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Ganchev, Alexander

Tsenov Academy of Economics

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MODELING THE YIELD CURVE OF SPOT INTEREST RATES UNDER THE CONDITIONS IN BULGARIA

Assist. Prof. ALEXANDER GANCHEV
„Dimitar A. Tsenov" Academy of Economics,
Department of Finance and Credit

The empirical research conducted in the last few years has shown that the bonds are playing an increasingly important role and are becoming increasingly popular on the global stock market. The total nominal value of the bonds issued in the USA and Japan exceed by far the market value of all shares traded on the stock markets in these countries.[1] The great importance of bond markets in the countries with developed stock markets has stirred the interest of the international academic community towards their investment characteristics. Traditionally, the foreign investment theory assigns an important place on the study of bond returns, their yield curves, quantitative characteristics[2] and term structure models.[3] The reason is related to the fact that the bond yield curve and its dynamics and behavior are considered the main indices for the status, characteristics and trends of development of the bond markets[4] and the economy as a whole.

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[1] **Левинсън, М.** Финансови пазари – пътеводител. София, 2004, p. 83-133 and p. 181-233 as well as **Mishkin, F. S.** The Economics of Money, Banking and Financial Markets. Harper Collins Publishers, 1992, p. 41.

[2] For more details see **Litterman, R., J. Scheinkman.** Common Factors Affecting Bond Returns. Journal of Fixed Income, 1991, p. 54-61; **Barber, J. R., M. Cooper.** Immunization Using Principal Component Analysis. Journal of Portfolio Management, 1996, p. 99-105; **Iimanen, A.** Time-varying expected returns in international bond markets. Journal of Finance, 1995, 50, p. 481-506; **Smith, K. L.** Government Bond Market Seasonality, Diversification and Cointegration: International Evidence. Journal of Financial Research, 2002, 25, p. 203-221.

[3] For more details see **Choudhry, M.** Bond and Money Markets: Strategy, Trading, Analysis. Bloomberg Press, 2005; **Cox, J. C., J. E. Ingersoll, S. A. Ross.** A Theory of Term Structure of Interest Rates. Econometrica, 1985, 53:385-407; **Heath, R., A. Jarrow, A. Morton.** Bond Pricing and the Term Structure of Interest Rates: A New Methodology for Contingent Claim Valuation. Econometrica, 1992, 60, p.77-105; **Vasicek, O.** An Equilibrium Characterization of the Term Structure. Journal of Financial Economics, 1977, 5, p. 177-188.

[4] **Choudhry, M.** Analysis and Interpreting the Yield Curve. John Wiley & Sons, 2004, p. 55.

On the other hand, the existing early studies in this financial field in Bulgaria[5] are focused only on the theoretical aspects of the bonds, their yield and term structure. The research conducted by A. Angelov in 2002[6] was the first major step in the process of defining the quantitative characteristics of the bond markets in Bulgaria under the conditions of the currency board. Nevertheless, it actually did not full clarify the issues related to the structure of the bond yield curve. This is why we may claim that although the problems related to the structure of the bond yield curve are studied thoroughly in the foreign scientific literature, in Bulgaria they haven't been studied in depth. This fact predetermines to a large extent the need for the present study and its topicality.

The article reviews the bond yield curve in Bulgaria. The subjects of research are the quantitative models for modeling the yield curve and especially the spot interest rate models. The main objective of the article is to present the methodology for modeling the spot interest rates curve to the investment community in the country.

Structurally the article is organized as follows: part one presents the subject of the research, the data used and the main restrictive conditions applied for data selection; part two presents the research methodology; part three presents the analysis of the empirical research results; part four summarizes the results and defines the trends for further research.

1. Analytical data

Analysis was made with bond prices used by the pension security funds in Bulgaria to evaluate their bond portfolios.[7] The database contains information about the net and gross prices[8] by days of state bonds traded between January 2006 and December 2007 and denominated in BGN, USD, and Euro. The data is analyzed on a daily basis for state bonds with maturity of up to ten years.

This choice of a database was made because of the long intervals between the emissions of new state bonds, which practically makes the results of such data unsuitable for analyses. Another reason for restricting the period covered by the analysis is the suitability of the data collected in

[5] **Парушев, Сл.** Облигациите. Варна, 1995; **Адамов, В., Пл. Пътев, С. Проданов, А. Захариев.** Инвестиции, В. Търново, 1999; **Пътев, Пл.** Управление на портфейла. В. Търново, 1996; **Йорданов, Й.** Финансови инвестиции. Варна, 2001; **Пътев, Пл., Н. Канарян.** Управление на портфейла. В. Търново, 2008.

[6] **Ангелов, А.** Лихвени структури. В. Търново, 2002.

[7] The author would like to express his gratitude to the employees of AMC „Standard Asset Management" for the database used in the research.

[8] Also known as **clean price** and **dirty price.**

compliance with the methods for portfolio evaluation adopted by the pension security funds. A third reason is the fact that on 1 June 1997 Bulgaria implemented a Currency Board, which had a stabilizing effect on our financial system. This is an important condition for the objectivity of the empirical research. Another reason is the condition of the bond market in Bulgaria. Practically at present there is no secondary bond market in Bulgaria. Due to the fact that bonds are risk-free and issued in limited quantities, they are easily sold on the primary bond market and are practically not traded on the secondary bond market.[9] An undeniable proof for that is the lack of official bond market indices for neither the separate segments of the bond markets nor an official composite index published by the Bulgarian Stock Exchange in Sofia. At present the only indexes reflecting the debt securities market are calculated and published by TBI Asset Management.[10] Another important fact was also taken into consideration to narrow the range of research data to treasury bonds only. The existing and currently traded corporate, mortgage and municipal bonds are not included in a universal credit rating system, and thus their common yield curves cannot be estimated. Thus, despite the abundance of data, the estimation of a common yield curve for the above mentioned bonds is both theoretically and practically impossible, because such yield curves and their subsequent analysis would provide an inadmissibly distorted and unreal information for the bond markets.

2. Research methodology

The models of the spot interest rates curve are the second most popular aspect in the modeling of the yield curve of after the spline models used by McCullough[11] Vasicek and Fong,[12] Shea[13] and Steeley[14], Fisher,

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[9] In February 2008 the deals in debt securities amounted to barely 0.39 % of all transactions on the floor of BSE-Sofia and to about 0.03 % of all traded stocks. Source: the weekly bulletin of the Central Depository published on http://www.csd-bg.bg/ and calculations of the author.

[10] For more details see http://tbiambg.spnet.net/bg/tbiindex/values/.

[11] **McCulloch, J. H.** Measuring the Term Structure of Interest Rates. Journal of Business, 44, January 1971, p. 19-31; **McCulloch, J. H.** The Tax Adjusted Yield Curve. Journal of Finance, 30(3), 1975, p. 811-830.

[12] **Vasicek, O., H. G. Fong.** Term Structure Modelling using Exponential Splines. Journal of Finance, 37(2), 1982, p. 339-348.

[13] **Shea, G. S.** Pitfalls in Smoothing Interest Rate Term Structure Data: Equilibrium Models and Spline Approximations. Journal of Financial and Quantitative Analysis, 19(3), 1984, p. 253-269.

[14] **Steeley, J. M.** Estimating the Gilt-Edged Term Structure: Basis Splines and Confidence Intervals. Journal of Business, Finance and Accounting, 18(4), 1991, p.513-529.

Nychka and Zervosc,[15] Waggonerp[16], and Anderson and Sleath.[17] Their indisputable advantages have rated them high in the field of investment practice. A research conducted by the Hungarian Central Bank in 1998[18] shows that most central banks in the countries with developed stock markets at that time preferred these models. The origins of the spot interest rate curve models date back from the seventies - in a research conducted by Vasicek.[19] One of the numerous results of his fundamental work regarding the models of yield curve dynamics is the assumption that the spot interest rates can be modeled non-discretely using equation 1:

$$R_t^c = R_\infty - \left(R_\infty - r_t\right)\left[\frac{1-e^{-\alpha t}}{\alpha t}\right] + \frac{\sigma^2}{\alpha^2}\left[\frac{\left(1-e^{-\alpha t}\right)^2}{4\alpha t}\right] \text{ at } R_\infty = \mu - \frac{\lambda\sigma}{\alpha} - \frac{\sigma^2}{2\alpha^2}, \tag{1}$$

where:

r_t^c is the continuous spot interest rate at a moment t;

r_t - the spot interest rate with the shortest period to maturity;

α - the speed of return of r_t to their long-term value μ;

σ - standard deviation of r_t;

λ - the risk premium or the market price of the risk in the absence of arbitrary possibilities on the bond markets;

R_∞ - a fixed variable which can be defined as a spot interest rate at $t \to +\infty$.

However, Vasicek's model is predominantly theoretical in nature. In practice it may lead to some difficulties related to the estimation of the value of the risk premium λ. This could render its direct application to real market data rather difficult. Despite this, the model has a very important feature - its values R_∞, $R_\infty - r_t$, and $\left(\sigma^2 / \alpha^2\right)$ could be interpreted as long-term level, bias, and hump of the spot interest rates. Their combination with the

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[15] **Fisher, M., D. Nychka, D. Zervos.** Fitting the Term Structure of Interest Rates with Smoothing Splines. US Federal reserve Board, Finance and Economic Discussion Series 95-1, 1995.

[16] **Waggoner, D.** Spline Methods for Extracting Interest Rate Curves from Coupon Bond Prices. Federal Reserve Bank of Atlanta Working Paper, No 97-10, 1997.

[17] **Anderson, N., J. Sleath.** New Estimates of the UK Real and Nominal Yield Curves. Bank of England Quarterly Bulletin, November 1999.

[18] **Csajbok, A.** Zero-coupon Yield Curve Estimation from a Central Bank Perspective. NBH Working Paper №2, May 1998.

[19] **Vasicek, O.** An Equilibrium Characterization of the Term Structure. Journal of Financial Economics, 1977, 5, p. 177-188.

parameters α, β, λ, and r_t increase the theoretical and scientific importance of the model because thus it is compatible with the theory and characteristics of the yield curve in general - a feature which is not characteristic for the spline models. This is why we may conclude that Vasicek's model set a new trend in the field of the modeling of the spot interest rate curve, which was different and innovative compared to the approach which used the spline functions.

In 1987 Nelson and Siegel proposed a concept for modeling the curve of forward interest rates at $t \to 0$ using equation 2:[20]

(2) $$f_{t\to 0} = \beta_0 + \beta_1 e^{-t/\tau_1} + \beta_2 (t/\tau_1) e^{-t/\tau_1}$$

According to the principle mathematical concepts for estimamation of spot and forward interest rates,[21] the continuous spot interest rates at a moment t are practically the average value of all forward interest rates until the moment t (equation 3):

(3) $$r_t^c = \frac{1}{T}\int_0^T f_{(0,t)} dt ,$$

where:

$f_{(0,t)}$ - the forward interest rate between the moments 0 and t; dt - time interval at $t \to 0$.

The integration of equation 2 into the main formula of Nelson and Siegel' model for the spot interest rates curve results in equation 4:

(4) $$r_t^c = \beta_0 + \beta_1 \left[\frac{1 - e^{-t/\tau_1}}{t/\tau_1} \right] + \beta_2 \left[\frac{1 - e^{-t/\tau_1}}{t/\tau_1} - e^{-t/\tau_1} \right],$$

where the continuous spot interest rate r^c at a moment t is a function of the linear parameters $\beta_0, \beta_1, \beta_2$ and the non-linear parameter τ_1. The β_0 parameter has the meaning of $\lim\limits_{t\to +\infty} r^c(0,t)$, i.e. the continuous spot interest rate at $t \to +\infty$ or the long-term level of non-discrete spot interest rates. The β_1 parameter shows $\lim\limits_{t\to 0}\left(r^c(0,t) - \beta_0\right)$ and in fact measures the difference between the short-term and long-term continuous spot interest rates, i.e. it

[20] **Nelson, C.R., A. F. Siegel.** Parsimonious Modelling of Yield Curves. Journal of Business, 60(4), 1987, p. 473-489.

[21] For more details see **Ганчев, А.** Фундаментални характеристики на доходността на дълговите ценни книжа. Годишник на С. А. „Д. А. Ценов", том CIX, АИ „Ценов", 2007.

defines the slope of their curve. The β_2 parameter defines the nature of the curve[22] of continuous spot interest rates. If its value is positive, the curve will be convex, and if the value is less than zero, spot interest rate curve will be concave. The value of the parameter τ_1 defines the position of the spot interest rate hump.

The model proposed by Nelson and Siegel is rather simplified. Practically it defines the spot interest rates curve with only four parameters and at the same time can model its basic forms.[23] The model has several other advantages. Its parameters have clear and strictly defined economic meaning. The presence of parameters showing the level, bias, and hump of the spot interest rates allows the inclusion of this model in strategies for elimination of the interest risks related to the dynamics of the spot interest rates curve. This is why Barret, Gosnell, and Heuson,[24] and, later, Willner[25] developed adequate quantitative strategies for hedging of interest risks based on the model proposed by Nelson and Siegel. The inclusion of only one non-linear parameter - τ_1 - however, is the reason for the low level of responsiveness of Nelson and Siegel's model to market data and is indeed its main disadvantage.

Despite the advantages of Nelson and Siegel's model from theoretical and mathematical point of view, the practice of bond markets showed that the insufficient parameterization of the spot interest rates curve cannot describe some non-standard spot interest rate curves. This is why in 1994 Svensson extended the original model proposed by Nelson and Siegel in order to increase its flexibility and improve its capacity to model non-standard yield curves at relatively weak parameterization. Svensson added the expression $\beta_3(t/\tau_2)e^{-t/\tau_2}$ to the equation of the forward interest rates curve of the original Nelson and Siegel's model. Thus the model of the spot interest rates curve evolved to equation 5:[26]

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[22] Known as **hump**

[23] The characteristics of the traditional forms of yield curves goes beyond the scope of this research. They are well defined and used in Bulgarian investment theory. For more details see **Адамов, В., С. Проданов.** Инвестиции. В. Търново, 2004, p. 202-208, АНГЕЛОВ, А. Лихвени структури. В. Търново, 2002, p. 29-46.

[24] **Barret, W., T. Gosnell, A. Heuson.** Yield Curve Shifts and the Selection of Immunization Strategies. Journal of Fixed Income, 5(2), September 1995, p. 53-64.

[25] **Willner, R.** A New Tool for Portfolio Managers: Level, Slope and Curvature Durations. Journal of Fixed Income, 6(1), June 1996, p. 48-59.

[26] **Svensson, L. E.** Estimating and Interpreting Forward Interest Rates 1992-1994. IMF Working Paper No. 114, 1994.

(5) $$r_t^c = \beta_0 + \beta_1\left[\frac{1-e^{-t/\tau_1}}{t/\tau_1}\right] + \beta_2\left[\frac{1-e^{-t/\tau_1}}{t/\tau_1} - e^{-t/\tau_1}\right] + \beta_3\left[\frac{1-e^{-t/\tau_2}}{t/\tau_2} - e^{-t/\tau_2}\right],$$

where:

the linear parameter β_3 defines the form (convex or concave) of the second hump of the spot interest rates curve, and the non-linear parameter τ_2, like τ_1 in the original Nelson and Siegel's model, defines its position.

Despite its theoretical enhancement some research results show that Svensson's model, like Nelson and Siegel's model, has low sensitivity to modeled data.[27] This was an important hint for the academic society that the existing models of the a spot interest rate curves had intrinsic flaw - the probability for their lower accuracy. Svensson's model, however, had a rarely recognized advantage. A research conducted by Bjork and Christensen[28] proved that it is compatible with the Hull and White model of the dynamics of the yield curve. This means that the spot interest rate curves estimated following this model reflect the nature of the processes generating the yield of debt securities. This makes the analyses and investment strategies based on the Svensson's model methodologically more accurate and fundamental for the bond market in general.

The model proposed by El Karoui, Cherif, Dicoum, and Savidan[29] further develops Vasicek's model by removing the limitations imposed by its rigid mathematical specification. It models the spot interest rate curve using equation 6:

(6) $$r_t^c = L - S\left[\frac{1-e^{-\alpha t}}{\alpha t}\right] + \gamma\left[\frac{\left(1-e^{-\alpha t}\right)^2}{4\alpha t}\right],$$

where:

L is the long-term level of non-discrete spot interest rates;

S - the spread between the spot interest rates with the shortest and longest term to maturity;

γ - a parameter defining the hump of the spot interest rate curve.

[27] **Bolder, D. J. and S. Gusba.** Exponentials, Polynomials, and Fourier Series: More Yield Curve Modelling at the Bank of Canada. Bank of Canada Working Paper №29, 2002, p.15.

[28] **Bjork, T., B. J. Christensen.** Interest Rate Dynamics and Consistent Forward Rate Curves. Mathematical Finance, 9(4), 1999, p. 323-348

[29] **El Karoui, N., T Cherif, J. A. Dicoum, K. Savidan.** Modelisation de la Structure par Terme des Taux d'Interet: Tests et Applications. Cahiers de la Caisse Autonome de Refinancement, Paris, vol. 6, 1994.

The model following equation 6 has all merits of Vasicek's methodology, but with more loose input assumptions. The similar mathematical definition makes this model compatible and comparable to the original model proposed by Nelson and Siegel. Thus it practically has the same advantages - clear definition and interpretation of its parameters. In 2001 Martellini and Priaulet[30] developed a technique for hedging of interest risks based on this model - another proof of this compatibility with the original model of Nelson and Siegel and its advantages. Martellini and Priaulet went even further - the similarities between the models proposed by El Karoui, Cherif, Dicoum, and Savidan and the original model of Nelson and Siegel inspired the two scientists to enhance it. Following the logic of Svensson, Martellini and Priaulet proposed a model (equation 7) with two new linear (T and K) and one non-linear (β) parameters added to equation 6. The three new parameters improved its flexibility and capacity to model more efficiently the spot interest rate curve. This makes the model proposed by Martellini and Priaulet compatible with the model of Svensson and explains the similar advantages and disadvantages of these models:

$$r_t^c = L - S\left[\frac{1-e^{-\alpha t}}{\alpha t}\right] + \gamma\left[\frac{\left(1-e^{-\alpha t}\right)^2}{4\alpha t}\right] + T\left[\frac{\left(1-e^{-\beta t}\right)^2}{\beta t}\right] + K\left[\frac{\left(1-e^{-\beta t}\right)^2}{4\beta t}\right] \tag{7}$$

The review of the most popular models of the spot interest rate curve outlines their common advantages and disadvantages. A common advantage is their strong theoretical and methodological base. We have already pointed out and substantiated the economic sense and importance of their parameters. This makes the models of the spot interest rate curve practically irreplaceable in the field of management of debt investment portfolios. Moreover, the direct modeling of the spot interest rate curve with few and predominantly linear parameters, predetermines the more rapid estimation of their values. Due to their fixed mathematical specifications, the models of the spot interest rate curve do not carry the risk of misinterpretation, which is carried by the spline models, i.e. the implied risk related to the use of the spot interest rate curve models is minimal. However, the presence of few non-linear parameters also defines their main disadvantage, i.e. their theoretically smaller capacity for accurate definitions of various yield curves.

The quantitative methods for modeling the spot interest rate curve constitute the principal methodological toolbox of the research. Specifically,

[30] **Martellini, L., P. Priaulet.** Fixed Income Securities: Dynamic Methods for Interest Rate Pricing and Hedging. John Wiley and Sons, 2001, p. 83-87.

the techniques for their practical implementation in Bulgaria requires the estimation of their parameters for each business day within the investigated period. For this end we applied an optimization program which minimizes the sum of the weighted quadratic differences between the actual and the theoretical bond prices according to the corresponding quantitative method for modeling the spot interest rate curves:

$$\text{(8)} \qquad \sum_{i=1}^{n}\left[\left(P_{i,t}-\hat{P}_{i,t}\right)/w_i\right]^2 \to min,$$

where:

$P_{i,t}$ is the actual price of the i th bond at the moment t;

$\hat{P}_{i,t}$ is the theoretical price of the i th bond at the moment t estimated with equation 9;

w_i - the weight in the optimization program, associated with the i th bond.

$$\text{(9)} \qquad \hat{P}_{i,t}=\sum_{t=1}^{N} CF_{i,t}\times e^{\left(-r_t^c t\right)},$$

where:

$CF_{i,t}$ is the cash flow from the i th bond, starting at moment t.

The actual market price of each bond is equal to its clean price defined as the average value of the clean sell price and the clean buy price of treasury bonds for each business day within the investigated period.

The weighing factor w_i had to be included in the optimization program due to the fact that the prices of long-term bonds are markedly more sensitive to the changes in the levels of the spot interest rates than the prices of short-term bonds, i.e. bond prices are heteroskedastic. This is why an optimization program which does not weigh the quadratic differences between the actual and the theoretical bond prices will have the proclivity to "compensate" any greater fluctuation in the prices of long-term bonds. Thus the value of the long-term bonds would be systemically overestimated while that of the short-term bonds - systemically underestimated. This is highly undesirable because it may distort the empirical results of the research.

The investment theory provides various specifications of the weighing factor w_i. A research conducted by Vasicek and Fong proposes a definition given by equation 10[31], while the central banks of Belgium and Italy have developed and implemented specifications of w_i based on equations 11[32] and 12:[33]

(10) $w_i^{\,2} = \left[\left(D_{i,t}^2 P_{i,t} \right) / \left(1 + Y_{i,t} \right)^2 \right]$,

(11) $w_i = \left[\left(D_{i,t} P_{i,t} \right) / \left(1 + Y_{i,t} \right) \right]$,

(12) $w_i = \left[\left(1 / D_{i,t} \right) / \sum_{i=1}^{n} \left(1 / D_{i,t} \right) \right]$,

where:

w_i is the weight of the i th bond in the optimization algorithm, $D_{i,t}$, $P_{i,t}$ and $Y_{i,t}$ - are the duration, the market price and the yield to maturity of the i th bond.

A common disadvantage of the specifications of w_i provided by the three equations (10 through 12) is their complexity added to the complexity of the non-linear optimization algorithm represented by equation 8. This is why we used a significantly simplified and more convenient weighing factor. It is inversely proportional to the modified duration of each bond and could be expressed with equation 13:

(13) $w_i = 1 / MD_i$,

where MD_i is the modified duration of the i th bond.

Since the spot interest rates modeled using equations 1,4,5,6 and 7 are essentially non-discrete, we use equation 14 to obtain discrete spot interest rates:

(14) $r_t = e^{r_t^c} - 1$,

where r_t is the discrete spot interest rate at the moment t

Several criteria were used to evaluate the accuracy of the methods to model the spot interest rates curve efficiently. The functional criteria for bond price accuracy (and hence the accuracy of the spot interest rates) are shown in Table 1.

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[31] **Vasicek, O., H. G. Fong.** Term Structure Modelling using Exponential Splines. Journal of Finance, 37(2), 1982, p. 339-348.

[32] BANK FOR INTERNATIONAL SETTLEMENTS. Zero-coupon yield curves: technical documentation. BIS papers №25, October 2005, p. 1-2.

[33] Ibid, p. 12-14.

Table 1.

Functional criteria for modeling accuracy of spot interest rates curve

Average error	(15)	$AE = \frac{\sum_{i=1}^{N}\left(P_{i,t} - \hat{P}_{i,t}\right)}{N}$
Root of the mean square error	(16)	$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(P_{i,t} - \hat{P}_{i,t}\right)^2}{N}}$
Mean absolute error	(17)	$MAE = \frac{\sum_{i=1}^{N}\left\|P_{i,t} - \hat{P}_{i,t}\right\|}{N}$
Weighted average error	(18)	$WAE = \frac{\sum_{i=1}^{N}\left[\left(P_{i,t} - \hat{P}_{i,t}\right)/ MD_i\right]}{N}$
Weighted root of the mean square error	(19)	$WRMSE = \sqrt{\frac{\sum_{i=1}^{N}\left[\left(P_{i,t} - \hat{P}_{i,t}\right)/ MD_i\right]^2}{N}}$
Weighted mean absolute error	(20)	$WMAE = \frac{\sum_{i=1}^{N}\left[\left\|P_{i,t} - \hat{P}_{i,t}\right\| / MD_i\right]}{N}$

Next we reviewed the ability of the spot interest curve models to evaluate accurately the modeled bond prices and hence the spot interest rates curve. For this purpose we reviewed the number of theoretical bond prices larger, smaller, or equal to the assumed actual price (equations 21 through 23), and in greater detail (see Table 2) – the average number of theoretical bond prices within the intervals between: the actual sell and buy prices, the actual price and the buy price, and the actual price and the sell price.

Table 2.

Functional criteria for correctly modeled bond prices and spot interest rates curve

Average number of theoretical bond prices greater than the actual price	(21) $AN_{\hat{P}_{i,t} > P_{i,t}} = \frac{\sum_{i=1}^{N} card\left(\hat{P}_{i,t} > P_{i,t}\right)}{N}$
Average number of theoretical bond prices smaller than the actual price	(22) $AN_{\hat{P}_{i,t} < P_{i,t}} = \frac{\sum_{i=1}^{N} card\left(\hat{P}_{i,t} < P_{i,t}\right)}{N}$
Average number of theoretical bond prices equal to the actual price	(23) $AN_{\hat{P}_{i,t} = P_{i,t}} = \frac{\sum_{i=1}^{N} card\left(\hat{P}_{i,t} = P_{i,t}\right)}{N}$
Average number of theoretical bond prices within the interval b/n the actual buy and sell prices	(24) $AN_{P_{i,t}^{L} \le \hat{P}_{i,t} \le P_{i,t}^{U}} = \frac{\sum_{i=1}^{N} card\left(\hat{P}_{i,t} \in \left(P_{i,t}^{L} \le \hat{P}_{i,t} \le P_{i,t}^{U}\right)\right)}{N}$
Average number of theoretical bond prices within the interval b/n the actual price and the actual buy price	(25) $AN_{P_{i,t}^{L} \le \hat{P}_{i,t}} = \frac{\sum_{i=1}^{N} card\left(\hat{P}_{i,t} \in \left(P_{i,t}^{L} \le P_{i,t}\right)\right)}{N}$
Average number of theoretical bond prices within the interval b/n the actual price and the actual sell price	(26) $AN_{\hat{P}_{i,t} \le P_{i,t}^{U}} = \frac{\sum_{i=1}^{N} card\left(\hat{P}_{i,t} \in \left(P_{i,t} \le P_{i,t}^{U}\right)\right)}{N}$

Thirdly, the article evaluates the chronological stability of the spot interest curve models in terms of the dynamics of the above criteria.

3. Empirical results

The empirical results shown in Tables 3 and 4 show the ability of the spot interest curve models to model accurately the curve. The results show that in terms of the “average error” criterion Vasicek’ s model performs best in terms of modeling the spot interest rates curve for the selected period. The most accurate model in terms of the significantly more enhanced from a methodological point of view criterion “ root of the mean square error” is the model of Svensson. According to the “average absolute error” criterion Vasicek’s model ahs again the best performance in terms of the accuracy of modeling of the spot interest rates curve. The most inaccurate model considering all the three criteria is the model of El Karoui, Cherif, Dicoum, and Savidan.

Table 3.

Results from equations 15 through 17

Criteria	*Model of Vasicek*	*Model of Nelson and Siegel*	*Model of Svensson*	*Model of El Karoui, Cherif, Dicoum, and Savidan*	*Model of Martellini and Priaulet*
AE	***-0.184979084***	0.382078503	0.226874661	0.695474933	0.385570782
RMSE	5.969464796	5.989376705	***5.586003156***	6.866918161	5.890812259
MAE	***3.908752773***	4.072904572	3.803117715	4.651382333	4.002202142

Table 4 shows the values of the criteria „average error", „root of the mean square error", and „mean absolute error" weighted with the modified durations of all bonds. The results indicate roughly the efficiency of the process of finding the best solution of the optimization criterion expressed in equation 8. The most accurate model considering all the three criteria is the model of El Karoui, Cherif, Dicoum, and Savidan, followed by the model of Martellini and Priaulet. The results from the three criteria and, most of all, the „root of the mean square error", and „mean absolute error" applied to the models of Vasicek, Nelson and Siegel, and Svensson showed that there obviously were some serious problems with their application. The results, which were about twice as bad as the results obtained for the model of El Karoui, Cherif, Dicoum, and Savidan, showed that in certain situations the models of Vasicek, Nelson and Siegel, and Svensson cannot provide the same optimization. This indirectly proves that these models are more sensitive to the input data. Their performance may become unstable and thus practically compromise them. On the other hand the disparity of the data shown in Table 3 and Table 4 regarding the model of El Karoui, Cherif, Dicoum, and Savidan can be due to the fact that the model has a better performance using weighted bond prices than actual market data. This, in term, shows that in a „sterile" environment, i.e. in ideal conditions, the model of El Karoui, Cherif, Dicoum, and Savidan has undeniable advantages, although its application to actual data is problematic as well. In comparison the model of Martellini and Priaulet stands out with the relative stability of its performance regardless of the applied criteria.

Table 4.

Results from equations 18 through 20

Criteria	*Model of Vasicek*	*Model of Nelson and Siegel*	*Model of Svensson*	*Model of El Karoui, Cherif, Dicoum, and Savidan*	*Model of Martellini and Priaulet*
WAE	2.361080689	2.541510635	2.140671468	***0.207579564***	1.254015101
WRMSE	12.18571386	12.25296718	11.61993634	***4.797404894***	6.211794512
WMAE	3.736569458	3.780338898	3.468514939	***2.321077288***	2.365059713

Considering the data in Tables 3 and 4 we may come to a general conclusion about the condition of the Bulgarian stock market. The extremely high fluctuations of the models of spot interest curves (which, represented through the criterion „weighted root of the mean square error", and „mean absolute error" deviate between 5.59 and 6.87 at a nominal of the analyzed bonds of BGN 100) practically reveal the extreme inconsistence of the debt market in our country and the presence of market distortions in the field of price formation of our treasury bonds.

Table 5 presents the results obtained from the application of equations 21 through 23 to the analyzed data. We can see that during the analyzed period none of the models of spot interest curve did not model correctly the price none of the traded bonds. These results are logical taking into account the inconsistence of the debt market in our country and the presence of distortions of the analyzed data from the debt market in Bulgaria. More interesting are the results showing the extent to which the analyzed models underestimate or overestimate the assumed actual bond prices. The results show that only the model of Vasicek tends to systematically underestimate the modeled bond prices and hence to overestimate the theoretical spot interest rates compared to the actual ones. On the other hand the models of Nelson and Siegel, Svensson, El Karoui, Cherif, Dicoum, and Savidan, and Martellini and Priaulet tend to systematically overestimate the modeled bond prices, which means that the spot interest rate curves obtained using these models would be significantly underestimated compared to the actual prices. This effect is weakest with the model of Nelson and Siegel followed by the model of Martellini and Priaulet. The other two models – the model of Svensson and the model of El Karoui, Cherif, Dicoum, and Savidan, although slightly, the trend for overestimation of the modeled bond prices is stronger.

Table 5.

Results from equations 21 through 23

Criteria	*Model of Vasicek*	*Model of Nelson and Siegel*	*Model of Svensson*	*Model of El Karoui, Cherif, Dicoum, and Savidan*	*Model of Martellini and Priaulet*
21	13.4384858	14.07619048	14.46101695	14.48376623	14.39873418
22	14.55205047	13.87301587	13.5220339	13.55519481	13.57278481
23	0	0	0	0	0

Table 6 shows the results obtained from equations 24 through 26 with the analyzed data. They clearly prove that the model of Martellini and Priaulet has the highest capacity to evaluate correctly the modeled bond prices and hence the spot interest rates curve. It is followed closely by the model of Nelson and Siegel, the model of Svensson, and the model of El Karoui, Cherif, Dicoum, and Savidan. To a large extent the results for the model of El Karoui, Cherif, Dicoum, and Savidan are not logical taking into account the fact that it, together with the model of Martellini and Priaulet, is closely related to the model of Vasicek and that the three models are practically members of the same family. This means that to a certain extent the model of El Karoui, Cherif, Dicoum, and Savidan is not precise and practically cannot adequately benefit from its mathematical methodology, which is very close to that of Vasicek' model.

Table 6

Results from equations 24 through 26

Criteria	*Model of Vasicek*	*Model of Nelson and Siegel*	*Model of Svensson*	*Model of El Karoui, Cherif, Dicoum, and Savidan*	*Model of Martellini and Priaulet*
24	3.227129338	2.561904762	2.274576271	2.470779221	***3.424050633***
25	1.899053628	1.40952381	0.972881356	1.178571429	***1.908227848***
26	1.32807571	1.152380952	1.301694915	1.292207792	***1.515822785***

The performance stability analysis of the models of spot interest rates curve in Table 3 for the period from January 2006 to December 2007 is illustrated by Figures 1 through 5. On the grounds of the presented information we can draw the following conclusions:

- Despite its highest accuracy according to criteria "average error" and "average absolute error", the model of Vasicek suffers from lack of stability. The dynamics of its three accuracy criteria shows rapid booms and slumps, which means that in certain market situations the model would lack accuracy.
- The model of Svensson, despite its superiority compared to the rest of the models in terms of the "weighted root of the mean square error" criterion, also lacks performance stability.
- The performance of the model of El Karoui, Cherif, Dicoum, and Savidan is most unstable and thus provides the poorest results in terms of performance stability shown in Table 3.
- The models of Nelson and Siegel and Martellini and Priaulet (with a slight advantage of the model of Nelson and Siegel) demonstrate greatest temporal stability of their performance accuracy indices.

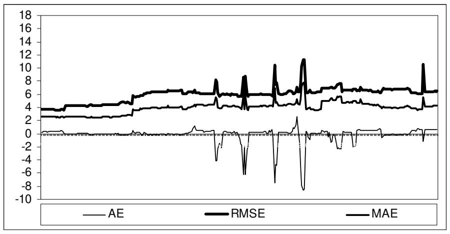

Fig. 1. **Model of Vasicek**

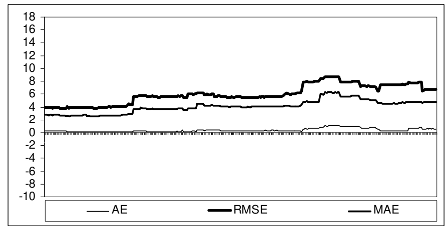

Fig. 2. **Model of Nelson and Siegel**

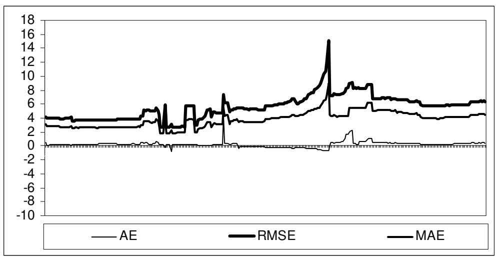

Fig. 3. **Model of Svensson**

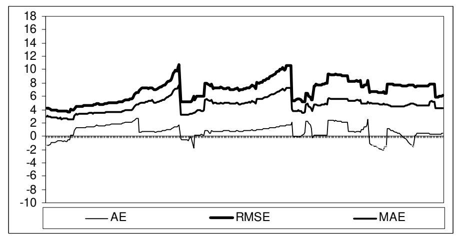

Fig. 4. **Model of El Karoui, Cherif, Dicoum, and Savidan**

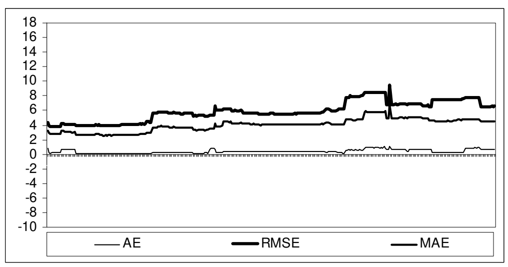

Fig. 5 **Model of Martellini and Priaulet**

The dynamics of the weighted accuracy indices described in Table 4 is illustrated by Figures 6 through 10. The data clearly indicates the reason why the models of Vasicek, Nelson and Siegel and Svensson failed - their inability in certain situations to produce optimal solutions for modeling the spot interest rates curve. On the other hand, the performance of the models of Martellini and Priaulet and El Karoui, Cherif, Dicoum, and Savidan is very similar, which, considering the deviations in the performance of the model of El Karoui, Cherif, Dicoum, and Savidan with non-weighted and weighted accuracy criteria, proves the superiority of the model of Martellini and Priaulet in terms of its capacity to model accurately and consistently the spot interest rates curve.

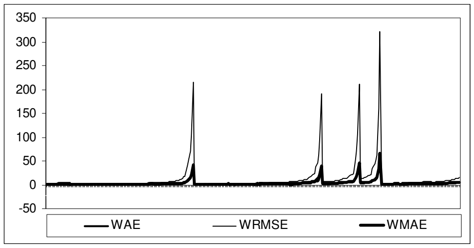

Fig. 6. **Model of Vasicek**

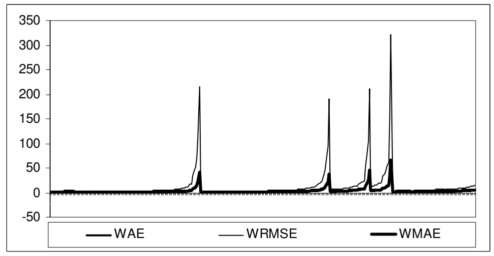

Fig. 7. **Model of Nelson and Siegel**

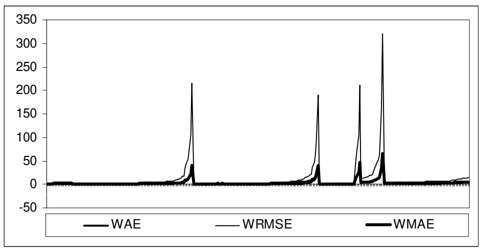

Fig. 8. **Model of Svensson**

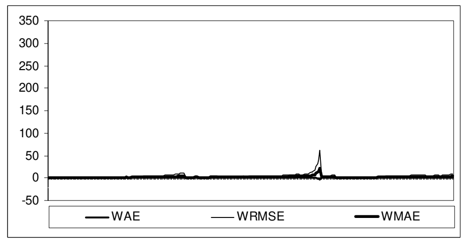

Fig. 9. **Model of El Karoui, Cherif, Dicoum, and Savidan**

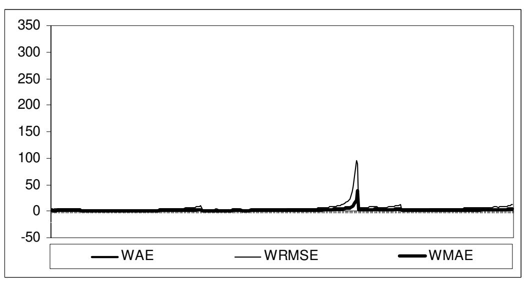

Fig. 10. **Model of Martellini and Priaulet**

Figures 11 through 15 show the temporal dynamics of the number of theoretical bond prices within the interval between the actual buy and sell bond prices (equation 24). The main reason why we considered the dynamics of this index only is because the other criteria are to a large extent its derivatives and their analysis would not contribute to the accuracy of the research.

The analysis clearly shows the unsatisfactory and temporally unstable performance of the models of Svensson и El Karoui, Cherif, Dicoum, and Savidan, compared the performance of the models of Martellini and Priaulet, Nelson and Siegel and Vasicek. The figures show that at times Vasicek’s model excels by far the model of Martellini and Priaulet, but in many cases demonstrated poorer performance as well. The model of Nelson and Siegel shows similar results, which means that the model of Martellini and Priaulet is not only the most accurate one, but also is more stable in terms of accurate modeling of bond prices and hence the spot interest rates curve.

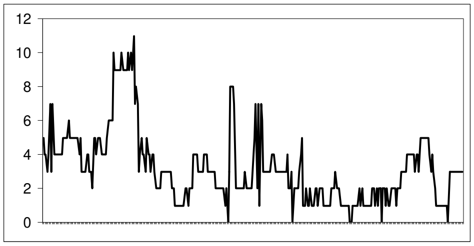

Fig. 11. **Model of Vasicek**

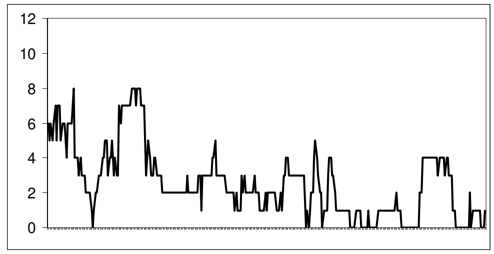

Fig. 12. **Model of Nelson and Siegel**

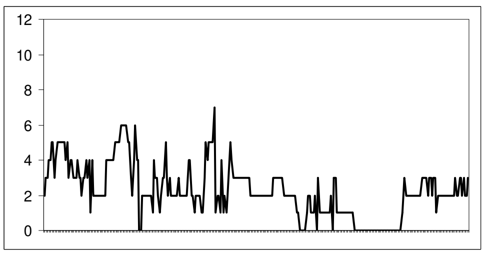

Fig. 13. **Model of Svensson**

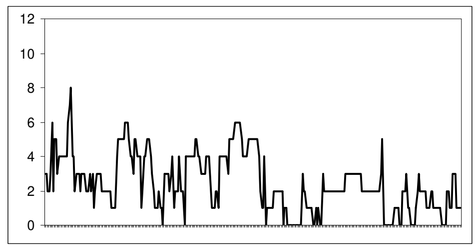

Fig. 14. **Model of El Karoui, Cherif, Dicoum, and Savidan**

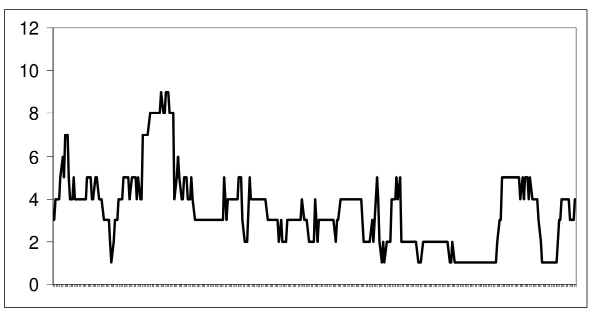

Fig. 15. **Model of Martellini and Priaulet**

4. Conclusions and directions for further research

The empirical analysis does not single out a single best model for modeling the spot interest curve although it shows that the model with the poorest performance considering all criteria is the one proposed by Nelson and Siegel. The analysis of the other models showed that none of them can be considered absolutely superior in terms of all quantitative criteria. This is why we need to a new comparison point for the various criteria. This in term requires the adoption of a certain compromise in terms of the performance of all models. The empirical results show that despite their accuracy the models proposed by Svensson, Vasicek, and El Karoui, Cherif, Dicoum, and Savidan lack stability and accuracy of estimation of the yield curve, and hence - the actual prices of traded bonds. In comparison the model proposed by Martellini and Priaulet stands out with its greater performance stability. The combination of its satisfactory stability and ability to model correctly the bond prices makes this model the most appropriate method for modeling the spot interest rates curve of the state bonds market in Bulgaria.

This study does not solve all problems related to the modeling of yield curve and its specific features in Bulgaria. The problems related to the compilation of debt investment portfolios and their protection against interest risk using the methods for modeling the yield curve remain unsolved. The relation between the forms and the dynamics of the bond yield curve in the context of our national economy (considering some major macroeconomic indicators such as GDP growth, inflation, and unemployment rates) are still not studied in depth. These are the two main directions for further research in this field.