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Abstract: This paper focuses on innovation for new product with exogenously determined horizontal difference from initial product which is provided either by a monopolist or by competitive firms. The innovator, no matter initially under monopoly or competition, will be unique producer of new product and need decide quality of new product which is correlated with investment for innovation. The paper through a model shows that for horizontally similar new product, competition is superior to monopoly to innovate. However, for typical horizontally differentiated product, a monopolist would choose higher quality and invest more than a competitive innovator does if innovation is complex, but brings about lower endogenous quality than the innovator initially under competition does if innovation is easy. Monopoly can support sales of new product with higher price of initial product, but also hamper product innovation to avoid erosion of initial profit. If it is presumed that complexity of innovation is always huge at the beginning, monopoly is more likely to generate innovation for horizontally different product while competition for similar product, respectively compared to each other.

Keywords: product innovation; horizontal difference; monopoly; complexity of innovation
1. Introduction

Since Schumpeter (1950) put forward that monopoly, with monopolistic profit, is helpful to innovation related investment while competition holds back innovation due to no extra profit to invest for innovation, impact of market structure on innovation, as a controversial topic, absorbs lots of discussion among economists. Process innovation and product innovation are included in meaning of innovation. This paper focuses on product innovation. Arrow (1962), by his seminal work, gave an opinion that for process innovation a competitive firm has stronger incentive than that of a monopolist because monopolistic profit only the monopolist initially acquires diminishes incremental profit from innovation. Spulber (2012) analyses both process innovation and product innovation in his paper. Chen and Schwartz (2013) suggest for product innovation, through which new product horizontally differentiated from initial product is provided in market, monopoly can generate higher incentive of innovator. About product innovation, see also Gilbert (2006), Tirole (1988), Shaked and Sutton (1983), and so on.

Difference between two products can be abstracted into two kinds of differences (see Phlips and Thisse, 1982), horizontal difference and vertical difference. The vertical difference can be described as difference in qualities of two products (see Jaskold-Gabszewicz and Thisse, 1979, 1980, 1982). Greenstein and Ramey (1998) analyze vertically differentiated product innovation and suggest monopoly has its advantage. Similar to work by Chen and Schwartz (2013), this paper pays attention to horizontal difference with model based on framework of Hotelling (1929) adapted to
allow existence of vertical difference. Moreover, quality of new product in this paper is an endogenous variable partly depending on complexity of innovation and horizontal difference between two products is determined exogenously. This paper also assumes that firms are black boxes, from which paper of Bassi et al. (2015) is different by analyzing product positioning problem of two competing vertical hierarchies with Hotelling model (about product positioning problem, see also Kotler and Keller, 2008; Christou and Vettas, 2005; Colombo, 2012).

The conclusion of this paper is that competition is superior to monopoly for product innovation when horizontal difference is small or complexity of innovation is low, but inferior to monopoly when new product is sufficiently differentiated from initial one and innovation is tough. With respect to typical horizontally similar new product, results of this paper is analogous to Arrow’s work, but in regard to typical horizontally differentiated new product, if innovation’s complexity is comparatively high, this paper is consisted with Chen and Schwartz(2013).

Entry is not considered and the secure monopolist doesn’t need to barrier entrant by innovation in this paper. The innovator is either an initial monopolist or one of initially competitive producers. And once innovation occurs, the innovator is the unique provider of new product due to perfect patent protection. Gilbert and Newbery (1982) take entry into consideration and show that a monopolist to foil entry has stronger incentive to innovate than a potential entrant. Arrow suggests that an inventor possessing information for innovation can only sell information to an enterprise, but not become an entrant himself, so rent paid for information has upper limit which is
exactly equivalent to incentive for innovation. Spulber (2012) allows an inventor become an entrepreneur and discusses games between inventor and incumbents. In this paper, new product’s position determined exogenously, i.e. the location on Hotelling line, if its information is possessed by an inventor, can be known by the innovator after paying rent and the inventor cannot entry market independently.

Aghion et al. (2001) come up with escape competition effect which makes a competitive firm have stronger motive to innovate in order to escape competition with “neck-and-neck” rivals and show Schumpeterian effect is almost always outweighed by it. Since competition has different effects on innovation, there may appear positive relationship (see Boone, 2000; Vives, 2008; Spulber, 2013), inverted-U relationship (see Aghion et al., 2005; Barbos, 2015) or U relationship (see Lee, 2005) between competition and innovation. And Aghion et al. (2001) and Aghion et al. (2005) discuss “step-by-step” innovation (see also Harris and Vickers, 1987; Budd et al. 1993). This paper differs and focuses on one-shot and deterministic innovation like Arrow’s work.

Wang and Shin (2012) discuss the impact of vertical structure on innovation with quality of product as an endogenous variable. Analogously, this paper analyzes impact of horizontal structure with endogenous quality as a measurement, which could avoid endogenous problem caused by exogenous quality discussed below. Bhaskaran and Krishnan (2009) discuss contracts problem in collaborating product innovation with endogenous innovation but without considering pricing problem.

Similar to terminology of utility function, the prospective incremental revenue resulting from future innovation with exogenous quality of new product can be called
direct incentive (DI) for innovation and that with endogenous quality called indirect incentive (IDI) for innovation. What really induces firms to innovate is, however, the potential incremental profit from innovation called here indirect net incentive (INI) for innovation which is the net value after subtracting cost for innovation from IDI. INI is upper limit of rent an innovator would like to pay for innovation related information and highest reward for the inventor of the information. Conclusion of comparison about INI under monopoly and competition is similar to that about endogenous quality of new product.

The organization of the remainder of this paper is shown as follows. Section 2 describes the frame of model, lists the possible situations about innovation under monopoly and initial competition, and gives revenue functions and cost function of innovation. Section 3 gives methods to solve the model and main results including comparison between under monopolistic structure and under competitive structure of revenue with exogenously given quality and of endogenous quality of new product. Section 4 discusses reasons of some results in section 3 and gives more conclusions. Section 5 repeats essential conclusions, does some remarking and gives some topics about future work.

2. Frame of Model

There is a linear market like Hotelling model, where there exists initial product on one point and will appear potential new product on one other position determined exogenously. Neither of their sales coverage can reach the end of the linear market which is large enough. Initial product is provided either by a monopolist under
monopoly or by many homogeneous firms under competitive structure. Presumed that potential new product is produced by an incumbent not entrant, the innovator, i.e. the monopolist or one of the initially competitive firms, may sell two products at the same time. Moreover, the innovator will be the unique provider of new product because of perfect patent protection. For simplicity, marginal cost of both products is zero and investment for the initial product has became sunk cost, which means the only cost in consideration is investment for potential new product to ensure quality’s being on some level.

With respect to product innovation, decision problems encountered by a potential innovator can be divided into three stages. First, facing given rent for the information about one kind of prospective new product, a firm chooses whether to occupy the information and innovate. In the second stage, an innovator chooses optimal quality for potential new product and invests for it. In the third stage, the provider of new product set optimal prices for his products. The stages will be analyzed reversely.

2.1 Revenue of an innovator

In the third stage, what needs to be solved is pricing problem. We uses $v$ to denote the utility from a unit of product a consumer gets without transportation cost which represents negative utility because of $l$, the horizontal differentiation of purchased product from his favorite position of product. In this model transportation cost is presumed as quadratic function of horizontal difference, i.e., $tl^2$, in which $t$ weighs influence of horizontal difference on consumers. If the producer is unique in the large linear market and sells product on price $p$, the output is $2k$ in which $k$, potential sales
radius, is defined by $k = \sqrt{\frac{v - p}{t}}$. We assume that marginal cost of both products is 0.

The optimal price set by a monopolist, which we call monopolistic price in this paper, is $p = \frac{2}{3}v$. By setting $v$ as 3 and $t$ as 1, we have monopolistic price being 2 and profit being 4, which shows that difficulty to enlarge sales makes the provider choose to set a higher price. In this paper qualities of initial product and new product are denoted respectively by $v_1$ and $v_2$; prices by $p_1$ and $p_2$.

By setting transportation cost as quadratic function, equilibrium problem can be avoided, see D’Aspremont et. al (1979) and Neven (1985). Under extreme condition where $l$ is tiny and verges to 0, results in this paper is approximately equivalent to that with no horizontal difference between two products, which shows quadratic transportation cost function is robust.

Under monopolistic structure, a monopolist can sell both of products or single product: new or initial one. Under competitive structure, one of the incumbent firms can choose to provide new product or not, and whether initial product will still be provided by the innovator doesn’t matter since it brings about zero profit.

Under the competitive structure, we need consider 6 kinds of situation as follows:
(a) There is only initial product being sold in the market with no new product sold because of too much similarity between two products or comparatively low quality of new product. In this situation it is called that quality of new product is 0.
(b) New product is sold on a monopolistic price because it’s sufficiently differentiated from initial product relative to given quality of two products then.
(c) Both products are sold in the market without overlap segment or buy-none
segment of market between their position and the marginal consumer get zero surplus. The marginal consumer means one consumer who is indifferent about buying initial product or new one.

(d) Both products are sold in the market with an overlap segment of market between their position and the marginal consumer gets positive surplus.

(e) New product exactly replaces entirely the initial product with a non-monopolistic price, and a higher or lower price leads to smaller revenue.

(f) There is only new product sold in the market on a monopolistic price because of its high quality.

Under monopolistic structure, taking into consideration that a monopolist will set a monopolistic price for new product if it is single product sold in market, (e) involved in competitive situations is invalid. Other 5 kinds of situations need to be considered under monopolistic structure. Some figures in appendix show the segments corresponding to different situations respectively under monopoly and competition, see appended drawings 3, 4, 7 and 8.

If the market has been entirely covered by a monopolistic firm, the appearance of new product makes it set prices on which marginal consumer get no surplus. In comparison, if the market has not been covered entirely, which will be discussed in this paper, the monopolist, to induce more consumers, may not set such high prices and marginal consumer could have positive surplus sometimes.

Situation (d), usually, is regarded as most regular situation and we analyze it at first. Use $x$ to denote the location of marginal consumer and then have following
\[ v_1 - p_1 - tx^2 = v_2 - p_2 - t(l - x)^2. \]

Then we have \( x \) as follow:
\[ x = \frac{l}{2} + \frac{v_1 - v_2 - p_1 + p_2}{2tl}. \]

Now we have equilibrium quantities of two products as follow:
\[ q_1 = \sqrt{\frac{v_1 - p_1}{t} + \frac{v_1 - v_2 - p_1 + p_2}{2tl} + \frac{l}{2}}, \]
\[ q_2 = \sqrt{\frac{v_2 - p_2}{t} + \frac{v_2 - v_1 - p_2 + p_1}{2tl} + \frac{l}{2}}. \]

To avoid the radical form which makes derivative complicated, we replace the square roots by
\[ k_1 = \sqrt{\frac{v_1 - p_1}{t}}, \quad k_2 = \sqrt{\frac{v_2 - p_2}{t}}. \]

For simplicity we will eliminate \( t \) by setting it as 1, because the transport cost is not a major topic considered in this paper. Firstly we set:
\[ u_1 = \frac{v_1}{t}, \quad u_2 = \frac{v_2}{t}. \]

And then we have:
\[ p_1 = t(u_1 - k_1^2), \quad p_2 = t(u_2 - k_2^2); \]
\[ q_1 = k_1 + \frac{k_1^2 - k_2^2}{2l} + \frac{l}{2}, \quad q_2 = k_2 + \frac{k_2^2 - k_1^2}{2l} + \frac{l}{2}; \]
\[ R = t \left[ (u_1 - k_1^2)(k_1 + \frac{k_1^2 - k_2^2}{2l} + \frac{l}{2}) + (u_2 - k_2^2)(k_2 + \frac{k_2^2 - k_1^2}{2l} + \frac{l}{2}) \right]. \]

With given \( k_1 \) and \( k_2 \), firms’ revenue, in fact consumer surplus and social welfare as well, can be calculated by multiplying by \( t \) what their values are when \( t \) is 1. Now we eliminate \( t \) and have:
\[ p_1 = u_1 - k_1^2, \quad p_2 = u_2 - k_2^2; \quad (1) \]
\[ R = (u_1 - k_1^2)(k_1 + \frac{k_1^2 - k_2^2}{2l} + \frac{l}{2}) + (u_2 - k_2^2)(k_2 + \frac{k_2^2 - k_1^2}{2l} + \frac{l}{2}); \quad (2) \]

Again for simplicity of numerical simulation mentioned below, we will set \( u_1 \) as 3 then.

\( R \) above can represent, fortunately, both industrial profit and innovator’s profit both under monopolistic structure and under competitive structure.

Expressions above are proper only when following conditions are satisfied:

(i) \( k_1 + k_2 \geq l \), (ii) \( k_1 - k_2 \leq l \), (iii) \( k_2 - k_1 \leq l \).

Otherwise situation (d) fails to describe the results with given values of other parameters and other situations need be considered. The clue of computer program to solve the pricing problem is shown below. When which situation the result belongs to with given parameters can be known at first, it is easy to give optimum price and corresponding revenue. When it cannot be made sure at first, all possible situations need be tested and respective values of revenue need be calculated, among which the biggest is generally biggest revenue value. Then corresponding values of other variables can be calculated.

### 2.2 The cost for innovation

In the second stage, cost of innovation need be considered. There is no financial constraints taken into consideration and, then, a monopolistic firm and a initially competitive one have no difference in financing cost. Cost function depending on quality of new product is \( C(v) = \frac{1}{6} av_2^3 \). Quadratic cost function is also applicable here but slow to convergent to optimum quality value, so in this model cubic cost
function is used for shorter numerical matching process, which generates a rising incremental cost as the quality is improved. The parameter $a$ is complexity coefficient of innovation and exogenously depends on technological factors. Cost function can be rewritten as $C(u_2) = \frac{1}{6} ar^3u_2^3$. By setting $t$ as 1, we have

$$C(u_2) = \frac{1}{6} au_2^3. \tag{3}$$

After setting $u_2$ as 3, if only new product is sold in market, a monopolistic provider of new product will get zero profit when $a = \frac{8}{9}$. Optimum value of $u_2$ is exactly 3 when $a = \frac{4}{9}$. If initial product is in market with $u_1$ being 3 at the beginning and new product has no horizontal differentiation from initial one, a monopolist will replace initial product by new one when $a < \frac{2}{9} (=0.2222)$, see appended drawing 7.

After setting $u_1$ as 3, profit of an initially competitive innovator from new product is positive when $a < \frac{16\sqrt{3}}{81} (=0.3421)$, see figure 4 and appended drawing 8. The looser condition of innovation, in one aspect, shows that the competitive structure prevails in race of innovation for product only vertically differentiated from initial product.

Complexity coefficient here, for simplicity, is independent from $l$. And the investment for initial product is sunk cost and the corresponding equipment cannot be updated or changed for producing new product (there is no economy of scope).

### 3. Solution and main results

#### 3.1 Analysis about revenue under monopolistic structure

Consider the third stage. Firstly, we solve (d) under monopolistic structure. With given expression (2), we have first order conditions as follow:
\[-3k_1^2 - \frac{2k_1^3}{l} + \frac{2k_1k_2^2}{l} - lk_1 + \frac{u_1-u_2}{l}k_1 + u_i = 0, \quad (4)\]

\[-3k_2^2 - \frac{2k_2^3}{l} + \frac{2k_2k_1^2}{l} - lk_2 + \frac{u_2-u_1}{l}k_2 + u_i = 0. \quad (5)\]

It is difficult to deduce precise solution of simultaneous equations (4) and (5). After setting $u_1$ as 3, given a numerical combination of $u_2$ and $l$, however, we can with computer calculate proper values of $k_1$ and $k_2$ satisfying both (4) and (5). To analyze characters of a monopolist’s conduction and acquire matching process with low algorithm complexity as well, before giving results of comparison graphically, we pay attention to equations above.

When $k_1$ is not 0, from (4) we have

\[2k_2^2 = 2k_1^2 + 3lk_1 + l^2 + u_2 - u_1 - \frac{lu_1}{k_1}. \quad (6)\]

The formula on the left side increases with $k_2$. Provided that the variables are positive, the formula on the right side increases with $k_1$. So when $k_1$ decreases, $k_2$ does also. What this means is that $p_2$ needs to rise if a monopolist set a higher $p_1$ and vice versa by symmetry.

Like (6), from (5) we have

\[2k_1^2 = 2k_2^2 + 3lk_2 + l^2 + u_1 - u_2 - \frac{lu_2}{k_2}. \quad (7)\]

From (6) and (7), we know that $k_2$ is a monotone increasing function of $k_1$ and vice versa, which are good attributes for designing matching process below with low algorithm complexity. Set $k_2$ as 0 at first. Through (6) calculate corresponding value of $k_1$ with which new value of $k_2$ can be calculated by (7). Recycle the process until
variations of both \( k_1 \) and \( k_2 \) are within error range set in advance. The convergence problem can be practically solved by the computer programs run for matching solution, which can report the number of errors that the value being tested is beyond right range and cannot end if there is no convergent value. So if programs give results with zero error reported, proper convergent values have been acquired. In each cycle, given that \( k_1 \) is positive and smaller than square root of \( u_2 \), with a value of \( k_2, k_1 \) can be calculated through (6) by dichotomy (which shall not be substituted by Newton iteration which generates no convergence due to uncertainty of concave-convex feature of related function). By the same method, \( k_2 \) can be calculated through (7).

Variables \( k_1, k_2, R, CS, W \) and so on, can be depicted by the dense results of numerical simulation, with inputs including the value of \( u_1 \) as 3 and a large number of numerical combinations of \( l \) and \( u_2 \).

When the results lead to \( k_1+k_2 \leq l \) but two products both on monopolistic price still have overlapped segment of market, a monopolist would set such prices about which the consumer being indifferent between two products gets zero utility, see situation (c). So the optimum solution satisfies \( k_1+k_2 = l \). Since \( k_2 \) can be replaced by \( l - k_1 \) in revenue function, first order condition is given as follow:

\[
2u_1 - 2u_2 + 6l^2 - 12k_1l = 0. \tag{8}
\]

It is easy to give the optimal solution:

\[
k_1 = \frac{l}{2} + \frac{u_1 - u_2}{6l}, \quad k_2 = \frac{l}{2} + \frac{u_2 - u_1}{6l}.
\]

Maximum revenue then can be given by:

\[
R_{ml} = \frac{(u_1 - u_2)^2}{6l} + l(u_1 + u_2) - \frac{l^3}{2}.
\]
When new product can entirely replace initial product on a monopolistic price, we have:

\[ R_{M} = \frac{4}{3} u_{2} \sqrt{\frac{u_{2}}{3}}. \]

### 3.2 Analysis about revenue under initially competitive structure

In competitive market, the price of initial product equals to marginal cost, i.e. 0. So we have \( k_{1} = \sqrt{u_{1}}. \)

We firstly consider the regular situation (d). From expression (2) we have:

\[ R_{c} = (u_{2} - k_{2}^2)(k_{2} + \frac{k_{2}^2 - u_{1}}{2l} + \frac{l}{2}). \] (9)

Also, the expression above is right only when conditions ( i ), ( ii ) and ( iii ) are satisfied. According to (9), we have first order condition

\[ u_{2} - 3k_{2}^2 + \frac{u_{1} + u_{2}}{l}k_{2} - \frac{2k_{2}^3}{l} - lk_{2} = 0. \] (10)

The formula on the left side is a concave function of \( k_{2}. \) Variable \( k_{2} \) in situation (d) is positive and smaller than square root of \( u_{2}, \) or means otherwise new product cannot be sold on a nonnegative price. After matching solution of (10) by dichotomy, we can easily have corresponding numerical values of other variables such as \( R. \)

Here is some extensive analysis. The monopolistic price of new product is \( \frac{2}{3} u_{2} \) and corresponding \( k_{2} \) is \( k_{2} = \sqrt{\frac{u_{2}}{3}}. \) The partial derivative of an initially competitive innovator’s revenue on \( k_{2} \) when setting a monopolistic price is

\[ \frac{\partial R_{c}}{\partial k_{2}} \bigg|_{k_{2} = \sqrt{\frac{u_{2}}{3}}} = \frac{3u_{1} + u_{2} - 3l^{2}}{3l} \sqrt{\frac{u_{2}}{3}}. \] (11)

The formula above is negative when value of \( l \) is large, which means, compared to a
monopolistic price, a competitive innovator would sometimes set a higher price and maintain a lower \( k_2 \). So the price of new product could sometimes be higher under competitive structure than under monopolistic structure, see figure 2.

When new product is on a non-monopolistic price which exactly makes it replace initial product entirely, we have: \( k_2 = l + \sqrt{u_1} \) and then \( R_c = 2k_2(u_2 - k_2^2) \).

When the innovator sells new product on a monopolistic price and there is buy-none segment of market between consumers purchasing initial product and that purchasing new one, we have \( R_c = \frac{4}{3}u_2\sqrt{u_2} \).

When the innovator sells new product on such a price that marginal consumer gets no surplus, we have \( k_2 = l - \sqrt{u_1} \) and then \( R_c = 2k_2(u_2 - k_2^2) \).

3.3 Results of comparison with exogenous quality of new product

Here think about only the third stage. When we set \( u_1 \) as 3, incremental revenue due to innovation, i.e. DI for innovation, under monopolistic and competitive structure can be respectively computed as numerical values corresponding to different numerical combinations of \( l \) and \( u_2 \). Two points need to be noticed here. First, realized revenue by potential innovator is highest among possible revenues in respective situations (for more detail about revenue functions, see table 1 and table 2 in appendix). Second, to get DI for innovation under monopoly, values of realized revenue need to be subtracted by initial revenue of a monopolist. By comparing values of DI for innovation, we get figure 1 and proposition 1. Capital letter “L” on horizontal axis and “U2” on vertical axis means respectively value of \( l \) and \( u_2 \). We use \( DI_M \) and \( DI_C \) to denote DI for innovation respectively under monopolistic structure and competitive
structure.

**Proposition 1**: If \( u_2 > u_0(l) \), \( DI_C > DI_M \); if \( u_2 < u_0(l) \), \( DI_C < DI_M \). Here the value of \( u_0(l) > 0 \) depends on the given value of \( l \).

![Figure 1. Comparison of DI for product innovation](image1.png)

![Figure 2. Comparison between two structures about price of new product](image2.png)

Figure 1. Comparison of DI for product innovation

E means there are equivalent values under monopoly and competition. M means DI is higher under monopoly while C means under competition. N means no product innovation occurs. (To avoid frequent fluctuation, difference of which value is within 0.1 is overlooked, so some detail of this figure is different from others.)

Figure 2. Comparison between two structures about price of new product

E means there are equivalent values under monopoly and competition. M means price of new product is higher under monopoly while C means under competition. N means no product innovation occurs.

Results of comparison about \( CS \) and \( W \) are more likely to show competition is superior to monopoly, see appended drawing 1 and drawing 2. Ex-post \( CS \) under competition is usually bigger than or equivalent to that under monopoly except that if \( l \) is large and \( u_2 \) is high \( CS \) under monopoly sometimes is higher even when a monopolist set a monopolistic price for new product, see appended drawing 5. Similar thing occurs about ex-post \( W \).

It is interesting that the price of new product under competitive structure could be higher than monopolistic structure if the horizontal difference between new product and initial product is comparatively large, see figure 2.
Though price of new product could be higher sometimes, the competitive structure, compared to monopoly, with lower price of initial product usually results in higher CS. Even after taking into consideration the revenue of industry, monopoly often results in lower social welfare especially when \( l \) is small.

In situation (f) under both monopolistic and competitive structure, revenue from new product no longer depends on \( l \), which means that when new product is sufficiently prominent on quality or comparatively similar with initial one, horizontal difference exerts of no effect and only quality, then vertical difference, matters.

### 3.4 Results of comparison with endogenous quality of new product

Now think about the second stage. With cost function (3) and revenue functions derived before (for more detail, see table 1 and table 2 in appendix), we now focus on the endogenous values of \( u_2 \), \( R \), \( CS \) and other variables. It is not difficult to calculate—by successive comparison with computer—the optimum numerical values of \( u_2 \) and other values of related variables corresponding to numerical combinations of parameters \( l \) and \( a \), under either monopolistic or competitive structure. To get precise numerical value of endogenous \( u_2 \), a small interval (like 0.1) between two contiguous elements in sequence of \( u_2 \) is necessary and for shorter time to run programs we could set the upper limit for \( u_2 \) not too high (like as 90) which also needs to be high enough compared with \( u_1 \) as 3 when taking into consideration realistic economic environment. Under both structure, the endogenous quality of new product and related investment for it both rise as \( a \) falls. If new product is sufficiently differentiated from initial one or comparatively easily innovated for high quality, two kinds of structure generate
equal \( u_2 \). The result of comparison between endogenous \( u_2 \) under monopoly and that under initial competition (denoted respectively by \( u_{2M}^* \) and \( u_{2C}^* \)) is depicted in figure 3 and abstracted by proposition 2. Capital letter “A” on vertical axis means value of \( a \).

**Proposition 2**: When \( l<l_0 (\approx 0.27) \), \( u_{2C}^* \geq u_{2M}^* \). When \( l>l_0 \), \( u_{2C}^* \geq u_{2M}^* \) if \( a<a_0(l) \); \( u_{2M}^* \geq u_{2C}^* \) if \( a>a_0(l) \). Here the value of \( a_0(l)>0 \) depends on the given value of \( l \).

Shown by figure 3, optimum \( u_2 \) under initially competitive structure is higher than or equal to that under monopoly when \( l \) is small. For bigger \( l \), when complexity of innovation is high, a monopolist chooses a higher \( u_2 \), and a competitive firm does when complexity coefficient is low, compared to each other respectively. Conclusions above can be seen more clearly in figure 4 where \( l \leq 2 \).

**Figure 3.** Comparison about endogenous quality of new product

E means there are equivalent values under monopoly and competition. M means endogenous quality is higher under monopoly while C means under competition. N means no product innovation occurs.

**Figure 4.** Comparison about endogenous quality of new product (details when \( l<2 \))

Meaning of capital letters is the same with that in Figure 3.

When \( l \) is small, incremental revenue from innovation under monopoly is independent of \( l \) as long as it belongs to situation (f). When \( a \) is 2/9, under monopoly endogenous value of \( u_2 \) is 4.7622 which brings about exactly zero incremental profit.
for the innovator. With \( u_2 \) being 4.7622, if \( l \) is small, the added revenue of a monopolist from innovation is smaller than a competitive firm, see figure 1. As a result, given \( l \) is small, with the same cost function, an initially competitive firm chooses a higher optimal value of \( u_2 \) than that by a monopolist when \( a \) is 2/9 and a higher or equivalent value when \( a \) is larger.

With given value of \( a \), according to cost function, the investment under one kind of structure is larger than under the other if and only if its endogenous value of \( u_2 \) is larger. So the result of comparison about optimum investment between two structures can also be shown by figure 3. An initially competitive innovator would invest no less resource than a monopolist on new product similar to initial one or without large difficulty to possess high quality. Monopoly results in larger \( CS \) only when \( l \) is large and \( a \) is low simultaneously, and larger \( W \) when \( l \) is large, see appended drawing 11 and drawing 12. Consider the circumstances where \( l \) is small (smaller than 2, for example), competitive structure is superior to monopolistic structure with respect to consumer surplus and social welfare.

**3.5 IDI and INI for product innovation**

Now think about the first stage. IDI is incremental revenue from product innovation corresponding to endogenous quality of new product. By subtracting cost of innovation from IDI, we get INI for innovation respectively under monopoly and competition, see figure 5 and figure 6. With given \( l \) and \( a \), a monopolist will innovate if the rent for related information is lower than the surface in figure 5, and a initially competitive firm will if the rent is below the surface in figure 6. Under competition
when $a$ is comparatively high, as $l$ rises at the beginning, INI for product innovation falls because replacing initial product entirely, most advantageous pricing method and only method by which new product can exist in market then, is increasingly challenging because horizontal difference turns larger.

The comparisons between monopoly and competition about IDI and INI are shown respectively in appended drawing 9 and drawing 10, which are both highly consistent with the result of comparison about optimum $u_2$ except that when both $l$ and $a$ are small competitive structure generates larger IDI and INI for innovation than monopoly does despite equal value of optimum $u_2$ because a monopolist has initial profit.

Competitive firms, compared with a monopolist, have higher or at least equivalent bid for information of potential new product if it is similar to initial one. For significantly horizontally differentiated product, higher bid by a competitive firm appears only when the expectant complexity of innovation is low.

4. Discussion

4.1 Barrier effect of limited market and competition
At first we need to look at the reason why in an entirely covered market monopolistic structure is often superior to competitive structure about innovation issues (Chen and Schwartz, 2013; Tirole, 1988). In an entirely covered market, given exogenous quality of products, output cannot be enlarged any more due to market limit as a barrier of sales, which, called here barrier effect of covered market, leaves lifting price an only method to increase revenue. A monopolist has stronger power to control price and suffer less from barrier effect than a competitive firm does in the covered market. In a comparatively large market, however, besides price, a producer can also increase revenue by enlarging output, which does not so depend on market power like setting high price. Specifically, in a covered market with new product and initial product respectively on its two ends, if under competitive structure, no matter how high is the quality of new product, competition from initial product could prevent innovator from setting a high price on which there exist consumers getting less surplus than purchasing initial product whose price is equal to marginal cost. This paper calls it location effect that the existence of initial product horizontally differentiated from new product holds back rise of new product’s price with its low price. So with barrier effect covered market is prone to higher price, and with location effect competitive providers of initial product diminishes potential revenue of innovator from new product. While in an enough large market with contestable initial product, there exists the monopolistic price set for new product with sufficiently high quality, which protects the new product from competition and leads to failure of the location effective from initial product.
When initial product and new one are both sold in market, a monopolist, compared with an initially competitive innovator, can coordinate the prices of these two products to acquire larger revenue which may lead to larger added profit in spite of initial monopoly profit. Covered market emphasizes price which endows a monopolist advantages while sufficiently large market emphasizes both price and output which give competitive firms opportunity to maintain more reward from product innovation.

Figure 2 describes the comparison about prices of new product between under competitive structure and under monopolistic structure respectively denoted by $p_{2C}$ and $p_{2M}$. For some large $l$, $p_{2C}$ is higher than $p_{2M}$. It possibly occurs due to one of three reasons. The first reason shown by expression (11) is that a competitive innovator in situation (d) sometimes would for new product set higher price than the monopolistic price a monopolist sets in situation (b). The second reason is competitive innovator choose a price in situation (c) which is higher than monopolistic price. The third is competitive innovator choose a price in situation (d) which sometimes is higher than the price a monopolist set in situation (c). Plotting figure 2 with appended drawing 3 or drawing 4 as background could be a useful tactic to observe the results, see appended drawing 6.

What could make the initially competitive innovator set a higher price for new product is that the competition from initial product decreases the profit of new product’s provider when lowering price induces more consumers but not enough to offset the depreciation of product. Competition then has barrier effect similar to
limited market, which in some degree bars the increase of output and makes the provider of new product opt to raise its price. Particularly, in situation (c) of competitive structure, fierce competition of initial product absolutely prevents sales extension of new product as if new product is provided for limited market so that the marginal consumer acquires no surplus.

4.2 Necessity to analyze endogenous quality of new product

Spulber(2012) suggests that the monopolist has a bigger incentive to innovate than that of an entrant under ex-post duopoly when the new product is sufficiently different from initial one (proposition 4). And Chen and Schwartz (2013) suggest that in a covered market the incentive to innovate under monopoly is larger than under ex-post duopoly when the quality of new product is higher than initial one. These both imply that it is under some conditions that monopoly could give rise to a larger incentive to innovate which then has a positive impact on innovation. Different from comparison between monopoly and ex-post duopoly, the results of comparison between monopolistic structure and competitive structure in this paper are more likely to lead to an embarrassing conclusion that a competitive firm has stronger incentive to innovate if the quality of new product is high. Combined with common sense that higher quality of product needs bigger investment, it could be deduced that if large investment is needed, a competitive firm compared with a monopolist is more like to innovate, which is reversed while it is easy to innovate new product with high quality without huge investment. Even though financial discrimination is eliminated ideally, it is difficult to comprehend the positive correlation between competition and big
innovations with high investment, or, between market concentration and small innovation with low investment.

But this conclusion will be explained from a perspective of endogenous innovation represented here by quality of new product. In fact, there exists an endogenous problem with respect to correlation between innovation and incremental revenue from it when taking into consideration that the provider of new product would choose an optimum value of quality by pondering over both revenue function and cost function. So observed innovation as a result of business decision depends not only on revenue it may bring about but also on the investment for it.

4.3 Diversion effect and coordination effect

By using the terms from Chen and Schwartz (2013), now we analyze the diversion effect and coordination effect under monopolistic structure compared to under competitive structure, which denote the differences between monopoly and ex-post duopoly by Chen and Schwartz.

![Figure 7. DE on initial product](image1)

![Figure 8. CE on new product](image2)

Under monopoly in situation (c) and (d), i.e., when both products are sold in market without both being on the monopolistic price, the price of initial product is
higher than monopolistic price which leads to less revenue from initial product compared with initial revenue before innovation. Appearance of new product erodes the initial profit from initial product, which is diversion effect (DE) of new product on initial product under monopoly. Since a competitive firm has zero initial profit, without DE it sometimes has stronger incentive to innovate for new product. We use \( R_{M0}, R_{1M}, R_{2M}, \) and \( R_C \) to denote respectively initial revenue, revenue from initial product after innovation and that from new product of a monopolist, and revenue from new product of initially competitive innovator here. The value of DE can be given by \( DE=R_{M0} - R_{1M} \). Because of symmetry, under monopoly the initial product also has the diversion effect on new product which erodes the revenue from new product compared to potential revenue acquired when only new product is sold on a monopolistic price. Nevertheless, compared to competitive firms, a monopolist could set a proper price for initial product and manage to acquire higher profit from new product, which is coordination effect (CE) of initial product under monopoly on new product. In situation (c), (d), and (e), a competitive firm with new product, without CE, has to face fierce competition from initial product. The value of CE can be given by \( CE=R_{2M} - R_C \). For a monopolist, compared to a competitive producer, DE hampers product innovation while CE promotes innovation.

The difference of DI for product innovation between of a monopolist and of an initially competitive firm then can be given by \( DI_M - DI_C = (R_{1M} + R_{2M} - R_{M0}) - R_C \). It is easy to get \( DI_M - DI_C = CE - DE \) (for similar equation, see Chen and Schwartz, 2013). The DI for product innovation under monopoly is larger if and only if the value of CE
is bigger than of DE. And with computer we can calculate precise values of CE and DE, see figure 7 and figure 8. When potential new product is similar to initial one and possesses high quality, a monopolist has a lower DI for product innovation than an initially competitive firm does because of prominent DE which will erode initial profit. While if prospective new product is comparatively differentiated from initial one, a monopolist could have a higher DI for product innovation compared to a competitive firm partly because strong CE will provide enough incremental profit.

4.4 Further discussion about impact of market structure on product innovation

With respect to the rent for the information relating to product innovation, provided that $a$ falls, generally, as time goes by, a competitive firm would, compared with a monopolist, pay a higher price when the prospective new product is similar with initial one, but a lower price for innovation about new product significantly differentiated from the initial one, since high $a$ is encountered even though the information has been occupied by the firm.

However, sometimes there exists new product without huge difficulty to be innovated with high quality and popularity among consumers, which often appears in some occasional cases where existing technology or natural resource is found capable to be utilized effectively in a way unknown before. For a much differentiated new product with low $a$, a competitive firm will also pay more for buying relative information, invest more on its application, and realize higher quality than a monopolist does. Even though having acquired the information, monopoly results in less investment on and lower quality of new product since high quality would erode
more of the profit from initial one. Then with respect to a kind of typical horizontally differentiated new product, there is a description for a phenomenon that a monopolist, compared with competitive firms, would like to invest to handle some extraordinarily tough problems related to new product but is reluctant to apply some simple technologies efficiently to innovate new product with high quality, for which diversion effect is the reason.

Therefore, for a kind of typical horizontally same or similar new product, a competitive firm often prevails in innovation race. A monopolist, however, would pay more than a competitive producer for a kind of typical horizontally different new product if original foundation work is needed to innovate it so that \( a \) is large. One exception to the disadvantage of monopoly is that existing technology which can be utilized easily brings about low complexity of innovation. It can also be easily explained by an extreme example that complexity coefficient is so sufficiently low that endogenous quality of new product is sufficiently high and capable to entirely substitute initial product on a monopolistic price, which leads to comparative unwillingness of a monopolist to innovate new product with high quality because of diversion effect.

Variables \( l \) and \( u_2 \) depend on and influence each other, so do variables \( l \) and \( a \). High quality of new product leading to violent substitution on initial product also means comparatively low horizontal difference, or high horizontal similarity, between them. Then the exception above on the other side shows that competitive firms have advantages to innovate similar product. From this perspective, monopoly would be
conductive to product innovation only when the potential new product is comparatively differentiated from initial one, which agrees with conclusion of Chen and Schwartz (2013).

Since complexity of innovation is a dynamic variable, with some given horizontal differentiation of potential new product, monopoly may be favorable for product innovation at beginning but to disadvantage of it when the complexity coefficient is sufficiently small. So with respect to innovation the past reason for monopoly probably will no longer be reasonable in the future, especially when difficulty of innovation is eased exogenously.

5. Concluding Remarks
Since monopoly would like to results in more investment on product innovation when innovation complexity is large but will be inferior to competition once improving quality of new product turns easy, then there appears a question whether an initially monopolistic enterprise should be demerged to promote subsequent upgrade of new product after the monopolist has done lots of work for innovation. Often, answer is prone to “yes” if game has been over yet, since monopoly also damage consumer surplus and social welfare. Nevertheless, demerger may hamper another product innovation from dynamic perspective, for example, in this model, on the other side of initial product’s position there may exist another potential new product. It is needed to consider and measure more factors when many innovative projects are conducted by a monopolist at the same time, which occurs sometimes.

According to results of the model, monopolistic structure often leads to lower
welfare than competitive structure. Without purpose to provoke goodness of monopoly, here we consider more factors. Financial constraints ignored in the model may change conclusion about innovation and social welfare. About impact of financial constraints, see Brown et al. (2011). Moreover, static welfare depreciates the value of innovation which yields a long-term welfare in a dynamic system. Besides exerting spill-over effect (see Caballero and Jaffe, 1993), R&D investment and research outputs accelerate the process of appearance of new products and application of new technology. From long-term perspective, technological progress composed of and derived from accumulated innovations can improve the social welfare in a nonlinear way, with which considered comparison of static welfare may not give a conclusive answer about merits of market structure.

Here repeat some essential results in preceding parts. For product innovation, in a sufficiently large market monopoly is superior to competition only if the potential new product is comparatively horizontally differentiated from initial product and complexity of innovation is high. Static consumer surplus and social welfare under monopoly are often lower than that under competition. The perspective this paper analyzes innovation from is impact of market structure on revenue from innovation, and diversion effect and coordination effect are analyzed as factors influencing difference of innovation between two structures. Schumpeter effect and coordination effect are in the same direction when having impact on product innovation, while replacement effective, escape competition effect and diversion effect are in the same direction. When complexity of innovation is high, an innovator needs more R&D
funds, impetus for innovation and enough power to occupy the benefits from innovation, Schumpeter effect is strong which endows monopoly advantage to promote innovation. However, for a competitive firm facing easy innovative conduction, constraints of capital and power to possess as much the benefits from innovation as possible are loose, which reinforces the role of escape competition effect and results in superiority of competition. So for horizontally differentiated new product, results in this paper can be explained partly by Schumpeter effect and escape competition effect. For horizontally similar new product, competition shows larger advantage than monopoly does because of significant diversion effect under monopoly, which can also be explained from the perspective of replacement effect or escape competition effect.

For future research, comparison of innovation between incumbent and entrant could be analyzed. An entrant confronting an incumbent monopolist can innovate for new product and enter market then. So can an entrant encountering competitive incumbents. Conductions relating to innovation of an incumbent monopolist facing entrant, of an entrant facing the monopolist, and of an entrant facing competitive incumbents, and ranking order of relative results of them, are all interesting topics. What impact complexity of innovation has on relationship between innovation and market concentration also needs empirical research and further theory study. Conducts and performance in limited or entirely covered market are also good points to do more research about innovation.
References


## Appendix

### Table 1. Functions under monopolistic structure with exogenous qualities

<table>
<thead>
<tr>
<th>situation</th>
<th>Functions under monopolistic structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>( p_1 = \frac{2}{3} u_1, q_1 = 2 \sqrt{\frac{u_1}{3}}, q_2 = 0, R = \frac{4}{3} u_1 \sqrt{\frac{u_1}{3}}, CS = \frac{4}{3} \left( \frac{u_1}{3} \right)^2, W = \frac{16}{3} \left( \frac{u_1}{3} \right)^2. )</td>
</tr>
<tr>
<td>(b)</td>
<td>( p_1 = \frac{2}{3} u_1, q_1 = 2 \sqrt{\frac{u_1}{3}}, p_2 = \frac{2}{3} u_2, q_2 = 2 \sqrt{\frac{u_2}{3}}, R = \frac{4}{3} u_1 \sqrt{\frac{u_1}{3}} + \frac{4}{3} u_2 \sqrt{\frac{u_2}{3}}, CS = \frac{4}{3} \left( \frac{u_1}{3} \right)^2 + \frac{4}{3} \left( \frac{u_2}{3} \right)^2, W = \frac{16}{3} \left( \frac{u_1}{3} \right)^2 + \frac{16}{3} \left( \frac{u_2}{3} \right)^2. )</td>
</tr>
<tr>
<td>(c)</td>
<td>( p_1 = u_1 - \left( \frac{l}{2} + \frac{u_1-u_2}{6l} \right)^2, p_2 = u_2 - \left( \frac{l}{2} + \frac{u_1-u_2}{6l} \right)^2, q_1 = \frac{l + u_1 - u_2}{3l}, q_2 = \frac{l + u_1 - u_2}{3l}, R = \frac{(u_1-u_2)^2}{6l} + l(u_1+u_2) - \frac{l^3}{2}, CS = \frac{l^3}{3} + \frac{(u_1-u_2)^2}{9l}, W = \frac{5(u_1-u_2)^2}{18l} + l(u_1+u_2) - \frac{l^3}{6}. )</td>
</tr>
<tr>
<td>(d)</td>
<td>( p_1 = u_1 - k_1^2, p_2 = u_2 - k_2^2, q_1 = k_1 + \frac{k_1^2 - k_2^2}{2l} + \frac{l}{2}, q_2 = k_2 + \frac{k_2^2 - k_1^2}{2l} + \frac{l}{2}, R = (u_1 - k_1^2)(k_1 + \frac{k_1^2 - k_2^2}{2l} + \frac{l}{2}) + (u_2 - k_2^2)(k_2 + \frac{k_2^2 - k_1^2}{2l} + \frac{l}{2}), CS = \frac{2}{3} (k_1^2 + k_2^2)^2 + \frac{(k_1^2 - k_2^2)^2}{12} + l^3, W = u_1 k_1 + u_2 k_2 - \frac{(k_1^2 + k_2^2)^2}{3} + \frac{l(u_1+u_2)}{2} + \frac{(u_1-u_2)(k_1^2 - k_2^2)}{2l} - \frac{(k_2^2 - k_1^2)^2}{12} - \frac{l^3}{12}. )</td>
</tr>
<tr>
<td>(f)</td>
<td>( q_1 = 0, p_2 = \frac{2}{3} u_2, q_2 = 2 \sqrt{\frac{u_2}{3}}, R = \frac{4}{3} u_2 \sqrt{\frac{u_2}{3}}, CS = \frac{4}{3} \left( \frac{u_2}{3} \right)^2, W = \frac{16}{3} \left( \frac{u_2}{3} \right)^2. )</td>
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### Table 2. Functions under competitive structure with exogenous qualities

<table>
<thead>
<tr>
<th>Situation</th>
<th>Functions under competitive structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>( p_1 = 0, q_1 = 2 \sqrt{u_1}, q_2 = 0, R = 0, CS = \frac{4}{3} u_1 \frac{1}{3}, W = \frac{4}{3} u_1 \frac{1}{3}. )</td>
</tr>
<tr>
<td>(b)</td>
<td>( p_1 = 0, q_1 = 2 \sqrt{u_1}, p_2 = \frac{2}{3} u_2, q_2 = 2 \sqrt{\frac{u_2}{3}}, R = \frac{4}{3} u_1 \sqrt{\frac{u_1}{3}}, CS = \frac{4}{3} u_1 ^\frac{1}{3} + \frac{4}{3} u_2 ^\frac{1}{3}, W = \frac{4}{3} u_1 ^\frac{1}{3} + \frac{16}{3} u_2 ^\frac{1}{3}. )</td>
</tr>
<tr>
<td>(c)</td>
<td>( p_1 = 0, q_1 = 2 \sqrt{u_1}, k_2 = l - \sqrt{u_1}, p_2 = u_2 - k_2^2, q_2 = 2k_2, R = 2k_2(u_2 - k_2^2), CS = \frac{4}{3} u_1 ^\frac{1}{3} + \frac{4}{3} k_2 ^\frac{1}{3}, W = R + CS. )</td>
</tr>
<tr>
<td>(d)</td>
<td>( p_1 = 0, p_2 = u_1 - k_2^2, q_1 = \sqrt{u_1} + \frac{u_1 - k_2^2}{2l} + \frac{l}{2}, q_2 = k_2 + \frac{k_2^2 - u_2}{2l} + \frac{l}{2}, R = (u_1 - k_2^2)(k_2 + \frac{k_2^2 - u_2}{2l} + \frac{l}{2}), CS = \frac{2}{3} (u_1^\frac{1}{3} + k_2^\frac{1}{3})^2 + \frac{u_1 - k_2^2}{4l} + \frac{l(u_1 + k_2^2)}{2} - \frac{l^3}{12}. )</td>
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</table>
\[ W = \frac{2}{3} u_1^2 + u_2 k_z - \frac{k_z^2}{3} + \frac{l (u_1 + u_2)}{2} + \frac{(u_i - u_e)(u_i - k_z^2)}{2l} - \frac{(u_i - k_z^2)^2}{4l^2} - \frac{l^2}{12}. \]

(e) \[ q_i = 0 \ , k_z = l + \sqrt{u_1} \ , p_z = u_i - k_z^2 \ , q_z = 2k_z \ , R = 2k_z(u_i - k_z^2) \ , CS = \frac{4}{3} k_z^3 \ , W = R + CS. \]

(f) \[ q_i = 0 \ , p_z = \frac{2}{3} u_2 \ , q_z = 2 \sqrt{\frac{u_2}{3}} \ , R = \frac{4}{3} u_2 \sqrt{\frac{u_2}{3}} \ , CS = \frac{4}{3} \left( \frac{u_2}{3} \right)^2 \ , W = \frac{16}{3} \left( \frac{u_2}{3} \right)^2. \]

Appended drawing 1. Comparison about CS with exogenously determined \( u_2 \).
E means there are equivalent values under monopoly and competition. M means CS is higher under monopoly while C means under competition.

Appended drawing 2. Comparison about W with exogenously determined \( u_2 \).
E means there are equivalent values under monopoly and competition. M means W is higher under monopoly while C means under competition.

Appended drawing 3. Segments corresponding to 5 situations under monopoly with exogenous \( u_2 \).

Appended drawing 4. Segments corresponding to 6 situations under competition with exogenous \( u_2 \).
Appended drawing 5.

Appended drawing 6.

Appended drawing 7. Segments corresponding to 5 situations under monopoly with endogenous $u_2$.

Appended drawing 8. Segments corresponding to 6 situations under competition with endogenous $u_2$. 
Appended drawing 9. Comparison of IDI
E means there are equivalent values under monopoly and competition. M means IDI is higher under monopoly while C means under competition. N means no product innovation occurs.

Appended drawing 10. Comparison of INI
E means there are equivalent values under monopoly and competition. M means INI is higher under monopoly while C means under competition. N means no product innovation occurs.

Appended drawing 11. Comparison about CS with endogenous u2
E means there are equivalent values under monopoly and competition. M means CS is higher under monopoly while C means under competition.

Appended drawing 12. Comparison about W with endogenous u2
E means there are equivalent values under monopoly and competition. M means W is higher under monopoly while C means under competition.