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**International Business Cycles and Risk Sharing with  
Uncertainty Shocks and Recursive Preferences**

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This paper analyzes the effects of output volatility shocks on the dynamics of consumption, trade flows and the real exchange rate, in a two-country, two-good world with consumption home bias, recursive preferences, and complete financial markets. When the risk aversion coefficient exceeds the inverse of the intertemporal substitution elasticity, then an exogenous rise in a country's output volatility triggers a wealth transfer to that country, to compensate for the greater riskiness of the country's output stream. This risk sharing transfer raises the country's consumption, lowers its trade balance and appreciates its real exchange rate. In the recursive preferences framework here, volatility shocks account for a non-negligible share of the fluctuations of net exports, net foreign assets and the real exchange rate. These shocks help to explain the high empirical volatility of the real exchange rate and the disconnect between relative consumption and the real exchange rate.

Key words: international business cycles, international risk sharing, external balance, exchange rate, volatility, consumption-real exchange rate anomaly.

JEL codes: F31, F32, F36, F41, F43.

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## 1. Introduction

The turmoil triggered by the recent global financial crisis has stimulated much research on the macroeconomic effects of economic volatility (uncertainty) shocks. Most theoretical models with volatility shocks are closed economy models (see survey by Bloom (2014)). This paper provides simple analytics and simulation results for the effect of output volatility shocks in open economies. A two-country world with two traded goods is considered. Each country is inhabited by a representative household who receives an exogenous endowment of *one* of the goods. Households consume both goods, but there is a preference bias toward the domestic good. Households have recursive preferences of the Epstein and Zin (1989) and Weil (1989, 1990) type. Under these preferences, the coefficient of risk aversion may differ from the inverse of the intertemporal elasticity of substitution. A well-developed closed economy literature shows that this flexibility helps to reconcile the joint stochastic behavior of consumption and asset returns (Swanson (2015)); however, the international macroeconomics literature has only recently begun to consider models with recursive preferences (see below).

The model assumes complete financial markets, so that consumption risk is efficiently shared across countries. The paper shows that, under recursive preferences, output volatility shocks can account for a non-negligible share of the fluctuations of net exports, net foreign assets and the real exchange rate. Volatility shocks help to explain the observed high volatility of the real exchange rate and the disconnect between relative consumption and the real exchange rate.

With complete markets, the ratio of the Home intertemporal marginal rate of substitution (IMRS) to the Foreign IMRS is equated to the (gross) rate of appreciation of the Home real exchange rate. Under standard time-separable preferences, this condition implies that the log Home/Foreign consumption ratio is inversely proportional to the log Home real exchange rate, defined as the ratio of the Home CPI to the Home currency-equivalent of the Foreign CPI. This static risk sharing condition guarantees that equilibrium domestic and foreign consumption and the real exchange rate at a given date depend solely on *contemporaneous* domestic and foreign endowments; with time-separable utility, consumption and the real exchange rate are thus not affected by uncertainty about *future* endowments.

A potentially powerful channel for the transmission of output volatility shocks emerges when agents have recursive preferences. If the coefficient of risk aversion (CRA) differs from the inverse of the intertemporal elasticity of substitution (IES), then a household's IMRS depends on her (future) lifetime utility. As volatility shocks affect lifetime utility, these shocks impact the IMRS when  $CRA \neq 1/IES$ , and thus volatility shocks affect equilibrium consumption, trade flows and the real exchange rate. Under the common assumption that  $CRA > 1/IES$  (e.g. Swanson (2014)), which implies a preference for the early resolution of uncertainty over future consumption (Weil (1990)), a fall in future lifetime utility *raises* the household's IMRS. Consider an idiosyncratic positive shock to the volatility of country 'Home' output. That shock lowers Home lifetime utility, which raises the Home IMRS, when  $CRA > 1/IES$ . Foreign lifetime utility falls less, as the bulk of Home output is consumed locally. Efficient risk sharing under recursive utility ( $CRA > 1/IES$ ) implies that the shock to Home output volatility triggers a wealth transfer from the rest of the world to the Home country, to compensate for the greater riskiness of the Home output stream. That risk sharing transfer raises Home's relative consumption and Home net imports; it improves Home's terms of trade and *appreciates* its real exchange rate. The risk sharing transfer thereby aligns the relative Home/Foreign IMRS and the appreciation rate of the Home real exchange rate.<sup>1</sup>

The model here helps resolve the widely discussed 'consumption-real exchange rate anomaly' (Obstfeld and Rogoff (2000)). Standard models with time-separable utility predict that a rise in a country's relative output depreciates its terms of trade and its real exchange rate, while it increases its relative consumption (Kollmann (1991, 1995, 2012), Backus and Smith (1993), Devereux and Kollmann (2012), Küçük and Sutherland (2015)). Qualitatively similar real exchange rate and consumption responses to output level shocks are generated under recursive utility.<sup>2</sup> Model variants with just output shocks

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<sup>1</sup>Consumption home bias is a key ingredient of the transmission mechanism of volatility shocks: if both countries consumed the same basket of Home and Foreign goods, the real exchange rate would be constant, and aggregate consumption and welfare would be perfectly correlated across countries, under complete markets. Without home bias, each country's consumption at a given date would thus solely be a function of contemporaneous endowments, i.e. uncertainty about future endowments would not affect consumption and the terms of trade.

<sup>2</sup>When  $CRA > 1/IES$ , the Home real exchange rate depreciates more strongly in response to a Home output increase (than with time-separable utility), as the shock raises Home lifetime utility, which lowers the Home IMRS. The sharper Home real exchange rate depreciation is accompanied by a more muted rise in Home consumption.

thus predict that the growth of a country's relative consumption and the rate of appreciation of its real exchange rate are (almost) perfectly **negatively** correlated. Yet, empirically, the growth rate of relative consumption and the rate of appreciation of the real exchange rate are (essentially) uncorrelated. The recursive preferences model with *simultaneous* output level and volatility shocks offers a possible solution to this puzzle. The key mechanism is that (as discussed above), the model predicts that output volatility shocks induce **positively** correlated responses of the growth of a country's relative consumption and of the rate of appreciation of its real exchange rate. A recursive preferences model with the appropriate mix of shocks to the level and the volatility of output can therefore generate a realistic correlation between relative consumption and the real exchange rate, i.e. a correlation that is close to zero.

The work here is related to several recent papers that study open economy macro models with recursive preferences; see, e.g., Kollmann (2009, 2015b), Colacito and Croce (2011, 2013), Dou and Verdelhan (2015), Lustig and Verdelhan (2015), Lewis and Liu (2014), Gourio et al. (2013, 2015), Tretvoll (2013), Caporale et al. (2014) and Sauzet (2015). These papers do not study the effect of volatility shocks. Backus et al. (2015), Benigno et al. (2012) and Mumtaz and Theodoridis (2015) study the effect of volatility shocks in open economy models with recursive preferences, but the focus of these papers is different.<sup>3</sup> The paper here is also complementary to research by Fogli and Perri (2014) and by Hoffmann et al. (2014) who study the effect of output volatility shocks in *one-good* models with *time separable* preferences and international financial markets that are *incomplete*, because just a riskless bond can be traded internationally.<sup>4</sup> In contrast to these papers, the analysis here centers on efficient risk sharing, consumption-real exchange rate co-movements and related open economy stylized facts.<sup>5</sup>

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<sup>3</sup>Backus et al. (2015) explore the dynamics of Pareto weights in a two-country RBC production economy. Benigno et al. (2012) and Mumtaz and Theodoridis (2015) use a two-country New Keynesian model to study the effect of volatility shocks in the presence of nominal rigidities.

<sup>4</sup>Other related studies include Fernandez-Villaverde et al. (2011) and Born and Pfeifer (2014) who model the effect of foreign interest rate volatility on a small open economy; these authors also assume time-separable preferences, and a bonds-only structure.

<sup>5</sup>A number of papers consider time-separable preferences in conjunction with incomplete financial markets to study international business cycles (e.g., Corsetti et al. (2008), Baxter and Crucini (1995), Kollmann (1996), Benigno and Thoenissen (2008) and Chen and Crucini (2014)). See the working paper version (Kollmann (2015c)) for an analysis of a recursive-preferences model with incomplete markets.

Section 2 describes the model. Section 3 discusses empirical regularities about international business cycles. Section 4 presents simulation results and Section 5 concludes.

## 2. A two-country model

### 2.1. Preferences, endowments, volatility shocks, risk sharing

I consider a world with two symmetric countries, referred to as ‘Home’ ( $H$ ) and ‘Foreign’ ( $F$ ), respectively. Each country is inhabited by a representative infinitely-lived household. All agents observe current and past realizations of all variables (full information). At date  $t$ , country  $i=H,F$  receives an exogenous endowment of  $Y_{i,t}$  units of a perishable tradable output good  $i$ . The country  $i$  household combines local and imported output into aggregate consumption, using the technology:

$$C_{i,t} \equiv (y_{i,t}^i/\alpha)^\alpha (y_{i,t}^j/(1-\alpha))^{1-\alpha}, \quad j \neq i, \quad (1)$$

where  $y_{i,t}^j$  is the amount of input  $j$  used by country  $i$ . There is consumption home bias:  $0.5 < \alpha < 1$ . At  $t$ , country  $i$ 's consumption price index is  $P_{i,t} = (p_{i,t})^\alpha (p_{j,t})^{1-\alpha}$ ,  $j \neq i$ , where  $p_{j,t}$  is the price of good  $j$ . The Home terms of trade and real exchange rate are defined as

$$q_t \equiv p_{H,t}/p_{F,t} \quad \text{and} \quad RER_t \equiv P_{H,t}/P_{F,t}, \quad (2)$$

respectively, i.e. a rise in  $q$  represents an improvement in the Home terms of trade, and an increase in  $RER$  represents an *appreciation* of the Home real exchange rate. Note that  $RER_t = (q_t)^{2\alpha-1}$ ; due to consumption home bias ( $2\alpha-1 > 0$ ), an improvement of the Home terms of trade induces a Home real exchange rate appreciation. Input demands are:

$$y_{i,t}^i = \alpha P_{i,t} C_{i,t} / p_{i,t}, \quad y_{i,t}^j = (1-\alpha) P_{i,t} C_{i,t} / p_{j,t} \quad \text{for } j \neq i. \quad (3)$$

Market clearing requires  $y_{H,t}^i + y_{F,t}^i = Y_{i,t}$  for  $i=H,F$ .

The country  $i$  household has a recursive intertemporal utility function inspired by Epstein and Zin (1989) and Weil (1989, 1990):

$$U_{i,t} = \{(1 - \beta_{i,t}) C_{i,t}^{1-\sigma} + \beta_{i,t} [E_t U_{i,t+1}^{1-\gamma}]^{(1-\sigma)/(1-\gamma)}\}^{1/(1-\sigma)}, \quad (4)$$

where  $U_{i,t}$  is lifetime utility at date  $t$ .  $0 < \beta_{i,t} < 1$  is the household's subjective discount factor between periods  $t$  and  $t+1$ ,  $1/\sigma$  is the intertemporal elasticity of substitution (IES).

$\gamma$  indexes the household's aversion against uncertainty in future lifetime utility. Note that time-separable utility obtains when  $\gamma=\sigma$ . Epstein, Zin and Weil assume a version of (4) in which  $\beta_{i,t}$  is a constant parameter. In order to ensure that the model has a unique deterministic steady state and an equilibrium in which the consumption/output and net foreign assets/output ratios are stationary, I assume that the subjective discount factor of household  $i$  is a decreasing function of its consumption, normalized by domestic output:  $\beta_{i,t}=\bar{\beta}-b\cdot\ln(C_{i,t}/Y_{i,t})$ , with  $b>0$ . In the simulations,  $b$  is set at a very small value.

Country  $i$ 's intertemporal marginal rate of substitution (IMRS) between aggregate consumption at dates  $t$  and  $t+1$  is:

$$\rho_{i,t+1} \equiv \beta_{i,t} \cdot \frac{\Psi_{i,t+1}}{\Psi_{i,t}} \cdot \left( \frac{C_{i,t+1}}{C_{i,t}} \right)^{-\sigma} \cdot \left( \frac{U_{i,t+1}}{(E_t U_{i,t+1}^{1-\gamma})^{1/(1-\gamma)}} \right)^{\sigma-\gamma}, \quad (5)$$

with  $\Psi_{i,t} \equiv 1 - \beta_{i,t} - b \cdot (1/\beta_{i,t}) \cdot [(U_{i,t}/C_{i,t})^{1-\sigma} - 1]/(1-\sigma)$ . Note that the term  $\beta_{i,t} \Psi_{i,t+1}/\Psi_{i,t}$  in (5) equals the steady state subjective discount factor  $\bar{\beta}$  when the slope parameter of the discount factor is zero ( $b=0$ ). For values of  $b$  close to zero, the term  $\beta_{i,t} \Psi_{i,t+1}/\Psi_{i,t}$  makes a negligible contribution to the high frequency dynamics of the IMRS.

The model assumes complete international financial markets. In equilibrium, the Home/Foreign IMRS ratio is thus equated to the (gross) rate of appreciation of the Home real exchange rate (Kollmann (1991, 1995), Backus and Smith (1993)):

$$\rho_{H,t+1}/\rho_{F,t+1} = RER_{t+1}/RER_t. \quad (6)$$

The market value of country  $i$ 's net foreign assets at the end of period  $t$ , denoted by  $NFA_{i,t+1}$ , equals the present value of  $i$ 's future net imports (e.g., Kollmann (2006), Evans (2014)):  $NFA_{i,t+1} = E_t \sum_{k=1}^{\infty} \rho_{i,t+k} \cdot (P_{i,t}/P_{i,t+k}) \cdot (-NX_{i,t+k})$ , where  $NX_{i,\tau} \equiv p_{i,\tau} Y_{i,\tau} - P_{i,\tau} C_{i,\tau}$  are net exports at date  $\tau$ .<sup>6</sup>

Empirical fluctuations of output and of *relative* output are highly persistent. I assume that output has a unit root, while relative output is stationary but highly serially

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<sup>6</sup>The market value of net foreign assets differs from cumulated past current accounts as defined in national accounts statistics (e.g., Kollmann (2006), Coeurdacier et al. (2010)).

correlated. Specifically, Home and Foreign output are assumed to follow an error-correction mechanism with heteroskedastic disturbances:

$$\Delta \ln(Y_{i,t+1}) = -\kappa \cdot [\ln(Y_{i,t}) - \ln(Y_{j,t})] + e^{s_{i,t}} \lambda_i \varepsilon_{i,t+1}^Y, \quad s_{i,t} = \rho_i s_{i,t-1} + \phi_i \varepsilon_{i,t}^S \quad \text{for } i=H,F \text{ and } j \neq i, \quad (7)$$

with  $\kappa, \lambda_i, \phi_i > 0$  and  $0 < \rho_i < 1$ .  $\varepsilon_{i,t+1}^Y$  and  $\varepsilon_{i,t}^S$  are exogenous  $N(0,1)$  white noise processes;  $\varepsilon_{i,t+1}^Y$  is independent of  $\varepsilon_{j,t+1}^S$  for  $i, j=H, F$ .  $s_{i,t}$  is time-varying output volatility. The unconditional standard deviation of the output innovation is  $\lambda_i e^{2V_{s,i}}$ , where  $V_{s,i} \equiv (\phi_i)^2 / (1 - \rho_i^2)$  is the variance of  $s_{i,t}$ . The parameter  $\lambda_i$  indexes thus the standard deviation of output innovations;  $\rho_i$  determines the persistence of output volatility, while  $\phi_i$  is the standard deviation of innovations to output volatility.

Asymmetric consumption baskets *and* recursive utility are both necessary for the transmission of output volatility shocks to the real exchange rate, consumption and net exports. Note that when both countries consume the same basket,  $\alpha=0.5$ , the real exchange rate equals unity, so that the risk sharing condition (6) entails equalization of the Home and Foreign IMRS:  $\rho_{H,t+1} = \rho_{F,t+1}$ . If the subjective discount rate is constant ( $b=0$ ), and both countries have initial wealth, this condition implies that aggregate consumption is equated across countries in all periods:  $C_{H,t} = C_{F,t}$ , so that each country consume half the endowment of each good; consumption is thus unaffected by changes in uncertainty about *future* endowments. With time-separable utility ( $\sigma=\gamma$ ), and a constant subjective discount factor ( $b=0$ ), the risk sharing equation (6) implies the familiar condition  $-\sigma \ln(C_{H,t}/C_{F,t}) = \ln(RER_t)$  (see Kollmann (1991, 1995), Backus and Smith (1993)). This *static* equation implies again that date  $t$  consumption and the real exchange rate depend solely on *contemporaneous* endowments, so that (once more) uncertainty about *future* endowments fails to affect the real exchange rate and consumption.<sup>7</sup>

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<sup>7</sup>With an endogenous discount factor ( $b>0$ ), output volatility shocks affect the IMRS, consumption and the real exchange rate when  $\gamma \neq \sigma$ , as  $\Psi_{i,t}$  (see (5)) depends on lifetime utility if  $b>0$ ; however, for small values of  $b$  (as used in the simulations) that effect is negligible, and thus the impact of volatility shocks on consumption and the real exchange is minimal when  $\gamma = \sigma$ . (With  $\alpha=0.5$ , consumption fails to be equated across countries, and depends on output volatility, if  $b>0$ ; however, the cross-country consumption correlation is close to unity, and the effect of volatility shocks is very weak, when  $b$  is small.)



A powerful channel for the transmission of country-specific output volatility shocks emerges when *both* consumption home bias *and* recursive preferences are assumed. Intuitively, home bias implies that country-specific changes in output volatility induce country-specific fluctuations in lifetime utility (when output is largely consumed locally, then a shock to a country's output uncertainty affects domestic welfare more strongly than foreign welfare, even under efficient risk sharing). With recursive preferences, those country-specific welfare changes affect the relative Home/Foreign IMRS. The risk sharing condition (6) then implies that the real exchange rate, consumption and net exports respond to output volatility shocks. With complete markets, volatility shocks induce cross-country risk sharing transfers that ensure that the relative Home/Foreign IMRS tracks the rate of appreciation of the real exchange rate, as prescribed by (6) (see further discussion in Section 4).

## 2.2. Numerical solution method

As output is assumed non-stationary, but cointegrated across countries, I reformulate the model by normalizing country  $i$  consumption, net exports, net foreign assets and utility by  $i$ 's output. The reformulated model is solved using a third-order approximation around the symmetric deterministic steady state. The Dynare toolbox is used for that purpose (Adjemian et al. (2014)). I simulate the model and compute moments of endogenous variables using the pruned state-space representation of the third-order accurate model solution (Kollmann (2005, 2015a)).

## 2.3. Calibration

### 2.3.1. Preference and technology parameters

One period represents one quarter. The steady state subjective discount factor and the slope parameter of the subjective discount factor are set at  $\bar{\beta}=0.99$  and  $b=0.001$ , respectively.<sup>8</sup> The intertemporal substitution elasticity  $1/\sigma$  is set at 1.5 in line with standard values of that parameter used in the macroeconomics literature (results are robust to assuming other values of  $1/\sigma$  in the same range). Following the macro-finance

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<sup>8</sup> That small value of  $b$  implies that the short term dynamics of the model are similar to those generated by a (non-stationary) model variant with a constant subjective discount factor ( $b=0$ ).

literature that assumes recursive preferences (e.g., Dew-Becker (2014), Swanson (2014)), I consider high risk aversion coefficients:  $\gamma=10$  and  $\gamma=40$ .

I calibrate the endowment process to quarterly 1973-2014 data for the US and for an aggregate of 23 other OECD economies (henceforth referred to as ‘rest of the world’, ROW) for which quarterly national accounts data for that period are available in the OECD national accounts database.<sup>9</sup> In 1973-2014, the mean US trade share ( $0.5*(\text{exports}+\text{imports})/\text{GDP}$ ) was 10%. Thus, I set the home bias parameter at  $\alpha=0.5$ .

### 2.3.2. Endowment process

The model abstracts from physical investment and government purchases. Like other papers that use models of endowment economies (without government) to study the dynamics of the external balance (e.g., Engle and Rogers (2006)), I consider an empirical ‘net output’ measure that equals private consumption plus net exports:  $GDP_{i,t}^{net} = (C_{i,t}^{nom} + X_{i,t}^{nom} - M_{i,t}^{nom}) / P_{i,t}^{GDP}$ , where  $C_{i,t}^{nom}$ ,  $X_{i,t}^{nom}$  and  $M_{i,t}^{nom}$  are nominal consumption, exports and imports (in domestic currency) respectively, while  $P_{i,t}^{GDP}$  is the GDP deflator. ROW net GDP is computed as a geometric weighted average of real net GDP indices for the 23 countries included in the ROW aggregate.<sup>10</sup>

I estimated the parameters  $\lambda_t, \rho_i, \phi_i$  of the endowment process (7) with quarterly US and ROW net GDP data (1973-2014), using Ruiz’ (1994) quasi-maximum likelihood method. For the US, the parameter estimates (standard errors in parentheses) are:  $\lambda_t=0.56\%$  (0.14%);  $\rho_i=0.99$  (0.02);  $\phi_i=5.82\%$  (3.76%). Thus, the standard deviation of US output volatility innovations ( $\phi_i$ ) is large, and volatility is highly persistent. For the sake of symmetry, I set the parameters of both countries’ endowment processes in the model at the estimates for the US.<sup>11</sup> The sample correlation between estimated US and

<sup>9</sup>The ROW countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, Mexico, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey and the United Kingdom.

<sup>10</sup> I use time-varying real GDP weights (based on countries’ real GDP at fixed PPPs, in USD of base year 2010) to compute the ROW aggregates.

<sup>11</sup>Parameter estimates of the ROW endowment process are similar to the US estimates (see web Appendix). An asymmetric calibration that uses the US [ROW] parameter estimates reported in the web Appendix for the country H [F] output process gives predictions that are close to those obtained for the symmetric

ROW output volatility series is close to zero (0.018). In the numerical simulations, I thus set the cross-country correlation of volatility innovations  $(\varepsilon_{H,t}^s, \varepsilon_{F,t}^s)$  at zero. Other empirical research also finds modest cross-country correlations of output volatility. For G7 countries, Antonakakis and Badinger (2016) report cross-country correlations of industrial production volatility that are in the range of 0.25 (sample period: 1986-2013). A sensitivity analysis with respect to the cross-country volatility correlation is presented below.

An Augmented Dickey-Fuller test fails to reject the hypothesis that relative US/ROW net GDP has a unit root. To ensure stationarity of the normalized model, I set the output error correction parameter (see (7)) at a very small positive value,  $\kappa=0.002$ .<sup>12</sup> The empirical correlation between US and ROW net output growth is 0.19. I set the correlation of  $\varepsilon_{H,t}^Y$  and  $\varepsilon_{F,t}^Y$  at 0.22, as this reproduces the empirical cross-country correlation of net output (given the calibrated values of  $\lambda_i, \rho_i, \phi_i$  and  $\kappa$ ).

### 3. Empirical regularities

Table 1 reports historical business cycle statistics (1973-2014) for US and ROW net GDP, consumption, net exports and net foreign assets, and the effective real exchange rate. Net exports are divided by quarterly net output, while net foreign assets are normalized by annual net output. The statistics pertain to quarterly first differenced data, with the exception of (normalized) net foreign assets for which first differences of annual data are used. Net GDP, consumption and the real exchange rate are logged before first differencing. The standard deviation of net GDP growth (about 0.7%) is very similar across the US and the ROW. Consumption and net exports are less volatile than net GDP. Net foreign assets and the real exchange rate are markedly more volatile than net GDP. Consumption and net exports are positively correlated with domestic net GDP. Net foreign assets and the real exchange rate are only weakly correlated with net GDP. The

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calibration (contributions of volatility shocks to fluctuations in net exports and the real exchange rate are similar etc.).

<sup>12</sup>The output process (7) implies that the autocorrelation of logged relative Home/Foreign net output is  $1-2\kappa$ . The 1973-2014 sample autocorrelation of logged relative US/ROW net output is 0.996. Setting  $\kappa=0.002$  matches that sample autocorrelation.

cross-country correlation of consumption growth (0.30) is higher than that of net output growth (0.19). The correlation between the growth of relative US/ROW consumption and the rate of appreciation of the US real exchange rate is 0.15 (not reported in Table), i.e. relative consumption is weakly positively correlated with the real exchange rate.

#### 4. Model predictions

Table 2 reports predicted standard deviations and cross-correlations of key variables generated by different model parameterizations. Predicted moments of output, consumption and the real exchange rate pertain to log first differenced variables, while moments for net exports pertain to first differenced net exports normalized by domestic GDP; moments for NFA pertain to annual differences of net foreign assets normalized by annual GDP.<sup>13</sup> The Tables also reports the Hansen-Jagannathan (1991) bound [‘HJ bound’] generated by the model, i.e. the ratio of the standard deviation of the intertemporal marginal rate of substitution (IMRS) divided by the mean IMRS. That statistic allows us to evaluate whether the model has the potential to generate realistic risk premia on financial assets. In equilibrium, the Sharpe ratio of any traded risky asset is bounded above by the HJ bound. The historical Sharpe ratio of US quarterly real equity returns was 0.22 in 1973-2014.<sup>14</sup> Thus, a model-generated HJ bound below 0.22 indicates that the model cannot generate a realistic equity premium.

Cols. (2)-(4) of Table 2 show predicted moments generated by a model variant with time-separable utility, i.e. in which the risk aversion coefficient is set at the inverse of the intertemporal elasticity of substitution (IES):  $\gamma=1/IES=0.66$ ; this specification corresponds to standard time-separable utility (recall that  $IES=1.5$ ). Columns labelled ‘Y,s’ show predicted model moments under simultaneous output level and volatility shocks. To disentangle the effect of level and volatility shocks, I also show predicted moments that are generated when just output level shocks are fed into the equilibrium decision rules (holding the volatility shock constant at  $s_{i,t}=0$ ), as well as predicted

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<sup>13</sup>Net exports and net foreign assets are normalized by output (e.g.,  $NX_{i,t}/(p_{i,t}Y_{i,t})$ ) before quarterly/annual differences are computed.

<sup>14</sup>The Sharpe ratio is the ratio of an asset’s average excess return (relative to a risk-free return), divided by the standard deviation of the excess return. The historical Sharpe ratio reported in the text was constructed using returns data from K. French’s web page. The quarterly equity Sharpe ratio was 0.19 in 1926-2014.

moments that obtain when just volatility shocks are fed into the decision rules (see Cols. labeled ‘Y’ and ‘s’, respectively).<sup>15</sup>

Cols. (2)-(4) of Table 2 show that the model variant with time-separable utility ( $\gamma=1/\text{IES}$ ) exhibits well-known shortcomings of first generation International Real Business Cycle models (e.g., Backus et al. (1992)): the predicted volatility of the real exchange rate, net exports and net foreign assets is much smaller than the counterparts in the data. The growth of relative consumption is perfectly negatively correlated with the rate of real exchange rate appreciation, which likewise is inconsistent with the data. Also, the HJ bound generated by the model variant with  $\gamma=1/\text{IES}$  is close to zero, i.e. that variant cannot generate a realistic equity premium. In this model variant, volatility shocks have no effect on consumption, the real exchange rate, net exports and net foreign assets (see Col. (3)), as discussed in Section 2.1.

Cols. (5)-(10) of Table 2 consider two model variants in which the risk aversion coefficient  $\gamma$  exceeds  $1/\text{IES}$ :  $\gamma=10$  and  $\gamma=40$ . These recursive-preferences model variants generate sizable IMRS fluctuations, as the IMRS is affected by shocks to expected lifetime utility when  $\gamma > 1/\text{IES}$  (see (5)). The model variants with  $\gamma=10$  and  $\gamma=40$  generate HJ bounds of 0.05 and 0.21, respectively, i.e. the IMRS is much more volatile than in the model variant with time-separable utility. Substantial risk aversion is thus needed to generate a realistic HJ bound. Due to the greater volatility of the IMRS, the model variants with  $\gamma=10$  and 40 generate more volatile real exchange rates, net exports and net foreign assets than the model variant with time-separable utility. Predicted volatility is greater, the higher the risk aversion coefficient. Importantly, the model variants with  $\gamma > 1/\text{IES}$  predict that *volatility shocks* have a non-negligible effect on consumption, the real exchange rate, net exports and net foreign assets. For example, when  $\gamma=40$ , the predicted standard deviations of these four variables are 0.17%, 0.60%, 0.09% and 3.78%, respectively, when just volatility shocks are fed into the equilibrium decision rules (see Col. (10)). The corresponding predicted standard deviations are 0.54%, 1.59%, 0.16% and 7.55%, respectively, with simultaneous level and volatility shocks (see Col.

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<sup>15</sup> A simulated path with just volatility shocks represent a path in which *realized* output innovations equal zero ( $\varepsilon_{i,t}^y=0$ ) for all periods, but in which volatility  $s_{i,t}$  fluctuates randomly.

(8)). Thus, the predicted volatility of the real exchange rate and of net foreign assets (with both shock types) is roughly in the range of empirical volatility (see Col. (1)); however, net exports remain insufficiently volatile.

With just output level shocks (and constant volatility), the model versions with  $\gamma > 1/\text{IES}$  generate a high positive cross-country consumption correlation, and a large negative correlations between relative consumption and the real exchange rate (e.g., when  $\gamma=40$ , these two correlations are 0.92 and -0.99, respectively). By contrast, volatility shocks induce consumption fluctuations that are perfectly negatively correlated across countries, and fluctuations in relative consumption that are perfectly positively correlated with the real exchange rate. However, with *simultaneous* output level and volatility shocks, the predicted correlation between relative consumption and the real exchange rate is closer to the empirical correlation, for  $\gamma=40$  (predicted correlation: -0.17). When  $\gamma=40$ , the predicted cross-country consumption correlation (0.73) remains too high, compared to the data. Higher risk aversion is required to generate a more realistic predicted cross-country consumption correlation.

The baseline model assumes a zero cross-country correlation of output volatility shocks. The contribution of volatility shocks to fluctuations of the real exchange rate, net exports and net foreign assets is smaller when volatility shocks are positively correlated across countries (these variables do not respond to common international movements in volatility shocks). Cols. (11)-(12) of Table 2 report predicted moments for a model variant with risk aversion  $\gamma=40$ , in which the cross-country correlation of volatility shocks is set at 0.50, i.e. at more than 2 times the assumed cross-country correlation of output level shocks (0.22). In that model variant, volatility shocks continue to have a non-negligible effect. When just volatility shocks are fed into the decision rules, the predicted standard deviations of the real exchange rate and of net foreign assets are now 0.42% and 2.67%. With simultaneous output level and volatility shocks, the predicted correlation between relative consumption and the real exchange rate (-0.42) remains markedly closer to the data than the correlation with just level shocks (-0.99).<sup>16</sup>

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<sup>16</sup>When just output shocks are fed into the decision rules, the model variant with  $\gamma=40$  and correlated volatility shocks generates virtually the same moments as the variant with independent shocks; see Col. (9).

Panel (a) of Figure 1 shows dynamic effects of a one-standard deviation (0.56%) Home output level shock in the model variant with time separable utility ( $\gamma=0.66=1/\text{IES}$ ). The shock triggers a persistent rise in Home output, and it leads to a very gradual rise in Foreign output. International risk sharing implies that Home sends part of its higher output to Foreign. However, due to the strong Home bias in consumption, Home consumption rises much more than Foreign consumption. There is a modest Home real exchange rate depreciation (-0.33%).

Panel (b) of Figure 1 shows dynamic responses to one-standard deviation Home output level and volatility shocks, for recursive utility with risk aversion  $\gamma=40$ . A positive innovation to the Home output level raises Home lifetime utility which reduces the Home IMRS between the period preceding the shock and the period of the shock (see equation (5)). This implies that the output level shock triggers a much more muted rise in Home consumption than in the model variant with  $\gamma=1/\text{IES}$ , while Foreign consumption rises much more. The more muted rise in Home consumption when  $\gamma>1/\text{IES}$  implies that the Home real exchange rate depreciates more strongly. On impact, a 0.56% Home output level innovation depreciates the Home real exchange rate by 1.0%, i.e. the depreciation is 3 times stronger than under time-separable utility. Home net exports rise persistently (by 0.08% of output), and thus Home net foreign assets (the present value of Home net imports) fall sharply (by -2.1% of annual output). The strong rise in Foreign consumption explains why the cross-country consumption correlation is so high. Note that the effects of the output level shock on consumption, the real exchange rate and net foreign assets are highly persistent.

Panel (b) of Figure 1 also shows dynamic responses to a positive one standard deviation (5.81%) innovation to Home output volatility. An unexpected rise in volatility  $s_{H,t}$  lowers Home lifetime utility; when  $\gamma>1/\text{IES}$ , this raises the Home IMRS between the period preceding the volatility shock and the date of the shock. Foreign lifetime utility falls too, but less than Home welfare (the bulk of Home output is consumed domestically, and so the rise in Home output volatility hurts the Home household more than the Foreign household). This implies that the Home IMRS rises relative to the Foreign IMRS. Hence, the Home output volatility shock triggers a surprise *appreciation* of the Home real exchange rate (see (6)), and an improvement of the Home terms of trade.

Goods market clearing requires, hence, that Home relative consumption rises.<sup>17</sup> At unchanged output levels, this implies that Home consumption increases, while Foreign consumption falls, so that Home net exports drop. Under recursive utility ( $\text{CRA} > 1/\text{IES}$ ), efficient risk sharing implies thus that the rise in Home output volatility triggers a wealth transfer from the rest of the world to the Home country (i.e. Home net foreign assets increase), to (partially) compensate for the greater riskiness of the Home output stream. The responses of consumption, the real exchange rate and net exports to a volatility shock are persistent. This explains why Home net foreign assets rise strongly (and persistently).

These dynamic responses also help to understand why, under recursive preferences, volatility shocks induce consumption fluctuations that are perfectly negatively correlated across countries, and fluctuations in the growth of *relative* consumption that are perfectly positively correlated with the real exchange rate appreciation rate (see above).

## 5. Conclusion

This paper has analyzed the effects of output uncertainty shocks on the dynamics of consumption, trade flows and the real exchange rate, in a two-country world with recursive preferences and complete financial markets. When the risk aversion coefficient exceeds the inverse of the intertemporal substitution elasticity, then an exogenous rise in a country's output volatility triggers a wealth transfer to that country, in equilibrium; this raises its consumption, lowers its trade balance and appreciates its real exchange rate. In the recursive preferences framework here, volatility shocks account for a non-negligible share of the fluctuations of net exports, net foreign assets and the real exchange rate. These shocks help to explain the high empirical volatility of the real exchange rate and they provide a possible solution for the consumption-real exchange rate puzzle.

To focus sharply and simply on the role of output volatility shocks for international risk sharing, this paper has assumed that output level and output volatility

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<sup>17</sup>The goods demand functions (3) imply that relative world demand for good H (divided by demand for good F) is a decreasing function of the Home terms of trade  $q$ , and increasing in Home relative consumption  $c \equiv C_H/C_F$ :  $d \equiv (y_H^H + y_F^H)/(y_H^F + y_F^F) = (l/q)(\alpha c q^{2\alpha-1} c + 1 - \alpha) / ((1 - \alpha) c q^{2\alpha-1} + \alpha) \equiv f(q, c)$ , with  $\partial f / \partial q < 0$  and  $\partial f / \partial c > 0$  as  $0.5 < \alpha < 1$ . Market clearing requires  $Y_H/Y_F = d$ . Holding output constant, any shock that improves the Home terms of trade has thus to be accompanied by a *rise* in Home relative consumption.



shocks are exogenous and independent. Empirical research suggests that a rise in uncertainty triggers a domestic output contraction (e.g., Basu and Bundick (2015), Caldara et al. (2015)). In the model here, the predicted effects of a country-specific positive volatility shock on the real exchange rate, net exports and net foreign assets are qualitatively unchanged, but stronger, if one (mechanically) assumes that the volatility shock lowers the domestic endowment. One useful avenue for future research would be to endogenize output, in the present two-country setting. Explanations for negative output effects of volatility shocks center on nominal and real frictions in goods and labor markets (Basu and Bundick (2015), Bloom et al. (2014), Leduc and Liu (2015)).

## References

- Adjemian, Stéphane, Houtan Bastani, Frédéric Karamé, Michel Juillard, Junior Maih, Ferhat Mihoubi, George Perendia, Johannes Pfeifer, Marco Ratto and Sébastien Villemot, 2014. *Dynare: Reference Manual, Version 4.4.3*, CEPREMAP, Paris.
- Antonakakis, Nikolas and Harald Badinger, 2016. Economic Growth, Volatility, and Cross-Country Spillovers: New Evidence for the G7 countries. *Economic Modelling* 52, 352-365.
- Backus, David, and Gregor Smith, 1993. Consumption and Real Exchange Rates in Dynamic Economies with Non-traded Goods. *Journal of International Economics* 35, 297-316.
- Backus, David, Patrick Kehoe, and Finn Kydland, 1992. International Real Business Cycles. *Journal of Political Economy*, 100, 745-775.
- Backus, David, Chase Coleman, Axelle Ferriere and Spencer Lyon, 2015. Risk and Risk-sharing in Two-Country Models. Mimeo, NYU.
- Basu, Susanto and Brent Bundick, 2015. Uncertainty Shocks in a Model of Effective Demand. Working Paper, Boston College.
- Baxter, Marianne and Mario Crucini, 1995. Business Cycles and the Asset Structure of Foreign Trade. *International Economic Review* 36, 821-54.
- Benigno, Gianluca and Christoph Thoenissen, 2008. Consumption and Real Exchange Rates with Incomplete Markets. *Journal of International Money and Finance* 27, 926-48.
- Benigno, Gianluca, Pierpaolo Benigno and Salvatore Nisticò, 2012. Risk, Monetary Policy, and the Exchange Rate. NBER Macroeconomics Annual Volume 26, 247-309.
- Bloom, Nicholas, 2014. Fluctuations for Uncertainty. *Journal of Economic Perspectives* 28, 153-176.
- Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten and Stephen Terry, 2014. Really Uncertain Business Cycles. Working Paper, Stanford University.
- Born, Benjamin and Johannes Pfeifer, 2014. Risk Matters: The Real Effects of Volatility Shocks: Comment. *American Economic Review* 104, 4231-4239.
- Caldara, Dario, Cristina Fuentes-Albero, Simon Gilchrist and Egon Zakrajšek, 2015. The Macroeconomic Impact of Financial and Uncertainty Shocks. Working Paper, Federal Reserve Board.
- Caporale, Guglielmo Maria, Michael Donadelli and Alessia Varani, 2014. International Capital Markets Structure, Preferences and Puzzles: The US-China Case. CESifo Working Paper 4669.
- Chen, Kan and Mario Crucini, 2014. Trends and Cycles in Small Open Economies. Globalization and Monetary Policy Institute (Dallas Fed), Working Paper 217.
- Coeurdacier, Nicolas, Robert Kollmann and Philippe Martin, 2010. International Portfolios, Capital Accumulation and Foreign Assets Dynamics. *Journal of International Economics* 80, 100-112
- Colacito, Riccardo and Mariano Croce, 2011. Risks for the Long Run and the Real Exchange Rate. *Journal of Political Economy* 119, 153-182.
- Cole, Harold and Maurice Obstfeld, 1991. Commodity Trade and International Risk Sharing. *Journal of Monetary Economics* 28, 3-24.

- Corsetti, Giancarlo, Luca Dedola and Sylvain Leduc, 2008. International Risk Sharing and the Transmission of Productivity Shocks. *Review of Economic Studies* 75, 443–473.
- Devereux, Michael B. and Robert Kollmann, 2012. Symposium on International Risk Sharing: Introduction. *Canadian Journal of Economics* 45, 373-375.
- Dew-Becker, Ian, 2015. Bond Pricing with a Time-Varying Price of Risk in an Estimated Medium-Scale Bayesian DSGE Model. *Journal of Money, Credit and Banking* 46, 837-888.
- Dou, Winston Wei and Adrien Verdelhan, 2015. The Volatility of International Capital Flows and Foreign Assets, Working Paper, MIT.
- Engel, Charles and John Rogers, 2006. The U.S. Current Account Deficit and the Expected Share of World Output. *Journal of Monetary Economics* 53, 1063-1093.
- Epstein, Larry and Stanley Zin, 1989. Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57, 937-969.
- Evans, Martin, 2014. External Balances, Trade Flows and Financial Conditions. *Journal of International Money and Finance* 48, 271-290.
- Fernandez-Villaverde, Jesus, Pablo Guerron-Quintana, Juan Rubio-Ramirez and Martin Uribe, 2011. Risk Matters: The Real Effects of Volatility Shocks. *American Economic Review* 101, 2530-2561.
- Fogli, Alessandra and Fabrizio Perri, 2014. Macroeconomic Volatility and External Imbalances. *Journal of Monetary Economics* 69, 1-15.
- Gourio, François, Michael Siemer and Adrien Verdelhan, 2013. International Risk Cycles. *Journal of International Economics* 89, 471-484.
- Gourio, François, Michael Siemer and Adrien Verdelhan, 2015. Uncertainty and International Capital Flows. Working Paper, MIT.
- Hansen, Lars Peter and Ravi Jagannathan, 1991. Implications of Security Market Data for Models of Dynamic Economies. *Journal of Political Economy* 99, 225-262.
- Hoffmann, Mathias, Michael Krause and Peter Tillmann, 2014. International Capital Flows, External Assets and Output Volatility. Working Paper, Deutsche Bundesbank.
- Kollmann, Robert, 1991. Essays on International Business Cycles. PhD Dissertation, Economics Department, University of Chicago.
- Kollmann, Robert, 1995. Consumption, Real Exchange Rates and the Structure of International Asset Markets. *Journal of International Money and Finance* 14, 191-211.
- Kollmann, Robert, 1996. Incomplete Asset Markets and the Cross-Country Consumption Correlation Puzzle. *Journal of Economic Dynamics and Control* 20, 945-962.
- Kollmann, Robert, 2005. Solving Non-Linear Rational Expectations Models: Approximations based on Taylor Expansions. Working Paper, University Paris XII.
- Kollmann, Robert, 2006. International Portfolio Equilibrium and the Current Account, CEPR Discussion Paper 5512.
- Kollmann, Robert, 2009. EZW in a World Economy. Working Paper, ECARES, Université Libre de Bruxelles.
- Kollmann, Robert, 2012. Limited Asset Market Participation and the Consumption-Real Exchange Rate Anomaly. *Canadian Journal of Economics* 45, 566-584.

- Kollmann, Robert, 2015a. Tractable Latent State Filtering for Non-Linear DSGE Models Using a Second-Order Approximation and Pruning. *Computational Economics* 45, 239-260.
- Kollmann, Robert, 2015b. Exchange Rate Dynamics with Long-Run Risk and Recursive Preferences. *Open Economies Review* 26, 175-196.
- Kollmann, Robert, 2015c. Risk Sharing in a World Economy with Uncertainty Shocks. CEPR Discussion Paper 10940.
- Küçük, Hande and Alan Sutherland, 2015. International Risk Sharing and Portfolio Choice with Non-Separable Preferences. CEPR Discussion Paper 10724.
- Leduc, Sylvain and Zheng Liu. 2015. Uncertainty Shocks are Aggregate Demand Shocks. Working Paper 2012-10, Federal Reserve Bank of San Francisco
- Lewis, Karen and Edith Liu, 2014. Evaluating International Consumption Risk Sharing Gains: An Asset Return View. Forthcoming, *Journal of Monetary Economics*.
- Lustig, Hanno and Adrien Verdelhan, 2015. Does Incomplete Spanning in International Financial Markets Help to Explain Exchange Rates? Working Paper, MIT.
- Mumtaz, Haroon and Konstantinos Theodoridis, 2015. Common and Country Specific Economic Uncertainty. Working Paper 752, Queen Mary College, London.
- Obstfeld, Maurice and Kenneth Rogoff, 2000. The Six Major Puzzles in International Macroeconomics: Is There a Common Cause? *NBER Macro Annual* 15, 339-390.
- Ruiz, Esther, 1994. Quasi-Maximum Likelihood Estimation of Stochastic Volatility Models. *Journal of Econometrics* 63, 289-306.
- Sauzet, Maxime, 2014. Great Retrenchment, Financial Contagion and International Risk-Sharing. Working Paper, Sciences Po, Paris.
- Swanson, Eric, 2014. A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt. Working Paper, UC Irvine.
- Tretvoll, Håkon, 2013. Real Exchange Rate Variability in a Two-Country Business Cycle Model. Working Paper, BI Norwegian Business School.
- Weil, Philippe, 1989. The Equity Premium Puzzle and the Risk-Free Rate Puzzle. *Journal of Monetary Economics* 24, 401-421.
- Weil, Philippe, 1990. Unexpected Utility in Macroeconomics, *Quarterly Journal of Economics* 105, 29-42.

**Table 1. Historical statistics (1973q1-2014q4)**

	US	ROW
<b>Standard deviations (in %)</b>		
Net GDP	0.69	0.70
Consumption	0.64	0.61
Real exchange rate	2.42	n.a.
Net exports/(net GDP)	0.52	0.45
Net foreign assets/(net GDP)	7.44	n.a.
<b>Correlations with domestic net GDP</b>		
Consumption	0.73	0.73
Real exchange rate	-0.12	n.a.
Net exports/(net GDP)	0.36	0.58
Net foreign assets/(net GDP)	0.16	n.a.
<b>Cross-country correlations</b>		
Net GDP		0.19
Consumption		0.30

Notes: Empirical statistics are shown for macroeconomic variables in the US and in an aggregate of 23 other OECD economies ('ROW'). Net GDP is the sum of consumption and net exports (deflated using the GDP deflator). Net exports are normalized by quarterly net output, while net foreign assets are normalized by annual net output. The statistics pertain to first differenced quarterly data, with the exception of net foreign assets (normalized by annual net GDP) for which first differences of annual data are used. Net GDP, consumption and the real exchange rate are logged before first differencing. ROW aggregate consumption is a weighted geometric average of real consumption in the 23 ROW countries. ROW net exports/(net GDP) is constructed as the sum of nominal net exports in the 23 ROW countries (in current dollars), divided by the sum of nominal net GDP in the 23 countries (in current dollars, based on the current nominal exchange rate). Due to limited data availability, statistics for the effective real exchange rate and net foreign assets are only shown for the US. The real exchange rate is an effective rate (CPI based), from OECD MEI. US Net foreign assets data are from the Bureau of Economic Analysis. Other series are from OECD quarterly national accounts.

**Table 2. Predicted moments for different values of the risk aversion coefficient ( $\gamma$ )**

Data	$\gamma=1/\text{IES}$			$\gamma=10$			$\gamma=40$			$\text{Corr}(\varepsilon_{H,t}^s, \varepsilon_{F,t}^s)=0.5$ $\gamma=40$		
	Shocks to:			Shocks to:			Shocks to:			Shocks to:		
	$Y,s$	$Y$	$s$	$Y,s$	$Y$	$s$	$Y,s$	$Y$	$s$	$Y,s$	$s$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
<b>Standard deviations (in %)</b>												
Y	0.69	0.65	0.56	0.00	0.65	0.56	0.00	0.65	0.56	0.00	0.65	0.00
C	0.64	0.63	0.54	0.00	0.53	0.45	0.03	0.54	0.45	0.17	0.53	0.12
RER	2.42	0.49	0.42	0.00	1.31	1.09	0.12	1.59	1.24	0.60	1.51	0.42
NX	0.52	0.03	0.02	0.00	0.10	0.08	0.02	0.16	0.11	0.09	0.14	0.06
NFA	7.44	0.97	0.83	0.00	5.20	4.42	0.71	7.55	5.56	3.78	6.95	2.67
<b>Correlations with domestic output</b>												
C	0.73	0.99	0.99	--	0.92	0.92	--	0.83	0.89	--	0.86	--
RER	-0.12	-0.64	-0.63	--	-0.63	-0.63	--	-0.58	-0.63	--	-0.60	--
NX	0.36	-0.64	-0.63	--	0.60	0.62	--	0.49	0.61	--	0.53	--
NFA	0.16	0.48	0.48	--	-0.46	-0.46	--	-0.39	-0.45	--	-0.41	--
<b>Cross-country correlations</b>												
Y	0.19	0.19	0.21	--	0.19	0.21	--	0.19	0.21	--	0.19	--
C	0.30	0.26	0.29	--	0.82	0.83	-1.00	0.73	0.92	-1.00	0.82	-1.00
<b>Correlation between <math>C_H/C_F</math> and RER</b>												
	0.15	-1.00	-1.00	--	-0.95	-0.99	1.00	-0.17	-0.99	1.00	-0.42	1.00
<b>Hansen-Jagannathan bound</b>												
		0.004	0.004	0.000	0.05	0.04	0.004	0.21	0.18	0.06	0.21	0.08

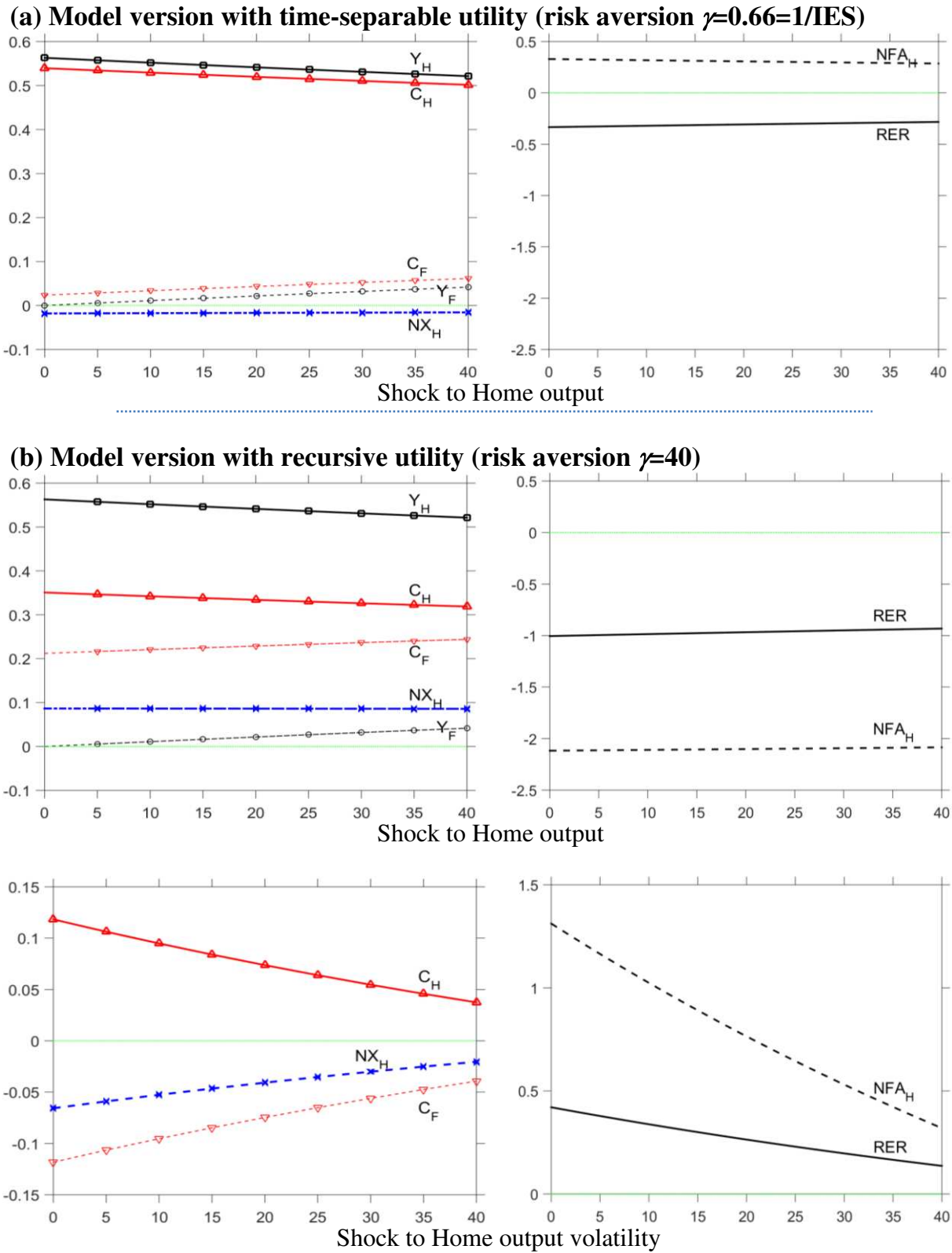
Notes: Column (1) shows empirical statistics for the US (from Table 1). The remaining Cols. show predicted model statistics. Cols. (2)-(4), (5)-(7), and (8)-(10) assume risk aversion of  $\gamma=1/\text{IES}(=0.66)$ ,  $\gamma=10$  and  $\gamma=40$ , respectively (zero cross-country correlation of volatility shocks). Col. (11)-(12) assume risk aversion  $\gamma=40$  and a 0.50 cross-country correlation of volatility shocks.

Cols. labelled 'Y,s' (see Cols. (2),(5),(8),(11)) show moments that obtain when Home and Foreign level and volatility shocks are simultaneously fed into the equilibrium decision rules. Cols. labelled 'Y' show moments that obtain when just output level shocks are fed into the decision rules (while output volatility is set at its unconditional mean). Cols. labelled 's' show moments that obtain when just volatility shocks are fed into the decision rules.

Variables are listed in the left-most column. Statistics for output (Y), consumption (C), the real exchange rate (RER) pertain to log growth rates of these variables. Moments for net exports (NX) pertain to the first difference of net exports normalized by quarterly GDP; moments for net foreign assets (NFA) pertain to annual first differences of net foreign assets normalized by annual GDP. A rise in RER represents an appreciation.

The Hansen-Jagannathan bound is defined as the ratio of the unconditional standard deviation of the intertemporal marginal rate of substitution (IMRS), divided by the unconditional mean of the IMRS.

**Figure 1. Dynamic responses to Home country innovations (1 standard deviation)**



**Note:** Panel (a) assumes time separable utility (risk aversion  $\gamma=0.66$ ) and shows responses to a 1 standard deviation Home output shock. Panel (b) assumes recursive utility ( $\gamma=40$ ) and shows effects of 1 stand. dev. Home output and volatility shocks. Effects on *expected* paths of Home and Foreign output ( $Y_H, Y_F$ ), consumption ( $C_H, C_F$ ), Home real exchange rate (RER), Home net exports ( $NX_H$ ) and Home net foreign assets ( $NFA_H$ ) are plotted. A rise in RER is an appreciation. Net exports are normalized by quarterly Home GDP. Net foreign assets are normalized by annualized Home GDP ( $4Y_H$ ). Responses of GDP, consumption and RER are shown as % deviations from unshocked paths. Responses of normalized net exports and net foreign assets expressed as percentage point differences from unshocked paths. Abscissa: periods after shock. Predetermined state variables are set at their unconditional mean, in the period of the shock ( $t=0$ ).

# WEB APPENDIX

## International Business Cycles and Risk Sharing with Uncertainty Shocks and Recursive Preferences

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**Table A.1. Estimates of the parameters of endowment processes for US and ROW (1973-2014)**

$\lambda_{US}$	$\rho_{US}$	$\phi_{US}$	$\lambda_{ROW}$	$\rho_{ROW}$	$\phi_{ROW}$
0.56% (0.14%)	0.99 (0.02)	5.81% (3.76%)	0.62 (0.07)	0.91 (0.10)	8.98% (8.43%)

Note: the Table reports estimates of the parameters of the US and ROW endowment processes (standard errors of parameter estimates) are shown in parentheses. Data: quarterly net GDP series for US and ROW. Estimation method: Ruiz (1994).

Recall that the endowment process is given by (see (7) in paper)

$$\Delta \ln(Y_{i,t+1}) = -\kappa \cdot [\ln(Y_{i,t}) - \ln(Y_{j,t})] + e^{s_{i,t}} \lambda_i \varepsilon_{i,t+1}^Y, \quad s_{i,t} = \rho_i s_{i,t} + \phi_i \varepsilon_{i,t}^S \quad \text{for } i=US, ROW \text{ and } j \neq i,$$

with  $\kappa, \lambda_i, \phi_i > 0$  and  $0 < \rho_i < 1$ .  $\varepsilon_{i,t+1}^Y$  and  $\varepsilon_{i,t}^S$  are exogenous  $N(0,1)$  white noise processes;

$\varepsilon_{i,t+1}^Y$  is independent of  $\varepsilon_{j,t+1}^S$ .  $s_{i,t}$  is time-varying output volatility.



**Table A2. Predicted moments for model variant in which volatility shocks are correlated across countries (risk aversion  $\gamma=40$ )**

Data	$Corr(\varepsilon_{H,t}^s, \varepsilon_{F,t}^s)=0.25$			$Corr(\varepsilon_{H,t}^s, \varepsilon_{F,t}^s)=0.50$			
	Shocks to:			Shocks to:			
	$Y,s$	$Y$	$s$	$Y,s$	$Y$	$s$	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
<b>Standard deviations (in %)</b>							
Y	0.69	0.65	0.56	0.00	0.65	0.56	0.00
C	0.64	0.53	0.45	0.14	0.53	0.45	0.12
RER	2.42	1.55	1.24	0.52	1.51	1.24	0.42
NX	0.52	0.15	0.11	0.08	0.14	0.11	0.06
NFA	7.44	7.25	5.56	3.27	6.95	5.56	2.67
<b>Correlations with domestic output</b>							
C	0.73	0.85	0.89	--	0.86	0.89	--
RER	-0.12	-0.59	-0.63	--	-0.60	-0.63	--
NX	0.36	0.51	0.61	--	0.53	0.61	--
NFA	0.16	-0.40	-0.45	--	-0.41	-0.45	--
<b>Cross-country correlations</b>							
Y	0.19	0.19	0.21	--	0.19	0.21	--
C	0.30	0.77	0.92	-1.00	0.82	0.92	-1.00
<b>Correlation between <math>C_H/C_F</math> and RER</b>							
	0.15	-0.27	-0.99	1.00	-0.42	-0.99	1.00
<b>Hansen-Jagannathan bound</b>							
		0.21	0.18	0.07	0.21	0.18	0.08

Notes: Column (1) shows empirical statistics for the US (from Table 1). The remaining Cols. show predicted model statistics. Cols. (2)-(4) and (5)-(8) assume that the cross-country correlation of volatility shocks is 0.25 and 0.50, respectively. The risk aversion coefficient is set at  $\gamma=40$ .

Cols. labelled 'Y,s' (see Cols. (2),(5),(8)) show moments that obtain when Home and Foreign level and volatility shocks are simultaneously fed into the equilibrium decision rules. Cols. labelled 'Y' show moments that obtain when just output level shocks are fed into the decision rules (while output volatility is set at its unconditional mean). Cols. labelled 's' show moments that obtain when just volatility shocks are fed into the decision rules.

See Table 2 in paper for further information.