s \geq \hat{T}-T \end{array}\right\} \text{ at all } \tilde{T} \leq T \leq \hat{T}$$

$$y^e(T+s/T) = y_0 \qquad \forall s \geq 0 \ \text{ at } \ T \geq \hat{T}.$$

This implies (using (3)) that:

$$w_t(T) = y_1 - \frac{z+i}{\pi_t(T)}.C \equiv w_t^1 \qquad \text{for all } T < \tilde{T}$$

$$w_t(T) = [1 - e^{-(z+i).(\hat{T}-T)}].y_1 + e^{-(z+i).(\hat{T}-T)}.y_0 - \frac{z+i}{\pi_t(T)}.C \equiv w_t^2 \quad \text{for } \tilde{T}\leq T\leq\hat{T}$$

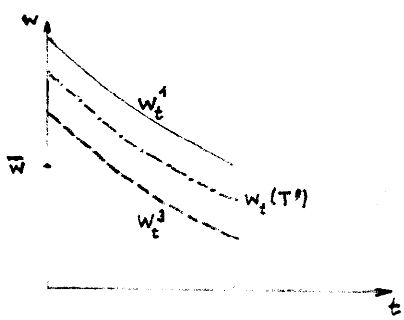
$$w_t(T) = y_0 - \frac{z+i}{\pi_t(T)}.C \equiv w_t^3 \quad \text{for } T \geq \hat{T}.$$

We note that the wage which obtains in the period between $\tilde{T}$ and $\hat{T}$ can be written as a weighted average of the wage which obtains before $\tilde{T}$ and the wage which obtains after $\hat{T}$, the weight attached to the latter increasing as the moment of the actual fall in gross productivity gets closer:

$$w_t(T) = [1 - e^{-(z+i).(\hat{T}-T)}].w_t^1 + e^{-(z+i).(\hat{T}-T)}.w_t^3 \quad \text{for } \tilde{T} \leq T \leq \hat{T};$$

as $w_t^3 < w_t^1$ the wage schedule shifts downwards during the time span $[\tilde{T},\hat{T}]$:

Fig. VI



The wage schedule at 3 points in time: before $\tilde{T}$ (w_t^1), at $\hat{T}$ (w_t^3) and at T' (T < T' < T).

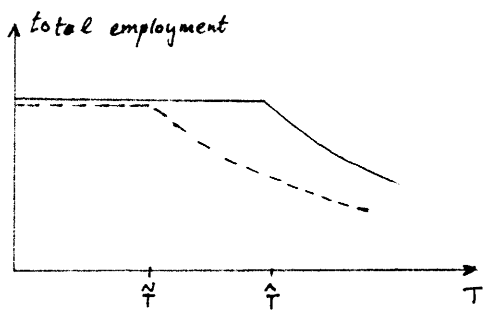
This implies that the cut-off duration of unemployment (which is determined by the intersection of the downward shifting wage schedule and the horizontal line through w) falls during the same time span.

Consequently the markets in the interval $[\bar{t}(y_0),\bar{t}(y_1)]$ close even before the actual fall in gross productivity takes place at $\hat{T}$.

As in the case of an unanticipated fall in y, the long run employment level corresponding to the lower level of gross productivity is only reached after all workers who were hired in markets which close under the impact of the fall in gross productivity have died away.

As these markets however begin to close <u>before</u> the actual fall in gross productivity at $\hat{T}$, the adjustment of total employment to its long-run level is quicker if the fall in gross productivity is anticipated by the firms.

Fig. VII

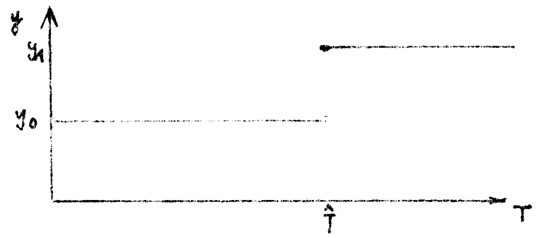


continuous line: time path of total employment if fall in y at T is unanticipated.

broken line: " " if fall in y is anticipated.

3.2 EFFECTS OF A PERMANENT INCREASE IN GROSS PRODUCTIVITY

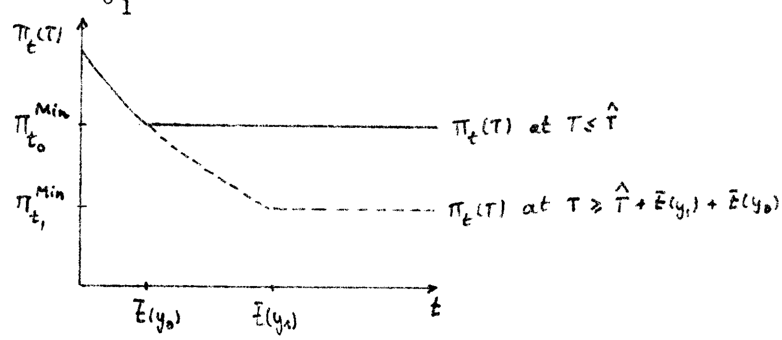
Fig. VIII



We first assume that the increase in gross productivity which takes place is unforseen by the firms. Before $\hat{T}$ the "age-profile" of the group of type-A unemployed, the wage schedule w_t and the π_t schedule are similar to those which are depicted in Fig. IIa-Fig. IIc.

It follows from (7) that the increase in gross productivity at $\hat{T}$ immediately leads to a fall in π^{Min}, i.e. in the minimal probability of conducting successful interviews which is required to prevent labour markets from closing (see Fig. IX; in what follows π^{Min}_t will be denoted by $\pi^{Min}_{t_0}$ for $T < \hat{T}$ and by $\pi^{Min}_{t_1}$ for $T \geqslant \hat{T}$).

Fig. IX

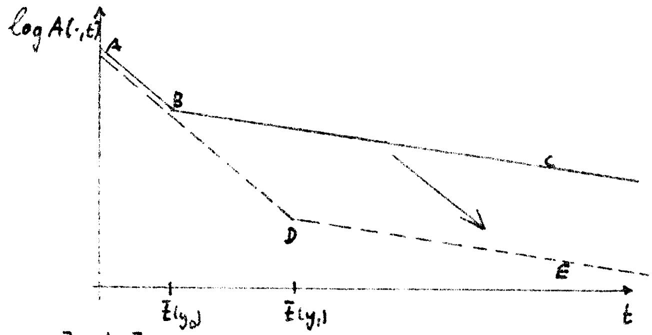


$\bar{t}(y_0)$ is the cut-off duration of unemployment before the increase in gross productivity.

This implies that at $\hat{T}$ in all labour market the probability of conducting a successful interview is not smaller than $\pi^{Min}_{t_1}$. Hence the increase in gross productivity "opens" all those labour markets which before $\hat{T}$ are closed. These markets remain "open" so long as the probability of conducting successful interviews in them is not smaller than $\pi_{t_1}^{Min}$.

Consider the "profile" of type-A unemployed at $\hat{T}$:
(continuous line in Fig. X)

Fig. X



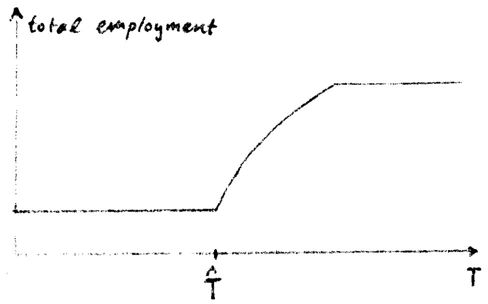
As before let $\bar{t}(y_1)$ denote the length of the time span during which interviews can be conducted in a cohort until the probability of successfully conducting interviews in the cohort has fallen to $\pi_{t_1}^{Min}$. As cohorts aged t with $t > \bar{t}(y_0)$ at $\hat{T}$ are before $\hat{T}$ interviewed during $\bar{t}(y_0)$ periods, they are interviewed during the first $\bar{t}(y_1) - \bar{t}(y_0)$ periods which follow $\hat{T}$.

Graphically the evolution of the age profile can be described as a shift to the South-East of the segment BC in Fig. VIII'. This shift stops at $\hat{T} + \bar{t}(y_1) - \bar{t}(y_0)$ when markets with $t > \bar{t}(y_1)$ close again.

If we assume that $\bar{t}(y_1)$ is finite (i.e. that there exists a cut-off duration of unemployment after the increase in gross productivity), the adjustment of employment which follows an increase in gross productivity is of finite length. (A)

(A) An argument similar to the one presented on p.18 shows that the adjustment path which follows an increase in gross productivity is concave, i.e. the "rapidity" of the adjustment decreases over time.

Fig. XI

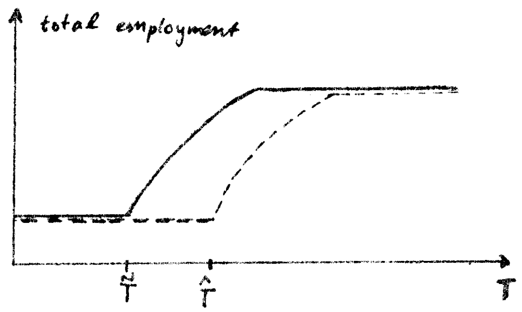


Assume now that the increase in gross productivity which takes place at $\hat{T}$ is anticipated by firms from a point in time $\tilde{T}$ onwards ($\tilde{T} < \hat{T}$). It follows from (7) that during the time span $[\tilde{T},\hat{T}]$, the minimal probability π^{Min} decreases.

Using the arguments developed above, we can see that this implies that all markets in which before $\tilde{T}$ no interviews are conducted, open at $\tilde{T}$. As interviews are conducted in these markets, the probability of conducting a successful interview in them falls. If the fall in π_t^{Min} after $\tilde{T}$ is more rapid than the fall in the probability of conducting a successful interview[(A)], interviews continue to be conducted in these markets after $\hat{T}$ (i.e. when the reduction in π^{Min} has stopped). As these interviews start before $\hat{T}$, the time span during which they are conducted after $\hat{T}$ is shorter than in the case of an unforseen increase in y; hence the new long term level of total employment is reached earlier than in the latter case; one thus obtains the following adjustment path:

(A) This is the case if $\hat{T}-\tilde{T}$ is small, i.e. if firms begin to anticipate the increase in y only a short time before it takes place.

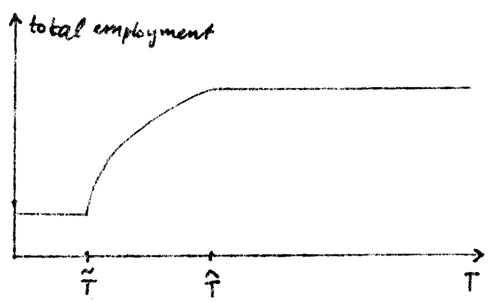
Fig. XI'



continuous line: time path of total employment if increase in y at T is anticipated.
broken line: time path if increase in y is unanticipated.

However if the rapidity with which the interviews which are conducted in the markets which open after $\tilde{T}$ tend to drive down the probability of conducting successful interviews in these markets is greater than the rapidity of the fall in π^{Min}, the adjustment of total employment to its long term value is finished at $\hat{T}$ (see Fig. XI").

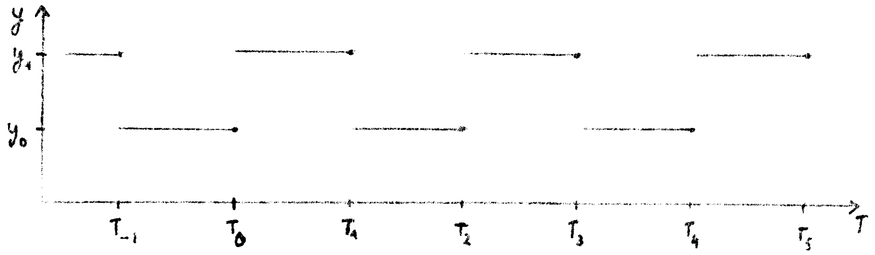
Fig. XI"



3.3 EFFECTS OF A GROSS PRODUCTIVITY CYCLE(A)

We will consider a gross productivity cycle of the following type:

Fig. XII



As before let $\bar{t}(y_1)$ $[\bar{t}(y_0)]$ denote the length of the time span during which interviews can be conducted in a cohort of unemployed workers until the probability of conducting a successful interview has fallen to the value of π_t^{Min} (as defined in (7)) which corresponds to y_1 (y_0) (as previously we denote this value of π_t^{Min} by $\pi_{t_1}^{Min}$ $(\pi_{t_0}^{Min})$).

3.3.1 INCREASES IN THE FREQUENCY OF THE GROSS PRODUCTIVITY CYCLE

We shall start the analysis of a gross productivity cycle of the type depicted in Fig. XII by studying the case in which each high (low) productivity phase is $\bar{t}(y_1) - \bar{t}(y_0)$ periods long, i.e. in which $T_i - T_{i-1} = \bar{t}(y_1) - \bar{t}(y_0)$.

We will thereafter increase the frequency of the cycle by letting $T_i - T_{i-1} = \frac{1}{n} \cdot [\bar{t}(y_1) - \bar{t}(y_0)]$ with n = 2,3,...,.

I. $\quad T_i - T_{i-1} = \bar{t}(y_1) - \bar{t}(y_0)$

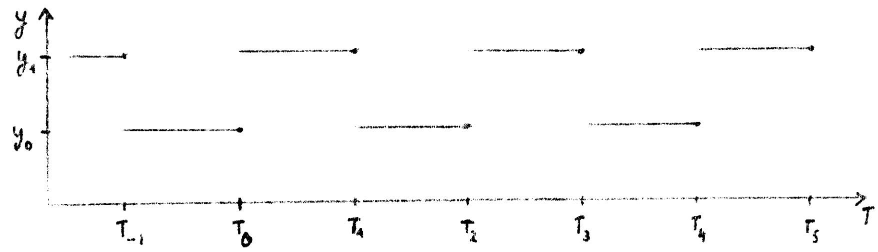
Consider the "age-profile" of the type-A unemployed: at T_1, i.e.

(A) Throughout this section it will be assumed that the shocks to gross productivity are unanticipated.

3.3 EFFECTS OF A GROSS PRODUCTIVITY CYCLE(A)

We will consider a gross productivity cycle of the following type:

Fig. XII



As before let $\bar{t}(y_1)$ $[\bar{t}(y_0)]$ denote the length of the time span during which interviews can be conducted in a cohort of unemployed workers until the probability of conducting a successful interview has fallen to the value of π_t^{Min} (as defined in (7)) which corresponds to y_1 (y_0) (as previously we denote this value of π_t^{Min} by $\pi_{t_1}^{Min}$ $(\pi_{t_0}^{Min})$).

3.3.1 INCREASES IN THE FREQUENCY OF THE GROSS PRODUCTIVITY CYCLE

We shall start the analysis of a gross productivity cycle of the type depicted in Fig. XII by studying the case in which each high (low) productivity phase is $\bar{t}(y_1) - \bar{t}(y_0)$ periods long, i.e. in which $T_i - T_{i-1} = \bar{t}(y_1) - \bar{t}(y_0)$.

We will thereafter increase the frequency of the cycle by letting

$$T_i - T_{i-1} = \frac{1}{n} \cdot [\bar{t}(y_1) - \bar{t}(y_0)] \text{ with } n = 2,3,\ldots,.$$

I. $T_i - T_{i-1} = \bar{t}(y_1) - \bar{t}(y_0)$

Consider the "age-profile" of the type-A unemployed: at T_1, i.e.

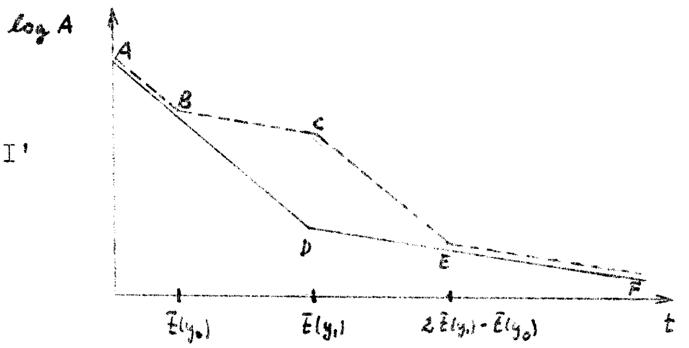
(A) Throughout this section it will be assumed that the shocks to gross productivity are unanticipated.

at the end of a phase of high productivity, the "profile" is ABDEF (see broken line in Fig. XII').

The subsequent fall in gross productivity implies that no interviews are conducted in markets $t > \bar{t}(y_0)$; this shifts the segment BD in South-Easterly direction (see the discussion on p.17).

By assumption the low productivity phase T_1-T_2 lasts $\bar{t}(y_1)$-$\bar{t}(y_0)$ periods; this implies that at its end the segment BD has reached CE.

Fig. XII'



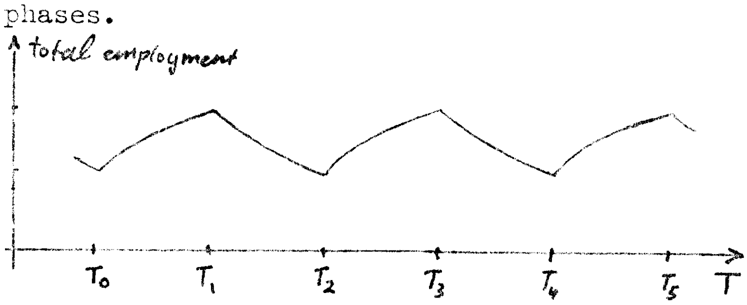
At the end of the low productivity phase T_1-T_2 the "profile" is thus ABCEF.

During the phase T_2-T_3 the evolution of the age profile can be described by shifting to the South-East the segment BC; $\bar{t}(y_1)$-$\bar{t}(y_0)$ periods after T_2, i.e. at T_3, the segment BC has reached DE(see the analysis on p.22).

Hence the profile at T_3 is again ABDE. During T_3-T_4 the evolution of the profile is the same as during T_1-T_2 etc. ... The age profile of type-B workers evolves in a similar way.

In the cycle which has just been described, total employment is highest at the end of high productivity phases and lowest at the end of low productivity phases.

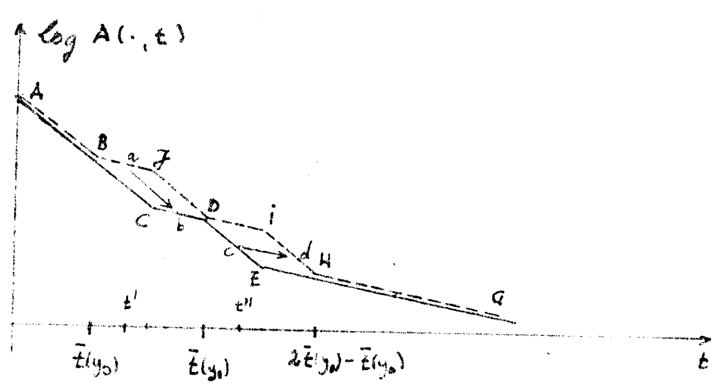
Fig. XII'''



II. $$T_i - T_{i-1} = \frac{1}{2}(\bar{t}(y_1) - \bar{t}(y_o))$$

We now increase the frequency of the gross productivity cycle by letting $T_i - T_{i-1} = \frac{1}{2}(\bar{t}(y_1) - \bar{t}(y_0))$. In this new cycle the age profile of type-A unemployed evolves in the following way (see Fig. XIII):

Fig. XIII



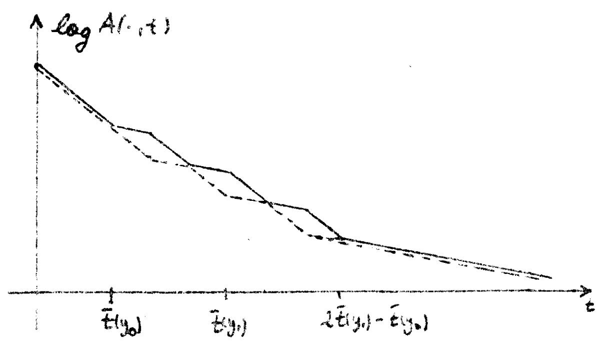
At the end of a low productivity phase- eg. at T_2 -the age profile is described by the broken line in Fig.XIII.In the subsequent phase of high productivity the age profile shifts in the direction of the profile ACDEG (the arrow pointing from a to b describes the evolution of the number of type-A unemployed in the cohort aged t' at T_2).

At T_3 the age profile is ACDEG; as after T_3 interviews cease to be conducted in all markets for which $t > \bar{t}(y_0)$, the age profile shifts to ABJDIHG (the arrow pointing from c to d describes the evolution of the number of type-A unemployed aged t" at T_3) during the interval $[T_3, T_4]$.

During the high productivity phase from T_4 to T_5 the profile shifts again to ACDEG etc...

For a cycle with $T_i - T_{i-1} = \frac{1}{3}(\bar{t}(y_1) - \bar{t}(y_0))$, we obtain the following age profiles:

Fig. XIV



while for $T_i - T_{i-1} = \frac{1}{4}[\bar{t}(y_1) - \bar{t}(y_0)]$ we obtain:

Fig. XV

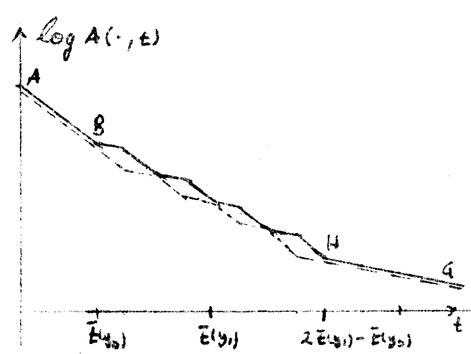
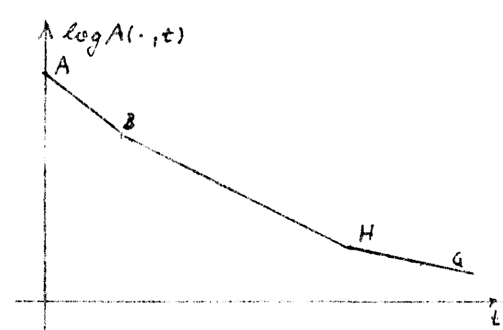


Fig. XV'



In Figs. XIV and XV the broken (continuous) line corresponds to the end of a high (low) productivity phase.

We observe that the profiles 'cycle'around ABHG in Fig. XV'. By increasing the frequency of the gross productivity cycle the age profiles in the age span $[\bar{t}(y_0), 2\bar{t}(y_1)-\bar{t}(y_0)]$ at the end of low and high productivity phases come closer and closer to a line which links the points *B* and *H* in Fig. XVII.

This will now be shown more "systematically" by comparing levels of total employment in the cycle with $T_i - T_{i-1} = (\bar{t}(y_1)-\bar{t}(y_0))$ and in the cycle with $T_i - T_{i-1} = \frac{1}{2}(\bar{t}(y_1) - \bar{t}(y_0))$; to do this we superimpose the age profiles depicted in Fig. XII' and in Fig. XIII at the end of a phase of low productivity (Fig. XVa) and at the end of a phase of high productivity (Fig. XVb).

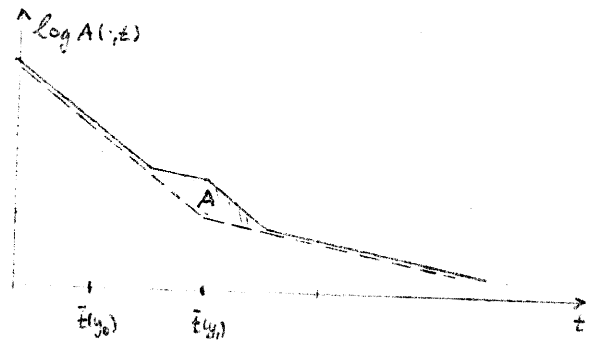


Fig. XVa

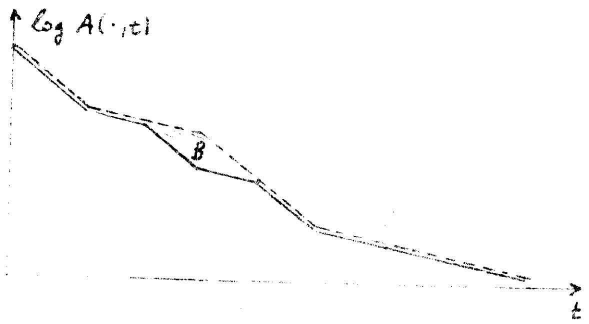


Fig. XVb

In Fig. XIVa and Fig. XIVb the broken lines represent age profiles in the "low frequency cycle" while the continuous lines represent age profiles corresponding to the "high frequency cycle".

Comparing the superficies of the area below the age profile in the "low frequency cycle" and that below the age profile in the "high frequency cycle" in Fig. XIVa and b, it appears that the former exceeds the latter at the end of a period of high productivity while it falls below it at the end of a period of low productivity.

Hence the increase in the frequency of the gross productivity cycle leads to a reduction in the amplitude of the employment cycle which it generates. (A) Total employment exhibits thus "inertia". The intuition behind this phenomenon is that as employment exhibits "persistence", the amount by which it falls (rises) during a phase of high (low) gross productivity decreases as the length of the high (low) productivity phases is reduced (i.e. as the frequency of the gross productivity cycle increases).

It is furthermore easy to see that the increase in the frequency of the gross productivity cycle increases the trough of employment by more than it reduces the peak of employment. (B)

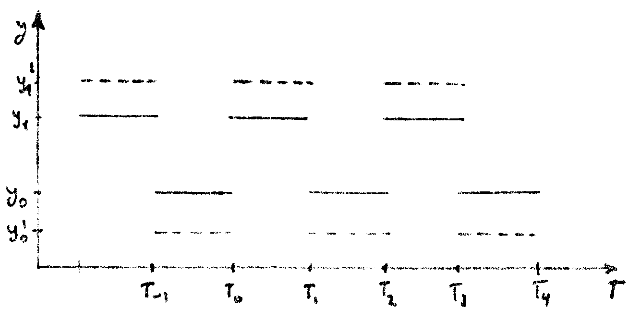
3.3.2 THE EFFECT OF AN INCREASE IN THE AMPLITUDE OF THE GROSS PRODUCTIVITY CYCLE

In what follows we shall increase the amplitude of the gross productivity cycle holding constant its mean and its frequency (see Fig. XVI).

(A) The argument which has been presented above was concerned with what happens to type-A unemployment over the cycle; the evolution of type-B unemployment and hence of total unemployment is similar to that of type-A unemployment.

(B) In Fig.XIVa-XIVb the superficies of A is equal to the superficies of B. Note however that what is measured on the vertical axis is the logarithm of type-A employment. Hence the volume of employment "corresponding" to A is greater than that corresponding to B.

Fig. XVI



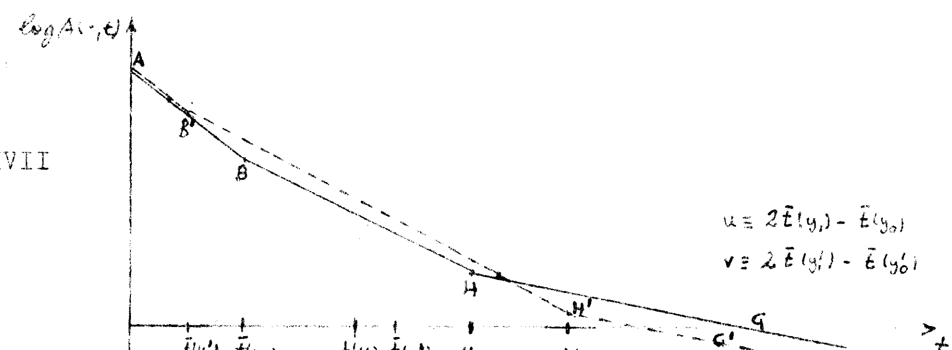
y_1' (y_0') is the maximal (minimal) level of gross productivity after the increase in the amplitude of the gross productivity cycle.

We shall show that the increase in the amplitude of the gross productivity cycle increases employment if the workers' death probability (z) is sufficiently small.

When the amplitude of the gross productivity cycle increases, the cut-off duration of unemployment corresponding to the low level of gross productivity decreases, while that corresponding to the high level of gross productivity increases. Denoting the cut-off duration corresponding to y_1' (y_0') by $\bar{t}(y_1')$ ($\bar{t}(y_0')$) we have thus $\bar{t}(y_1') > \bar{t}(y_1)$ and $\bar{t}(y_0') < \bar{t}(y_0)$. (A)

Let **ABHG** (AB'H'G') in Fig. XIX denote the age profile around which the age profile of type-A unemployed "cycles" before (after) the increase in the amplitude in gross productivity.

Fig. XVII



(A) This follows from equations (7) and (5). It can furthermore be shown that $\bar{t}(y_0) - \bar{t}(y_0') > \bar{t}(y_1') - \bar{t}(y_1)$.

One sees from Fig. XVII that for $t = 2\bar{t}(y'_1) - \bar{t}(y'_0)$ the total number of unemployed type-A workers is smaller after the increase in the amplitude than before it. Denote the first number by $\bar{\bar{A}}$ and the latter by $\bar{A}$ ($\bar{\bar{A}} < \bar{A}$).

As a market with $t > 2\bar{t}(y'_1) - \bar{t}(y'_0)$ is closed at all stages of the high and the low amplitude gross productivity cycle , we have that the total number of type-A unemployed in that market after (before) the increase in the amplitude is $\bar{\bar{A}}.e^{-z(t-(2\bar{t}(y_1)-\bar{t}(y_0)))}$ ($\bar{A}.e^{-z(t-(2\bar{t}(y_1)-\bar{t}(y_0)))}$). The difference in total type-A unemployment in all markets with $t > 2\bar{t}(y'_1) - \bar{t}(y'_0)$ which follows from the increase in the amplitude in gross productivity is thus

$$\Delta \equiv \int_{2\bar{t}(y'_1)-\bar{t}(y'_0)}^{\infty} \bar{\bar{A}}.e^{-z(t-(2\bar{t}(y'_1)-\bar{t}(y'_0)))}dt - \int_{2\bar{t}(y'_1)-\bar{t}(y'_0)}^{\infty} \bar{A}.e^{-z(t-(2\bar{t}(y'_1)-\bar{t}(y'_0)))}dt$$

with $\Delta < 0$.

One easily sees that for small values of z the absolute value of Δ is very large.

If $\bar{t}(y'_1)$ is finite, $2\bar{t}(y'_1) - \bar{t}(y'_0)$ too is finite and hence the change in (average) type-A unemployment in all markets with $0 \leq t \leq 2\bar{t}(y'_1) - \bar{t}(y'_0)$ which follows from the increase in the amplitude of gross productivity is smaller than some constant $\tilde{\Delta}$. For z sufficiently small we have thus $\Delta + \tilde{\Delta} < 0$, i.e. type-A unemployment decreases when the amplitude of the gross productivity cycle goes up. The same thing can be shown for type-B unemployment. This establishes that the increase in the amplitude of the gross productivity cycle increases total employment if the workers' death probability is sufficiently small.

APPENDIX:

In order to derive (5), we shall first consider a discrete time version of the model. In this version births and deaths only take place at the end of intervals of fixed length.

Set this length equal to $\tau = \frac{1}{n}$ $(n > 1, n \in N)$.

At the end of each subinterval, workers die with a probability of $z.\tau$. Workers who die are instantaneously "replaced" by newborn workers who immediately after their birth enter unemployment.

In order to obtain (5), we will "follow" the evolution of a cohort of unemployed workers through time.

Assume that during the time span of length $\hat{t}$ after the birth of the cohort, interviews are conducted amongst its workers. In the discrete time version of the model, a cohort of newborn workers consists of $z.\tau.N$ workers (N being the total workforce) of which $a.z.\tau.N$ workers are type-A workers and $b.z.\tau.N$ are type-B workers.

In equilibrium the number of interviewees equals the number of firms in each labour market.

Interviewees are randomly allocated to firms.

Firms can conduct one interview every period of unitary length. It seems plausible that a similar "capacity constraint" exists for workers too;

we assume that each worker can be interviewed once every period of unitary length. Under the assumption that workers' interviews are evenly distributed over time, a fraction τ of all unemployed in each cohort is interviewed in every subinterval of length τ.

We will approximate this situation by assuming that at the beginning of each subinterval a fraction τ of the unemployed of each cohort are randomly selected for interviews. This implies that $(\tau.p_A).(a.z.\tau.N)$ type-A and $(\tau.p_B).(b.z.\tau.N)$ type-B workers are hired at the beginning of the first period of the life of a cohort of newborns. Amongst those who are not hired, a fraction $(1-z\tau)$ enters market 2.

Denote the point in time at which the cohort is born by $\bar{T}$ and the number of unemployed type-A (type-B) workers in that cohort at $T = \bar{T} + t$ (with $t \geq 0$), i.e. when the age of members of the cohort is t, by $A(T,t)$ $(B(T,t))$.

We have $A(\bar{T} + \tau,\tau) = (1-z.\tau).(1-p_A.\tau).(a.z.\tau.N)$

and $B(\bar{T} + \tau,\tau) = (1-z.\tau).(1-p_B.\tau).(b.z.\tau.N)$.

Similarly one obtains

$$A(\bar{T} + 2.\tau,2.\tau) = ((1-z.\tau).(1-p_A.\tau))^2.(a.z.\tau.N)$$

$$B(\bar{T} + 2.\tau,2.\tau) = ((1-z.\tau).(1-p_B.\tau))^2.(b.z.\tau.N)$$

and - generalizing -

(c) $$A(\bar{T}+i.\tau,i.\tau) = ((1-z.\tau).(1-p_A.\tau))^i.(a.z.\tau.N)$$

$$B(\bar{T}+i.\tau,i.\tau) = ((1-z.\tau).(1-p_B.\tau))^i.(n.z.\tau.N)$$

for all i such that $i.\tau < \hat{t}$ (by assumption interviews are conducted in the cohort during the time span of length $\hat{t}$ after its birth).

In the discrete time version of the model which has just been presented, unemployed have to wait τ units of time after being interviewed (without being hired) before they can be interviewed again.

In continuous time this waiting time tends to zero; i.e. a worker who has been interviewed at an arbitrary point in time can be interviewed almost immediately later although the probability that this happens is infinitesimal. Hence in continuous time the number of unemployed type-A (type-B) workers in the cohort at $T = \bar{T} + t \leqslant \bar{T} + \hat{t}$ is

$$(7) \qquad A(\bar{T}+t,t) = \lim_{\tau\to 0} ((1-z\tau).(1-p_A.\tau))^{L(t,\tau)-1} \underbrace{.a.z.\tau.N}_{= A(\bar{T},0)}$$

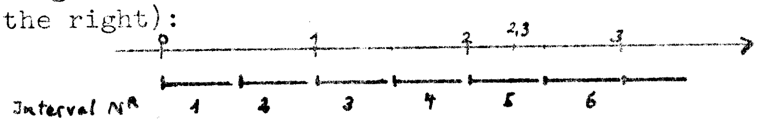
$$= e^{-(p_A+z).t}.A(\bar{T},0)$$

and similarly $B(\bar{T}+t,t) = e^{-(p_B+z).t}\, B(\bar{T},0)$, where $L(t,\tau)$ is the number of the interval in which t lies if the positive real line is divided in intervals of length τ and if these intervals are numbered from the left to the right .(A)

By definition $a(T,t) = \dfrac{A(T,t)}{A(T,t)+B(T,t)} = \dfrac{A(\bar{T}+t,t)}{A(\bar{T}+t,t)+B(\bar{T}+t,t)}$

$$= \frac{1}{1+\dfrac{B(\bar{T},0)}{A(\bar{T},0)}.e^{(p_A-p_B).t}}$$

(A) Example: Let $T_o = 2.3$, $T_1 = 1$ and $\tau = 0.5$ then $L(T_o,\tau) = 5$ and $L((T_1,\tau) = 3$ (w.l.o.g. we assume that the intervals are closed to the left and open to the right):



Note that $t/\tau-1 < L(t,\tau) \leqslant t/\tau$; hence
$((1-z\tau)(1-p_A.\tau)^{t/\tau-1} < ((1-z\tau)(1-p_A.\tau))^{L(t,\tau)} \leqslant ((1-z\tau).(1-p_A.\tau))^{t/\tau}$.
We have $\lim_{t\to 0}((1-z\tau)(1-p_A.\tau)^{t/\tau-1} = \lim_{\tau\to 0}((1-z\tau).(1-p_A.\tau))^{t/\tau}$
Hence $\lim_{\tau\to 0}((1-z\tau).(1-p_A.\tau)^{L(t,\tau)} = \lim_{\tau\to 0}((1-z\tau).(1-p_A.\tau))^{t/\tau} = e^{-(p_A+z).t}$

and hence $a(T,t) = \dfrac{1}{1+\dfrac{b}{a}.e^{(p_A-p_B).t}}$ as $a = \dfrac{A(\bar{T},0)}{A(\bar{T},0)+B(\bar{T},0)}$ and

$$b = \frac{B(\bar{T},0)}{A(\bar{T},0)+B(\bar{T},0)}$$

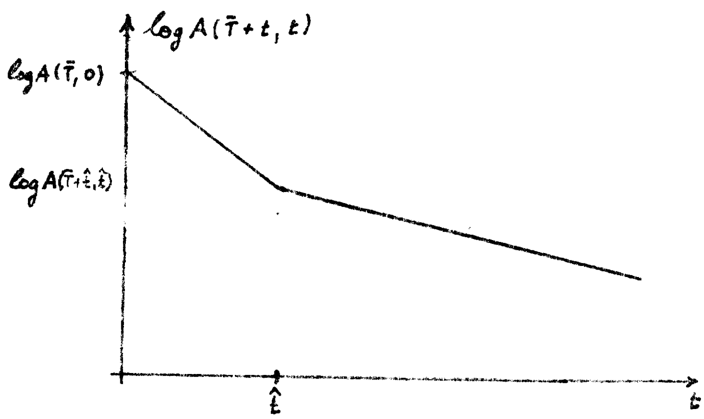
By assumption interviews cease to be conducted in the cohort at $\bar{T}+\hat{t}$. One easily sees (using arguments similar to those which have just been used) that

$$\text{(III)} \quad \begin{aligned} A(T,T-\bar{T}) &= A(\bar{T}+\hat{t},\hat{t}).e^{-z(T-(\bar{T}+\hat{t}))} \\ B(T,T-\bar{T}) &= B(\bar{T}+\hat{t},\hat{t}).e^{-z(T-(\bar{T}+\hat{t}))} \end{aligned}$$

$$\forall T > \bar{T} + \hat{t}.$$

Illustration: evolution of $A(\bar{T} + t,t)$ over time (the slope of the segment AB is $-(z+p_A)$, while that of the segment BC is $-z$).

Fig. 1.



Assume now that after $\bar{T}+\hat{t}$ interviews are conducted intermittently, e.g. in the following age intervals: $(\hat{t}_1,\hat{t}_2)$, $(\hat{t}_2,\hat{t}_3)$, $(\hat{t}_3,\hat{t}_4)$,... with $\hat{t} < \hat{t}_1 < \hat{t}_2 < \hat{t}_3 < \hat{t}_4 \ldots$

At any age t, the proportion of type-A unemployed in the cohort is a function of the total number of periods during which interviews were conducted in it before it reached age t, only.

This is due to the fact that when no interviews are conducted in the cohort, death is the only reason for the reduction of the number of its members; by assumption the death probability is however identical for type-A and type-B workers. Hence the proportion of type-A unemployed at e.g. $\tilde{t} \in [\hat{t}_1, \hat{t}_2]$ is equal to that in a cohort in which interviews are conducted in the age span $(0, \hat{t} + ((\tilde{t} - \hat{t}_2))$.

Denote the length of this age span by t' (in accordance with the notation used in (5)); graphically t' can be determined by measuring the total length of the age intervals for which the slope of the graph of $\log A(\tilde{T}+t, t)$ is $-(p_A+z)$ in the interval $[0, \tilde{t}]$. Once t' is known $a(\tilde{T}+\tilde{t}, \tilde{t})$ can easily be determined: $a(\tilde{T}+\tilde{t}, \tilde{t}) = \dfrac{1}{1+b/a.e^{(p_A-p_B).t'}}$. This proves (5).