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Abstract

This paper evaluates the predictability of monthly stock return using out-of-sample (multi-step ahead and dynamic) prediction intervals. Past studies have exclusively used point forecasts, which are of limited value since they carry no information about the intrinsic predictive uncertainty associated. We compare empirical performances of alternative prediction intervals for stock return generated from a naive model, univariate autoregressive model, and multivariate model (predictive regression and VAR), using the U.S. data from 1926. For evaluation free from data snooping bias, we adopt moving sub-sample windows of different lengths. It is found that the naive model often provides the most informative prediction intervals, outperforming those generated from the univariate model and multivariate models incorporating a range of economic and financial predictors. This strongly suggests that the U.S. stock market has been informationally efficient in the weak-form as well as in the semi-strong form, subject to the information set considered in this study.

Keywords: Autoregressive Model, Bootstrapping, Financial Ratios, Forecasting, Interval Score, Market Efficiency

JEL Classification: G12, G14.

1 Introduction

Stock return predictability has been an issue of profound importance in empirical finance. It has strong implications to investment decisions and strategies, as well as to the fundamental concepts such as market efficiency. The empirical literature is extensive, ranging from the seminal works of Campbell and Shiller (1988) and Fama and French (1988) to the notable recent contributions such as Welch and Goyal (2008) and Neely et al. (2014). While a number of recent studies evaluate out-of-sample predictability of stock return, they rely exclusively on point forecasting (e.g., Welch and Goyal, 2008; Campbell and Thompson, 2008; Lettau and Van Nieuwerburgh, 2008; Rapach et al., 2010; Westerlund and Narayan, 2012). A point forecast is a single number as an estimate of the unknown future value. Although it may represent the most likely outcome from a predictive distribution, it carries no information about the degree of intrinsic uncertainty or variability associated. For this reason, one may justifiably argue that comparison of point forecasts only is of limited value for assessing predictability. As Chatfield (1993) and Christoffersen (1998) argue, interval forecast (or prediction interval) is of a higher value to decision-makers, allowing for a more complete and informative evaluation of predictability (see also De Gooijer and Hyndman, 2006; Pan and Politis, 2016). This is particularly so for stock returns which show a high degree of volatility over time. This paper contributes to the extant literature on stock return predictability by evaluating out-of-sample prediction intervals.

A prediction interval consists of an upper and a lower limit between which the future value is expected to lie with a prescribed probability (Chatfield, 1993). As an estimate of possible future scenario, it is substantially more informative than a single value. It shows the possible direction of future value, also giving a clear indication about the extent of uncertainty associated. A tight interval is informative to decision-makers, since they can be highly confident about the future outcome, given the prescribed probability content. In contrast, a wide one carries little information about the future outcome, indicating a high degree of uncertainty. Prediction intervals can be generated from popular linear forecasting models available in many econometric packages, including the predictive regression models for stock return (see, for example, Welch and Goyal, 2008; Amihud et. al, 2004; and Kim, 2014). Conventionally, a prediction interval is constructed based on an asymptotic (normal) approximation to the predictive distribution, ignoring estimation uncertainty. An alternative is the bootstrap method, which provides a non-parametric approximation to the predictive distribution based on data resampling (see Pan and Politis, 2016). It is capable of generating prediction intervals which take full account of estimation uncertainty and without resorting to the normality assumption.

In this paper, we consider prediction intervals based on a range of linear models, which are widely used in practice to predict stock return at the monthly frequency. For the univariate case, an autoregressive (AR) model is used. For the multivariate case, the predictive regression and vector autoregressive (VAR) model are used. The AR model is constructed with an assumption that the stock return depends on its own past only. The AR(0) model represents a naive model where the stock return has no dependency on its own past. The predictive regression specifies that the stock return depends on the past of a predictor such as financial and macroeconomic variables (e.g., Welch and Goyal, 2008; Campbell and Thomson, 2008; Lettau and Van Nieuwerburgh, 2008; Rapach et al., 2010; Westerlund and Narayan, 2012). The VAR model represents a general linear model which specifies the stock return as a function of its own past and the past of its predictor. For the predictive regression and VAR models, we employ bias-corrected parameter estimation to construct prediction intervals free from small sample estimation bias (see Stambaugh, 1999). We mainly consider prediction intervals generated based on the conventional normal approximation to the predictive distribution, but a bootstrap alternative is also considered. As a means of comparison, we use the coverage rate and interval score (Gneiting and Raftery, 2007; p.370). While the former is a dichotomous measure as to whether prediction interval covers the true value or not, the latter is a quality-based measure which captures both accuracy and variability of prediction.

To the best of our knowledge, this paper is the first study examining the stock return predictability using prediction intervals. As already mentioned, the previous studies exclusively evaluate point forecasts, often accompanied by predictive ability tests. Our study represents the emphasis of estimation over testing, which more directly address the effect size of prediction. In light of the recent warnings and concerns expressed by the American Statistical Association (Wasserstein and Lazar, 2016) for the scientific findings entirely based on the p-value approach to statistical significance, our study is unique and novel in the literature of stock return predictability. The main question of our study is whether the informative quality of prediction interval improves as additional information is incorporated into the model. If the AR(0) model is found to generate the prediction interval of the highest quality, this is an indication that the additional information such as the past values of stock return or those of the predictors adds little value to the predictability of stock return. If a multivariate model with a particular predictor appears to be the clear winner, it serves as evidence that the predictor has a strong predictive power for stock return. We use the monthly data set compiled by Welch and Goyal (2008) for the U.S. stock market, which contains stock return and a range of potential predictors from 1926 to 2014, including the dividend yield, dividend-payout ratio, book-to-market ratio, price-earnings ratio, inflation rate, and risk-free rate. We extent these predictors by considering two macroeconomic variables (the industrial production growth and the output gap), because they are found to be informative about expected business conditions (Cooper and Priestley, 2009; Schrimpf, 2010); and the index of economic policy uncertainty proposed by Baker et al. (2015) because the economic uncertainty is found to affect financial markets (see, e.g., Bekaert et al., 2009; Brogaard and Detzel, 2015; Bali and Zhou, 2016). Evaluation of alternative out-of-sample prediction intervals is conducted in a purely empirical setting by calculating the mean coverage rate and interval score using the realized future values. For evaluation free from data snooping bias and possible structural changes, we apply moving sub-sample windows with a set of different window lengths.

The main finding of the paper is that the prediction intervals from the naive AR(0) model often outperform those generated from the models with additional information content. The univariate and multivariate models show little evidence of generating more accurate and informative prediction intervals than the AR(0) model. This suggests that the U.S. stock return has been unpredictable and that the market has been efficient in the weak and semi-strong forms, subject to the information content considered on this study. The next section presents a brief review of the relevant literature. Section 3 presents the methodologies, and Section 4 presents the data and computational details with illustrative examples. The empirical results are discussed in Section 5, and the conclusion is drawn in Section 6.

2 A Brief Literature Review

Given that the empirical literature of stock return predictability is broad and expansive, we provide a brief review of past studies focusing on those that evaluate out-of-sample predictability. We also point out the limitations of the past studies, and highlight the contribution of our study in the context of extant literature.

Whether stock return is predictable from an economic fundamental has been an issue of much interest and contention in empirical finance. The literature on return predictability has brought more questions than answers. In the first models, such as Samuelson (1965, 1969) and Merton (1969), excess returns were assumed to be unpredictable. However, the empirical literature in the 1980s has found variables with predictive power to explain stock returns (see, e.g., Keim and Stambaugh, 1986; Campbell and Shiller, 1988; Fama and French, 1988, 1989). After strong evidence in favor of return predictability on the aggregate level in the 1990s and 2000s, recent empirical evidence considers that the predictability of stock return is rather weak (see, e.g., Ang and Bekaert, 2007; Cochrane, 2008; Lettau and Van Nieuwerburgh, 2008; Welch and Goyal, 2008). More precisely, the evidence for U.S. stock return predictability seems to be predominantly in-sample, but it is not robust to out-of-sample evaluations.[1]

The previous studies on stock return predictability evaluate the out-of-sample (OOS) forecasting using various approaches (see Table 1). A number of studies assess the OOS performance of the predictive regression models by comparing the OOS $\overline{R}^2$ suggested by Campbell and Thompson (2008) and/or the root mean square errors (RMSE). Obviously, simply comparing RMSE does not take into account the sample uncertainty underlying observed forecast differences. Therefore, recent studies use predictive ability tests (Welch and Goyal, 2008; Rapach et al., 2010; Westerlund and Narayan, 2012). Westerlund and Narayan (2012) use the equality predictive ability test proposed by Diebold and Mariano (1995) and the forecast encompassing test developed by Harvey et al. (1998) which compare OOS forecasts from non-nested models. A drawback of these tests is that they have a nonstandard distribution when comparing forecasts from nested models (see Clark and McCracken, 2001; McCracken, 2007). In order to account for the nested mod-

[1]Some studies addresses the issue of parameter instability by estimating regime-switching (e.g., Pastor and Stambaugh, 2001; Paye and Timmermann, 2006; Lettau and Van Nieuwerburgh, 2008; Dangl and Hallin, 2012), but this issue is out the scope of this paper.

els Welch and Goyal (2008) and Westerlund and Narayan (2012) apply the MSEF and ENCNEW statistics of McCracken (2004), and Clark and McCracken (2001), respectively. The McCracken (2004) test statistic is a variant of the Diebold and Mariano (1995) statistic, while the Clark and McCracken (2001) test statistic is a variant of the Harvey et al. (1998) statistic. Rapach et al. (2010) use the MSPE-adjusted statistic of Clark and West (2007) that is an adjusted version of the Diebold-Mariano statistic allowing to compare forecasts from nested linear models.

Methodologically, there are two limitations of the above-mentioned past studies that examine out-of-sample predictability. First, as mentioned in the previous section, the analysis based on point forecasts only is of limited value since the variability of prediction is not fully taken account (see Chatfield, 1993; and Christoffersen, 1998). Our paper contributes to the extant literature as the first study that adopts the prediction interval as a means of assessing stock return predictability. Secondly, the recent statement made by the American Statistical Association (Wasserstein and Lazar, 2016) expresses a serious concern on the use of p-value with an arbitrary threshold of 0.05 in many scientific research. In particular, they warn that "the widespread use of statistical significance (generally interpreted as p-value less than 0.05) as a license for making a claim of a scientific finding (or implied truth) leads to considerable distortion of the scientific process." We note that the past studies on return predictability (both in-sample and out-of-sample) rely heavily on the statistical significance based on p-value in establishing their findings. Our study based on prediction interval represents an estimation-based investigation which directly addresses the effect size of out-of-sample forecasting, which Wasserstein and Lazar (2016) suggest as a desirable alternative to statistical significance solely based on the p-value criterion.

3 Methodology

In this section, we present the models for stock return prediction and the methods for generating out-of-sample prediction interval. These models have simple linear structures and their specifications can be automatically determined by a fully data-dependent method without an intervention of a researcher. Throughout the paper, we use AIC (Akaike's information criterion) to determine the unknown model orders. Let Y_t denote the stock return and X_t a predictor at time t. From the sample of size n $(t = 1, ..., n)$, we generate a point forecast $Y_n(h)$ for the h-period ahead future value Y_{n+h} of Y. A h-step ahead prediction interval with probability content 100(1-2θ)% is denoted as $PI_n(h, \theta)$.

3.1 Univariate autoregression

We consider the $AR(p)$ model of the form

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + ... + \alpha_p Y_{t-p} + u_t \tag{1}$$

where u_t is an identically and independently (IID) distributed error term with zero mean and fixed variance. The model specifies that the stock return is predictable purely from its own past. An $AR(0)$ model with $\alpha_i = 0$ for all i $(i = 1, ..., p)$ is used as a naive model where past returns have no predictive power for the future return.

The unknown parameters are estimated using the least-squares (LS) method. The LS estimators for $(\alpha_0, \alpha_1, ..., \alpha_p)$ are denoted as $(\widehat{\alpha}_0, \widehat{\alpha}_1, ..., \widehat{\alpha}_p)$ and the LS residuals $\{\widehat{u}_t\}_{t=p+1}^n$. The point forecast for Y_{n+h} $(h = 1, 2, ..., H)$ is generated using the LS estimators as

$$Y_n(h) = \widehat{\alpha}_0 + \widehat{\alpha}_1 Y_n(h-1) + ... + \widehat{\alpha}_p Y_n(h-p) \tag{2}$$

where $Y_n(j) = Y_{n+j}$ for $j \leq 0$. The 100(1-2θ)% prediction interval for Y_{n+h} can be constructed based on the prediction mean-squared error $(MSE(Y_n(h)))$, obtained using the delta method[2] with the AR parameter estimators $(\widehat{\alpha}_0, \widehat{\alpha}_1, ..., \widehat{\alpha}_p)$ and assuming the normality of prediction error distribution, as

$$PI_n(h, \theta; AR) \equiv [Y_n(h) - z_\tau MSE(Y_n(h)), Y_n(h) + z_\tau MSE(Y_n(h))], \tag{3}$$

where z_τ is the $100(1-\tau)\%$ percentile of the standard normal distribution with $\tau = 0.5\theta$.

3.2 Bootstrap prediction intervals

The prediction intervals given in the previous subsection are constructed based on the assumption that the predictive error distribution follows a normal distribution.

[2]The delta method is a method of approximating the variance of the limiting distribution of a statistic (see, for example, Lutkepohl, 2005).

The prediction MSE is calculated based on the delta method (Lutkepohl, 2005), which approximates the true variability based on asymptotic approximation. One may argue that the normality assumption is difficult to justify for stock return and that the delta method may provide inaccurate estimation of the true variability of the predictive distribution. In addition, in constructing the prediction interval, the conventional method does not account of estimation uncertainty. Hence, it is sensible to consider a non-parametric alternative which does not require the assumption of normality and asymptotic approximations.

The bootstrap is a method approximating the true sampling distribution of a statistic using the repetitive re-sampling the observed data, without imposing normality or resorting to asymptotic approximation (Thombs and Schucany, 1990; Pan and Politis, 2106). For the univariate AR model, the bootstrap method can be described as follows:

Generate the artificial set of data as

$$Y_t^* = \widehat{\alpha}_0 + \widehat{\alpha}_1 Y_{t+1}^* + ... + \widehat{\alpha}_p Y_{t+p}^* + u_t^* \tag{4}$$

where $(\widehat{\alpha}_0, ..., \widehat{\alpha}_p)$ are the LS estimators for $(\alpha_0, ..., \alpha_p)$ and u_t^* is random draw with replacement from the LS residuals $\{\widehat{u}_t\}_{t=p+1}^n$. Note that we follow Thombs and Schucany (1990) to generate $\{Y_t^*\}_{t=1}^n$ based on the backward AR model using the last p observation as the starting values. This is to accommodate the conditionality of the AR parameter estimators on the last p values of the series. Using $\{Y_t^*\}_{t=1}^n$, the unknown AR parameters $(\alpha_0, ..., \alpha_p)$ are re-estimated, which are denoted as $(\widehat{\alpha}_0^*, ..., \widehat{\alpha}_p^*)$. The bootstrap replicates of the AR forecast for Y_{n+h}, made at time period n, are generated recursively as

$$Y_n^*(h) = \widehat{\alpha}_0^* + \widehat{\alpha}_1^* Y_n^*(h-1) + ... + \widehat{\alpha}_p^* Y_n^*(h-p) + u_{n+h}^* \tag{5}$$

where $Y_n^*(j) = Y_{n+j}$ for $j \leq 0$ and u_t^* is random draw with replacement from $\{\widehat{u}_t\}_{t=p+1}^n$.

Repeat (4) and (5) many times, say B, to yield the bootstrap distribution for the AR forecast $\{Y_n^*(h;j)\}_{j=1}^B$. This distribution is used as an approximation to the predictive distribution for Y_{n+h}. The 100(1-2θ)% prediction interval for Y_{n+h} can be constructed by taking appropriate percentiles from the bootstrap distribution. That is,

$$PI_n(h, \theta; AR^*) \equiv [Y_n^*(h, \tau), Y_n^*(h, 1-\tau)], \tag{6}$$

where $Y_n^*(h, \tau)$ is 100τ% percentile from $\{Y_n^*(h;j)\}_{j=1}^B$ and $\tau = 0.5\theta$.

3.3 Predictive regression: IARM

We consider a predictive model for stock return Y as a function of a predictor X with lag order p, which can be written as

$$Y_t = \beta_0 + \beta_1 X_{t-1} + ... + \beta_p X_{t-p} + v_{1t} \tag{7}$$

$$X_t = \delta_0 + \delta_1 X_{t-1} + ... + \delta_p X_{t-p} + v_{2t}. \tag{8}$$

It is assumed that the error terms are IID with fixed variances and covariances: $Var(v_{1t}) \equiv \sigma_1^2, Var(v_{2t}) \equiv \sigma_2^2$ and $Cov(v_{1t}, v_{2t}) \equiv \sigma_{12}$.

It is well-known that the LS estimators for (β_1,..., β_p) are biased in small samples, as long as $\sigma_{12} \neq 0$. It is because the LS estimators completely ignore the presence of σ_{12}, as Stambaugh (1999) points out. Amihud et al. (2004, 2008, 2010) propose a bias-correction method based on a augmented regression, called the augmented regression method (ARM), which is subsequently improved by Kim (2014).

The method assume that the error terms in (7) and (8) are linearly related as $v_{1t} = \phi v_{2t} + e_t$ where e_t is an independent normal error term with a fixed variance. It involves running the regression for Y against lagged X's as in (7), augmented with the bias-corrected residuals from the predictor equations (8). That is, we run the regression of the form

$$Y_t = \beta_0 + \beta_1 X_{t-1} + ... + \beta_p X_{t-p} + \phi \hat{v}_{2t}^c + e_t \tag{9}$$

where $\widehat{v}_{2t}^c \equiv X_t - \widehat{\delta}_0^c - \widehat{\delta}_1^c X_{t-1} - ... - \widehat{\delta}_p^c X_{t-p}$, while $(\widehat{\delta}_0^c, \widehat{\delta}_1^c, ..., \widehat{\delta}_p^c)$ are the bias-corrected estimators for δ_i's. Amihud et al. (2010) use the asymptotic formulae derived by Shaman and Stine (1988) to obtain these bias-corrected estimators. The bias-corrected estimators $(\widehat{\beta}_0^c, \widehat{\beta}_1^c, ..., \widehat{\beta}_p^c)$ for $(\beta_0, \beta_1, ..., \beta_p)$ are obtained by regressing the augmented regression (9).

Kim (2014) proposes three modifications to the ARM of Amihud et al. (2010). The first is the bias-correction method of a higher order accuracy than the one used by Amihud et al. (2010). The second is the use of stationarity-correction (Kilian, 1998), which ensures that the bias-corrected estimators satisfy the condition of stationarity. This correction is important because bias-correction often makes the parameter estimates of the model (7) and (8) imply non-stationarity of stock return. The third is the use of matrix formula for bias-correction, which makes the implementation of the ARM for a higher order model computationally easier. According to the Monte Carlo study of Kim (2014), the improved ARM (IARM) provides more accurate parameter estimation and statistical inference than its original version in small samples.

The point forecast for stock return based on the IARM is generated jointly with that of the predictor as

$$Y_n(h) = \widehat{\beta}_0^c + \widehat{\beta}_1^c X_n(h-1) + ... + \widehat{\beta}_p^c X_n(h-p) \tag{10}$$

where $X_n(h) = \widehat{\delta}_0^c + \widehat{\delta}_1^c X_n(h-1) + ... + \widehat{\delta}_p^c X_n(h-p)$ and $X_n(j) = X_{n+j}$ for $j \leq 0$. The 100(1-2θ)% prediction interval for Y_{n+h} can be constructed based on the prediction mean-squared error ($MSE(Y_n(h))$) obtained using the delta method with IARM parameter estimators and assuming the normality of prediction error distribution. That is,

$$PI_n(h, \theta; IARM) \equiv [Y_n(h) - z_\tau MSE(Y_n(h)), Y_n(h) + z_\tau MSE(Y_n(h))]. \tag{11}$$

3.4 Vector Autoregressive Model

The predictive model given (7) and (8) specifies that the stock return depends only on the past value of a predictor. This means that the model allows for only one-way causality from the predictor to stock return, and that the stock return does not depend on the past value of its own. These restrictions may deliver a simple and parsimonious model, but they completely exclude the possibility stock return depending on its own past and the potential feedback effect from stock return to the predictor. A more general model can be specified by resorting to the vector autoregressive (VAR) model, which can be written as

$$Y_t = \tau_0 + \alpha_1 Y_{t-1} + ... + \alpha_p Y_{t-p} + \beta_1 X_{t-1} + ... + \beta_p X_{t-p} + u_{1t} \tag{12}$$

$$X_t = \tau_0 + \gamma_1 Y_{t-1} + ... + \gamma_p Y_{t-p} + \delta_1 X_{t-1} + ... + \delta_p X_{t-p} + u_{2t}. \tag{13}$$

The model is widely used for modeling and forecasting stock return dynamically inter-related with other predictors (see, for example, Engsted and Pedersen, 2012).

The LS estimator for the unknown parameters in (12) and (13) are biased in small samples, which can provide biased prediction intervals. In this paper, we employ the bias-correction based on the asymptotic formula given by Nicholls and Pope (1988), which is also used by Engstead and Pedersen (2012). We also apply

Killian's (1998) stationarity correction in case the bias-correction provides parameter estimates which imply non-stationarity. Using these bias-corrected estimators, the point forecasts are generated recursively as

$$Y_n(h) = \widetilde{\tau}_0 + \widetilde{\alpha}_1 Y_n(h-1) + ... + \widetilde{\alpha}_p Y_n(h-p) + \widetilde{\beta}_1 X_n(h-1) + ... + \widetilde{\beta}_p X_n(h-p) \quad (14)$$

$$X_n(h) = \widetilde{\tau}_0 + \widetilde{\gamma}_1 Y_n(h-1) + ... + \widetilde{\gamma}_p Y_n(h-p) + \widetilde{\delta}_1 X_n(h-1) + ... + \widetilde{\delta}_p X_n(h-p). \quad (15)$$

where $X_n(j) = X_{n+j}$ for $j \leq 0$, $Y_n(j) = Y_{n+j}$ for $j \leq 0$, and the parameters with tilde indicates the bias-corrected estimator for the corresponding parameters.

The 100(1-2θ)% prediction interval for Y_{n+h}, constructed based on the prediction mean-squared error ($MSE(Y_n(h))$) obtained using the delta method with the VAR bias-corrected parameter estimators and assuming the normality of prediction error distribution, is denoted as

$$PI_n(h, \theta; VAR) \equiv [Y_n(h) - z_\tau MSE(Y_n(h)), Y_n(h) + z_\tau MSE(Y_n(h))]. \quad (16)$$

4 Data and Computational Details

In this section, we provide the data and computational details, along with the simple illustrative examples in relation to interval forecasting and their assessment.

4.1 Data

We use the financial variables compiled by Welch and Goyal (2008) for the U.S. stock market, available from Amit Goyal's website. The precise definitions of these variables are given in Welch and Goyal (2007). For stock return, we use the CRSP NYSE value-weighted return, which is widely used as a benchmark for investment and academic research. These financial variables (monthly from 1926 to 2014, except for NTIS which starts from 1927) are listed as below:

- Dividend-Yield (DY)
- Dividend-Price Ratio (DP)
- Earnings-Price Ratio (EP)
- Dividend Payout Ratio (DE)
- Book-to-Market (BM)

- Risk-free rate (RF)
- Inflation (INF)
- Stock Variance (SVAR)
- Long Term Yield (LTY)
- Long Term Return (LTR)
- Net Equity Expansion (NTIS)
- Default Return Spread (DFR)
- Default Yield Spread (DFY)
- Term Spread (TMS)

We add three economic variables (monthly from 1927 to 2014) to those proposed by Welch and Goyal (2008):

- Industrial production growth (IPG)
- Output gap (GAP)
- Economic policy uncertainty (EPU)

The data used to construct the industrial production growth and the output gap are downloaded from the FRED database of St Louis Fed. Following Schrimpf (2010) we construct the output gap measure by applying the filter by Hodrick and Prescott (1997) to the logarithmic series of industrial production. The smoothing parameter is set to 128,800 (monthly data). The cyclical component of the series is taken as the output gap. The index of economic policy uncertainty (EPU) is proposed by Baker et al. (2015), built on three components: (i) the frequency of newspaper references to economic policy uncertainty, (ii) the number of federal tax code provisions set to expire, and (iii) the extent of forecaster disagreement over future inflation and government purchases.[3]

4.2 Computational Details

Evaluation of alternative out-of-sample prediction intervals is conducted in a purely empirical setting using the realized future values. For evaluation free from data

[3]See Baker et al. (2015) for a detailed description of the historical EPU index.

snooping bias, we apply moving sub-sample windows to the above data set, adopting a grid of different estimation window lengths ranging from 24 months to 240 months (with an increment of 24 months). From each estimation window, 12-step ahead (out-of-sample) prediction intervals are generated from a predictive model. For example, when the window length of 24 months, we take the first 120 observations from January 1926 to estimate the model, and generate 12-step ahead forecasts. And then we move to the next set of 120 observations from February 1926 to estimate the model and generate forecasts. This continues until the end of the data set is reached.

As a means of comparison, we use the coverage rate and the interval score proposed by Gneiting and Raftery (2007, p.370). Let a $100(1-2\theta)\%$ h-step ahead prediction interval be given by $[L_h, U_h]$. The coverage rate is calculated as the proportion of the true values covered by the prediction interval, i.e.,

$$C(h) = \frac{\sharp(L_h \le Y_h \le U_h)}{N},$$

where Y_h is the true future value, N is the total number of prediction intervals for forecast horizon h, and $\sharp$ indicates the frequency at which the condition inside the bracket is satisfied. A $100(1-2\theta)\%$ prediction interval is expected to have $C(h)$ value of $(1-2\theta)$ in repeated sampling.

The interval score for a $100(1-2\theta)\%$ interval $[L_h, U_h]$, it is given by

$$S_\theta(L_h, U_h; Y_h) = (U_h - L_h) + \frac{1}{\theta}(L_h - Y_h)I(Y_h < L_h) + \frac{1}{\theta}(Y_h - U_h)I(Y_h > U_h)$$

where $I(\cdot)$ is an indicator function which takes 1 if the condition inside the bracket is satisfied and 0 if otherwise; and Y_h is the true future value. If the interval covers Y_h, the score takes the value of its length; if otherwise, a penalty term is added to the value of length, which is how much the interval misses Y_h scaled by $1/\theta$. In the event that the interval misses Y_h by a small (large) margin, a light (heavy) penalty is imposed. The interval score measures the quality of the probabilistic statement implied by a prediction interval. We note that the interval score is far more informative than the coverage rate, since it takes full account of the accuracy and riskiness of a prediction interval. In fact, the dichotomous nature of the coverage rate may deliver misleading assessment of predictive accuracy, as the examples in the next subsection show. Hence, in this paper, we use both measures, but giving more importance to the interval score.

4.3 Examples

In this subsection, we present three simple illustrative examples. The first highlights the reason as to why evaluation of point forecasts only is an incomplete exercise in assessing predictability; while the other two explain why the interval score S_θ is a more informative measure of predictive quality than the coverage rate $C(h)$ presented in Section 4.2.

Consider a set of future values $(y_1, y_2, y_3, y_4, y_5) = (0, 0, 0, 0, 0)$, along with two sets of 80% prediction intervals PI_1 and PI_2 generated from two alternative models (called Model 1 and Model 2):

Example 1: Point Forecasting versus Interval Forecasting

$$PI_1 = [(-1, 1), (-1, 1), (1, 2), (-1, 1), (-1, 1)]$$
$$PI_2 = [(-2, 2), (-2, 2), (0.5, 2.5), (-2, 2), (-2, 2)]$$

For the purpose of simplicity, suppose that Models 1 and 2 generate the identical point forecasts, which means that the two appear to show the predictive accuracy of the same degree if evaluation is carried out using the point forecasts only. However, Model 2 generates prediction intervals twice wider than Model 1, indicating that its prediction is twice riskier than that of Model 1. In this case, Model 1 should be clearly preferred for the purpose of forecasting. If Model 2 includes an information set additional to that of Model 1, the extra information does not improve the quality of prediction but only increases its variability. An important point is that comparison based on point forecasts is not capable of capturing the difference in forecast variability.

Example 2: Coverage Rate versus Interval Score

$$PI_1 = [(-1, 1), (-1, 1), (1, 2), (-1, 1), (-1, 1)]$$
$$PI_2 = [(0.1, 0.2), (-0.2, -0.1), (0.1, 0.2), (-0.2, -0.1), (0.1, 0.2)]$$

PI_1 covers the true values (y) four out of five times, with the correct coverage rate of 0.8; while PI_2 never covers the true value and its coverage rate is 0. If one adopts the coverage rate as a means of comparison, the Model 1 should be preferred. However, the mean interval score of PI_1 is 3.8 and that of PI_2 is 1.1, which means that Model 2 delivers a more informative and higher-quality prediction. This is because PI_2's are much tighter, only narrowly missing the true values. In contrast, PI_1's are too wide and uninformative. In this case, the coverage rate fails to capture the richer informative quality of PI_2.

Example 3: Coverage Rate versus Interval Score

$$PI_1 = [(-0.5, 0.5), (-0.5, 0.5), (1.5, 2.5), (-0.5, 0.5), (-0.5, 0.5)]$$

$$PI_2 = [(-0.5, 0.5), (-0.5, 0.5), (0.1, 1.1), (-0.5, 0.5), (-0.5, 0.5)]$$

PI_1 and PI_2 have the correct coverage rate of 0.8 and identical lengths. However, the mean interval score of PI_1 is 4 while that of PI_2 is 1.2. This is because PI_1 misses by big margin when it fails to cover the true value, while PI_2 misses it only with a small margin. As before, the coverage rate does not fully reflect the informative quality of prediction interval because it is unable to capture the effect of a big miss (which can be costly economically), while the interval score is capable of including this costly miss in its evaluation.

5 Empirical Results

Given the large number of possible predictors for stock return and the prediction models being considered, we report only a set of selective but representative results. This is to simplify the exposition and to present the results in a manageable way. However, we note that qualitatively similar results are obtained from those unreported. Figure 1 plots the examples of 50% and 95% prediction intervals for stock return, 1-step ahead from 1936:01 to 2014:12 generated with rolling window of length 120. The first figure plots those from the AR(0) model and the second those from the IARM with the dividend yield (DY) as a predictor. As might be expected, 95% intervals are wider but less informative, while 50% intervals are tighter but riskier with a higher chance of missing the true values. The width of the intervals changes over time, wider during the periods of higher volatility. The main question of the paper is whether additional information included in the predictive model can improve the informative quality of prediction intervals for stock return.

We first check whether the prediction intervals provide reasonable coverage properties by employing the likelihood ratio (LR) test for the conditional efficiency of prediction interval proposed by Christoffersen (1998). It is based on the property that, for 100(1-2θ)% prediction intervals to be efficient (conditionally on the past information), the indicator variable for their coverage should follow the independent Bernoulli distribution with parameter 1-2θ. Christoffersen (1998) develops the LR test for the joint null hypothesis for the coverage rate (1-2θ) and independence, which asymptotically follows the chi-squared distribution with 2 degrees of freedom. Figure 2 plots the LR statistics for the prediction intervals from AR(0) model given in Figure 1, using the rolling sub-sample window of 120 observations. For the 50% intervals, the joint hypothesis is rejected at 1% level of significance only occasionally over time; while for the 95% intervals, the hypothesis cannot be rejected at the 1% level over the entire period. Testing at the 5% level provides similar results. This indicates that the prediction intervals generated from the AR(0) model show reasonable coverage properties over time. We note that those generated from the other models (such as the IARM with the predictor DY) provide similar results.

We now pay attention to the interval score properties of alternative prediction intervals. As we have seen in the previous section, the interval score is a more complete measure for the quality of prediction interval than the coverage rate. Figure 3 reports the mean interval score of all prediction intervals for forecast horizon h from 1 to 12. The multivariate models (IARM and VAR) have the predictor DY. When the window length is 24, the mean score of the AR(0) and AR(p) models are the smallest for all forecast horizons, for both cases of 50% and 95% prediction intervals. When the window length is 120, again the univariate prediction intervals show better performance than the multivariate ones in most cases. Hence, there is no clear evidence that inclusion of DY improves the predictability of stock return. In fact, the VAR model (which has the most general dependency structure) provides prediction intervals with the lowest quality in terms of the interval score.

Figure 4 reports the mean interval score averaged across all forecast horizon (median) for all prediction intervals. These median of mean interval scores are plotted against the window length from 24 to 240. As before, the multivariate models have the DY as a predictor. Again, the prediction intervals generated from the univariate models outperform those from the multivariate models for nearly all window lengths. Hence, the evidence suggests that the use of DY as a predictor does not improve predictability of stock return. It can also be observed that the accuracy improves with the sample size only to a certain point. For example, when the nominal coverage is 0.95, the mean score nearly hits the bottom when the sample size (or window length) is around 100, for both cases of 50% and 95% prediction intervals. In addition, as is also clear from Figure 3, we find no evidence that the bootstrap prediction intervals perform better than those generated from AR(0) or AR(p) models. This suggests that the prediction intervals based on the conventional normal approximation to the predictive distribution perform adequately for monthly stock return.

In Figure 5, the mean interval scores of the AR(0) model are compared with those from the IARM with different predictors including DY, DP, EP, BM, PE, inflation rate, and risk-free rate, for forecast horizons 1, 4, 8, and 12. For h = 1, the AR(0) model shows smaller mean interval scores than the IARM for most cases, especially when the length of rolling window is short. When forecast horizon is long (h = 8 or 12), there are occasions where the prediction intervals from IARM beat those from the AR(0), but the margins are fairly small. When the rolling window length is greater than 120, the performances of the alternative prediction intervals are almost indistinguishable. That is, there is no compelling evidence that the IARM with a range of predictors beats AR(0) model in terms of the interval score.

Figure 6 plots the mean interval scores from the economic variables (IPG, GAP, and EPU) based on the IARM for the forecast horizons 1, 4, 8, and 12, in comparison with the score from the AR(0) model. Again, there is little evidence that the economic indicators help generate prediction intervals that are of higher quality than those from the AR(0) model. There are occasions where the mean interval score from an economic variable is lower than that of AR(0) model, but the marginal improvement is fairly small. As also observed in Figure 5, when $h = 1$ and the window length is short, the AR(0) model is the clear winner. This means that, for short-term and short-horizon prediction of stock return, the naive AR(0) model provides the prediction intervals of the highest quality.

Figure 7 plots time variation of interval score when the window length is 120 and $h = 1$, for the AR(0) model and IARM with selected predictors. The spikes represent the failure of prediction interval in predicting the future stock return. It appears that all prediction intervals show similar pattern over time, showing the spikes at the times of stock market volatility, such as late 30's, early 60's, oil shock, and stock market crashes (1987, 2008). There is no clear evidence that any predictors from the IARM generate more accurate than AR(0) model. In comparison with the NBER recession and boom dates, we observe that the times of predictive failure are not related with the business cycle.

The empirical results shown that there is no clear indication that the multivariate model with predictors beats the most simple and naive AR(0) model, in terms of predictive accuracy and informative quality of prediction intervals. This finding points to the conclusion that the stock return has been unpredictable in the U.S. market and that the stock market has been informationally efficient in the weak and semi-strong form, subject to the information set under investigation in this study.

6 Concluding Remarks

This paper contributes to the extant literature of stock return predictability, as the first study that adopts prediction interval as a measure of out-of-sample predictability. Past studies exclusively used point forecasts, which are of limited value in assessing predictability of stock return which often shows a high degree of volatility over time. A point forecast is an estimate of the mean of the predictive distribution, which carries no information about its variability. A more complete analysis of predictive distribution can be achieved by evaluating prediction intervals (see Chatfield, 1993; Christoffersen, 1998, and Pan and Politis, 2015). We consider prediction intervals for monthly stock return generated from a range of linear models with different degrees of information content. They include a naive model, simple linear univariate autoregressive models, and multivariate (predictive regression and vector autoregressive model). For the latter, we use a range of economic and financial variables as possible predictors for stock return. We also consider the bootstrap prediction interval which relies on a non-parametric method and does not require the assumption of normality. In view of the recent statement made by the American Statistical Association which expresses serious concerns on the research practice heavily based on statistical significance (Wasserstein and Lazar, 2016), our study represents an attempt to address the issue of stock return predictability based on an estimation-based approach. In contrast, the past studies rely heavily on statistical significance using the p-value as an indicator.

Using the data set compiled by Welch and Goyal (2007) with three additional economic variables, we evaluate and compare out-of-sample and multi-step prediction intervals from alternative models in a purely empirical setting, using movingsubsample windows of different lengths. The mean coverage rate and interval score are used as the measures for predictive accuracy and quality of prediction intervals. We find that all models considered provide prediction intervals with reasonable coverage properties. In terms of the interval score, we find that the AR(0) model, which is the most naive model, provides the prediction intervals that often outperform those generated from its univariate and multivariate alternatives. We find no clear indication that univariate autoregression and multivariate models provide prediction intervals of higher quality than those from the AR(0). That is, we find little evidence that predictability of stock return is improved by incorporating the past history of its own and that of its predictors. The evidence suggests that the U.S. stock market has been efficient in the weak-form as well as in the semi-strong form, subject to the information set considered in this study.

There are two further issues that future studies may explore. First, the predictors not considered in this study may be examined. The universe of possible predictors for stock return is expansive, and we are calling for additional future studies to evaluate their predictive power in the context of interval forecasting. For example, recent studies (based on point forecasting) report that technical indicators show a higher degree of predictability than financial ratios (see, for example, Neely et al., 2014). Since we are limited by data availability for technical indicators due to the historical span of the dataset in this study, future studies may assess the predictive power of technical indicators for interval forecasting of stock return. Second, only the prediction intervals generated from linear time series models are considered in this study. It is possible that stock returns show non-linear dependence on past information (see Hinich and Patterson, 1985), while this possibility has not been extensively investigated in the empirical literature of stock return predictability. In addition to the difficulty of finding a suitable non-linear model for stock return, we note that construction of prediction interval from a non-linear model is a technically and computationally challenging exercise (see, for example, Frances and van Dijk, 2000). On this basis, this line of research is left as a possible future study.

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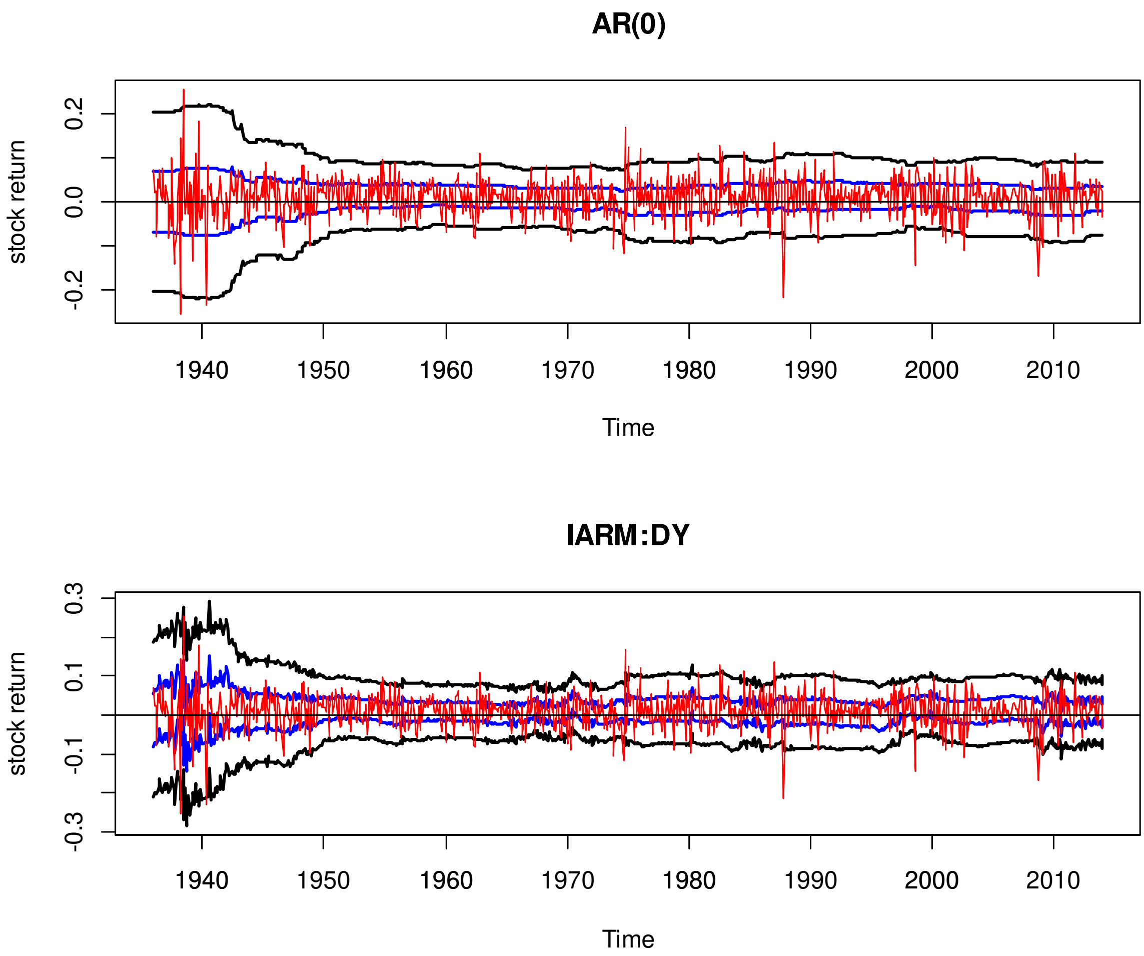
Table 1: Selected studies on the US stock return predictability.

Studies	Sample	Dep. Variables	Indep. Variables	Methodologies		
				Estimator	In-sample	Out-of-sample
Welch and Goyal (2007)	1926-2005 (M)	ER	d/e, svar, lty, dfr, ltr, inf, tms, dfy, tbl, d/p, e/p, d/y, ntis, eqis, b/m, e10/p, csp, cay	OLS with bootstrapped F-stat	$\overline{R}^2$	$\overline{R}^2$, ΔRMSE MSE-F & ENC-NEW tests
Campbell and Thompson (2008)	sample start 2005 (M)	ER	d/e, lty, inf, tms, dfy, tbl, d/p, e/p, ntis, b/m, e10/p, cay, roe	OLS	$\overline{R}^2$	$\overline{R}^2$
Lettau and Van Nieuwerburgh (2008)	1926-2004 (Y)	R, DG	d/p	OLS	$\overline{R}^2$	$\overline{R}^2$
	1946-2004 (M)	R	d/p, e/p, b/m			
Rapach et al. (2010)	1947-2005 (Q)	ER	d/e, svar, lty, dfr, ltr, inf, tms, dfy, tbl, d/p, e/p, d/y, ntis, b/m, i/r	OLS		$\overline{R}^2$, MSPE & MHLN tests
Westerlund and Narayan (2012)	1871-2008 (M)	ER	d/e, d/p, d/y, e/p	OLS, AOLS, FGLS		Theil U and DM, MSE-F & ENC-NEW tests

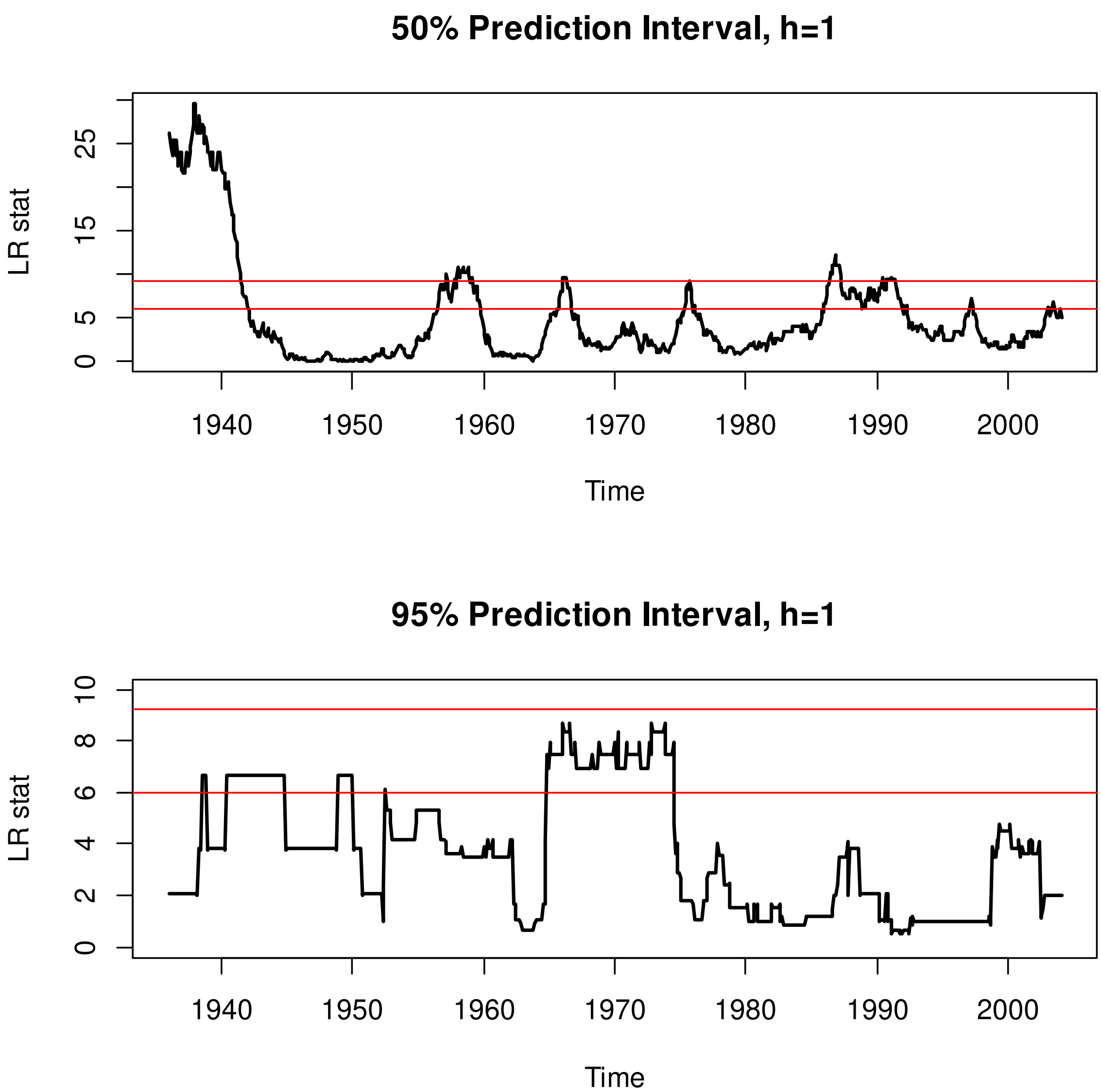
Dependent variables: Excess stock returns (ER), Stock returns (R), Dividend growth (DG).
Independent variables: Dividend Payout Ratio (d/e), Stock Variance (svar), Long Term Yield (lty), Default Return Spread (dfr), Long Term Return (ltr), Inflation (inf), Term Spread (tms), Default Yield Spread (dfy), T-Bill Rate (tbl), Dividend Price Ratio (d/p), Earning Price Ratio (e/p), Dividend Yield (d/y), Net Equity Expansion (ntis), Percent Equity Issuing (eqis), Book to Market (b/m), Earning(10Y) Price Ratio (e10/p), Cross-Sectional Premium (csp), Consumption, wealth, income ratio (cay), Investment-to-capital ratio (i/k), real Return-on-Equity (roe).
Methodologies: Estimators: the OLS estimator, the bias-adjusted OLS (AOLS) estimator of Lewellen (2004), the feasible generalized least squares (FGLSs) estimator of Westerlund and Narayan (2015).
Out-of-sample (OOS) tests: ENC-NEW tests by Clark and McCracken (2001), MSE-F test by McCracken (2007), MSPE test by Clark and West (2007), DM test by Diebold and Mariano (1995), MHLN test by Harvey, Leybourne, and Newbold (1998).

Figure 1. Examples of Prediction Intervals (50% and 95%, h=1, rolling window size =120)



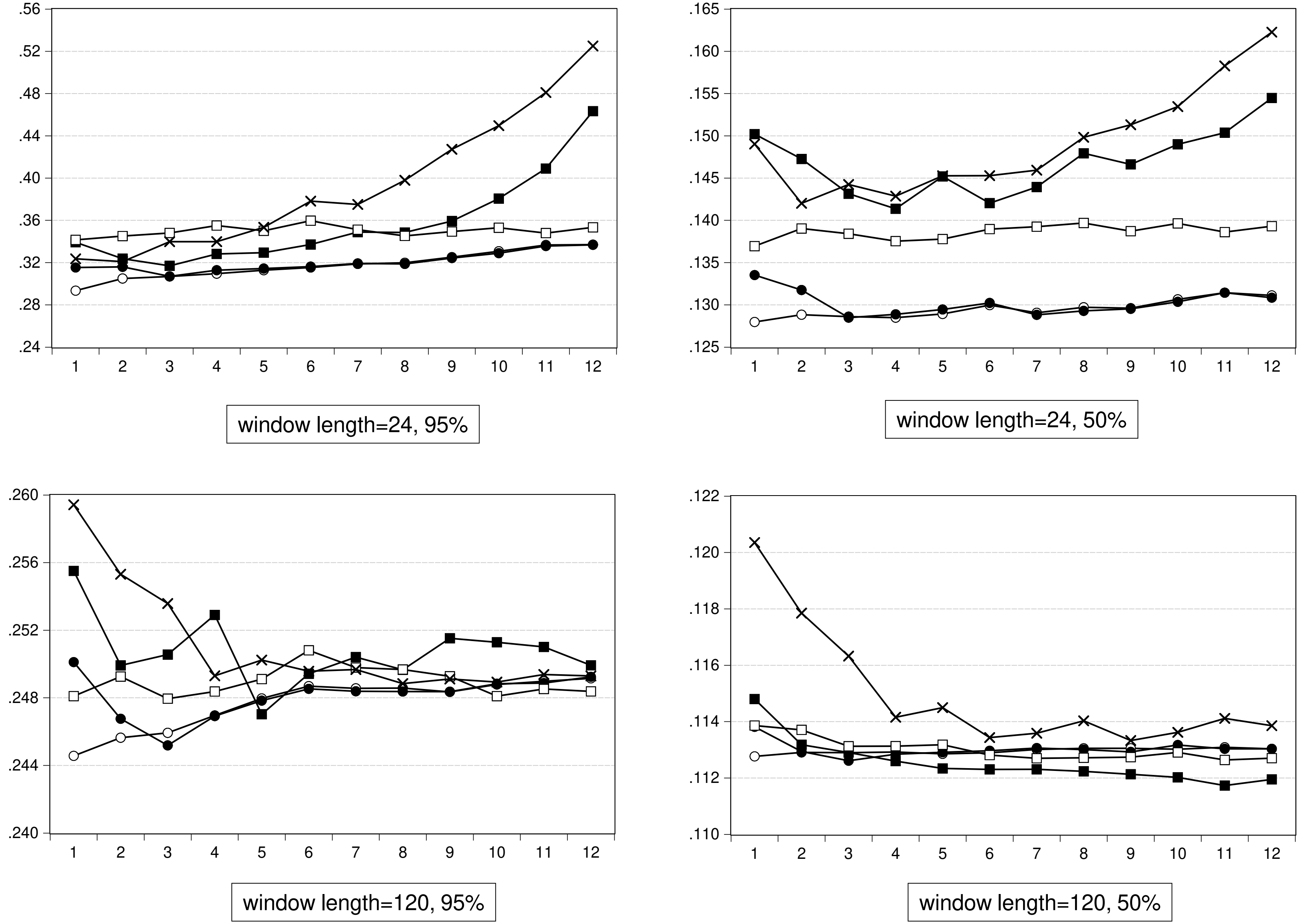
The black lines represent 95% prediction intervals, the blue lines 50% intervals, and the red line indicates the stock return.

Figure 2. Likelihood Ratio Test Statistic for Prediction Intervals from AR(0) model (rolling window size =120)



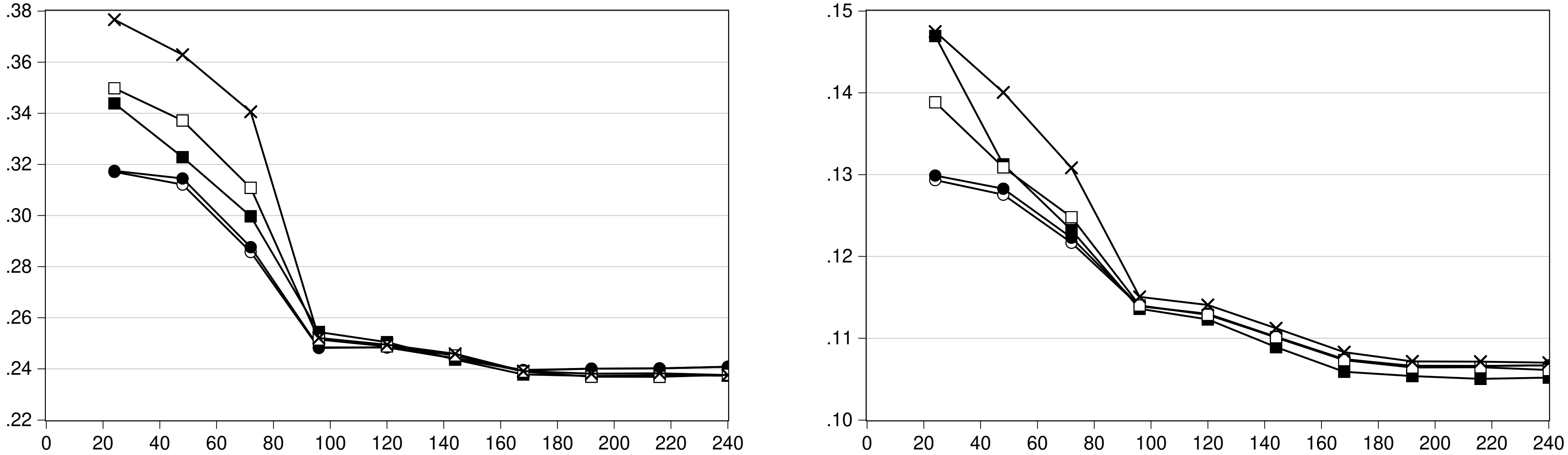
The red horizontal lines correspond to 5.99 and 9.21, which are 5% and 1% critical values from the chi-squared distribution with two degrees of freedom.

Figure 3. Mean Score of Prediction Intervals



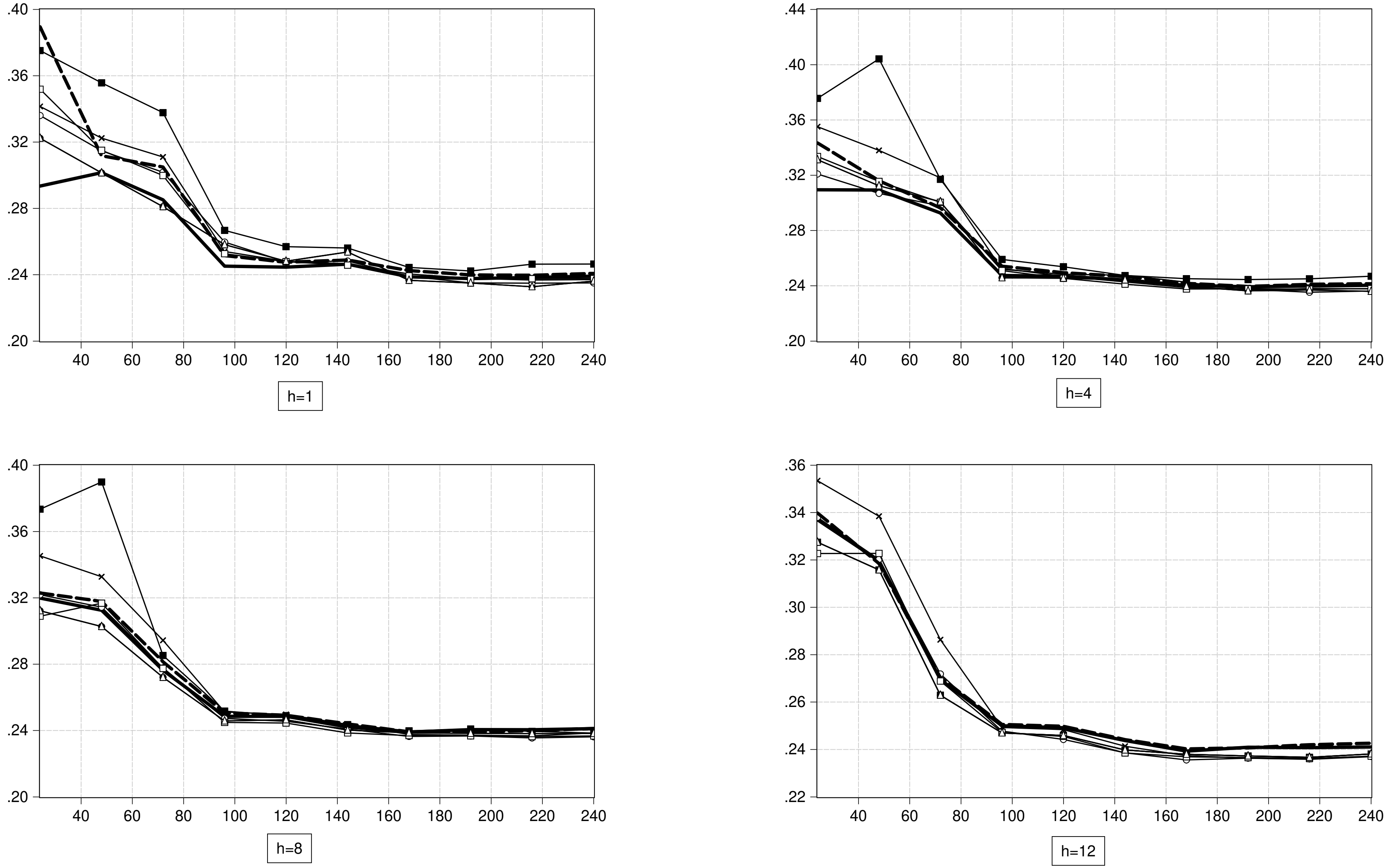
Open Circle: AR(0), Dark Circle: AR(p), Dark Square: AR(p) Bootstrap; Open Square: IARM; Cross: VAR
The predictor used in the IARM and VAR is DY (dividend-yiled).

Figure 4. Mean Interval Scores against Rolling Windows Length



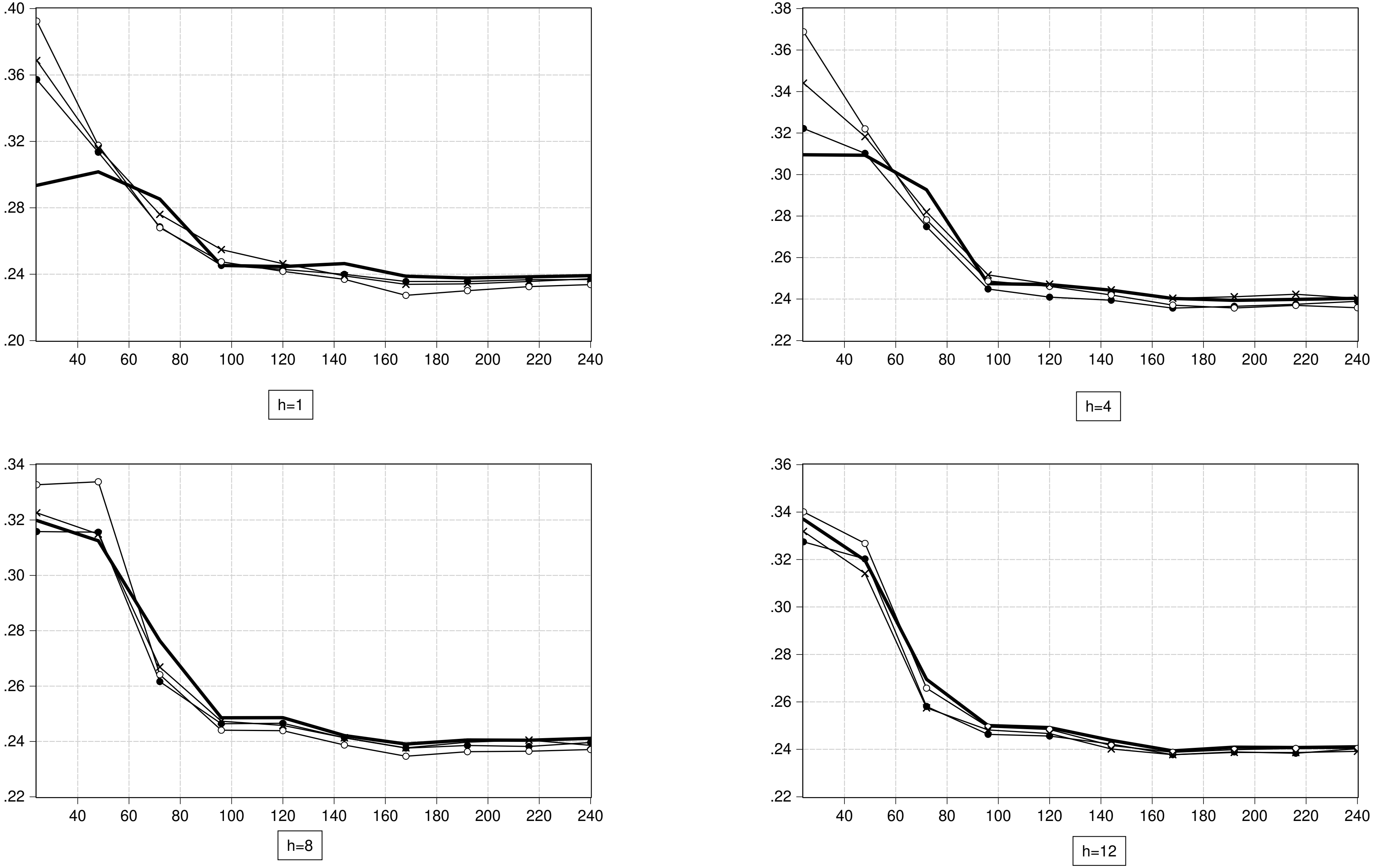
Open Circle: AR(0), Dark Circle: AR(p), Dark Square: AR(p) Bootstrap; Open Square: IARM; Cross: VAR
The predictor used in the IARM and VAR is DY (dividend-yiled).
The mean score values plotted are median value over the forecast horizon 1, ...,12.

Figure 5. Mean Interval Score of Alternative 95% Prediction Intervals (Financial Predictors)



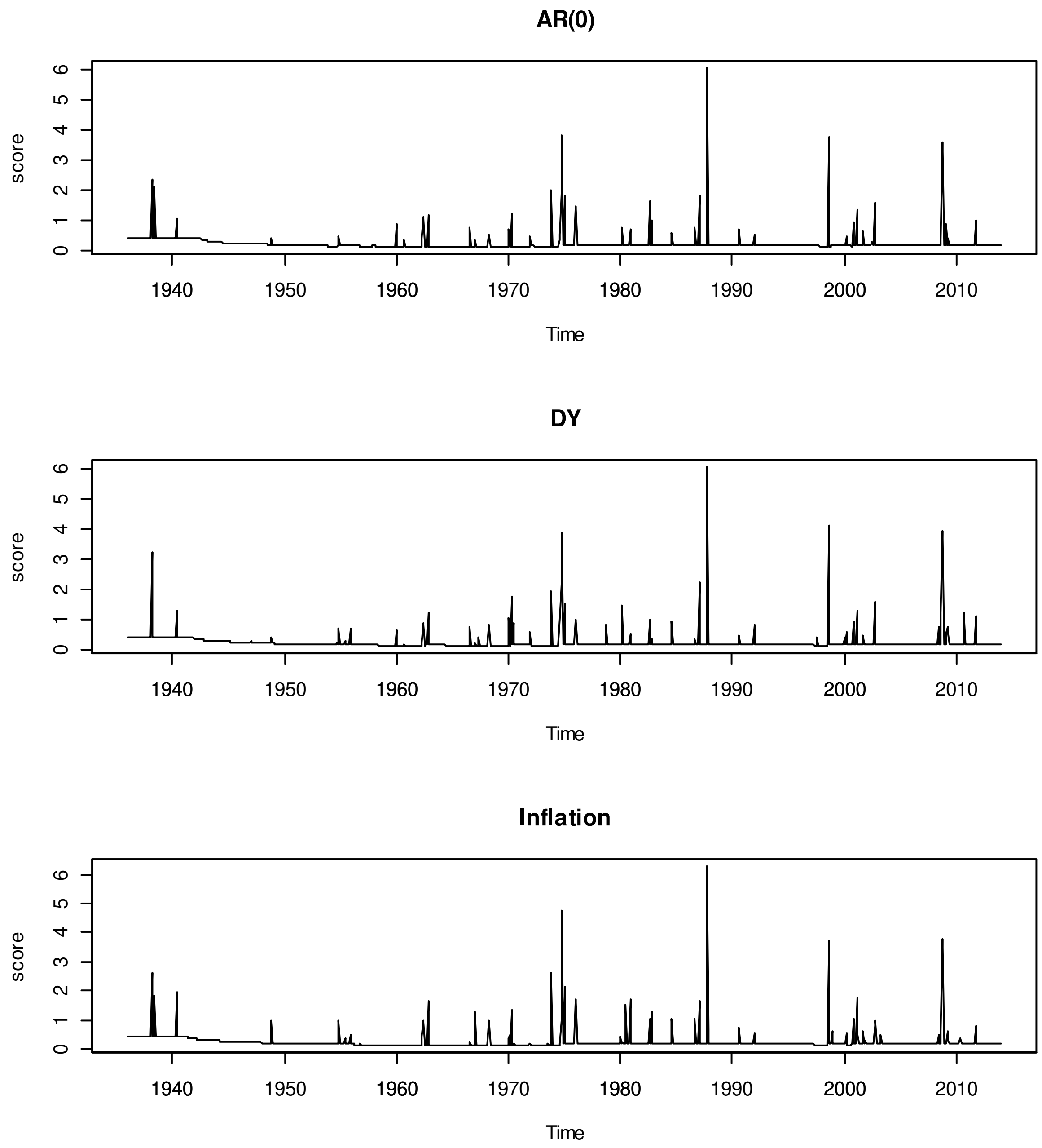
Solid Line: AR(0); dark circile: BM; open circle: DP; cross: DY;
Dark Square: EP; Broken line: inflation; Open Sqaure: PE; Triangle: RF
The IARM is used for all predictors.

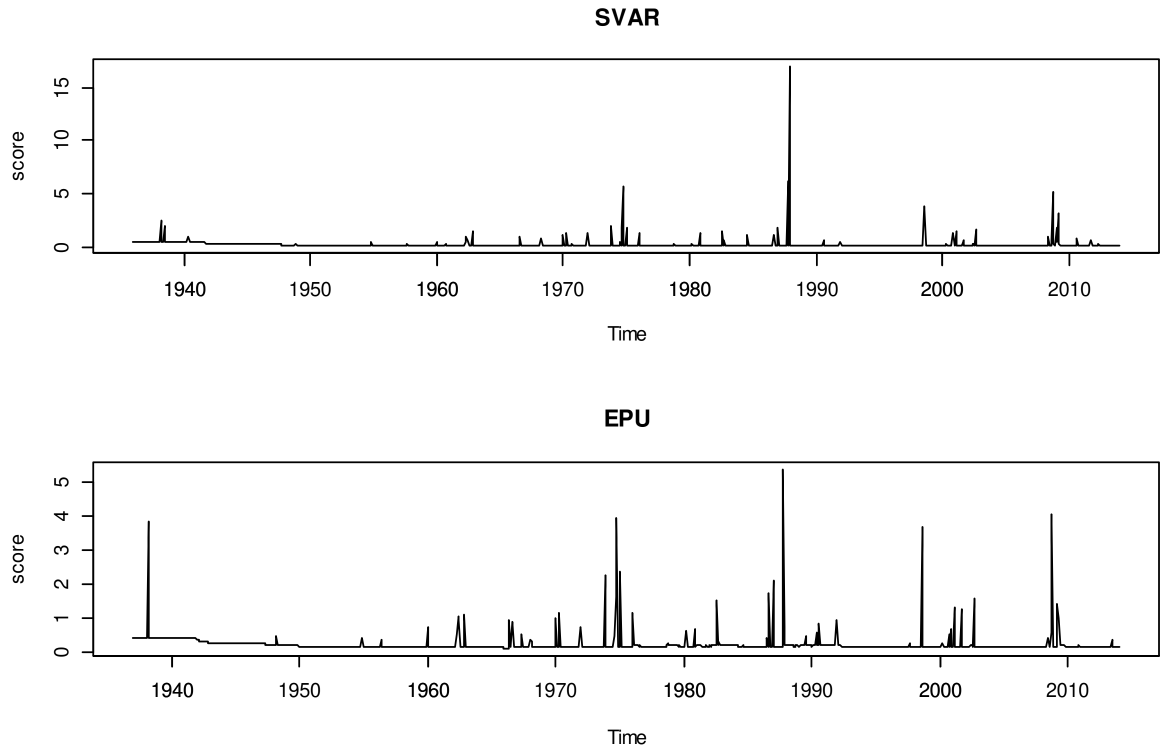
Figure 6. Mean Interval Score of Alternative 95% Prediction Intervals (Economic Predictors)



Solid Line: AR(0); dark circile: EPU open circle: GAP; cross: IPH;

Figure 7. Plot of Interval Scores over time (h=1, rolling window length =120)





Note: The IARM is used for the model with predictors