Law of one price and optimal consumption-leisure choice under price dispersion

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Law of One Price and Optimal Consumption-Leisure Choice

Under Price Dispersion

Abstract
If the demand under price dispersion is formed by consumers with zero search costs and consumers with positive search costs, the law of one price holds at the equilibrium price level, where the lowest willingness to pay between consumers with zero search costs meets the willingness to accept or to sell of consumers with positive search costs. Consumers with positive search costs maximize their utility with respect to their optimal decisions when marginal losses in labor income during the search are equal to marginal savings on purchase. Optimal decisions move their willingness to accept to the equilibrium price level. Suboptimal decisions of consumers with positive search costs result in willingness to accept below the lowest willingness to pay of consumers with zero search costs and arbitrage takes place. Arbitrage drops down the equilibrium price to the level where willingness to accept of consumers with positive search costs meets the new lowest willingness to pay of consumers with zero search costs and where purchasing decisions of consumers with positive search costs become optimal.

Key words: equilibrium price, consumption-leisure choice, cost of search, price dispersion, willingness to pay, willingness to accept

JEL Classification: D11, D83.

Introduction
introduced that phenomenon with the proposition that empirical studies had revealed the fact that
dispersion was the rule rather than the exception in many homogeneous product markets.
In addition, the authors strengthened their opinion by very famous statement: “Economists have
belatedly come to recognize that the “law of one price” is no law at all” (Varian 1980,p.651).
The persistence of price dispersion is supported by consumers’ heterogeneity. Usually analytical
approaches to consumers’ heterogeneity envisage two groups of consumers. There are
consumers that do not search, i.e., price-takers, and there are consumers that search for low
price: “Some consumers have zero search costs, while all others have a positive search cost”
(Stahl 1989,p.700). The model of the optimal consumption-leisure choice under price dispersion
(Malakhov 2013,2014a,2014b,2014c,2015) also uses this dual approach. Examining shoppers
with zero search costs and searchers with positive search costs, the model proposes some
additional reasoning to the question whether Walras’ law holds or not under price dispersion in
homogeneous product markets.
The model of the optimal consumption-leisure choice under price dispersion argues that market
transforms everyday satisficing buying decisions into optimal consumption-leisure choices that
equalize marginal costs of search with its marginal benefits. The model describes the analytical
framework that demonstrates why an explicit satisficing decision becomes optimal. Observing
behavior of searchers, this paper specifies the role of optimization of search costs in the
establishment of the equilibrium price level.

**Willingness to pay, equilibrium price, and willingness to accept**
The optimal consumer choice under price dispersion represents the result of the trade-off
between consumption and leisure with respect to two constraints – the wage rate $w$ and marginal
savings on purchase, i.e., the price reduction with regard to the time of search at the moment of
purchase $\partial P/\partial S<0$. The trade-off between consumption $Q$ and leisure $H$ is provided by the
propensity to search $\partial L/\partial S<0$, i.e., the propensity to substitute labor for search as for another
source of income. When the problem of the maximization of consumption-leisure utility $U(Q,H)$
is constrained by the equality of marginal values of search $w\partial L/\partial S=Q\partial P/\partial S$, where the left side
of the equation represents the value of marginal loss in labor income during the search and the
right side represents the value of marginal benefit of search, the marginal rate of substitution of
leisure for consumption takes the following form (Fig.1):
The analysis of the propensity to search discovers the time-based structure of this apparently complex psychological variable with respect to the time horizon $T=L+S+H$ of the consumption-leisure choice (Malakhov 2013, 2015):

$$w \frac{\partial L}{\partial S} = -w \frac{L+S}{T} \quad (2)$$

And the derivative of the propensity to search with respect to leisure time simplifies the presentation of the $MRS(H \text{ for } Q)$:

$$-\frac{L+S}{T} = \frac{H-T}{T} \Rightarrow \frac{\partial^2 L}{\partial S \partial H} = \frac{1}{T} \quad (3)$$

And we see that this consideration gives us another form of the constraint for the utility maximization problem:

$$w \frac{\partial L}{\partial S} = Q \frac{\partial P}{\partial S} \Rightarrow w(L+S) = -TQ \frac{\partial P}{\partial S} \quad (5)$$

The static maximization problem simply requires the equality of marginal values of search $w \partial L/\partial S = Q \partial P/\partial S$. It tells us that any optimal choice should respect this equality. However, a common consumer choice usually represents the result of some dynamic sequential search for the predetermined quantity $Q$ along the dispersion of prices that produce different marginal savings on purchase $\partial P/\partial S$. And the final decision, for example, the choice of the first quote below the reservation level of labor income $wL < wL_0$ that could be spent and restored after the purchase, represents the choice of the price of purchase $P_P = wL$ for the given quantity $Q$ with corresponding marginal savings $Q \partial P/\partial S$. If we follow step by step a common purchasing decision, it gives us the following picture (Fig.2):
We see the graphical presentation of consumer decision develops Equations 4 and 5. It gives us some price $P$ at the zero-search-costs level. And this price is greater than the willingness to pay $WTP=wL_0$.

To understand better this hypothetical price let us take home production, say, preparing a meal, as a particular form of search where the purchase price is equal to the price of inputs for home production or meal’s ingredients, or $QP_{p}=wL$, and costs of production are equal to their opportunity costs, or to the $wS$ value. And the value of total costs $w(L+S)$ should give us the price of the prepared meal, or $w(L+S)=QP$. This assumption gives us an understanding that the value of our hypothetical price is equal to the willingness to accept or to sell the prepared meal.

The same thing happens when a consumer decides at what price he should sell the bought item. If a consumer decides to sell an item he should recover not only labor costs $wL$ but also search costs $wS$. Hence, the marginal rate of substitution of searcher’s consumption to his leisure takes the final form:

$$\frac{\partial U}{\partial H} = -w \frac{\partial L}{\partial S} - \frac{w}{\partial P/\partial S} = -\frac{w}{\partial P/\partial S} = \frac{w}{P} \quad (6)$$

However, searchers are not willing to pay this price and they start the search with the reservation level $wL_0$. Who can pay this price? Obviously, there are consumers that are not interesting in search. Really, zero search costs don’t eliminate the propensity to search. Equation (2) simply takes the following form:

$$w \frac{\partial L}{\partial S} = -\frac{L}{T} \quad (7)$$

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1 The model presented here simplifies the vision of the allocation of time. While it takes into account only labor, leisure, and search, the $\partial L/\partial S<0$ rule takes the search as any form of activity that reduces price of purchase. However this simplification doesn’t look methodologically inconsistent because even the detailed analysis of the allocation of time can assume that “the price of time is the same for the shopper and for the home producer” (Aguiar and Hurst 2007, p.1536). And that “price of time” here really “does not necessarily equal a market wage (ibid.). Being compared with marginal savings on purchase, or $\mu=QP/\partial S$, it gives us $\mu=w\partial L/\partial S$. 
The search is not interesting for *shoppers* because at this price level, where all *shoppers* are still price-takers and therefore *price-reduction-takers*, the search, let’s take for illustrative simplicity a single unit purchase, produces absolute marginal savings that are not greater than absolute marginal losses in labor income, or:

$$\left| -\frac{wL}{T} \right| \geq \left| \frac{\partial P}{\partial S} \right| \quad (8)$$

It means that the zero-search-costs level collects all buyers with willingness to pay higher or equal to the price that represents the full attractiveness of an item. However, the inequality of marginal values of search is not stable. *Shoppers* with very high wage rate need less time to restore their cash balances and they reduce the expected time-horizon. The cut in the expected time horizon by saving in labor time decreases the absolute value of propensity to search (Fig.3):

$$wL_0 \gg P; \left| \frac{L_0}{T_0} \right| > \left| \frac{\partial P}{\partial S} \right| \Rightarrow wL_0 > -T_0 \frac{\partial P}{\partial S};$$

$$L_1 < L_0; T_1 = T_0 - dL;$$

$$\left| \frac{L_1}{T_1} \right| = \frac{\partial P}{\partial S} \Rightarrow wL_1 = P = -T_1 \frac{\partial P}{\partial S} \quad (9)$$

The process of adjustment of time horizon of consumers with different high willingness to pay eliminates the inequality of marginal values of search in Equation (8) and all consumers with high willingness to pay equalize their marginal losses in labor income with marginal savings at this price level:

$$w_aL_a = w_bL_b = ... = w_nL_n = -T \frac{\partial P}{\partial S} = P_e \quad (10)$$

However, when the value of price reduction is given, we see that at this price level market adjusts different perceptions of time horizons and makes itself really homogenous with the unique time horizon.

This is the level of equilibrium price. If we do not take into account for the moment the existence of upper price niche where consumers with high willingness to pay, suffering from the “snob effect” at the equilibrium price level, can search and make ambitious purchases, we can say that
the equilibrium price level is equal to the lowest willingness to pay between high-income consumers with zero search costs.

For the moment, these considerations follow the assumption that “the price in the high-price stores is the reservation price of shoppers with high willingness to pay, not their maximum willingness to pay for the good” (Diamond 1987, p.434). However, the possibility to adjust time horizon attracts to this price level or to the high-price store also some low-income consumers. Impatient low-income consumers can compensate at this price level the low wage rate by high propensity to search that results in earlier and more intensive consumption. The acceleration of consumption changes the time horizon. And impatient low-income consumers should accept not only the equilibrium price but also the equilibrium time horizon, which is shorter than the time horizon of their easy-going low-income neighbors. The reduction in the time horizon transforms the initial inequality of marginal values of search that encourages easy-going low-income consumers to search, into the optimal equation for their impatient low-income neighbors that eliminates the need to look or to wait for low price:

\[
\left| w \frac{\partial L}{\partial S} \right| < \left| \frac{\partial P}{\partial S} \right|; -w \frac{L}{T_0} < \frac{\partial P}{\partial S} < T_0; -w \frac{L}{T_1} = \frac{\partial P}{\partial S}
\]

In fact, the famous example of tourists, looking for a restaurant (Salop and Stiglitz 1977), can be revised under this assumption of impatience. Even low-income tourists don’t want to waste time, they choose the restaurant for a lunch on their way, and in the evening they become hungry earlier and they are ready to take a dinner.

Generally, low-income consumers do not accept the equilibrium price level, which is appropriate for their high-income or low-income impatient neighbors. Easy-going low-income consumers can take an advantage of their low wage rates and low propensity to search with respect to great marginal savings produced by the equilibrium price level. Searchers begin to look for low prices with regard to their willingness to pay. Of course, they esteem the total aller-et-retour time of search, i.e. the time to get in and out. Some of them search in out-of-town commercial centers and some of them at factories’ outlets where purchase prices \( P_p \) are really different (\( P_a, P_b, \ldots P_n \)). However, wherever they make purchases their willingness to accept comes to the equilibrium price level (Fig.4), or:
Fig. 4. Equilibrium price, purchase prices, and WTA of heterogeneous searchers

\[ w_a (L_a + S_a) = -T_a \frac{\partial P_a}{\partial S_a} = w_b (L_b + S_b) = -T_b \frac{\partial P_b}{\partial S_b} = \ldots = w_n (L_n + S_n) = -T_n \frac{\partial P_n}{\partial S_n} = P_e \]  

(12)

What happens when price dispersion is distorted and some prices don’t result in corresponding marginal savings? This is the same thing that takes place when **satisficing decision seems to be suboptimal**. Generally, **searchers** begin to look for low prices when the search is interesting, or the marginal loss in labor income is less than the marginal saving:

\[ \left| w \frac{\partial L}{\partial S} \right| < \left| \frac{\partial P}{\partial S} \right| \]  

(13)

Let us suppose that the satisficing choice of the first quote below the reservation level \((\partial^2 L / \partial S^2 < 0)\) stays suboptimal in accordance with Equation (13). However, if it is suboptimal, the **searchers’** willingness to accept or to sell stays below the equilibrium price level, more definitely, below the lowest zero-search-costs willingness to pay (Fig. 5), or:

\[ w \frac{\partial L}{\partial S} < \frac{\partial P}{\partial S} \Rightarrow w (L + S) < -T \frac{\partial P}{\partial S} = P_e \]  

(14)

If it happens, **searchers will sell their purchases to shoppers**. This extra supply drops the equilibrium zero-search-costs price level down to the level where arbitrage becomes unprofitable for **searchers**, i.e., to the level where they equalize marginal values of search, and to its turn this equality matches their willingness to accept with a new equilibrium price. In addition, this new equilibrium price level reinforces the team of **shoppers** by newcomers from lower income...
bracket of searchers that makes the high-price store noisy and the equilibrium price level really becomes intolerable for snobs among shoppers.

The same effect takes place when a searcher finds an unexpected great discount, which results in unexpected low price. And, facing Equation (13), either the searcher adjusts the time horizon of his choice according to Equation (11), for example, due to shorten shelf-life of a product (Malakhov 2014a), or he makes an arbitrage. Adjustments of time horizon, i.e., decision to cut or to extend products’ lifecycles at the moment of purchase reduce possibilities of arbitrage.² It means that we can expect resale to be a common economic phenomenon in markets with fixed time horizons where adjustments are not possible. And it really takes place in markets of tickets for events (Courty 2003).

Hence, arbitrage and adjustment of time horizon transform suboptimal decisions of searchers into satisficing optimal choices. Finally, the equilibrium price level collects different willingness to accept of searchers with different wage rates and different propensities to search.

Thus, the equilibrium price is equal to the willingness to accept of searchers, which is equal to the lowest willingness to pay of shoppers with zero search costs and where all individuals equalize their marginal losses in labor income with their marginal savings on purchases.

This assumption takes us back to the classical optimal consumption-leisure choice:

\[
\frac{\partial U}{\partial H} = -\frac{w}{T\partial P/\partial S} - \frac{\partial^2 L/\partial S \partial H}{w} = \frac{w}{T \partial P/\partial S} = \frac{w}{w(L + S)} = \frac{w}{P_e} \quad (15)
\]

We see, that slight modifications in Stigler’s revolutionary equation do not change the general economic sense of exchange. The detailed answer to the question, whether sellers agree with that conclusion or not, stays beyond the scope of this paper but it might be framed by some comments.

In the model presented here sellers meet very different willingness to pay. They try to discriminate shoppers and to propose additional services to consumers with very high willingness to pay that could suffer from the “snob effect” at the equilibrium price level in order to separate them. The discrimination might be explicit when sales are made on high streets where consumers get a positive externality of prestige purchases, or implicit, like it happens in web ‘clearinghouses’ where a set of different prices does not exhibit the total sellers’ heterogeneity. However, searchers can adjust their propensity to search to different quotes when they reconsider time horizons of their purchases with regard to seller’s reputation, post-purchase services, etc. When this uncertainty begins to worry shoppers they become searchers. They

² The analysis of the increase in the time horizon with the increase in quantity to be purchased when the quote is dissatisfying is presented in Malakhov (2014b).
either begin to investigate seller’s reputation, or they look for a new market with more guarantees. The appearance of stable upper price niche, i.e., the organization of a new market, does not change the logic of consumers’ decision-making. The search model presented here slightly decorates a Walrasian market. The stable upper price niche can be considered as a new market if there a new group of zero-search-costs consumers emerges. If such a group appears, other consumers with high willingness to pay become searchers and they either make satisficing optimal purchases when they search for prestigious items, or their purchases are suboptimal and these new searchers immediately find some shopper who can buy at zero search costs an item that has been already found and bought. And numerous web sites for resale of luxuries demonstrate that it is possible. If such a group does not appear, all consumers with high willingness to pay can make resale only at the original equilibrium price level and a consumer who has overpaid for an item can sell it only with a loss at this level to other zero-search-costs consumers.³

The sellers’ tactics in front of searchers is definitely artless – they redistribute transaction costs in order to charge consumers’ costs of search, especially when search entails travel costs. The idea to sell for searchers at the equilibrium price level, like it is presumed by the Diamond’s Paradox, is not fruitful even if sellers have a monopoly power to reinforce consumers to pay a monopoly price. Thus, producers incorporate all transaction costs and they sell at the zero-search-costs level in the high-price store. If searchers should buy at this equilibrium price, they will bring to labor market all time of search. This extra labor supply decreases wage rates and makes the equilibrium price level unattainable. Hence sellers also stay heterogeneous. And information clearinghouses, newspapers and web sites, “discount” different terms of sale to some list of price quotes. In practice, readers see only the vertical axis of Fig.4., where the price dispersion is evident and stable. In addition, some sellers can also expose their attempts to discriminate shoppers and they set prices above the equilibrium price level.

**Conclusion**

The analysis of propensity to search that optimizes satisficing purchasing decisions shows that the “law of one price” holds in an imperfect homogenous market if there are consumers with zero search costs. These consumers have different willingness to pay but they make purchases at the level of the lowest zero-search-costs willingness to pay. Consumers with positive search costs are also heterogeneous but they have the same willingness to accept or to sell that matches the

³ The satisficing purchases are made within the “common model” of behavior even on markets of luxuries. This is not true for the “leisure model” of behavior that produces Veblen effect (Malakhov 2015)
lowest willingness to pay of consumers with zero search costs at the equilibrium price level. Arbitrage adjusts not only prices but also the propensity to search that equalizes marginal losses in labor income during the search with marginal savings on purchase on a new equilibrium price level. When purchase decisions of consumers with positive search costs are optimal arbitrage doesn’t take place.

The equilibrium price level does not eliminate price dispersion. Consumers have different willingness to pay that meet heterogeneous sellers. Sellers try to discriminate consumers and, if they find a zero-search-costs demand, the new market is organized.

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