How to decode Unemployment Persistence: An econometric framework for identifying and comparing the sources of persistence

Møller, Niels Framroze

Technical University of Denmark, University of Copenhagen, Department of Economics

March 2016

How to decode Unemployment Persistence:

An econometric framework for identifying and comparing the sources of persistence

Niels Framroze Møller[1]

Technical University of Denmark,

[1]nfmo@dtu.dk

Abstract

Most econometric analyses of persistence focus on the existence of non-stationary unemployment but not the origin of this. The present research contains a multivariate econometric framework for identifying and comparing different sources of unemployment persistence (e.g. hysteresis versus a slowly moving equilibrium rate). A small example, considering historical data (1988-2006) for the UK, demonstrates how the method can be applied in practice. Although this primarily serves as an illustration, the evidence clearly suggests that persistence was due to a slowly moving equilibrium (driven by the price of crude oil) and not to hysteresis mechanisms.

1   Introduction

Witnessing the course of European unemployment during the 1970s and through the mid-80s led economists to think about why high unemployment tends to be so persistent - a phenomenon which was henceforth dubbed hysteresis (Blanchard and Summers 1986).[1] Although the interest in the topic probably peaked in the late 1980s/early 90s (Røed 1997), renewed focus followed in the wake of the more recent worldwide economic crisis (see e.g. Ball 2009, Andersen 2010, Amable and Mayhew 2011, O'Shaughnessy 2011, and Delong and Summers 2012). At that time some of these economists stressed that our understanding of the mechanisms giving rise to hysteresis was still limited despite the amount of research conducted in the preceding two decades. To quote Ball (2009), "..hysteresis is an important phenomenon, but one that is not well understood. This means more research is needed".[2] Today, long after the onset of the crisis, it is clear that in the meantime many European countries have experienced a prolonged period of high unemployment, suggesting a continued importance of the topic and a need for explaining the phenomenon.

Despite a voluminous literature, the majority of studies have focussed primarily on detecting the empirical existence of hysteresis as measured by the non-stationarity of unemployment rates (see e.g. the surveys in Røed (1997), Gustavsson and Österholm (2009) and Arestis and Sawyer (2009). In particular, following Blanchard and Summers (1986), many empirical papers have been based on Dickey-Fuller type tests (Dickey and Fuller 1981) for unit root non-stationarity in *univariate* autoregressive models for unemployment, which have proven useful for that purpose. However, given the abundant evidence of unit roots, there has been far less research based on methods which aim more directly at econometrically identifying and comparing different explanations of hysteresis and persistence. For this purpose, *multivariate* or system-based econometric modelling seems to show more promise. Although the system aspect of unemployment hysteresis was emphasized long ago (see e.g. Amable, Henry, Lordon, and Topol 1995 and Göcke 2002), only few multivariate econometric analyses exist (see Jacobson, Vredin, and Warne 1997, Dolado and Jimeno 1997 and Hansen and Warne 2001). Nevertheless, these studies represent a great step forward when it comes to gaining insights into the sources of unemployment persistence, which is important since different sources have different policy implications (Røed (1997), Andersen (2010) and Delong and Summers (2012)).

The purpose in this paper is therefore to present a multivariate econometric framework for analyzing and comparing different sources of unemployment persistence and show how

---

[1] See e.g. Røed (1997), and more recently, O'Shaughnessy (2011), for surveys on hysteresis.
[2] Other examples are Andersen (2010) and Delong and Summers (2012).

this may be used in a given application. By nesting hypotheses of persistence in the same structural model, a main advantage of the framework is that it allows one to distinguish between persistence (more generally) and hysteresis, which is a special case of the former (see below). In particular, it makes it possible to compare hypotheses implying a persistent equilibrium versus hypotheses implying hysteresis. This is important since only in the latter case, temporary (demand) shocks may have permanent (i.e. long-lasting) effects.

The framework builds on Davidson and Hall (1991) and Mosconi and Giannini (1992), and is based on a general Structural VAR (SVAR) with exogenous variables. That is, it is represented in block matrices, thereby applying to a wide range of generic economic models. However, in order to demonstrate how the methodology may be applied in practice, a *fully specified* SVAR, i.e. with restrictions on the block matrices, is used to derive and test different statistical hypotheses of unemployment persistence, in the form of different cointegration restrictions. The model is confronted with UK macroeconomic quarterly time series data.

The present research relates to the studies, Jacobson et al. (1997) and Dolado and Jimeno (1997), in that it builds on a SVAR with cointegration. However, these studies are primarily concerned with hysteresis, whereas here, the focus is on unemployment persistence more generally, and in particular on identifying and comparing different sources of this.[3] Distinguishing between "persistence" and "hysteresis" seems in place here:[4] In this paper *persistence* is said to be present if the estimated characteristic roots, corresponding to the estimated VAR, are "close" to 1 (with moduli always outside the unit circle). When the SVAR has exogenous variables, there can in general be three reasons for persistence: slow adjustment of the endogenous variables for fixed values of the exogenous variables, persistence in the exogenous variables, or a combination of the two (see below). *Hysteresis* is a *special case* of persistence which relates to slow adjustment. In order to formulate testable statistical hypotheses about persistence, among these hysteresis, exact real unit roots (i.e. cointegration in the multivariate model) are imposed in the statistical model.[5] This imposition, which is analogous to the univariate Dickey-Fuller approach, should therefore generally be viewed as a statistical *approximation*.

The SVAR considered in the empirical analysis has four endogenous variables, output prices, wages, output, and unemployment, and one exogenous variable, the price of crude

---

[3]Additional differences relative to Jacobson et al. (1997) are that their analysis is based on a *partial* VAR and a so-called Common-Trends approach. Moreover, in their model, equilibrium unemployment depends on unobserved variables, whereas here all variables are observable.

[4]Each of these terms have been used in more than one sense throughout the literature (see Arestis and Sawyer (2009), for discussions).

[5]Hence, as a statistical hypothesis, hysteresis has the property, by which this concept is commonly defined, namely that *transitory* impulses have *permanent* effects (see e.g. Røed 1997). For other concepts of hysteresis see Amable, Henry, Lordon, and Topol (1995) and Göcke (2002).

oil, influencing equilibrium unemployment via price setting. Although the example primarily serves as an illustration, the evidence seems to favor a persistent equilibrium over hysteresis as an explanation for the persistence of UK unemployment.

The remainder of the paper is organized as follows: As a motivation for a multivariate approach to analyzing unemployment persistence, Section 2 first emphasizes the systemic nature of persistence. The general Structural VAR with exogenous variables is then introduced and represented using block matrices, where the partitioning resembles the endogenous-exogenous dichotomy. This section analyzes the SVAR process as relating to two sub-processes (see Mosconi and Giannini 1992): That is, the process of the endogenous variables for fixed values of the exogenous variables, and the process of the exogenous variables, respectively. Beyond forming the basis for the subsequent analysis this analysis suggests a conceptual framework for systematically categorizing persistence (and the associated statistical hypotheses) by its source, which, by construction, is not possible with univariate methods. Using the framework from Section 2, Section 3 then considers the five-dimensional SVAR, and from this, derives and tests different hypotheses of persistence against each other based on the UK macro-data. Section 4 concludes.

2   Unemployment persistence in a multivariate context

Following Blanchard and Summers (1986), the majority of empirical analyses of unemployment hysteresis have been based on *univariate* autoregressive models, i.e. including unemployment only.[6] The focus has been on "testing for hysteresis" in the form of unit root non-stationarity in unemployment. Many different approaches have been developed, but most of them have been rooted in the well-known Dickey-Fuller type tests, with the more recent studies focusing on increasing the test power, using e.g. panel data models.[7] Typically, evidence of non-stationary unemployment rates is found for the EU countries while this is not the case for US data.[8] Evidence of non-stationarity is often being ascribed to mechanisms that imply sluggishness in wage formation such as those suggested by Blanchard and Summers (1986) and Lindbeck and Snower (1986). In contrast, when unit root tests reject in favor of stationarity (perhaps conditional on a few level breaks), this is taken as evidence of a constant equilibrium (i.e. *Natural*) rate of unemployment.

Although this literature has doubtlessly been contributing valuable insights of great im-

---

[6]See e.g. the surveys in Røed (1997), and more recently, Gustavsson and Österholm (2009) and Arestis and Sawyer (2009).

[7]See e.g. De Lee, Lee, and Chang (2009), p. 326, for a survey.

[8]See the survey in Røed's paper from 1997 and note that, there is relatively less evidence to review for the period beyond this (Ball 2009).

portance for the general debate and for policy guidance, it remains unclear to what extent univariate econometric methods can be used to draw such conclusions. In particular, whether these methods can be used to identify and compare different explanations of persistence (among these, hysteresis). Given that unemployment is determined endogenously in a complicated dynamic interaction with other macroeconomic variables (prices, wages, GDP etc.) conditional on exogenous variables, there may be many competing explanations of persistence and hence for finding non-stationarity: Fundamentally, the rate of unemployment may display persistence because it adjusts slowly (jointly with other endogenous variables) to changes in its underlying exogenous determinants. In contrast, persistence may also be due to a slowly moving exogenous determinant of equilibrium unemployment, even though actual unemployment may in fact be quick to adjust to changes in such a determinant. Digging further into slow adjustment dynamics, this generally results from a slow interaction between goods-, financial- and labor markets (see e.g. Section 3). The theories of hysteresis, Blanchard and Summers (1986) and Lindbeck and Snower (1986), have emphasized labor markets and sluggish wage formation, i.e. that wages respond slowly to unemployment, as a main reason for this. Other well-known rigidities, e.g. menu costs, sticky wages and labor hoarding, as well as costs associated with adjusting production to accommodate demand (e.g. Danziger 2008), of course also play a role in this interaction.[9]

Hence, even when adhering to mainstream macroeconomic reasoning there are many potential sources of non-stationarity, and these remain econometrically unidentified within a univariate approach. Because different sources of persistence will require different policy measures, it seems important that the adopted econometric method is in fact capable of identifying the different sources. Moreover, it should provide a framework for comparing the different explanations (hypotheses) of unemployment persistence on an equal footing, for example by nesting these in the same statistical model. For this purpose multivariate econometric models, such as a Structural VAR (SVAR), naturally suggest themselves.

The purpose is therefore now to present a general SVAR, which contains exogenous variables, and which is stated in block matrices. The SVAR forms the basis of the analysis in Section 3, which contains the actual derivations and testing of statistical hypotheses of persistence.

The point of departure is the SVAR with two lags for the full variable vector, x_t which is

---

[9]In addition, the matter is even further complicated in a multivariate context, since when a unit root is rejected, this is not necessarily inconsistent with hypotheses of hysteresis: For example, in open economies, other stabilizing mechanisms, such as real wage resistance, may be present and may dominate the destabilization caused by persistence generating mechanisms (see e.g. Carlin and Soskice 2006).

$p \times 1$.[10] This may be written in its Structural Error-Correction-Mechanism (ECM) form as,

$$A\Delta x_t = m_t + Fx_{t-1} - C\Delta x_{t-1} + \varepsilon_t, \tag{1}$$

where A is invertible and has ones on the diagonal, m_t includes deterministic terms and $\varepsilon_t \sim N(0, \Sigma)$, Σ being positive definite and diagonal.

The SVAR is based on a *theoretical model* that determines which variables are endogenous, x_{1t} $(p_1 \times 1)$, and which are exogenous, x_{2t} $(p_2 \times 1)$, where $p_1 + p_2 = p$. Hence, using the partitioning, $x_t \equiv (x_{1t}', x_{2t}')'$, (1) may be written as,

$$
\begin{aligned}
A_{11}\Delta x_{1t} &= -A_{12}\Delta x_{2t} + m_{1t} + F_{11}x_{1t-1} + F_{12}x_{2t-1} - C_{11}\Delta x_{1t-1} - C_{12}\Delta x_{2t-1} + \varepsilon_{1t}, \\
A_{22}\Delta x_{2t} &= m_{2t} + F_{22}x_{2t-1} - C_{22}\Delta x_{2t-1} + \varepsilon_{2t}.
\end{aligned}
\tag{2}
$$

See equation (18), Appendix A.1.

The theoretical model specifies the equations for x_{1t}, in particular, the parameter matrices, F_{11} and F_{12}, are usually fully specified by this. In addition to unemployment, x_{1t} could, for example, include prices, wages, output, long-term unemployment etc., whereas the exogenous variables may comprise unemployment benefits, interest rates, exchange rates, foreign prices, foreign output etc. (see e.g. Layard et al. 2005). The reduced form corresponding to (1) is,

$$\Delta x_t = \mu_t + \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \upsilon_t, \tag{3}$$

where $\mu_t \equiv A^{-1}m_t, \Pi \equiv A^{-1}F, \Gamma_1 \equiv -A^{-1}C$ and $\upsilon_t \equiv A^{-1}\varepsilon_t$. The partitioned form of this is,

$$
\begin{aligned}
\Delta x_{1t} &= \mu_{1t} + \Pi_{11}x_{1t-1} + \Pi_{12}x_{2t-1} + \Gamma_{11}\Delta x_{1t-1} + \Gamma_{12}\Delta x_{2t-1} + \upsilon_{1t}, \\
\Delta x_{2t} &= \mu_{2t} + \Pi_{22}x_{2t-1} + \Gamma_{22}\Delta x_{2t-1} + \upsilon_{2t},
\end{aligned}
\tag{4}
$$

where the parameters, as functions of the structural parameters in (2), are given in Appendix A.1.

An economic theoretical model can in most cases be thought of as a model for the endogenous variables *given* the exogenous variables, and it is assumed that this *conditional* model has a stable steady state. Although a theoretical model by construction does not explain how the exogenous variables have been generated, it is assumed here that these variables are determined in another larger system that also has a stable steady state. The result of these assumptions is that the SVAR has a stable equilibrium, and thus that *all* roots, $z \in \mathbb{C}$, of the characteristic equation corresponding to (4) or (3) have modulus, $|z| > 1$

---

[10] All results generalize straigthforwardly with more lags.

(see below). However, as mentioned, *persistence* means that some of these roots are "close" to $z = 1$.

The characteristic equation corresponding to the full process is, $\det(P(z)) = 0$, where the polynomial is defined as, $P(z) \equiv I(1 - z) - \Pi z - \Gamma_1(1 - z)z$, $z \in \mathbb{C}$. Due to the endogenous-exogenous dichotomy this splits into the product,

$$\det(P_1(z)) \det(P_2(z)) = 0, \tag{5}$$

where, $P_1(z) \equiv I_{p_1}(1 - z) - \Pi_{11}z - \Gamma_{11}(1 - z)z$ and $P_2(z) \equiv I_{p_2}(1 - z) - \Pi_{22}z - \Gamma_{22}(1 - z)z$, $z \in \mathbb{C}$. Hence, for the given structural model, *all* roots corresponding to the *full* system, and hence, *all* potential sources of unemployment persistence, can be analyzed based on the union of the roots, respectively of $\det(P_1(z)) = 0$ and $\det(P_2(z)) = 0$ (see below).

The dichotomous partitioning in (5) reflects that one can think of the full process x_t, in particular the endogenous variables, as being related to two processes: The first may be termed the *counterfactual* x_1-process (denoted x_{1t}^c henceforth). This is the process that would result if, counterfactually, *the x_2-process, were to be fixed at some level.* The second process is simply the exogenous x_2-process given by the second block line in (4).

The counterfactual process can be written as,

$$\Delta x_{1t}^c = \Pi_{11}x_{1t-1} + \Gamma_{11}\Delta x_{1t-1} + \mathcal{D}_{1t} + \upsilon_{1t}, \tag{6}$$

where $\mathcal{D}_{1t}$ is constant (see equation 4), and from which it appears that $P_1(z)$ in (5) is the corresponding characteristic polynomial (Mosconi and Giannini 1992). The other polynomial, $P_2(z)$, is the characteristic polynomial corresponding to the exogenous x_2-process.

Fundamentally, these two processes or sub-systems represent two pure sources of persistence in the system of the endogenous variables and thus in unemployment: That is, roots of $P_1(z)$ close to 1 reflect slow mutual adjustment (towards steady state) between the endogenous variables. If no other roots of (5) are close to unity, this can be referred to as the case of (pure) *slow adjustment*. In contrast, if the only roots of (5) that are close to 1, are roots of $P_2(z)$, this mirrors sluggishly evolving exogenous variables. In the context of unemployment the latter type of persistence may be denoted (pure) *equilibrium persistence*, to signify that the persistence of the endogenous variables originates from slowly evolving exogenous determinants of equilibrium unemployment.

The division into the two sub-systems thus implies that, for any economic model, as comprised by the very broad range of models nested in (2), unemployment persistence will always originate from either pure slow adjustment, pure equilibrium persistence, or a combination. Hence, this suggests a threefold categorization of unemployment persistence. However, when

formulating the corresponding statistical hypotheses about persistence, a further distinction can be made. In particular, since these hypotheses are based on *approximating* the roots that are close to unity by exact unit roots, i.e. $z = 1$, this means that the variables are assumed to be integrated, i.e. $x_t \sim \mathrm{I}(d)$, d being the order of integration, and in the multivariate case, possibly cointegrated (see Johansen 1996).[11],[12] Here, d is at most 2, since this is sufficient for the majority of macroeconomic applications (see e.g. Juselius 2006). Hence, a further distinction that can be made is with respect to the order of integration of the processes, x_{1t}^c and x_{2t}. An advantage of making this distinction is that the choice of the statistical methods depends on what approximation, say, $d = 1$ or $d = 2$, seems to be the most appropriate given the data at hand. In addition, to some extent a higher order of integration corresponds to an increasing degree of persistence.[13]

Table 1 summarizes the above analysis as an overall taxonomy or conceptual framework for classifying persistence (and the associated statistical hypotheses), by its type and "degree" (i.e. order of integration). As the table shows, in practice one can distinguish between eight *cases* of persistence, each of which corresponds to a class of testable parameter restrictions, that is, a class of restricted CVARs for x_t.

Table 1: A taxonomy for classifying statistical hypotheses of persistence (including hysteresis), according to the source of persistence (slow adjustment or equilibrium persistence). Each case represents a class of restricted CVARs corresponding to a specific type of persistence.

Exo. sub-system, x_2	Counterfactual sub-system, x_1^c		
	I(0)	I(1)	I(2)
I(0)	No persistence (stationarity)	I Pure slow adjustment, I(1)	VIII Pure slow adjustment, I(2)
I(1)	II Pure equil. pers., I(1)	III	VII
I(2)	IV Pure equil. pers., I(2)	V	VI

Given evidence of persistence, any economic model which can be written as a special case of the SVAR, (2), will offer a number of explanations for this, e.g. hysteresis, each of which fall into one of the categories in the table (see e.g. Section 3). Although general, the table should, to some extent, provide an overview of the relationship between non-stationarity on the one hand, and hypotheses of persistence as implied by the theoretical models, on the

---

[11] An example, for which this approximation is also derived explicitly form a theoretical model (as below), see Møller and Sharp (2013) or Klemp and Møller (2015).

[12] Recalling that the remaining roots obey $|z| > 1$.

[13] For example, often I(2) seems to be a useful statistical approximation for time series of nominal prices and wages whose time plots are very smooth or slowly changing.

other. As an example consider the Cases II and IV. In these cases there are no roots of $P_1(z)$ that are close to 1, but rather, it is the roots of $P_2(z)$ that are close to one, and hence approximated by $z = 1$. Moreover, conditions preclude $x_t \sim$ I(2) in Case II and $x_t \sim$ I(3) in Case II (see Møller 2013). Hence, these cases of pure equilibrium persistence may imply a non-stationary unemployment rate, but they have nothing to do with the restrictions implied by hysteresis theories, or any other hypothesis of slow adjustment. In contrast, all of the remaining cases involve slow adjustment, and in Cases I and VIII, this is the only source of persistence in unemployment.[14] In other words these classes of CVARs are compatible with the theories suggested by Blanchard and Summers (1986) and Lindbeck and Snower (1986), but, as mentioned, models with various sorts of rigidities, such as menu costs, labor hoarding, costs of adjusting production to accommodate demand (e.g. Danziger 2008) and real wage resistance etc. (Layard, Nickell, and Jackman 2005 and Carlin and Soskice 2006), also fit into this group. In contrast, an example of equilibrium persistence, whose pure form corresponds to Cases II and IV, could be a persistently evolving replacement ratio, which is often assumed to be an exogenous determinant of the natural rate of unemployment. In general, a distinguishing property between the CVAR classes corresponding to the pure cases of equilibrium persistence and slow adjustment, respectively, is that, only in the latter case, even temporary changes in the exogenous variables and shocks to the equations will have permanent influence, i.e. in practice, "long after they have disappeared". This is what economists usually mean by hysteresis (see e.g. Røed 1997, Papell, Murray, and Ghiblawi 2000 and Göcke 2002 and references therein), and is also what Jacobson et al. (1997) and Dolado and Jimeno (1997), are about. In particular, the restrictions they analyze can be related to the second column in the table.

Note that in the "pure" cases, I, II, IV and VIII, the order of integration for the full process, x_t, is simply the order of integration for the sub-process, x_{1t}^c or x_{2t}, with the highest order. This has been signified in the table. However, in the remaining (combined) cases, x_t can have a higher order of integration than those of x_{1t}^c and x_{2t}, and further conditions on the parameters need to be imposed to ensure $x_t \sim$ I(1) or $x_t \sim$ I(2), see e.g. Møller (2013). For example, $x_t \sim$ I(2) may arise in Case III, which was shown generally in Mosconi and Giannini (1992). Although complicated, such cases may be relevant for the present economic context. In particular, often exogenous determinants of equilibrium unemployment are real variables which can be approximated by I(1). At the same time nominal prices and wages can typically be described as I(2). In principle, this sort of empirical finding could be consistent

---

[14]In these remaining cases it is thus the roots of $P_1(z)$ that are approximated by $z = 1$ and parameter restrictions ensure that x_{1t}^c obeys I(1) or I(2), for the second and third column of the table respectively, etc. (see Møller 2013).

with hysteresis hypotheses (e.g. wage formation based) which imply I(1) in x_{1t}^c, as is normally the case.

As mentioned each case corresponds to a class of testable parameter restrictions, that is, a class of restricted CVARs for the full process, x_t. Using the respective assumptions about x_{1t}^c and x_{2t} that effectively define each of the cases, it is possible to derive the general block matrix representations of these CVARs based on the structural block matrices. This was done in Møller (2013) for the Cases I-III. However, as shown ibid., the block matrix algebra quickly becomes involved. Since in practice a SVAR is fully specified, a considerably simpler approach can be adopted as shown in the next section (see Section 3.3).

3   Testing persistence hypotheses in a multivariate model

Based on a fully specified SVAR, i.e. for which the block matrices are restricted by economic theory, the purpose is now to show how the above framework can be used in practice to formulate and test different hypotheses of unemployment persistence, among these wage-formation-based hysteresis hypotheses. The hypotheses derived from this model are then confronted with quarterly time series for the UK economy, for the period 1988-2006.

The SVAR builds on a small macroeconomic model containing four endogenous variables: prices, wages, output and unemployment, and one exogenous variable, the price of crude oil, which influences equilibrium unemployment via price setting (see below). The exogeneity captures that UK oil supply and demand is not big enough to influence the world price of crude oil. The overall structure of the model builds on the framework presented in the book Layard, Nickell and Jackman (2005, first ed. in 1991). This framework (henceforth *the LNJ framework*) has had a great influence on the work of applied macroeconomists and has often been regarded as a key reference representing the consensus view on European unemployment (e.g. Blanchard 2006 and Holden and Nymoen 2002). Moreover, the book provided the theoretical underpinnings of the policy recommendations in the influential OECD Jobs Study from 1994 (see Mitchell and Muysken 2008, p. 72). Hence, the LNJ framework seems to be a practically relevant point of departure to build on. Nevertheless, it should be underscored again that the analysis in this section first and foremost serves the purpose of illustrating how the framework from Section 2 can be used in practice for deriving the relevant hypotheses of unemployment persistence and hysteresis.

3.1 A SVAR for UK unemployment 1988-2006

The equations (8) through (11) below are consistent with a general equilibrium model of unemployment building on aggregate supply and demand, imperfect competition and adaptive expectations. On the supply side goods markets are characterized by monopolistic competition, and unions and firms bargain over wages in the labor markets. The demand side can be interpreted as a reduced form IS-Taylor Rule system. The short-run equilibrium is defined by the goods market equilibrium, whereas the long-run equilibrium (steady state), in addition, involves labor market equilibrium.[15]

At any given point in time, capital, technology, and expectations are given. Based on the expected aggregate levels of price and demand, monopolistically competitive firms choose a price and plan production in order to maximize expected short-run profits. The chosen price is realized, and stays fixed within the period due to positive costs associated with changing it. Given a realization of an aggregate demand shock, a level of aggregate demand will result at this price level. It is assumed that actual production accommodates this level fully. Given output, actual employment can then be determined from the production function. This determines actual unemployment since the labor force by assumption is given. Finally, unemployment and prices impact on nominal wage setting.

To elaborate, firms set prices as a markup over *expected* marginal costs. The markup is allowed to depend on activity and crude oil prices. The latter assumption is a shortcut of modeling the role of oil prices. Note that the model could be extended here, for example by letting the markup depend on foreign manufacturing prices, in addition to oil prices. It is assumed that the marginal product of labor is constant, which implies the aggregate production function,

$$y_t = a_t + n_t + \varepsilon_{yt}, \tag{7}$$

where y_t is the logarithm of output, n_t is the logarithm of employment and where the (logarithmic) technological state is described by an unsystematic component, ε_{yt}, and a deterministic labor augmenting part, a_t, evolving as $a_t = g_A t$ with $g_A > 0$. Log-linear relations are assumed throughout and these should be interpreted as log-linear approximations, made around an underlying stable and unique steady state similar to that described in footnote 15. Hence, as in (7) all variables in the following are stated in logarithms. The assumption of constant marginal returns captures that "under normal circumstances" the *average* utilization of capital is sufficiently below the capacity limit, so that for a realistic range

---

[15] For this type of model, the latter is not unique and is often referred to as a medium-run and not a long-run equilibrium. A "true" unique long-run equilibrium (steady state) would additionally entail a constant net-foreign asset-to-income ratio to ensure *external balance* (see Carlin and Soskice 1990, Layard et al. 2005, Carlin and Soskice 2006, Groth 2009). However, to keep this example simple this is disregarded here.

of variation in employment the marginal product does not diminish. Excess capacity *on average* is a reasonable assumption for many reasons. For example, it could represent an equilibrium state because firms use it to deter entrance (Fudenberg and Tirole 1983). Under this assumption the capital stock does not influence marginal costs and thus price setting. This is an advantage, both because it simplifies dynamics and since, as is well-known, capital stock data are in general of dubious quality. The aggregate pricing relation can therefore be written as,

$$
\begin{aligned}
p_t \;=\;& \rho_0 - g_A t + z_t^p + \rho_1 y_{t-1} + \rho_{21} u_{t-1} + \rho_{22} u_{t-2} + \rho_3 p_{t-1}^o \\
&+ \rho_4 p_{t-2}^o + \rho_5 w_{t-1} + \rho_6 w_{t-2} + \kappa_1 p_{t-1} + \kappa_2 p_{t-2} + \varepsilon_{pt},
\end{aligned} \tag{8}
$$

where p is aggregate output price, w denotes wages, u is unemployment and p^o, the price of crude oil (all in logs). The autoregressive terms (i.e. $\kappa_1 p_{t-1} + \kappa_2 p_{t-2}$) in this equation, and similar terms in the subsequent equations, have been added to make the dynamics of adjustment more flexible (realistic). It is assumed that $\kappa_1 + \kappa_2 \neq 1$ (similar conditions hold for the equations below), which implies that these parameters do not affect the unit roots of the model. The LNJ framework is not specific about how activity will influence the price markup and in the present analysis both output and unemployment enter. Although p^o could enter the other equations below, for example aggregate demand and wage setting, in order to keep the example simple this is not considered here. The rationale behind the lagged effect of p^o on p in relation (8) could be adaptive expectations, which is how the nominal wage rate enters. However, since the present exposition is merely a short-cut of including oil prices, there is no specific prior with respect to the coefficients ρ_3 and ρ_4, other than $\frac{\rho_3 + \rho_4}{1 - \kappa_1 + \kappa_2}$ > 0, and following Layard et al. (2005), there are no prior restrictions on the signs of ρ_1 and $\rho_{21} + \rho_{22}$. The term, $z_t^p + \varepsilon_{pt}$, captures the (combined) influence from various exogenous unmodelled variables, where ε_{pt} is an unsystematic stochastic error term, while z_t^p accounts for the more systematic changes in such variables. In the empirical implementation the latter will be approximated by some expression of deterministic components (e.g. impulse dummies, trends and level shift dummies etc.). Similar terms are added to the equations for the other observable variables below.

The relation in (8) determines p_t and can be referred to as the *short-run Price Setting relation* (PS), to be distinguished from the *long-run PS* relation obtained when expectations are correct, as implied by a long-run equilibrium. It is the latter relation that is usually referred to as the *Price Setting* relation or *feasible real wage*.

In the next step output, y_t, is determined according to,

$$y_t = \phi_1 + z_t^d + \phi_2 p_t + \phi_3 p_{t-1} + \phi_4 p_{t-2} + \kappa_3 y_{t-1} + \kappa_4 y_{t-2} + \varepsilon_{dt}, \tag{9}$$

This is the Aggregate Demand (AD) relation for which some dynamics have been added. It is a short-cut to be interpreted roughly as the reduced form that results from an IS relation and a simple Taylor Rule, the latter of which is based on trend-adjusted output and inflation relative to some target. It allows for an effect from both the price *level*, through the real exchange rate (IS), and the *inflation rate,* working through the Taylor Rule and the real interest rate. The long-run level effect is thus expected to be negative, whereas the sign of the latter inflation rate (short-run) effect is indeterminate. The term, z_t^d, may include trends and dummy variables proxying the unobserved (or unmodelled) exogenous variables that shift the Taylor Rule and the IS relation (see Layard et al. 2005).

When production is determined from (9), the production function, (7), gives the required amount of labor input, which in turn will be actual employment since firms have the right to manage. Using the definition of unemployment, assuming that the labor force is exogenous, and using (7) to eliminate n_t, it follows that,

$$u_t = \lambda_0 + z_t^u + \lambda_1 y_t + \lambda_2 y_{t-1} + \lambda_3 y_{t-2} + \kappa_5 u_{t-1} + \kappa_6 u_{t-2} + \varepsilon_{yt}, \tag{10}$$

where again some further adjustment has been added and where changes (trend-like and discrete shifts) in the size of the labor force, in aggregate capital and hours of work, as well as a_t (and shifts in a_t), are potentially captured by z_t^u. It is assumed that $\frac{\lambda_1 + \lambda_2 + \lambda_3}{1 - \kappa_5 - \kappa_6} < 0$.

Finally, the nominal wage, w_t, is determined. Wage formation is a central part of the LNJ framework and is where the hypotheses of unemployment persistence and in particular hysteresis, appear. Wages are determined according to the short-run Wage-Setting (WS) relation,

$$w_t = \omega_0 + z_t^w + \omega_1 w_{t-1} + \omega_2 w_{t-2} + \omega_3 u_t + \omega_4 u_{t-1} + \omega_5 p_{t-1} + \omega_6 p_{t-2} + \varepsilon_{wt}, \tag{11}$$

where $\omega_1 + \omega_2 \neq 1$ and where z_t^w may include changes in various "exogenous wage pressure variables" e.g. union power, and exogenous movements in the (expected) replacement ratio (Layard et al. (2005) eq. 21, p. 202). It is assumed that, $\frac{\omega_3 + \omega_4}{1 - \omega_1 + \omega_2} < 0$, for unemployment to have a stabilizing effect on the system. However, this term may be close to zero, which is exactly what hypotheses of unemployment hysteresis would imply. The LNJ framework emphasizes two dominating of such hypotheses: The first is the insider-outsider theory, claiming that when workers are fired, the remaining employed workers increase their wage

12

targets, and that due to e.g. collective agreement contracts the unemployed cannot underbid to get their jobs back (Blanchard and Summers 1986, Lindbeck and Snower 1986). The other hypothesis asserts that when total unemployment increases, so does its share of long-term unemployed, and this attenuates the downward pressure on wage inflation from higher unemployment needed to bring down unemployment again (Layard and Nickell (1987)). In the present context, both of these hypotheses imply that, $\omega_3 + \omega_4 = 0$, as an approximation.

Based on the empirical evidence below it is assumed that oil prices can be described by the following autoregressive model,

$$p_t^o = \pi_1 + z_t^o + \pi_2 p_{t-1}^o + \pi_3 p_{t-2}^o + \varepsilon_{ot}, \tag{12}$$

where z_t^o reflects the major exogenous *oil shocks* (see e.g. Blanchard and Galí 2007).

This completes the description of the structural VAR model. The next sections consider the hypotheses of unemployment persistence derived from this model.

3.2 The hypothetical sources of persistence

The SVAR defined by the equations (8) through (12) can be written as the SECM for the full process, i.e. (1), with the matrices defined in Appendix A.2. From these matrices the partitioned matrices, $A_{11}, A_{12}, , ...C_{22}$, (see (2)) may be read off immediately and the reduced form parameters of (4) can be computed, using the formulae in (19). The partitioned reduced form parameters can then be used to compute the characteristic polynomials corresponding to the counterfactual process and the exogenous x_2-process, i.e. $P_1(z)$ and $P_2(z)$, respectively (see equation 5).

To analyze the different cases of persistence that this macroeconomic model generates, consider the dichotomous partitioning corresponding to (5). As discussed above, persistence implies characteristic roots, z, close to 1, which may be tested as the parameter restrictions that imply exact unit roots, i.e. as $\det(P(1)) = 0$ or $\det(P_1(1))\det(P_2(1)) = 0$. Calculating this parameter restriction based on the partitioned matrices as implied by the matrices in Appendix A.2 leads to a cumbersome expression. However, this may be stated in a simpler way by introducing a notation based on accumulated dynamic multipliers. The notation is found in Table 5 in Appendix A.2, where $M_{x,y}$ denotes the partial accumulated (multiplier) effect on variable x from variable y. Based on this, the unit root restriction, $\det(P_1(1))\det(P_2(1)) = 0$, for this model becomes,

$$\underbrace{\left([M_{p,y} + (M_{p,u} + M_{p,w}M_{w,u})M_{u,y}]M_{y,p} + M_{p,w}M_{w,p} - 1\right)}_{\det(P_1(1))}\underbrace{\left(\pi_2 + \pi_3 - 1\right)}_{\det(P_2(1))} = 0. \tag{13}$$

13

What may first be noted is that, in contrast to its univariate counterpart, $\rho - 1 = 0$, where ρ is the sum of the autoregressive parameters in an AR model, this equation is a complicated expression depending on many parameters. Here, this reflects the fact that in a dynamic equation system, slow adjustment may come from many sources. In particular, note that the hypotheses of unemployment hysteresis concern the parameter, $M_{w,u}$, which constitute only a small part of this overall restriction, i.e. $\det(P_1(1)) = 0$. Equation (13) also shows how slow adjustment in the system is related to nominal wage rigidity (sticky wages), $M_{w,p} = 0$, rigidity in prices, for example, $M_{p,w} = 0$, excessive labor hoarding effects, i.e. that unemployment is very slow to adjust to output ($M_{u,y} = 0$). Hence, even within the framework of the simple structural model, (8) through (12), there are several sources of persistence. The picture is already relatively complicated and in particular hysteresis is in general neither sufficient nor necessary for non-stationarity.

However, given price-wage homogeneity, which is assumed in most applications, the expression does have a more clear interpretation. Homogeneity, is defined by the restrictions $M_{p,w} = 1$ and $M_{w,p} = 1$, and these imply that, the first term in (13), $\det(P_1(1))$, reduces to $[M_{p,y} + (M_{p,u} + M_{p,w}M_{w,u})M_{u,y}]M_{y,p}$. This is a price-quantity interaction term consisting of a product of two terms: $M_{y,p}$ the accumulated multiplier effect from prices on output, and $[M_{p,y} + (M_{p,u} + M_{p,w}M_{w,u})M_{u,y}]$ which is the total effect of output on prices consisting of the sum of the direct effect, $M_{p,y}$, and the indirect effect which propagates through the system, starting by affecting unemployment ($M_{u,y}$) which, in turn, affects prices directly ($M_{p,u}$) and indirectly effect via wages ($M_{p,w}M_{w,u}$). Under homogeneity it is thus reasonably clear why slow adjustment may occur: Either prices have no effect on output and/or vice versa. This expression also shows that even under homogeneity the hysteresis hypotheses, implying $M_{w,u} = 0$, are in general *not* sufficient for a non-stationary unemployment rate. It appears that what is needed for this to be the case is "a flat PS curve", i.e. that price setting is independent of activity ($M_{p,y} = -M_{p,u}M_{u,y}$).

For this SVAR, *equilibrium persistence* results when $\pi_2 + \pi_3 - 1 = 0$. When this is the case oil prices are I(1) and unemployment will inherit this I(1) stochastic trend (non-stationarity) via equilibrium unemployment. The interpretation is that non-stationary oil prices will make the (trend adjusted) PS curve shift up and down (in unemployment real wage space) in a non-stationary (i.e. I(1)) manner, which will translate into non-stationarity of equilibrium unemployment. How strongly this non-stationarity will transmit into unemployment also depend on the slope of the WS relation, as well as on the extent of slow adjustment. This is fully explicated below.

14

3.3 Testing the statistical hypotheses of persistence against the UK data

Based on the model above and the quarterly UK data, the purpose is now to derive and test some of the implied persistence hypotheses, i.e. restricted CVARs, against each other. The focus will be on testing whether the persistence of UK unemployment for this period, is consistent with *slow adjustment*, or whether this has been due to a slowly changing equilibrium rate of unemployment (*equilibrium persistence*). Moreover, it will also be assessed whether there is any support to slow adjustment of the form based on sluggish wage formation, as emphasized by hysteresis models.

In the empirical analysis below, the evidence suggested that oil prices can be assumed to be both exogenous and I(1). In particular, this series can be described as an independent random walk. Moreover, this I(1) stochastic trend seems sufficient as a description of the non-stationarity. I.e. there are no signs of explosive roots (i.e. with $|z| < 1$), and it is furthermore reasonable to disregard I(2). Hence, in the following, the focus will be on testing a special case of Case II versus special cases of Case III, for which the I(1) condition for x_t is fulfilled.[16]

The econometric analysis is conducted in the following way: A given case or hypothesis of persistence is first formulated as a parameter restriction, which imply that, $\det(P_1(1)) = 0$ and/or $\det(P_2(1)) = 0$, depending on the considered case (see below). From (5) this implies that $\det(P(1)) = 0$, and hence that, $\det(\Pi) = 0$, so that Π in (3) has reduced rank, $r < p$. As shown in Johansen (1996), *in general* when Π has reduced rank, r, this can be parameterized as,

$$\Pi = \alpha\beta', \tag{14}$$

where α is the adjustment matrix and β the cointegrating matrix both $p \times r$ of rank r, but otherwise unrestricted. Therefore, a given hypothesis of persistence will imply *constrained* values of r, α and β, which may be denoted, r_c, α_c and β_c. To compute α_c and β_c one can first compute Π_c, which is Π under the restriction $\det(P(1)) = 0$. Then the value of r_c follows and one can compute α_c as a column basis of Π_c. From this one can define $\overline{\alpha}_c \equiv \alpha_c(\alpha'_c\alpha_c)^{-1}$, which fulfill $\overline{\alpha}'_c\alpha_c = I_{r_c}$, (see Johansen 1996, p. 49). Pre-multiplying $\overline{\alpha}'_c$ on both sides of the restricted version of (14) implies that β_c can be found as, $\beta_c = \Pi'_c\overline{\alpha}_c$.[17] Subsequently, one may change the normalization of α_c and β_c.[18]

In order to test the statistical hypothesis of persistence, one starts with the inference

---

[16] This particular condition is analyzed in detail in Møller (2013).

[17] Maple was used to conduct these computations.

[18] See Johansen (1996), on normalization in general. Here, the normalization of the cointegrating matrix has simply been chosen for computational ease.

on r_c and if $r = r_c$ seems empirically supported one can proceed to testing α_c and β_c. In particular, one first identifies the I(1) model, $H(r_c)$, which is defined by (3) with (14) imposed with α and β both $p \times r_c$ but otherwise freely varying elements. $H(r_c)$ is a sub-model of the Unrestricted VAR (UVAR), i.e. (3) with no restrictions on the parameters, and the persistence restriction, $\alpha = \alpha_c$ and $\beta = \beta_c$, defines, in turn, a sub-model of $H(r_c)$. Therefore, $H(r_c)$ is tested against the UVAR, and if not rejected, the restrictions, $\alpha = \alpha_c$ and $\beta = \beta_c$, are tested in $H(r_c)$ subsequently.

To maintain statistically valid inference on r_c or $H(r_c)$ and eventually, α_c and β_c, a statistical adequate UVAR was therefore first specified based on residual-based misspecification tests. Then the Johansen trace test and other relevant aspects of the estimated model, e.g. the estimated modulus of the characteristic roots, the graphs of the cointegrating relations and the significance of individual coefficients in $\widehat{\alpha}$, were used to determine the value r.[19] As usual there was some uncertainty related to that but it seems safe to claim that a cointegration rank of $r = 3$ or $r = 4$ was clearly supported by the evidence. These are the values consistent with the hypotheses of persistence (see below).

The tables in the following report the likelihood ratio test statistic (and its p-value) corresponding to the restriction, $\alpha = \alpha_c$ and $\beta = \beta_c$, in $H(r_c)$, for the respective cases. The estimates of α_c and β_c are also given, and in light of these the identification of the structural parameters is considered. Note however that, the purpose of the present analysis is *not* to identify specific structural parameters, as such, but rather to identify the joint persistence restrictions. Finally, note also that, insignificant coefficients in $\widehat{\alpha}_c$ and $\widehat{\beta}_c$ were removed subsequently. All estimation is based on PcGive, OxMetrics 6.10 and CATS in RATS (see respectively, Doornik and Hendry 1998, Doornik 2010 and Dennis, Hansen, and Juselius 2006). The data are described in Table 6 in Appendix A.2.

3.3.1 Testing for slow adjustment of a general form

Table 2 reports the p-value and the estimates, $\widehat{\alpha}_c$ and $\widehat{\beta}'_c$ corresponding to the *general* restriction of slow adjustment, i.e. $\det(P_1(1)) = 0$ in (13). Note that, $\widehat{\beta}$ has been augmented with the trend term.[20] Also, note that price-wage homogeneity is not imposed, since this is not necessary for persistence to occur. Clearly, homogeneity could be imposed in a given application, but this is not likely to change the conclusions. This hypothesis is a special case of Case III. The restrictions are strongly rejected with a p-value of less than 0.1%.

---

[19]See the discussion in Juselius (2006).

[20]This is standard practice in the reduced rank regression method (see Johansen 1996).

Table 2: Testing slow adjustment of the general from, $\det(P_1(1)) = 0$, in equation 13 (a special case of Case III): The likelihood ratio test, with p-value and estimates of α_c and β_c, of the corresponding CVAR against an unrestricted CVAR with $r = 3$.

Test: $\chi^2(10) = 30.645$, p-value; 0.0007

	$\widehat{\alpha}_c$				$\widehat{\beta}'_c$					
						p_t	y_t	u_t	w_t	p^o_t	$Trend$
Δp_t	0.198 [3.536]	1.282 [5.518]	0.083 [5.134]	$\widehat{\beta}'_{c,1}$	1	0	0	2.886 [12.539]	0	−0.035 [−14.484]
Δy_t	−0.265 [−6.302]	−1.344 [−7.682]	−0.079 [−6.473]	$\widehat{\beta}'_{c,2}$	0	1	0	−0.789 [−35.124]	0	0
Δu_t	0.307 [3.212]	0	−0.065 [−2.047]	$\widehat{\beta}'_{c,3}$	0	0	1	6.44 [7.643]	0	−0.052 [−5.849]
Δw_t	0	0	−0.012 [−6.561]							
Δp^o_t	0	0	0							

Note: The brackets contain t-ratios.

The coefficients, $\widehat{\alpha}_{c,3,2}, \widehat{\alpha}_{c,4,1}, \widehat{\alpha}_{c,4,2}$ and trend coefficient in $\widehat{\beta}'_{c,2}$ were insignificant and thus removed

Under this restriction, the cointegrating coefficients on wages in $\beta'_{c,1}$, $\beta'_{c,2}$ and $\beta'_{c,3}$ are given as, $\frac{M_{p,w}}{(M_{u,y}M_{p,u}+M_{p,y})M_{y,p}-1}, \frac{M_{p,w}M_{y,p}}{(M_{u,y}M_{p,u}+M_{p,y})M_{y,p}-1}$, and $\frac{M_{p,w}M_{y,p}M_{u,y}}{(M_{u,y}M_{p,u}+M_{p,y})M_{y,p}-1}$, respectively. Note how the multipliers may be identified as ratios between these coefficients. Note also that, although the overall test rejects, the signs of the estimated α_c and β_c coefficients are in line with Table 5, in the Appendix A.2.

3.3.2 Testing for slow adjustment based on the hysteresis hypotheses

Next, consider the special case of Case III, that imposes $M_{w,u} = 0$, i.e. the main restriction consistent with the hysteresis theories à la Blanchard and Summers (1986), Lindbeck and Snower (1986) and Layard and Nickell (1987). As discussed above, for this restriction to imply a unit root in unemployment in this model, it is necessary to impose, in addition, homogeneity ($M_{p,w} = M_{w,p} = 1$) and a flat price setting relation ($M_{p,y} = -M_{p,u}M_{u,y}$). It seems plausible for the latter restriction to hold as a result of $M_{p,y} = 0$ and $M_{p,u} = 0$, which is thus imposed. Table 3 reports the estimation results under this restriction.

Table 3: Testing slow adjustment á la hysteresis theories (here, a special case of Case III): The likelihood ratio test, with p-value and estimates of α_c and β_c, of the corresponding CVAR against an unrestricted CVAR with $r = 3$.

Test: $\chi^2(15) = 72.017$, p-value; < 0.0001										
	$\widehat{\alpha}_c$				$\widehat{\beta}'_c$					
					p_t	y_t	u_t	w_t	p^o_t	$Trend$
Δp_t	−0.178 [−6.329]	0	0	$\widehat{\beta}'_{c,1}$	1	0	0	0.355 [2.269]	0	−0.009 [−5.247]
Δy_t	0	−0.185 [−4.130]	0	$\widehat{\beta}'_{c,2}$	0	1	0	0	0	−0.008 [−58.430]
Δu_t	0	0	−0.072 [−4.098]	$\widehat{\beta}'_{c,3}$	0	0	1	−11.513 [−4.380]	0	0.132 [4.653]
Δw_t	0	0.158 [5.390]	0							
Δp^o_t	0	0	0							

Note: The brackets contain t-ratios.
The coefficients, $\widehat{\alpha}_{c,2,1}, \widehat{\alpha}_{c,3,1}, \widehat{\alpha}_{c,3,2}, \widehat{\alpha}_{c,4,1}, \widehat{\alpha}_{c,4,3}$, and the wage coefficient in $\widehat{\beta}'_{c,2}$ were insignificant and thus removed.

Since this hypothesis imposes more restrictions on the parameter space than the more general one in Table 2, it is not surprising that it is also strongly rejected.

3.3.3 Testing for a persistent equilibrium

Given the lack of support to slow adjustment, and in particular, hysteresis hypotheses, let us now turn to testing whether, alternatively, the unit root evidence can be explained as a result of pure equilibrium persistence, cf. Case II. The results in Table 4 suggest that this is the case. The restriction defining this case, i.e. $\pi_2 + \pi_3 - 1 = 0$, while $M_{y,p}(M_{u,y}\rho_2 + \rho_1) \neq 0$, and the additional zeros imposed because of insignificance, were accepted with a p-value as high as 0.92.

Table 4: Testing for equilibrium persistence (a special case of Case II): The likelihood ratio test, with p-value and estimates of α_c and β_c, of the corresponding CVAR against an unrestricted CVAR with $r = 4$.

Test: $\chi^2(9) = 3.8266$, p-value; 0.9224											
	$\widehat{\alpha}_c$					$\widehat{\beta}'_c$					
						p_t	y_t	u_t	w_t	p_t^o	$Trend$
Δp_t	0	1.120 [6.167]	0.078 [5.408]	0.204 [2.367]	$\widehat{\beta}'_{c,1}$	1	0	0	0	−0.053 [−1.873]	−0.005 [−8.361]
Δy_t	−0.160 [−3.575]	−1.267 [−8.373]	−0.077 [−6.870]	−0.275 [−4.080]	$\widehat{\beta}'_{c,2}$	0	1	0	0	0.058 [3.747]	−0.009 [−30.384]
Δu_t	0	0	−0.090 [−4.673]	1.113 [5.679]	$\widehat{\beta}'_{c,3}$	0	0	1	0	−0.862 [−4.712]	0.027 [8.010]
Δw_t	0.097 [2.835]	0	−0.016 [−4.777]	−0.140 [−3.969]	$\widehat{\beta}'_{c,4}$	0	0	0	1	0	−0.010 [−61.485]
Δp_t^o	0	0	0	0							

Note: The brackets contain t-ratios.

The coefficients, $\widehat{\alpha}_{c,1,1}, \widehat{\alpha}_{c,3,1}, \widehat{\alpha}_{c,3,2}, \widehat{\alpha}_{c,4,2}$ and the oil price coefficient in $\widehat{\beta}'_{c,4}$ were insignificant and thus removed.

Under these restrictions, β_c and, to some extent, α_c, have relatively straightforward interpretations, and identify some of the key parameters. In particular,

$$\alpha_c = \begin{pmatrix} 0 & \rho_1 & \rho_2 & \rho_5 + \rho_6 \\ * & * & * & * \\ 0 & 0 & * & * \\ * & 0 & * & * \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \beta'_c = \begin{pmatrix} 1 & 0 & 0 & 0 & \frac{\rho_3 + \rho_4}{M_{y,p}(M_{u,y}\rho_2 + \rho_1)} \\ 0 & 1 & 0 & 0 & \frac{\rho_3 + \rho_4}{M_{u,y}\rho_2 + \rho_1} \\ 0 & 0 & 1 & 0 & \frac{M_{u,y}(\rho_3 + \rho_4)}{M_{u,y}\rho_2 + \rho_1} \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (15)$$

where $*$ denotes a complicated short-run adjustment coefficient and $\rho_2 \equiv \rho_{21} + \rho_{22}$. By computing the Structural Moving Average (SMA) representation of the full process, x_t, one will find that the coefficients in β'_c identify the *long-run effects* from crude oil prices on the respective variable corresponding to that row of β_c (see Møller 2013).[21] That is, in the first row of β'_c, $\frac{\rho_3 + \rho_4}{M_{y,p}(M_{u,y}\rho_2 + \rho_1)}$ is (minus) the long-run effect of a long-run change of one unit in crude oil (log) prices on the domestic price level etc.[22] As mentioned, there are no prior with respect to signs of ρ_1 and ρ_2, but the estimates from Table 4 suggest that both are positive. The estimate of $\frac{\rho_3 + \rho_4}{M_{u,y}\rho_2 + \rho_1}$ in the second row of $\widehat{\beta}'_c$ is positive, which implies that the remaining estimates in $\widehat{\beta}'_c$ are consistent with the expected signs, i.e. $M_{y,p} < 0$ and $M_{u,y} < 0$. The third row of $\widehat{\beta}'_c$, thus shows the long-run effect on unemployment. In this

---

[21] This is generally not the case (see Johansen 2005 and Lütkepohl 2005).

[22] The zero restrictions, which were exclusively motivated by insignificance, contributed to simplifying the estimated β_c matrix considerably.

case it is illuminating to compute the equation in the SMA representation corresponding to unemployment, which becomes,

$$u_t = \mathcal{U}_t - \frac{M_{u,y}(\rho_3 + \rho_4)}{M_{u,y}\rho_2 + \rho_1} \frac{1}{1 + \pi_3} \sum_{i=1}^{t} \varepsilon_{oi} \tag{16}$$

where $1 + \pi_3$, is positive which follows from the fact that all roots except unit roots are strictly outside the complex unit disc. This is empirically supported. The term, $\mathcal{U}_t$ comprises deterministic components and initial values and includes the *transitory* influence from the stochastic ε-shocks. Hence, in relation to persistence this term is not so interesting. Rather it is the second term which shows that the stochastic trend in unemployment (i.e. the approximated persistence) comes from the stochastic trend in oil prices, $\sum_{i=1}^{t} \varepsilon_{oi}$. The equation implies that oil prices influence unemployment permanently via influencing the Price Setting relation and thus equilibrium unemployment. To be precise, consider an isolated positive shock to crude oil prices of magnitude $1 + \pi_3$, i.e. simply normalized to produce a long-run unit change in the oil prices (that is the term $(1 + \pi_3)^{-1} \sum_{i=1}^{t} \varepsilon_{oi}$ changes by one unit). This will have a positive long-run effect on prices, of magnitude, $-\frac{\rho_3 + \rho_4}{M_{y,p}(M_{u,y}\rho_2 + \rho_1)} > 0$, which will then lower output by, $-M_{y,p}\left(-\frac{\rho_3 + \rho_4}{M_{y,p}(M_{u,y}\rho_2 + \rho_1)}\right) = \frac{\rho_3 + \rho_4}{(M_{u,y}\rho_2 + \rho_1)}$, which eventually will raise unemployment, by $-M_{u,y}\frac{\rho_3 + \rho_4}{(M_{u,y}\rho_2 + \rho_1)} = -\frac{M_{u,y}(\rho_3 + \rho_4)}{(M_{u,y}\rho_2 + \rho_1)}$, as identified in the third row of $\widehat{\beta}'_c$.

It is essential to note that, here the hypothetical change in oil prices is permanent (i.e. the long-run unit change). If, on the other hand, this were temporary the effect on unemployment would die out relatively fast. As mentioned, this is in contrast to hysteresis, for which *transitory* influences do have *permanent* effects.

4   Concluding remarks

The literature on unemployment persistence has focussed mostly on detecting the empirical existence of persistence as measured by the non-stationarity of unemployment. Univariate autoregressions, i.e. including unemployment only, and the related Dickey-Fuller type testing have proven useful in this context and hence widely applied. However, much less research has been directed towards the question why (as opposed whether) persistence occurs. In particular, whether this is the result of hysteresis, say caused by sluggish labor market dynamics, other kinds of slow adjustment, or in fundamental contrast, generated by slowly moving equilibrium unemployment. Such distinctions are of utmost importance since

different sources of persistence have different policy implications.[23]

In light of this, the present research has suggested a multivariate econometric framework for identifying and comparing different sources of unemployment persistence, and shown how this may be applied in practice. The basic premise for the analysis is that unemployment persistence is essentially a systemic outcome and should therefore be analyzed by multivariate methods. The framework is based on a general Structural VAR model which contains exogenous variables and which is represented in block matrices, thereby applying to a wide range of generic economic models. The endogenous-exogenous dichotomy implies that the SVAR process can be seen as composed by two sub-processes: That is, the process of the endogenous variables given the exogenous variables and the process of the exogenous variables, respectively. This in turn naturally suggests a conceptual framework for systematically categorizing persistence (and the associated statistical hypotheses) by its source, which, by construction, is not possible with univariate methods.

In order to demonstrate how this methodology can be applied in practice, a *fully specified* SVAR, i.e. with theoretical restrictions on the block matrices, is used to derive and test different statistical hypotheses of unemployment persistence, in the form of different cointegration restrictions. The SVAR includes four endogenous variables (prices, wages unemployment and output) and one exogenous variable (oil prices), which influences equilibrium unemployment via price setting. The cointegration restrictions are confronted with UK macroeconomic quarterly time series. Although this application is primarily meant as an illustration of the method, the evidence at face value seems to suggest that for the period under study, the persistence of UK unemployment was *not* the result of slow adjustment. In particular, it was not the result of sluggish wage formation as is often emphasized by the hysteresis theories. Instead, these data support the hypothesis that equilibrium unemployment has evolved persistently, in this case as a consequence of persistently changing oil prices shifting the Price Setting curve (in unemployment-real wage space). However, in order to draw more definitive conclusions, further robustness checks, other samples, other models, or extensions of the present model could also be considered.

---

[23]See for example Røed (1997) and more recently, Andersen (2010) and Delong and Summers (2012).

A Appendices

A.1 The partitioned Structural ECM and its reduced ECM form

The point of departure is the SECM for the full process, $x_t \equiv (x'_{1t}, x'_{2t})'$, i.e.,

$$A\Delta x_t = m_t + Fx_{t-1} - C\Delta x_{t-1} + \varepsilon_t. \tag{17}$$

This has the following block representation,

$$
\begin{pmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{pmatrix}
\begin{pmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{pmatrix}
=
\begin{pmatrix} m_{1t} \\ m_{2t} \end{pmatrix}
+
\begin{pmatrix} F_{11} & F_{12} \\ 0 & F_{22} \end{pmatrix}
\begin{pmatrix} x_{1t-1} \\ x_{2t-1} \end{pmatrix}
$$
$$
-
\begin{pmatrix} C_{11} & C_{12} \\ 0 & C_{22} \end{pmatrix}
\begin{pmatrix} \Delta x_{1t-1} \\ \Delta x_{2t-1} \end{pmatrix}
+
\begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix},
\tag{18}
$$

corresponding to (2).

The corresponding reduced form ECM becomes,

$$
\begin{pmatrix} \Delta x_{1t} \\ \Delta x_{2t} \end{pmatrix}
=
\begin{pmatrix} \mu_{1t} \\ \mu_{2t} \end{pmatrix}
+
\begin{pmatrix} \Pi_{11} & \Pi_{12} \\ 0 & \Pi_{22} \end{pmatrix}
\begin{pmatrix} x_{1t-1} \\ x_{2t-1} \end{pmatrix}
$$
$$
+
\begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ 0 & \Gamma_{22} \end{pmatrix}
\begin{pmatrix} \Delta x_{1t-1} \\ \Delta x_{2t-1} \end{pmatrix}
+
\begin{pmatrix} \upsilon_{1t} \\ \upsilon_{2t} \end{pmatrix},
\tag{19}
$$

with,

$$
\mu_t \equiv \begin{pmatrix} \mu_{1t} \\ \mu_{2t} \end{pmatrix}
= \begin{pmatrix} A_{11}^{-1}\left(m_1 - A_{12}A_{22}^{-1}m_{2t}\right) \\ A_{22}^{-1}m_{2t} \end{pmatrix},
$$

$$
\Pi \equiv \begin{pmatrix} \Pi_{11} & \Pi_{12} \\ 0 & \Pi_{22} \end{pmatrix}
= \begin{pmatrix} A_{11}^{-1}F_{11} & A_{11}^{-1}\left(F_{12} - A_{12}A_{22}^{-1}F_{22}\right) \\ 0 & A_{22}^{-1}F_{22} \end{pmatrix},
$$

$$
\Gamma_1 \equiv \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ 0 & \Gamma_{22} \end{pmatrix}
= \begin{pmatrix} -A_{11}^{-1}C_{11} & -A_{11}^{-1}\left(C_{12} - A_{12}A_{22}^{-1}C_{22}\right) \\ 0 & -A_{22}^{-1}C_{22} \end{pmatrix},
$$

and, Γ which is defined as $I - \Gamma_1$ is thus,

$$
\Gamma \equiv \begin{pmatrix} I - \Gamma_{11} & -\Gamma_{12} \\ 0 & I - \Gamma_{22} \end{pmatrix}
= \begin{pmatrix} I + A_{11}^{-1}C_{11} & A_{11}^{-1}\left(C_{12} - A_{12}A_{22}^{-1}C_{22}\right) \\ 0 & I + A_{22}^{-1}C_{22} \end{pmatrix}.
$$

A.2 The UK example

The matrices

$$F = \begin{pmatrix} \kappa_1 + \kappa_2 - 1 & \rho_1 & \rho_{21} + \rho_{22} & \rho_5 + \rho_6 & \rho_3 + \rho_4 \\ \phi_2 + \phi_3 + \phi_4 & \kappa_3 + \kappa_4 - 1 & 0 & 0 & 0 \\ 0 & \lambda_1 + \lambda_2 + \lambda_3 & \kappa_5 + \kappa_6 - 1 & 0 & 0 \\ \omega_5 + \omega_6 & 0 & \omega_3 + \omega_4 & \omega_1 + \omega_2 - 1 & 0 \\ 0 & 0 & 0 & 0 & \pi_2 + \pi_3 - 1 \end{pmatrix},$$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -\phi_2 & 1 & 0 & 0 & 0 \\ 0 & -\lambda_1 & 1 & 0 & 0 \\ 0 & 0 & -\omega_3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, C = \begin{pmatrix} \kappa_2 & 0 & \rho_{22} & \rho_6 & \rho_4 \\ \phi_4 & \kappa_4 & 0 & 0 & 0 \\ 0 & \lambda_3 & \kappa_6 & 0 & 0 \\ \omega_6 & 0 & 0 & \omega_2 & 0 \\ 0 & 0 & 0 & 0 & \pi_3 \end{pmatrix}, m_t = \begin{pmatrix} \rho_0 - g_A t + z_t^p \\ \phi_1 + z_t^d \\ \lambda_0 + z_t^u \\ \omega_0 + z_t^w \\ \pi_1 + z_t^o \end{pmatrix}.$$

Accumulated multiplier notation

Table 5: Accummulated multipliers corresponding to the UK example.

Definition of multiplier	Expected sign:
$M_{p,y} \equiv \frac{\rho_1}{1-\kappa_1-\kappa_2}$	?
$M_{p,u} \equiv \frac{\rho_{21}+\rho_{22}}{1-\kappa_1-\kappa_2}$	?
$M_{p,p^o} \equiv \frac{\rho_3+\rho_4}{1-\kappa_1-\kappa_2}$	> 0 ($= 1$ under homogeneity)
$M_{p,w} \equiv \frac{\rho_5+\rho_6}{1-\kappa_1-\kappa_2}$	> 0 ($= 1$ under homogeneity)
$M_{y,p} \equiv \frac{\phi_2+\phi_3+\phi_4}{1-\kappa_3-\kappa_4}$	< 0
$M_{u,y} \equiv \frac{\lambda_1+\lambda_2+\lambda_3}{1-\kappa_5-\kappa_6}$	< 0
$M_{w,p} \equiv \frac{\omega_5+\omega_6}{1-\omega_1-\omega_2}$	> 0 ($= 1$ under homogeneity)
$M_{w,u} \equiv \frac{\omega_3+\omega_4}{1-\omega_1-\omega_2}$	< 0

Description of the data

Table 6: Descrption of the UK data.

Name:	Details:	Source:
p	log. of GDP deflator	Office for National Statistics, UK (Ecowin)
y	log. of real GDP	Office for National Statistics, UK (Ecowin)
u	log. of unemployment rate	IMF - International Financial Statistics
w	log. of hourly wage rate	IMF - International Financial Statistics
p^o	log. of crude oil prices*	OECD

*Dubia Spot. Annual series were used to construct quarterly series (by simple interpolation).
This is likely to be of minor importance for the obtained results since these concern the
low-frequency co-movements (stochastic trend movements) in the data,
and not the short-term noise.

References

Amable, B., J. Henry, F. Lordon, and R. Topol (1995). Hysteresis revisited. In R. Cross (Ed.), *The Natural Rate of Unemployment: Reflections on 25 Years of the Hypothesis*, Chapter 9. Cambridge University Press.

Amable, B. and K. Mayhew (Summer 2011). Unemployment in the oecd. *Oxford Review of Economic Policy 27*(2), 207–220.

Andersen, T. (2010, 04). Unemployment persistence. *CESifo Forum 11*(1), 23–28.

Arestis, P. and M. Sawyer (Eds.) (2009). *Path Dependency and Macroeconomics*. Palgrave Macmillan.

Ball, L. M. (2009). Hysteresis in unemployment: Old and new evidence. NBER Working Papers 14818, National Bureau of Economic Research, Inc.

Blanchard, O. (2006, 01). European unemployment: the evolution of facts and ideas. *Economic Policy 21*(45), 5–59.

Blanchard, O. J. and J. Galí (2007, September). The macroeconomic effects of oil price shocks: Why are the 2000s so different from the 1970s? In *International Dimensions of Monetary Policy*, NBER Chapters, pp. 373–421. National Bureau of Economic Research, Inc.

Blanchard, O. J. and L. H. Summers (1986). Hysteresis and the european unemployment problem. *NBER Macroeconomics Annual 1*, 15–78.

Carlin, W. and D. Soskice (1990). *Macroeconomics and the Wage Bargain: A Modern Approach to Employment, Inflation, and the Exchange Rate*. Oxford University Press.

Carlin, W. and D. Soskice (2006). *Macroeconomics: Imperfections, Institutions and Policies*. Oxford University Press.

Danziger, D. (2008, 09). Adjustment costs, inventories and output. *Scandinavian Journal of Economics 110*(3), 519–542.

Davidson, J. E. H. and S. G. Hall (1991). Cointegration in recursive systems. *Economic Journal 101*(405), 239–51.

De Lee, J., C.-C. Lee, and C. P. Chang (2009). Hysteresis in unemployment revisited: Evidence from panel lm unit root tests with heterogeneous structural breaks. *Bulletin of Economic Research 61*(4), 325–334.

Delong, J. B. and L. H. Summers (2012). Fiscal policy in a depressed economy. *Brookings Papers on Economic Activity*, 233–274.

Dennis, J. G., H. Hansen, and K. Juselius (2006). *CATS in RATS. Cointegration analysis of time series, Version 2*. Evanston, Illinois, USA: Estima.

Dickey, D. A. and W. Fuller (1981). The likelihood ratio statistics for autoregressive time series with unit root. *Econometrica 49*(4), 1057–1072.

Dolado, J. J. and J. F. Jimeno (1997). The causes of spanish unemployment: A structural var approach. *European Economic Review 41*(7), 1281–1307.

Doornik, J. A. (2010). *OxMetrics version 6.10*.

Doornik, J. A. and D. F. Hendry (1998). *Givewin. An Interface to Empirical Modelling*. London: Timberlake Consultants Press.

Fudenberg, D. and J. Tirole (1983, December). Capital as a commitment: Strategic investment to deter mobility. *Journal of Economic Theory 31*(2), 227–250.

Groth, C. (2009). Macroeconomic analysis. mimeo, Department of Economics, University of Copenhagen.

Gustavsson, M. and P. Österholm (2009). The presence of unemployment hysteresis in the oecd: what can we learn from out-of-sample forecasts? *Empirical Economics*.

Göcke, M. (2002). Various concepts of hysteresis applied in economics. *Journal of Economic Surveys 16*(2), 167–188.

Hansen, H. and A. Warne (2001). The cause of Danish unemployment: Demand or supply shocks? *Empirical Economics 26*(3), 461–486.

Holden, S. and R. Nymoen (2002). Measuring structural unemployment: NAWRU estimates in the nordic countries. *Scandinavian Journal of Economics 104*(1), 87–104.

Jacobson, T., A. Vredin, and A. Warne (1997). Common trends and hysteresis in scandinavian unemployment. *European Economic Review 41*(9), 1781 – 1816.

Johansen, S. (1996). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Advanced Texts in Econometrics, Oxford University Press.

Johansen, S. (2005). Interpretation of Cointegrating Coefficients in the Cointegrated Vector Autoregressive Model. *Oxford Bulletin of Economics and Statistics 67*(1), 93–104.

Juselius, K. (2006). *The cointegrated VAR model: Econometric methodology and macroeconomics applications.* Oxford University Press.

Klemp, M. and N. F. Møller (2015). Post-malthusian dynamics in pre-industrial scandinavia. *The Scandinavian Journal of Economics*.

Layard, R. and S. Nickell (1987). The labour market. In R. Dornbusch and R. Layard (Eds.), *The performance of the British economy.* Clarendon.

Layard, R., S. Nickell, and R. Jackman (2005). *Unemployment. Macroeconomic Performance and the labour market* (2nd ed.). Oxford University Press.

Lindbeck, A. and D. J. Snower (1986, May). Wage setting, unemployment, and insider-outsider relations. *American Economic Review 76*(2), 235–39.

Lütkepohl, H. (2005). *New Introduction to Multiple Time Series Analysis.* Springer.

Mitchell, W. and J. Muysken (2008). *Full employment abandoned: shifting sands and policy failures.* Edward Elgar Publishing.

Mosconi, R. and C. Giannini (1992, August). Non-causality in cointegrated systems: Representation estimation and testing. *Oxford Bulletin of Economics and Statistics 54*(3), 399–417.

Møller, N. F. (2013, August). Understanding unemployment hysteresis: A system-based econometric approach to changing equilibria and slow adjustment. Discussion Papers 13-06, University of Copenhagen. Department of Economics.

Møller, N. F. and P. Sharp (2013). Malthus in Cointegration Space: Evidence of a Post-Malthusian pre-industrial England. *Journal of Economic Growth*, DOI:10.1007/s10887–013–9094–0.

O'Shaughnessy, T. (2011). Hysteresis in unemployment. *Oxford Review of Economic Policy 27*(2), 312–337.

Papell, D. H., C. J. Murray, and H. Ghiblawi (2000). The structure of unemployment. *The Review of Economics and Statistics 82*(2), 309–315.

Røed, K. (1997, December). Hysteresis in unemployment. *Journal of Economic Surveys 11*(4), 389–418.