Futures market approach to understanding equity premium puzzle

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Abstract

In this paper, another factor that affects equity risk premium is derived from a simple classical monetary model, which basically adds back labor-leisure to a simple consumption-only consumption-based asset pricing model. If every present/future good is traded at time $t = 0$, just as in traditional Arrow-Debreu general equilibrium models and understanding bonds as essentially trading labor with future goods, it is inevitable that risk-free bonds have lower interest rate than ideal risk-free bonds of classical monetary models.

1 Introduction: equity premium puzzle

In this paper, I will assume the infinite-life representative agent framework. The model presented in this paper is partly derived from Mehra/Prescott (1985) [1], which raised equity premium puzzle questions. In an economy, there are two agents: household and firm. The household obtains utility $u(C_t, N_t)$ at time t, where C_t is consumption and N_t is labor. Total utility of the household is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, N_t) \tag{1}$$

where β is time preference. In this economy, nominal factor can be ignored, and thus every variable will be a real variable. The household has budget constraint as follows:

$$C_t + S_{t+1} + B_{t+1} \leq R_{t,S} S_t + R_{t,B} B_t + W_t N_t \tag{2}$$

where S_t is stock, B_t is bond.
Optimality condition for B_t is given by:

$$1 = R_{t+1,B} \beta E_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \right] \tag{3}$$

Optimality condition for S_t is given by:

$$1 = \beta E_t \left[R_{t+1,S} \frac{u'(C_{t+1})}{u'(C_t)} \right] \tag{4}$$

Re-arranging,

$$\beta \left(E_t \left[R_{t+1,S} \right] - R_{t+1,B} \right) E_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \right] = -COV_t \left(R_{t+1,S}, \beta \frac{u'(C_{t+1})}{u'(C_t)} \right) \tag{5}$$

where COV_t refers to covariance at time t.
Equation 5 shows how equity risk premium may arise, but it has been shown that under plausible utility restrictions, actual equity risk premium is much greater than theoretical predictions.

2 An extra factor to equity risk premium

In this section, I will ignore S_t but will keep B_t. The conclusions reached in this section will not be affected by inclusion of S_t. The household budget constraint is re-written as

$$C_t + B_{t+1} \leq R_{t,B} B_t + W_t N_t + \Pi_t \tag{6}$$

where Π_t is firms' profits (basically, the household owns the firm completely).
Equation 3 remains as before.
For convenience, let us now specify the CRRA utility specification:

$$u(C_t, N_t) = \frac{{C_t}^{1-\sigma} - 1}{1-\sigma} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \tag{7}$$

This leads to the following optimality conditions:

$$W_t = {C_t}^{\sigma} {N_t}^{\varphi} \tag{8}$$

$$1 = R_{t+1,B} \beta E_t \left[\frac{{C_{t+1}}^{-\sigma}}{{C_t}^{-\sigma}} \right] \tag{9}$$

For first-order dominant effects (note that when first-order approximation is taken, $R_{t+1,B} \approx R_{t+1,S}$), log-linearization is sufficient. Log-linearizing Equation 8 and 9,

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{10}$$

$$c_t = E_t \left[c_{t+1} \right] - \frac{1}{\sigma} \left(r_{t+1,B} - \rho \right) \tag{11}$$

where $\rho = -\log \beta$ and lower-case, except u, refers to the log of upper-case.
The firm produces a consumption good using Cobb-Douglas technology:

$$C_t = A_t {N_t}^{1-\alpha} \tag{12}$$

Log-linearizing equation 12,

$$c_t = a_t + (1-\alpha)n_t \tag{13}$$

The firm maximizes profit

$$\Pi_t = C_t - W_t N_t \tag{14}$$

leading to the optimality condition:

$$w_t = a_t - \alpha n_t + \log(1-\alpha) \tag{15}$$

It is clear from the above that in equilibrium for any time t, $B_t = 0$. In a way, one can say risk-free bond return rate is derived from risk-free bonds that no one really buys. It is also clear that B_t really does not have any real good attached. Notice also that in the representative agent framework, the household is the one who buys goods and invests for the firm.

labor service ↔ money ↔ bond ↔ money ↔ good

Essentially, the household at time t provides labor service and obtains money, uses that money to buy bonds, and at time $t+1$, use money from bond payoffs to buy goods at $t+1$. One can thus abstract money away and form the following relationship:

labor service ↔ bond ↔ good

or more simply,

labor service ↔ firm bond/good

All DSGE models are rooted in general equilibrium models, and this means that these models can roughly be understood as representing a futures market. Thus at initial time $t = 0$, goods to be delivered in the future are sold, exchanged with labor service contracts with labor amount and wage specified.

Suppose everyone participates in a futures market. Output (consumption) quantity is determined in the futures market. However, there are productivity shocks to A_t, meaning that when time $t \neq 0$ is actually realized, the firm wants to produce different quantity of C_t to maximize its profit. In any case, the minimum wage demanded by the household for some output C_t at some time $t \neq 0$ is:

$$w_t = \sigma c_t + \varphi \frac{1}{1-\alpha}(c_t - a_t) \tag{16}$$

Effectively, this produces a model of sticky wage with renegotiation, naturally flowing from a basic real business cycle model without explicitly introducing any friction. Notice also that firms already received the money for selling c_t is the futures market, and thus cannot adjust this part.

Let contracted consumption be $C_{t,ac}$, contracted labor be $N_{t,ac}$ and contracted wage be $W_{t,ac}$, and assume that the firm does not default any futures contract.

$$A_{t,ac} = \frac{C_{t,ac}}{(N_{t,ac})^{1-\alpha}} \tag{17}$$

When there is negative technology shock compared to $A_{t,ac}$ (while this does not necessarily equal to $E_0\left[A_t\right]$, this should not qualitatively matter much), the firm needs more labor to produce the amount it is contracted for. But doing so requires incentives for the household to work, and thus the effect of Equation 16 kicks in with c_t replaced with $c_{t,ac}$ and a_t realized at time t.
When there is positive technology shock relative to $A_{t,ac}$, the standard optimality equilibrium may kick in:

$$(1-\alpha)A_t(N_t)^{-\alpha} = \left[A_t(N_t)^{1-\alpha}\right]^{\sigma} N_t{}^{\varphi} \tag{18}$$

Log-linearizing Equation 18,

$$(1-\sigma)a_t = -\log(1-\alpha) + (\alpha + \sigma(1-\alpha) + \varphi)n_t \tag{19}$$

If $\sigma > 1$, this implies that "pseudo-equilibrium" n_t of Equation 19 decreases when positive technology shock (to a_t) occurs. However, the contract already specifies for $n_{t,ac}$ - thus the firm would be stuck on $w_{t,ac}$ and $n_{t,ac}$, but $c_{t,ac}$ would increase. When there is negative technology shock with $\sigma > 1$, the firm is stuck on $c_{t,ac}$ but must increase $w_{t,ac}$ and $n_{t,ac}$ as given by Equation 16.
If $\sigma < 1$, then the "pseudo-equilibrium" n_t of Equation 19 increases when positive technology occurs.

$$\frac{(1-\sigma)a_t + \log(1-\alpha)}{\alpha + \sigma(1-\alpha) + \varphi} = n_t \tag{20}$$

$$w_t = \log(1-\alpha) + a_t - \alpha\left(\frac{(1-\sigma)a_t + \log(1-\alpha)}{\alpha + \sigma(1-\alpha) + \varphi}\right) \tag{21}$$

$$w_t = \log(1-\alpha) - \frac{\alpha\log(1-\alpha)}{\alpha + \sigma(1-\alpha) + \varphi} + \frac{\sigma + \varphi}{\alpha + \sigma(1-\alpha) + \varphi}a_t \tag{22}$$

If $\sigma + \varphi > 1$, then positive technology shock increases w_t, and this will be taken by the household. If $\sigma + \varphi < 1$, then positive technology shock decreases w_t. This will not be taken by the household, meaning that the firm is stuck on $w_{t,ac}$ and $n_{t,ac}$. When there is negative shock, the firm is stuck on $c_{t,ac}$ but must increase $w_{t,ac}$ and $n_{t,ac}$ by Equation 16.
This renegotiation analysis shows that the firm either must keep the wage as contracted or must increase the wage in order not to default. The important conclusion derived from this is that in the futures market, the firm must promise the consumption quantity less than it is expected to produce. Similarly, the household expects this renegotiation and acts appropriately.
Recall Equation 11:

$$E_t[c_{t+1}] - c_t = \frac{1}{\sigma}(r_{t+1,B} - \rho) \tag{23}$$

$$c_{t+1,ac} - c_t = \frac{1}{\sigma}(r_{t+1,B} - \rho) - F \tag{24}$$

where F is addition return rate deduction due to the futures market nature of a bond and c_t is assumed to be the "present" consumption. Again note that a bond never defaults - thus is risk-free. Thus, equity risk premium is now defined by:

$$\textbf{equity risk premium} = E_t[R_{t+1,S} - R_{t+1,B}] + F + \text{non-linear terms}$$

where non-linear terms are the terms dropped by the linear approximation used in this paper. Non-linear terms will not affect the result qualitatively.
Let c_t derived from Equation 20 be $\phi(a_t)$.
Now let us get the solution for $c_{t,ac}$. For $\sigma > 1$

$$z_1 = \int_{-\infty}^{\lambda} c_{t,ac} p(a_t) da_t \tag{25}$$

where $p(a_t)$ refers to probability density function of a_t. For taking expected value of c_t, it is not necessary to change $p(a_t)$ to probability density function of c_t. λ is defined by

$$\lambda = c_{t,ac} - (1-\alpha) n_{t,ac} \tag{26}$$

Let the c_t obtained from Equation 13 with a_t and n_t given be $\psi(a_t, n_t)$.

$$z_2 = \int_{\lambda}^{\infty} \psi(a_t, n_{t,ac}) p(a_t) da_t \tag{27}$$

$$E_0 c_t = z_1 + z_2 \tag{28}$$

For $\sigma < 1$, if $\sigma + \varphi < 1$, then Equation 28 holds. If $\sigma + \varphi > 1$,

$$z_4 = \int_{\lambda}^{\infty} \phi(a_t) p(a_t) da_t \tag{29}$$

$$E_0 c_t = z_1 + z_4 \tag{30}$$

Now let us consider the case when $\sigma = 1$. In this case, $n_t = n_{t,ac}$ in any circumstance. When negative technology shock hits, wage must be increased and $c_{t,ac}$ remains stuck. When positive shock hits, wage must increase according to Equation 18 and the household takes it, and c_t is given by $\phi(a_t)$. Thus, Equation 30 is what should be expected.

Thus, $c_{t,ac}$ should be set according to Equation 28 or 30. In any circumstance, $c_{t,ac} < E_0 c_t$.
The paper used linear approximations for analysis. But in general, one cannot assume that $E_0[A_t](E_0[N_t])^{1-\alpha} = E_0[C_t]$. In such a case, one needs to solve in terms of utility maximization and derive the appropriate $n_{t,ac}$ and $c_{t,ac}$. The result, of course is much more complicated than the analysis carried out in this paper, but qualitatively the result should not change as linear approximations show dominant effects.
Also, if not all agents participate in the futures market, then heterogeneity is inevitable, and this will affect how the macro F is different from the micro F. The paper will leave this question to future papers.

3 Conclusion

In a basic classical monetary model, no one buys a bond ($B_t = 0$) in equilibrium. This is because a bond itself does not give any utility. Similarly, fiat money itself intrinsically does not hold any value. The so-called new monetarists have been critical of this fact - the fact that interest rate is derived from a bond that no market actually exists - and have been trying to give money microfoundation. In one of new monetarist papers, it was concluded that equity premium puzzle is no longer a puzzle. [2]

This paper does not exclude possibility of new monetarist effects. Rather, what this paper intends to provide is that even without explicitly assigning a role for "money," it is possible to tackle equity premium puzzle using the fact that bonds in practice must have real values, which are corresponding goods in consumption models. Furthermore, the model of this paper is essentially the model macroeconomists have been using for thinking about macroeconomy. Without explicitly introducing frictions and money, one can introduce sticky wage in terms of futures contracts, and study how risk premium may arise from this stickiness.

References

[1] Mehra, R. and Prescott, E. (1985). The equity premium: a puzzle. Journal of Monetary of Economics 15, pp. 145-161.

[2] Waller, C. (2015). Microfoundations of money: why they matter. Federal Reserve Bank of St. Louis Review, Fourth Quarter 2015, 97(4), pp. 289-301.