Time Varying Volatility Modeling of Pakistani and leading foreign stock markets

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Abstract

This study estimates the volatility of Pakistani and leading foreign stock markets. Daily data are used from nine international equity markets (KSE 100, NIKKEI 225, HIS, S&P 500, NASDAQ 100, DOW JONES, GADXI, FTSE 350 and DFMGI) for the period of Jan, 2005 to Nov, 2014. The whole data set is used for modeling of time varying volatility of stock markets. Univariate GARCH type models i.e. GARCH and GJR are employed for volatility modeling of Pakistani and leading foreign stock markets. The residual analysis also employed to check the validity of models. Our study brings important conclusions for financial institutions, portfolio managers, market players and academician to diagnose the nature and level of linkages between the financial markets.

Key words: Volatility, Equity Market, GARCH and GJR.

1. Introduction

Volatility modeling is one of the leading issue now a days addressed by financial econometrics. Predictability of time varying volatility is the elementary purpose of financial econometric modeling. In financial markets risk is a synonym of volatility. The understanding and predictability of time varying volatility modeling is significantly important for asset allocation, strategies of global hedging and pricing of internal securities. In econometrics the main purpose of modeling of time series is to estimate the conditional mean, some theoretical models are used to estimate conditional variance, it is also known as volatility. These models are employed to analyze the historical behavior of volatility, future prediction of volatility, examining series of asset return by considering volatility clustering, leverage effect and persistence. The volatility clustering are piles of low and high values of financial asset return. The volatility clustering is commonly seen in financial time series. The persistence of shock is a measure to demonstrate how much time a price shock takes for decay in financial time series. Leverage effect illustrates the negative correlation between current asset return and future volatility of asset return. All this information can be obliging in portfolio allocation and hedging.

2. Literature review

This section briefly discusses previous studies. There's a long debate on volatility modeling between the intra and cross financial markets in financial Econometrics literature. Many researchers have presented an enormous empirical and theoretical work to validate their particular selected models. This review emphasizes on estimating time varying volatility, particularly, in case of Pakistani and foreign stock markets. The volatility modeling has been studied in the financial econometric literature in case of Pakistan. Most of the studies also the found volatility spillover effect transmitted from global financial markets to Pakistani financial markets, [see, e.g., Ali and Afzal (2012), Zia-Ur-Rehman et al. (2011), Attari and Safdar (2013), and Tahir et al. (2013)]. Sajid et al. (2012), employed ARMA-GARCH for measurement of inflation and inflation uncertainty. Jabeen and Khan (2014) employed GARCH model to find out "Exchange rate volatility by macroeconomic fundamentals in Pakistan". [see, e.g., Qayyum and Khan (2014), Qayyum and Kemal (2006), Khalil et al. (2013), Zia and Zahid (2011), Gomez and Ahmad (2014), and Bashir et al. (2014)]. investigated the volatility spillover effect and estimate volatility of foreign exchange market and Pakistan stock markets by using different econometric tools. Padhi and Lagesh (2012) estimated volatility by using DCC-GARCH and also found Information transmission mechanisms persists through return and volatility, it plays a significant role in determining the distribution and financial integration across the global financial markets. Yang and Doong (2004) estimated volatility and explore relationship between the stock market prices and the foreign exchange market prices in case of G-7countries. Choi et al. (2009) examined the volatility and integration between the exchange market and stock market in case of New Zealand. Sinha and Sinha (2010) investigated the volatility modeling and dynamic relationships between India, UK, Japan and USA, incorporating the structural change by using GARCH type modeling, concluded that the Japan and USA stock market's volatility impacted Indian stock markets. Sok-Gee and Karim (2010) examined volatility and volatility spillover between five countries of ASEAN, Japan and USA. Abou-Zaid (2011) estimated Volatility and Spillover Effects In Emerging MENA Stock Markets.

3. Econometric Methodology and Model Specification

To describe the variation of conditional variance with respect to time, Engle (1982) proposed Autoregressive conditional hetroscedastic (ARCH) model. Although ARCH model is a substantial contribution in econometric tools, it has some problems like long lag length and non-negativity restriction on parameters. Bollerslev (1986) introduced generalized autoregressive conditional heteroskedastic (GARCH) model, which improves the unique specification with the addition of lag value of conditional variance, which acts like smoothing term. GARCH model cannot analyze leverage effect. For this Glosten, Jagannathan & Runkle (1993) proposed GJR model. GJR model is a significant extension of standard GARCH model; it contains asymmetric term in conditional variance equation.

There are dozens of univariate and multivariate (ARCH) type model. To avoid any non-convergence problem in this study we employ appropriate univariate GARCH type model such as GARCH (p, q) and GJR (p, q) to estimate volatility of Pakistani and leading foreign stock markets. The GARCH (p, q) and GJR (p, q) Univariate models are capable of exploring better volatility dynamics.

The financial series at level are trendy in nature. It is impossible to estimate a robust model if the series is trendy. To deal with trend we used the log difference return.

$$R_t = log_e(l_t/l_{t-1})$$

l_t= Financial time series at level i.e. stock indices and exchange rates at the end of time t.

l_{t-1}= First lag of financial time series.

Granger and Andersen in (1978) anticipated that the conditional variance depends upon the predicted past value of return series.

$\gamma_t = \varepsilon_t r_{t-1}$ ………………………………………… (3.1)

The conditional variance is

$V\left(\frac{\gamma_t}{r_{t-1}}\right) = \sigma^2 r_{t-1}^2$ ………………………………………… (3.2)

There is no restriction for unconditional variance, either it is unspecified or zero. Then another famous approach came at front to find the ARCH effect in return series.

3.1 ARCH (q) Model

Robert F. Engle in (1982) introduced the Autoregressive conditional hetroscedastic (ARCH) model. This model overcomes all short comings which exist in previous models. In this model Engle, introduced conditional mean and conditional variance equations. Empirically the conditional mean equation follows ARMA (p, q) process and the conditional variance depends upon the square of past values of error process ε_t.

The general description of ARCH model is

Conditional mean equation

$R_t = \alpha_0 + \beta X_t + \varepsilon_t$ ………………………………………… (3.3)

Where $\varepsilon_t \sim N(0, \sigma_t^2)$

Conditional variance equation

$\sigma_t^2 = \theta_0 + \sum_{i=1}^{q} \theta_i \, \varepsilon_{t-1}^2$ ………………………………………… (3.4)

Where $\theta_0 > 0, \theta_i \geq 0$ $\quad i$= 1,2,…….., q

In conditional mean equation R_t represents the return which is linear function of X_t. $where\ \beta$ shows the vector of parameters. Empirically βX_t illustrates ARMA (m, n) process with different specifications. In some cases it may be ARMA (0, 0). According to the "Efficient Market Hypothesis (EMH)" R_t represents mean reversion behavior and it is unpredictable. In conditional variance equation the restriction on coefficients is that they must be non-negative. σ_t^2 Represents conditional variance, which depend upon lags of squared past value of ε_t process.

3.2 GARCH (p, q) Model

Linear ARCH (q) model has some problems first, sometime takes long lag length 'q' due to this number of parameters are going to increase as result loss of degree of freedom. Second, imposition of non-negativity condition on parameters of conditional variance equation. Bollerslev (1986) proposed generalized extension of ARCH (q) model Generalized autoregressive conditional hetroscedastic (GARCH) model.

The general description of GARCH model is

Conditional mean equation

$$R_t = \alpha_0 + \beta X_t + \varepsilon_t \qquad \text{... (3.5)}$$

Where $\varepsilon_t \sim N(0, \sigma_t^2)$

Conditional variance equation

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{q} \theta_i\, \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \varphi_j\, \sigma_{t-1}^2 \qquad \text{... (3.6)}$$

Where $\theta_0 > 0, \theta_i \geq 0, \varphi_j \geq 0$

In GARCH (p, q) model the conditional variance depends upon square of past values of processε_t and lag of conditional varianceσ_{t-1}^2. The condition of non-negativity of parameter also applied in this model.

3.3 Asymmetric GARCH models

Simple GARCH type models deal with the symmetric effect of bad and good news on volatility. These models do not take into account the asymmetries which are associated with the distribution. In financial econometrics literature Asymmetric GARCH type models consider the asymmetries of response to bad or good news. Asymmetric GARCH models account for leverage effect. The leverage effect indicates the negative correlation between the assets returns and the volatility of the assets return (Black 1976), means the magnitude of bad and good news are different.

Engle and Victor (1993) conducted a brief discussion on how univariate GARCH type model capture the impact of bad news. They have used Japan stock market data. They argued that the GJR model is the best model to capture the asymmetries. According to them EGARCH model capture the Asymmetries but when we employ the EGARCH model the standard deviation is going too high, as compare to GJR model. They also concluded that GJR model is best for capturing the asymmetries. Bollerslev and Mikkelsen (1996) The GARCH type models are easily deduced as ARMA type models for second order conditional moment and data generating process of conditional variance. GARCH type models commonly employed to quantify the persistence of the expected process of conditional variance.

3.3.1 GJR (p, q) Model

Glosten, Jagannathan and Runkle introduced (GJR) model in 1993. GJR model is a significant extension in simple GARCH model. This model also captures the asymmetries in ARCH process. GJR model also account for the leverage effect in a financial series.

The general representation of the GJR model is:

Conditional mean equation

$$R_t = \alpha_0 + \beta X_t + \varepsilon_t \qquad \text{..............................} \ (3.9)$$

Where $\varepsilon_t \sim N(0, \sigma_t^2)$

Conditional variance equation

$$\sigma_t^2 = \theta_0 + \sum_{i=1}^{q} \theta_i \, \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \delta_i \, \varepsilon_{t-i}^2 G_t + \sum_{i=1}^{p} \varphi_j \, \sigma_{t-j}^2 \qquad \text{..................} \ (3.10)$$

Where $\theta_0 > 0$, $\theta_i \geq 0$, $\varphi_i \geq 0$

$0 \leq \delta_i \geq 1$ Range of parameter of leverage effect.

Gt = 1 when $\varepsilon_{t-1} < 0$ and Gt = 0 when $\varepsilon_{t-1} \geq 0$

Gt = 1 when $\varepsilon_{t-1} < 0$ illustrates bad news or the negative shock and Gt = 0 when $\varepsilon_{t-1} \geq 0$ indicates good news or positive shock. GJR model also shows that bad news has more impact $(\theta_i + \delta_i)$. The good news has less impact (θ_i). If the $\delta_i > 0$ means that there is leverage effect and shows that response to shock is distinct. If the $\delta_i = 0$ means symmetric response to distinct shock (In other words both news have same impact). Condition $(\theta_i + \varphi_i + \frac{\delta_i}{2} < 1)$ shows the persistence of shock.

3.4 Residual Analysis

To identify the good fitness of employed model we use post estimation results (Residual analysis). The Jarque Bera test (Normality test) employs to check the null hypothesis that distribution of return series is normal. Q-stat (return series) employs to validate the null hypothesis, there is no serial autocorrelation in standardized residuals. Q^2-stat (return series) checks the null hypothesis, there is no serial autocorrelation in squared standardized residuals. LM-ARCH with the Null hypothesis, there is no ARCH effect in return series. Due to convergence problem we check Q-stat and Q^2-stat up to 10^{th} lag. LM-ARCH test up to 5^{th} lag.

3.5 Description of Data and sources

The daily data of stock market indices are used form 2005 to 2014. These stock markets are taken from ASIA, Europe, America and Gulf countries. From US S&P 500, DOW JONES (DJI), and NASDAQ 100 are used. From EU London (FTSE 350) and German (GDAXI) stock exchange data are taken. From Asia Pakistan (KSE 100), Japan (NIKKEI 225) and Hong Kong (HIS) stock market indices are used. Dubai financial market index (DFMGI) is taken from Gulf countries.

4. Estimations and Analysis

4.1 Graphical Analysis

Figure 4.1.1.a Graphs of series at level of stock indices

Figure 4.1.1.a and 4.1.1.b given above show in the beginning all series have upward trend than sharp decline and then again there is an upward trend continuously. This means that series are trendy at level. In figure 4.1.1.a the series are Karachi stock market (KSE 100), Nikkei 225,

Hang Seng (HIS), Standard and Poor (S&P 500) and Dow Jones. In figure 4.1.1.b the series are Nasdaq 100, FTSE 350, GDAXI, Dubai financial market (DFMGI). Daily data is used from 3rd Jan, 2005 to 28th Nov, 2014.

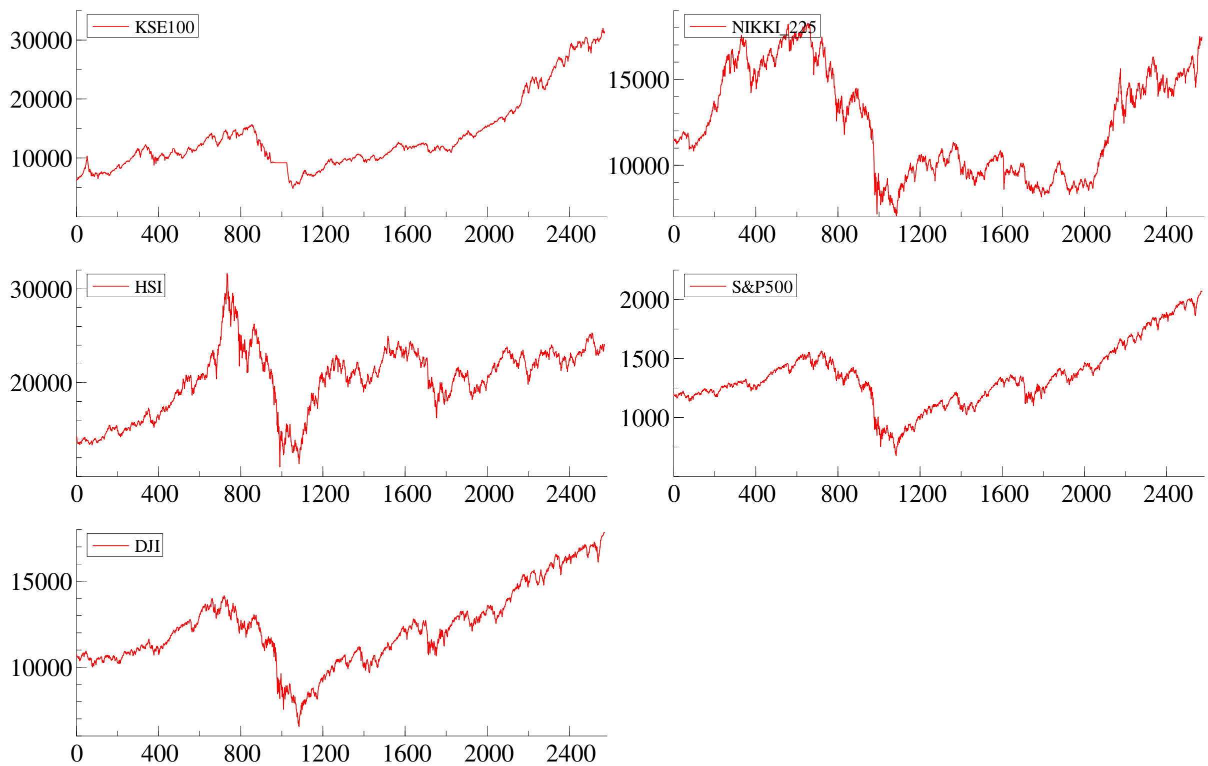


Figure 4.1.1.b Graphs of series at level

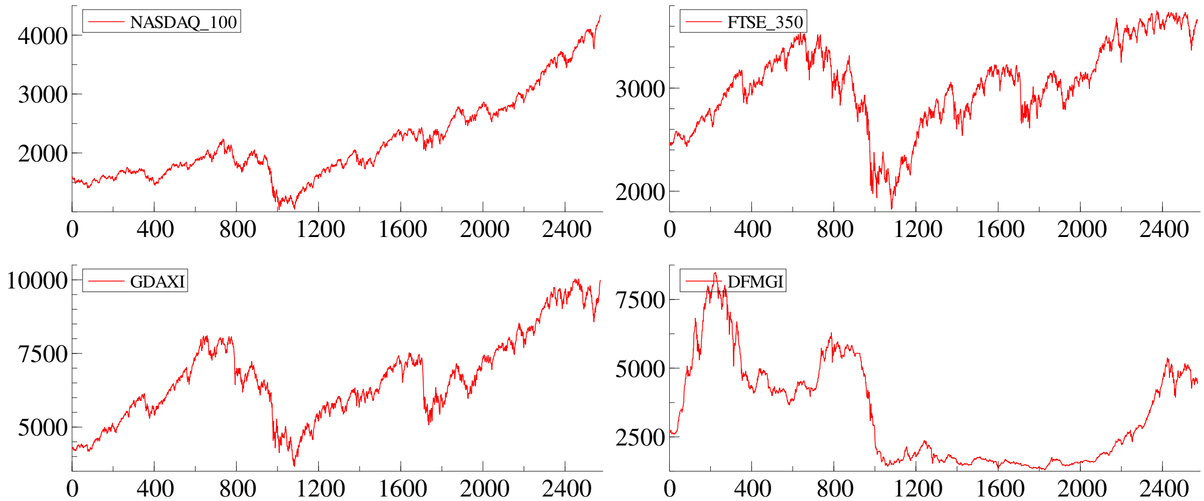
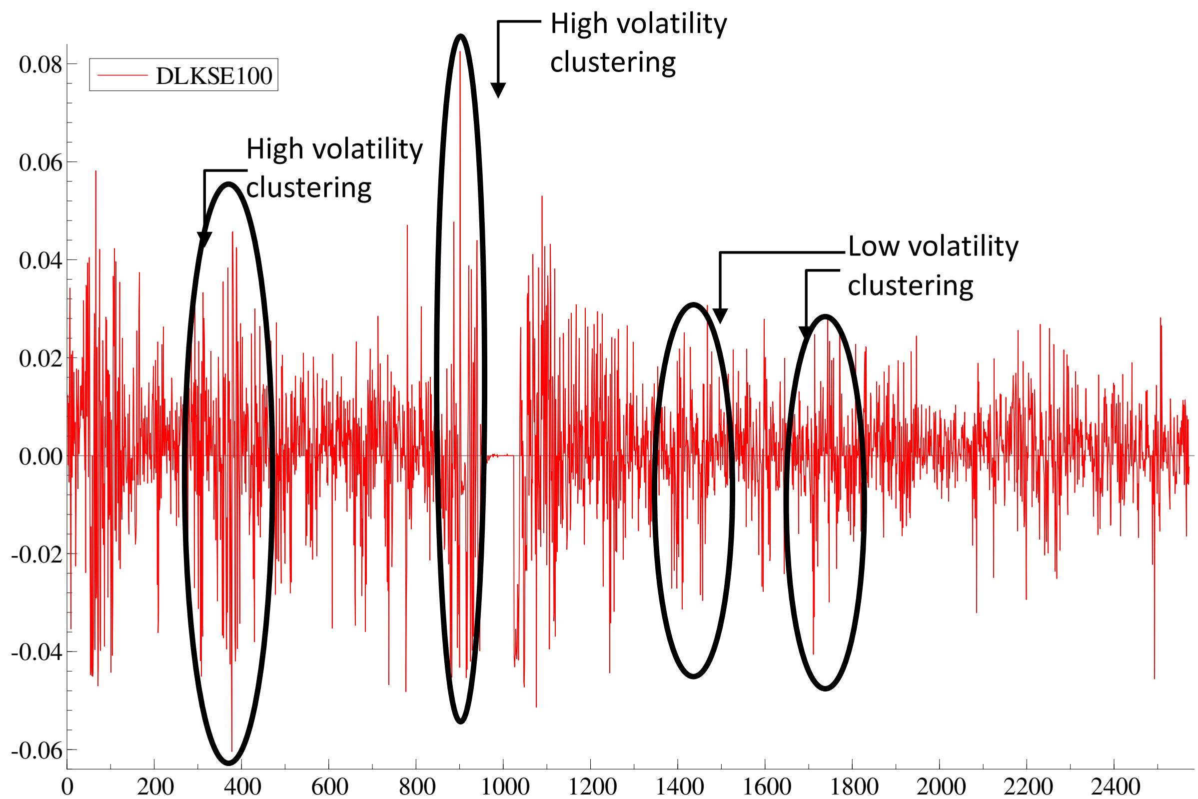


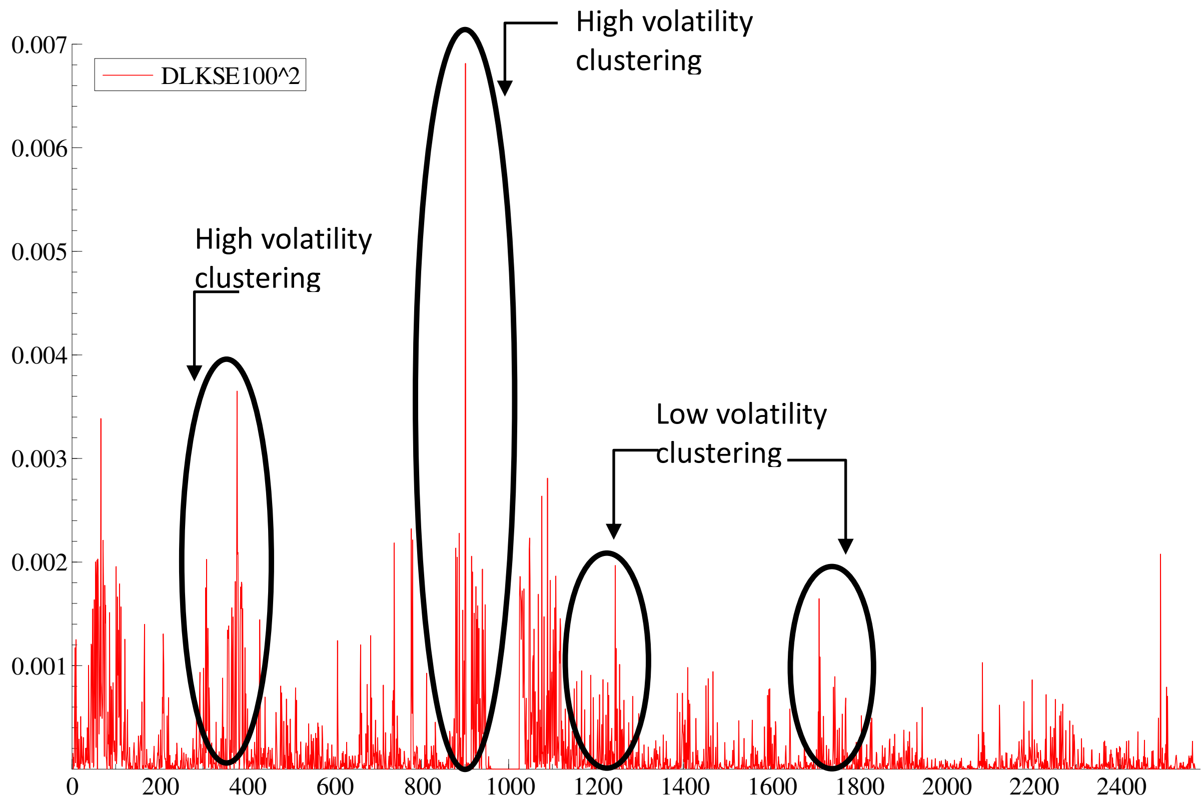
Figure 4.1.2 given below represents return series of Karachi stock market indices. It is impossible to find out robust model if the series is trendy, we use log difference return series to deal with trend. In financial econometrics, spread characterized as volatility. In return series spread does not remain constant, it is known as Hetroscedasticity. The circles in figure 4.1.2 are indicating the low and high volatility which denote the spread autocorrelation. According to "The Efficient Market Hypothesis (EMH) return are unpredictable and show mean reversion behavior". That's why all return series have mean reversion behavior. If we combine all effects it indicate ARCH (Auto-Regressive Conditional Hetroscedasticity) effect. We can easily distinguish between low volatility clustering and high volatility clustering period. The greater depreciation from constant level (mean of return series) indicates high volatility clustering and less depreciation illustrate low volatility clustering. In the same way we can plot and analyze return series of other stock markets.

Figure 4.1.2 Graph of given return series



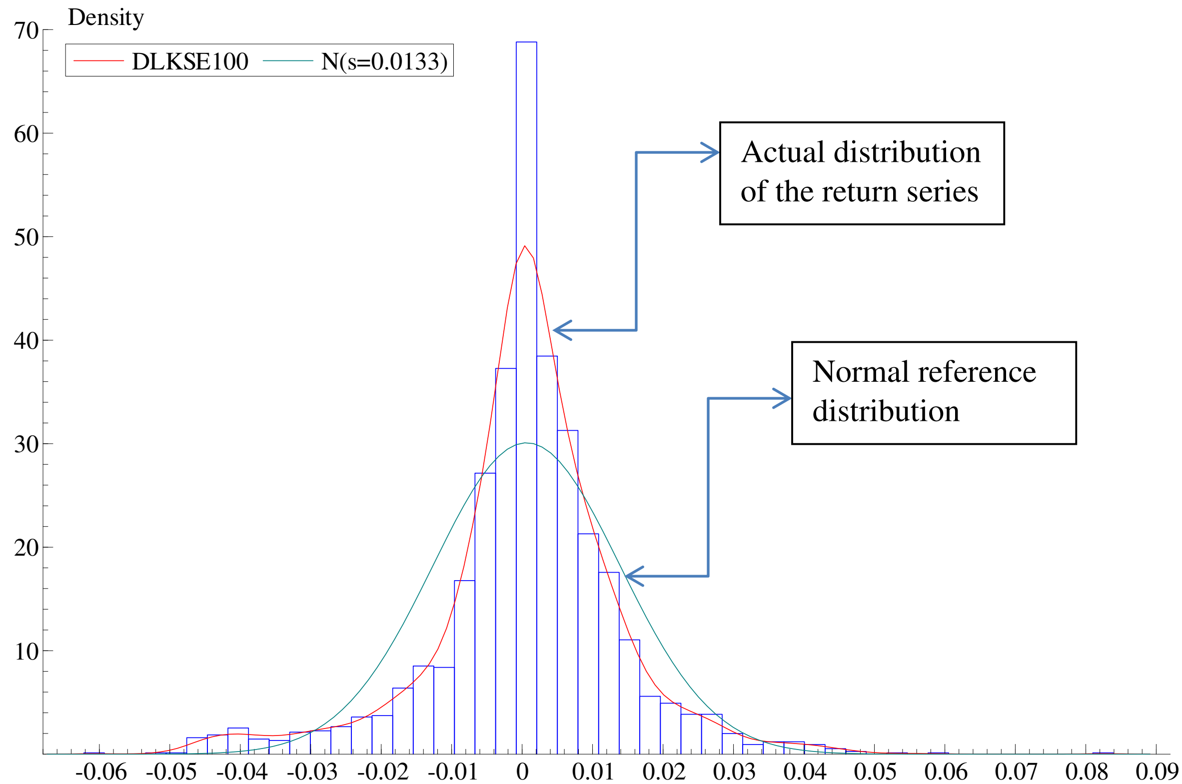
In figure 4.1.3 given below shows squared returns series of KSE 100. The graph of square return series have "spiky" look signifying variation in square return. Circles indicate high volatility and low volatility. It also shows that extreme values (outliers) of return series contribute more to the high volatility. Square of the return series is also known as variance of the return series means these graphs illustrate the dispersion. In the same way we can analyze square return series of other stock markets' indices.

Figure 4.1.3 Graph of Squared return series



The figure 4.1.4 given below illustrates the distribution of the return series. The distribution of return series is non-normal. In this graph blue line shows the normal reference distribution of return series. The red line indicates the actual distribution of the return series. Histograms describe the outliers (extreme values) in return series. The distribution of return series have heavy tails and is leptokurtic. This all is due to different response of market players by having same information from the same market.

Figure 4.1.4 Graphs distribution of the return series



In figure 4.1.5 given below presents ACF (Auto-correlation function) and PACF (Partial Auto-correlation function) of return series. The green straight lines in this graph show 95 percent confidence interval, if any bar of ACF and PACF outside these lines means at that lag the values are auto correlated in other words significantly vary from zero. The ARMA (p, q) process specify through the significant lags of ACF and PACF. The ACF specify the MA (q) process PACF specify the AR (p) process. In this graph 1st, 2nd, 3rd, 4th, 10th, 17th and 18th lags of ACF are significant and 1st, 3rd, 4th, 10th, 11th, 12th, 17th and 18th lags of PACF are significant, these lags format ARMA (p, q) process in conditional mean equation. It means auto correlation and partial autocorrelation exist in the return series. We can also analyze cyclical behavior in return series through ACF and PACF graphs.

Figure 4.1.5 Graphs of ACF and PACF of return series

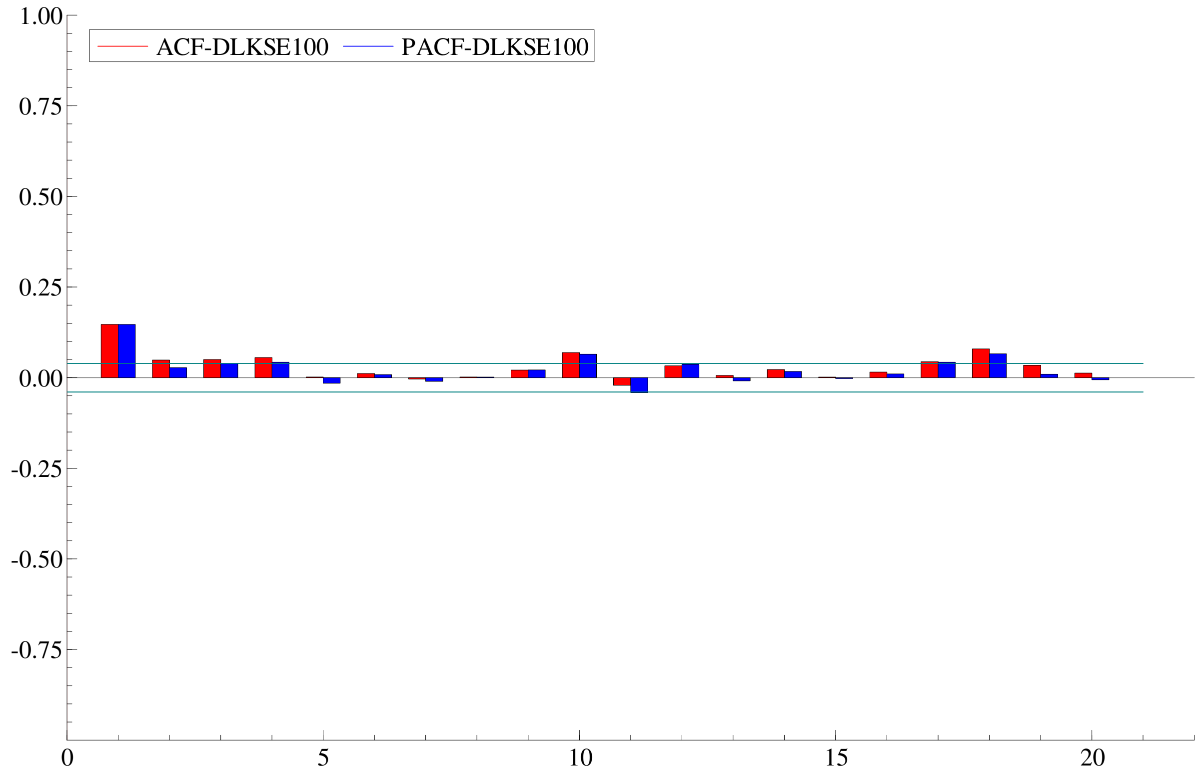
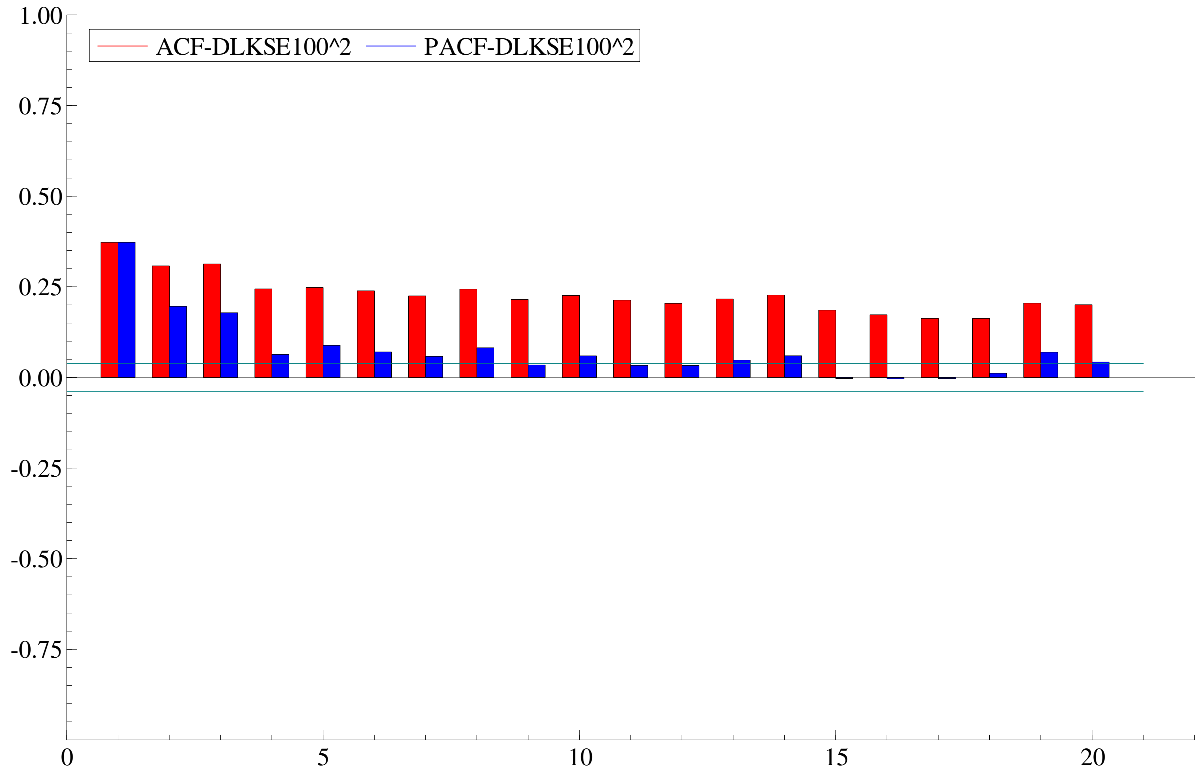


Figure 4.1.6 given below show the graph of ACF and PACF of square return series. 1st to 20th lags of ACF are significantly differ from zero and 1st……...8th, 10th, 13th, 14th, 19th and 20th lags of PACF are statistically significant. In the same manner square return series ACF and PACF may provide an indication about the critical lags in conditional variance equation structure of GARCH (p, q) model. Means there is autocorrelation and partial autocorrelation in the square return series.

Figure 4.1.6 Graphs of ACF and PACF of square of return series



The initial statistics of return series of stock markets indices are given below unveil some indications about the behavior of stock markets. The distributions of return are non-normal, heavy tails and leptokurtic. The mean of all return series are about zero which implies that return series show mean reversion behavior. Standard deviation of return series describe the dispersion from mean value which return series have greater standard deviation it means more deviation from mean value. The skewness deals with the asymmetry of the distribution. The distributions of KSE 100, S&P 500, NASDAQ 100, DJI, NIKKEI 225, FTSE 350 and DFMGI return series are negatively skewed which means that the return of these stock markets are less than average return. The distributions of HIS and GDAXI are positively skewed which imply the returns of these markets are more than average return. The Jarque-Bera test with null hypothesis of normal distribution is employed. Jarque-Bera statistics of all return series are significant means the distribution of all return series are non-normal.

Table 4.2 Summary statistics

Variables	Summary statistics								
	Mean	**Standard deviation**	**Skewness**	**Jarque Bera**	**Excess Kurtosis**	**Q-stat (5)**	**Q^2-stat (5)**	**ARCH 1-2**	**KPSS**
KSE 100	0.0006	0.0132	-0.3854 (0.000)	1098.1 (0.000)	3.1075 (0.000)	76.120 (0.000)	1167.51 (0.000)	266.88 (0.000)	0.2073
S&P 500	0.0002	0.0127	-0.3409 (0.000)	14088 (0.000)	11.448 (0.000)	45.484 (0.000)	1131.31 (0.000)	266.72 (0.000)	0.1965
NASDAQ 100	0.0003	0.0136	-0.1587 (0.000)	7985.9 (0.000)	8.6282 (0.000)	24.928 (0.000)	765.777 (0.000)	156.96 (0.000)	0.2005
DJI	0.0001	0.0116	-0.0851 (0.077)	14168 (0.000)	11.499 (0.000)	45.037 (0.000)	1123.85 (0.000)	283.89 (0.000)	0.1548
NIKKEI 225	0.0001	0.0153	-0.5737 (0.000)	8597.9 (0.000)	8.8850 (0.000)	10.564 (0.000)	1396.71 (0.000)	489.45 (0.000)	0.1994
HIS	0.0002	0.0156	0.0459 (0.000)	10971 (0.000)	10.120 (0.000)	8.3870 (0.000)	1361.38 (0.000)	361.66 (0.000)	0.0525
FTSE 350	0.0001	0.0118	-0.1879 (0.000)	7288.8 (0.000)	8.2401 (0.000)	39.367 (0.000)	1130.0 (0.000)	147.29 (0.000)	0.0569
GDAXI	0.0003	0.0137	0.0297 (0.537)	5510.5 (0.000)	7.1719 (0.000)	16.783 (0.000)	686.71 (0.000)	111.39 (0.000)	0.07411
DFMGI	0.0001	0.0183	-0.8778 (0.000)	13612 (0.000)	11.135 (0.000)	32.381 (0.000)	166.23 (0.000)	44.647 (0.000)	0.4874

Null Hypotheses **(All Null Hypotheses are for n^{th} order)**

KPSS H_0: Return series is level stationary, Asymptotic significant values 1% (0.739), 5% (0.463), 10% (0.347). Q-stat (return series) there is no serial autocorrelation. Q^2-stat (square return series) H0: there is no serial autocorrelation. Jarque-Bera H0: distribution of series is normal. LM-ARCH H_0: there is no ARCH effect. Use these Asymptotic Significance values of t-stat 1% (0.01), 5% (0.05), 10% (0.1) and compare these critical values with P-values (Probability values). P-values are in the parenthesis.

The Excess kurtosis of all returns series are significant which means that return series distributions are leptokurtic and also indicates that probability of large values is more than normal return series. Q-stat of return series are significant, rejecting the null hypothesis of no autocorrelation return series. This shows that there is serial autocorrelation in return series. Q-stat of squared return series are significant, rejecting the null hypothesis of no autocorrelation in squared return series. This shows that there is serial autocorrelation in square return series. LM-ARCH test validates that there is ARCH effect in return series. KPSS is a unit root test with null hypothesis of stationary series. KPSS test results of all variable show that the estimated values lies in acceptance region [less than given three significance values 1% (0.739), 5% (0.463), 10% (0.347)] means the null hypothesis is accepted, return series are level stationary.

4.3 Volatility Model specifications of Return Series

In this section volatility models of stock markets (area under study) are presented to understand the Data Generating Process of all financial return series (area under study). It will be helpful to understand the mean and volatility structure of financial return series (area under study). Volatility modeling is a striking issue for market players, portfolio managers, academicians and policy makers. A lot of empirical work on volatility modeling exists in financial econometrics, the predictability and modeling of volatility is still a challenge for researchers. Many researcher in their studies employed GARCH type model for volatility modeling. In these studies researchers employed different ARCH-GARCH family models to describe volatility modeling and volatility forecasting [Vijayalakshmi and Gaur (2013); Pasha et al. (2007); Kamal et al. (2011); Khan and Parvez (2013); Chand et al. (2013); Jabeen and Saud (2014); Sajid et al. (2012) and Faisal et al. (2012)]. In this study we employ GARCH (p, q) and GJR (p, q) model for volatility modeling and exploring spillover effect. The GARCH and GJR models mostly employed by the researchers due to unique characteristics of these models. These univariate models are best to give a better explanation of asset volatility modeling.

The GARCH model is employed for HIS volatility modeling. The estimated conditional mean equation (4.1) is from equation (3.5) and the estimated conditional dispersion equation (4.2) is from equation (3.6). The P-values are in parenthesis.

$$R_t = 0.0005 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.1)$$

(0.0000)

$$\sigma_t^2 = 0.0077 + 0.0523\varepsilon_{t-2}^2 + 2.0077\sigma_{t-1}^2 + 1.7204\sigma_{t-2}^2 + 0.6600\sigma_{t-3}^2 \ldots \quad (4.2)$$

(0.0830) (0.0000) (0.0000) (0.0000) (0.0000)

The GJR model is employed for KSE 100 volatility modeling. The estimated conditional mean equation (4.3) is from equation (3.9) and the estimated conditional dispersion equation (4.4) is from equation (3.10). The P-values are in parenthesis.

$$R_t = 0.0008 + 1.0000R_{t-1} - 0.9000\varepsilon_{t-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4.3)$$

(0.0000) (0.0000) (0.0000)

$$\sigma_t^2 = 0.0000 + 0.1460\varepsilon_{t-2}^2 + 0.3234\varepsilon_{t-1}^2 G_t + 0.8031\sigma_{t-1}^2 \ldots\ldots\ldots\ldots \quad (4.4)$$

(1.0000) (0.0000) (0.0000) (0.0000)

The employed models in table 4.3.1 given below describe the data generating process of the return series. The estimated parameters of the employed models are statistically significant. In KSE 100 model AR (1) term is statistically significant which means that current return of KSE 100 depends upon 1st lag. MA (1) term in this model is also differ from zero, shows relationship between past and current variations. The leverage effect term δ_1 in KSE 100 and NIKKEI 225 models are significant, indicates that the current return negatively correlated with future volatility, no leverage effect is found in HIS stock return series. Most of the parameters are statistically significant at 5% level of significance. ARCH and GARCH terms are also significant in three models means the return series are subject to ARCH effect. The persistence of shock of the return series are KSE 100, NIKKEI 225 and HSI all are close to 1 which means that the persistence of ARCH and GARCH effect take long time to decay.

Tables 4.3.1 also illustrate the post estimation results (Residual analysis). The Jarque Bera test (Normality test) results show non normal residuals. The Q-stat are insignificant up to 10th lags accept null hypothesis means no serial autocorrelation in the standardized residuals. The Q-stat on squared standardized residuals are insignificant up to 10th lags accept null hypothesis means no serial autocorrelation in squared standardized residuals. LM-ARCH test is also insignificant up to 5thlags accept null hypothesis means no ARCH effect remain in series residuals.

Table 4.3.1 Volatility models of Asian Stock markets Return series

Return series / **Parameters**	**KSE-100** ARMA(1,1) GJR (1,1)	**NIKKEI-225** ARMA(0,0) GJR (1,1)	**HIS** ARMA(0,0) GARCH (3,2)
Conditional Mean Equation			
Constant α_0	0.0009 (0.7538)	0.0006 (0.0093)	0.0006 (0.0013)
AR(1) ϑ_1	1.0000 (0.0000)	-----------	-----------
MA(1) $\emptyset_1$	-0.9000 (0.0000)	-----------	-----------
Conditional Variance Equation			
Constant θ_0	0.0000 (1.0000)	0.0475 (0.0007)	0.0070 (0.0830)
ARCH(1) θ_1	0.1460 (0.0003)	0.0273 (0.0240)	-----------
ARCH(2) θ_2	-----------	-----------	0.0523 (0.0000)
GARCH(1) φ_1	0.8032 (0.0000)	0.8839 (0.0000)	2.0078 (0.0000)

GARCH(2) φ_2	-----------	-----------	1.7205 (0.0000)
GARCH(3) φ_3	-----------	-----------	0.6600 (0.0000)
GJR(1) δ_1	0.3234 (0.0000)	0.1287 (0.0000)	-----------
Persistence of shock	1.1109	0.9755	0.9996

Null Hypotheses(All Null Hypotheses are for nth order)

AR (p) H0: ϑ_i= 0 No AR Process, MA (q) H0: $\emptyset_i$= 0 No MA Process, ARCH H0: θ_i= 0 No ARCH effect, GARCH H0: φ_i= 0 No GARCH effect, Leverage effect H0: δ_i= 0 No leverage effect. P-values are in the parenthesis.

Residual Analysis

Parameter / Series	Jarque Bera	Q-Stat (5)	Q-Stat (10)	Q^2-Stat (5)	Q^2-Stat (10)	LM -ARCH (1-2)	LM-ARCH (1-5)
HIS	128.04 (0.0000)	1.7793 (0.8787)	5.1972 (0.8776)	5.2051 (0.3913)	13.983 (0.5267)	0.3964 (0.6727)	0.5536 (0.7356)
NIKKEI 225	509.46 (0.0000)	2.0605 (0.8407)	6.0220 (0.8134)	0.6678 (0.8807)	5.1253 (0.7440)	0.2047 (0.8149)	0.1380 (0.9835)
KSE 100	7.0272 (0.0000)	0.0019 (0.9999)	0.0024 (1.0000)	0.0020 (0.9999)	0.0039 (1.0000)	0.0004 (0.9999)	0.0004 (1.0000)

Null Hypotheses(All Null Hypotheses are for nth order)

Q-stat (return series) there is no serial autocorrelation. Q^2-stat (square return series) H0: there is no serial autocorrelation. Jarque-Bera H0: distribution of series is normal. LM-ARCH H_0: there is no ARCH effect. P-values are in the parenthesis.

The employed models given below in table 4.3.2 describe the data generating process of the return series. The estimated parameters of the fitted models are statistically significant. In S&P 500 and DOW JONES models AR term is statistically significant which means that current return of these market are only depends upon lag values. In NASDAQ model AR term is insignificant which means current return of this market not depends upon lag values. MA (1) term in S&P 500 and DOW JONES model is differ from zero, shows relationship between past and current variations of return series. Most of the parameters are statistically significant at 5% level of significance. ARCH and GARCH terms are also significant in three models means these return series encompass ARCH and GARCH effect. The persistence of shock of the return series are S&P 500, NASDAQ 100 and DOW JONES all are close to 1 which means that the persistence of ARCH and GARCH effect take long time for decay.

Table 4.3.2 also illustrate the post estimation results (Residual analysis). The Jarque Bera test (Normality test) results show non normal residuals. The Q-stat are insignificant up to 10th lags accept null hypothesis means no serial autocorrelation in the standardized residuals. The Q-stat on squared standardized residuals are insignificant up to 10th lags accept null hypothesis means no serial autocorrelation in squared standardized residuals.

Table 4.3.2 Volatility models of American Stock markets Return series

Return series / **Parameters**	**S&P 500** ARMA(1,1) GARCH (1,2)	**NASDAQ 100** ARMA(0,0) GARCH (1,1)	**DOW JONES** ARMA(2,1) GARCH (1,1)
Conditional Mean Equation			
Constant α_0	0.0008 (0.0000)	0.0011 (0.0000)	0.0008 (0.0000)
AR(1) ϑ_1	0.7395 (0.0000)	-----------	-0.9511 (0.0000)
AR(2) ϑ_2	-----------	-----------	-0.0560 (0.0039)
MA(1) $\emptyset_1$	-0.7992 (0.0000)	-----------	0.8958 (0.0000)
Conditional Variance Equation			
Constant θ_0	0.0219 (0.0000)	0.0232 (0.0000)	0.0130 (0.0005)
ARCH(1) θ_1	-----------	0.0890 (0.0000)	0.1112 (0.0000)

ARCH(2) θ_2	0.1433 (0.0000)	-----------	-----------
GARCH(1) φ_1	0.8481 (0.0000)	0.8991 (0.0000)	0.8841 (0.0000)
Persistence of shock	0.9915	0.9883	0.9954

Null Hypotheses(All Null Hypotheses are for n^{th} order)

AR (p) H0: ϑ_i= 0 No AR Process, MA (q) H0: $\emptyset_i$= 0 No MA Process, ARCH H0: θ_i= 0 No ARCH effect, GARCH H0: φ_i= 0 No GARCH effect. P-values are in the parenthesis.

Residual Diagnostic Test

Parameter / Series	Jarque Bera	Q-Stat (5)	Q-Stat (10)	Q^2-Stat (5)	Q^2-Stat (10)	LM -ARCH (1-2)	LM-ARCH (1-5)
S&P 500	524.96 (0.0000)	4.5842 (0.2049)	7.4715 (0.4867)	1.2465 (0.5361)	9.7895 (0.2008)	0.2319 (0.7930)	0.2597 (0.9350)
NASDAQ 100	213.74 (0.0000)	2.8643 (0.2387)	5.1957 (0.6360)	5.8315 (0.1201)	14.408 (0.0717)	2.7272 (0.0656)	1.1573 (0.3279)
DJI	438.61 (0.0000)	5.2781 (0.0714)	8.9697 (0.2548)	8.5410 (0.0360)*	17.475 (0.0255)*	3.8500 (0.0214)*	1.6483 (0.1438)

Null Hypotheses(All Null Hypotheses are for n^{th} order)

Q-stat (return series) there is no serial autocorrelation. Q^2-stat (square return series) H0: there is no serial autocorrelation. Jarque-Bera H0: distribution of series is normal. LM-ARCH H_0: there is no ARCH effect. P-values are in the parenthesis.

LM-ARCH test is also insignificant up to 5^{th}lags accept null hypothesis means no ARCH effect remain in series residuals.

In table 4.3.3 given above describes the data generating process of the return series. The estimated parameters of the fitted models are statistically significant. In GDAXI and DFMGI models AR terms are statistically significant which means that current return of markets are only depends upon 1^{st} lag. In FTSE 350 model AR term is insignificant which means current return of this market not depends upon lag values. MA (1) term in GADXI and DFMGI models are differ from zero, shows relationship between past and current variations of return series. Most of the parameters are statistically significant at 5% level of significance. ARCH and GARCH terms are also significant in three models means these return series encompass ARCH and GARCH effect. The persistence of shock of the return series are FTSE 350 (0.99359), GDAXI (0.99321) and DFMGI (0.99417) all are close to 1 which means that the persistence of ARCH and GARCH effect take long time for decay.

Table 4.3.3 also illustrate the post estimation results (Residual analysis). The Jarque Bera test (Normality test) results show non normal residuals. The Q-stat are insignificant up to 10th lags accept null hypothesis means no serial autocorrelation in the standardized residuals. The Q-stat on squared standardized residuals are insignificant up to 10th lags accept null hypothesis means no serial autocorrelation in squared standardized residuals. LM-ARCH test is also insignificant up to 5thlags accept null hypothesis means no ARCH effect remain in series residuals.

Table 4.3.3 Volatility models of European and Gulf Stock markets Return series

Return series / Parameters	FTSE 350 ARMA(0,0) GARCH (1,1)	GDAXI ARMA(1,1) GARCH (1,1)	DFMGI ARMA(1,1) GARCH (1,1)
Conditional Mean Equation			
Constant α_0	0.0006 (0.0000)	0.0010 (0.0000)	0.0005 (0.2032)
AR(1) ϑ_1	-----------	0.9396 (0.0000)	0.8848 (0.0000)
MA(1) $\emptyset_1$	-----------	-0.9551 (0.0000)	-0.8330 (0.0000)
Conditional Variance Equation			
Constant θ_0	0.0139 (0.0025)	0.0218 (0.0025)	0.0313 (0.0633)
ARCH(1) θ_1	0.1145 (0.0000)	0.0988 (0.0000)	0.0630 (0.0000)

GARCH(1) φ_1	0.8790 (0.0000)	0.8943 (0.0000)	0.9311 (0.0000)
Persistence of shock	0.9935	0.9932	0.9941
Null Hypotheses(All Null Hypotheses are for nth order) AR (p) H0: ϑ_i= 0 No AR Process, MA (q) H0: $\emptyset_i$= 0 No MA Process, ARCH H0: θ_i= 0 No ARCH effect, GARCH H0: φ_i= 0 No GARCH effect, Mean spillover. P-values are in the parenthesis.			

Residual Analysis

Parameter / Series	Jarque Bera	Q-Stat (5)	Q-Stat (10)	Q^2-Stat (5)	Q^2-Stat (10)	LM -ARCH (1-2)	LM-ARCH (1-5)
FTSE 350	134.01 (0.0000)	3.5698 (0.6128)	5.2425 (0.8743)	3.7790 (0.2863)	5.0251 (0.7548)	0.6032 (0.5471)	0.7742 (0.5682)
GDAXI	309.27 (0.0000)	4.5445 (0.2083)	7.4357 (0.4904)	6.9227 (0.0744)	8.9998 (0.3423)	0.8606 (0.4230)	1.4246 (0.2121)
DFMGI	12835 (0.0000)	6.7335 (0.0808)	12.502 (0.1301)	3.4343 (0.3293)	8.3277 (0.4021)	0.2255 (0.7981)	0.6888 (0.6319)

Null Hypotheses(All Null Hypotheses are for n^{th} order)

Q-stat (return series) there is no serial autocorrelation. Q^2-stat (square return series) H0: there is no serial autocorrelation. Jarque-Bera H0: distribution of series is normal. LM-ARCH H_0: there is no ARCH effect. P-values are in the parenthesis.

Conclusion

This study has offered a framework to model the time varying volatility of equity markets by employing the risk models. On the basis of given data sets we employed symmetric GARCH and asymmetric GARCH models to estimate conditional mean equations follow ARMA process and conditional variance equations for risk (dispersion). For the validity of models the residual diagnostic test also employed. KSE 100 and NIKKEI 225 series have asymmetric effect while other series take symmetric effects. The persistence of shock is measure to specify the period of persistence of ARCH and GARCH effect in return series. The leverage effects are also quantified to check the effects of different news on volatility.

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