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Credence Goods, Risk Averse, and Optimal Insurance*

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Abstract

We analyze a credence goods market with risk averse consumers when the assumptions of both liability and verifiability hold. In the basic model, we show that the consumer's risk-aversion would induce expert's overtreatment behavior and thus cause social inefficiency. But the probability of overtreating decreases with the degree of consumer's risk-aversion or the coefficient of absolute risk aversion(CRRA). Furthermore, we extend the basic model with insurance option. We assume there exists a perfectly competitive insurance market where the consumer could purchase insurance. Two sets of equilibria indexed by expert's pricing strategy could be specified. The equilibrium outcome shows that social efficiency could always be achieved and the expert could obtain all the social surplus in the equilibrium.

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1 Introduction

When the consumers search for some repair services, the service providers have more information about the nature of consumer's problem and the corresponding treatments. The goods or services that bear this property that seller gets better information than the buyer is **credence goods**¹. The consumers barely know what they need ex ante, and even can not verify ex post. Credence goods is very common, such as medical service, education. Three types of fraudulent behaviours could be induced by the information superiority of service providers, providing unnecessary expensive treatment, charging the price for the treatment actually unprovided and provide insufficient treatment. The literature entitles these three types of frauds as *overtreatment*, *overcharging* and *undertreatment*, respectively. For instance, the car owners are recommended with unnecessary repairs by employees of Sears Automotive Centers in 90% of the test cases in the United States(see Emons (1997)). There still are numerous other examples about the expert's fraud²

Researches in credence goods mainly focus on the influences of three aspects on the market performance, the consumers' characteristics, the experts' features and market circumstances and regulations. As our paper focuses on the impact of consumer's risk-aversion on the credence goods market, the discussions about the researches into the later two aspects are deliberately omitted³. We direct all of our attentions to researches on consumer's characteristics. Within a model of strategic information transmission, Pitchik and Schotter(1987) study whether the amount of frauds would decrease as the consumer's and expert's preference become more similar. Fong(2005) considers heterogeneous consumers in terms of the valuation for repair of major problem and analyses who might be more susceptible to the expert's fraudulent behaviors. The results show that consumers with high valuation are more likely to be victims of fraud, which later is testified in Hyndman and Ozerturk(2011). Recently, Bester and Dahm(2014) find that first-best outcome

¹See the definition in Darby and Karni (1973)

²See Pesendorfer and Wolinsky(2003), Emons(1997), Dulleck and Kerschbamer (2006), Bester and Dahm(2014).

³The papers about expert's features: Dulleck and Kerschbamer (2009), Liu(2011), Beck, et al(2013); the papers about institutions: Ekelund and Jackson(2003), Dulleck and Kerschbamer (2006), Dulleck and Kerschbamer (2011), Fong, et al(2014).

can always be achieved when the consumer's delay cost or discount rate is zero, although the increase of discount rate would impose some restrictions on the parameters.

The other two papers are closely related to the problem under concern. Sulzle and Wambach(2005) investigate the influence of consumers' coinsurance rate on the expert's honesty. They add the insurance into the simplified version of credence goods model in Pesendorfer and Wolinsky(2003). In their model, there exists many profit-maximizing experts in the market and the consumer can search for second opinion whenever the current relationship is broken. The price for treatments is fixed and exogenously given. On contrast, in our model the expert is a monopoly and have pricing power and no second opinion is allowed. Besides, they assume that the types of treatment is not verifiable for the consumer, then the fraud takes the form of overcharging, while in our model we assume that both liability and verifiability holds and study the expert's overtreatment. More importantly, in Sulzle and Wambach's model, the coinsurance rate is exogenously given, while we assume that the insurance choice is endogenously decided by the consumer's optimal strategy. Then the consumer may deliberate on the coinsurance rate, and avoid the adverse effect of a higher coinsurance rate in their model.

Under a standard expert model of credence goods in Dulleck and Kerschbamer (2006) with risk-averse consumers, Bonroy, et al (2013)analyse the impacts of consumer's risk-aversion on expert's incentive to diagnosis, but no formal discussions about consumer's insurance are carried in their paper. Therefore, many interesting questions remains unanswered. How would the consumers' risk-aversion of consumer influence the experts treatment behavior? Whether social efficiency could be achieved or not? What would be consumers' optimal insurance choice? What is the consumer's insurance impact on expert's behavior? whether insurance could improve the social welfare or not?

This paper considers a standard model of credence goods with risk-averse consumer. Comparing to overcharging problem in Sulzle and Wambach(2005) and diagnosis problem in Bonroy, et al(2013), we focus on expert's overtreatment behaviour. We assume that both liability and verifiability hold and the expert could costless diagnose the nature of

the consumer's problem. The expert is a monopoly in the market, and he could price the treatments and propose a take-it-or-leave-it offer. The consumer is homogenous and risk averse. In extensive model, we extend that the consumer could purchase insurance after observing the expert's price list and insurance scheme posted by the insurer. We study the consumer's optimal insurance choice and its possible impact on expert's overtreatment and thus the market efficiency. Differing from the model in Sulzle and Wambach(2005), we model the expert-consumer relationship as a one-shot game and no second opinion is possible and the insurance is also endogenously chosen by consumer. In order to concentrate on the consumer's insurance choice, we assume that the insurance market is perfectly competitive and then the insurance is actually fair.

In basic model, we show that consumer's risk aversion could induce expert's overtreatment and then cause social inefficiency and the degree of consumer's risk-aversion would affect the expert's overtreatment behaviour. But the probability of overtreatment decreases with the degree of consumer's risk-aversion or the coefficient of absolute risk aversion(CRRA). Furthermore, when the consumer is extremely risk-averse, the expert would behave honestly and the market efficiency could be achieved.

In extensive model, the consumer could purchase actually fair insurance provided by a competitive insurance company. Two sets of equilibria indexed by expert's pricing strategy could be specified. The equilibrium outcome shows that social efficiency could always be achieved and the expert would obtain all the social surplus in the equilibrium. In one set of equilibria, the expert posts a price list with price margin from major treatment being larger than that from minor treatment. The expert's equilibrium profits equal to the price margin from minor treatment and the consumer would purchase as much insurance as possible. Interestingly, the expert could grab all the social surplus facilitated by the insurance purchased by the consumer, which acts as the tunnel that transports the consumer's surplus from major problem to minor treatment. Optimally, the consumer's insurance choice would make the expert honestly treat his problem, and in turn the expert post a optimal price list to capture all the social surplus, and the social efficiency could

be implemented in the equilibrium.

The other set of equilibria relates to the situation where the expert posts a price list with profits from major treatment being smaller than that from minor treatment. Under such a price list, the expert credibly commits to honestly repair consumer's problem, and then the consumer would purchase full insurance and ends up with the same utility level no matter which type his problem turns out to be. Again, the insurance acts as a surplus-grabbing device. Since the consumer would always purchase full insurance and gets the same utility in both states, leaving no rents for the consumer in one states (e.x. minor problem) means grabbing all the surplus in the two states. Therefore, the expert need only post a price list that make the consumer with minor problem is indifferent between accepting and rejecting the treatment offer, and then could obtain all possible surplus. Actually, the expert indeed has the incentive to post such a price list as a honesty signal because all possible surplus could be captured under the price list.

The remainder of the article is organized as follows. Section 2 presents the standard credence goods model with risk-averse consumer and analyses the equilibrium. Section 3 extends the basic model by introducing the insurance company and the equilibria are solved. Section 4 makes some discussions and then concludes.

2 Basic Model

2.1 Players and Payoff Function

There is a monopoly expert(she) and a consumer(he). The consumer has a problem r either being minor m or serious S , $r \in \{m, S\}$. $r = S$ with probability β , with $\beta \in (0, 1)$. If the consumer's problem $r \in \{m, S\}$ is resolved, the consumer could gain a gross of V_r , otherwise he gets 0. The consumer is risk-averse and his utility function follows a concave Von Neumann-Morgenstern form $U(x)$ which is twice differentiable with $U(0) = 0$, $U'(\cdot) > 0$ and $U''(\cdot) < 0$, where x being the consumer's net gain. So the consumer's utility is $U(V_b - P)$ if his problem b is resolved at price P , and $U(-P)$ if his problem is unresolved

with treatment at price P . The reserve utility of the consumer is $\bar{U} > 0$.

The expert can privately learn the nature of the consumer's problem by costless diagnosis and then make a prescription d for the consumer: major treatment H or minor treatment L , i.e., $d \in \{L, H\}$. Prescription $d = H$ repairs both types of problems while prescription $d = L$ only repairs a minor problem. The cost of performing a treatment d for the expert are C_d . Suppose the expert is a profit maximizer and her utility from performing a treatment at price P is $P - C_d$, otherwise her payoff is 0. Meanwhile, we also assume that overtreatment will induce the same cost of treating the major problem for the expert⁴.

Suppose that it is socially efficient to repair both problem with proper treatment while not efficient to repair minor problem by major treatment, i.e., $0 < C_L < V_m < C_H < V_S$. Suppose major treatment is potentially more profitable than minor treatment for the expert⁵. Also, we assume that it is not optimal for the consumer to always accepting major treatment, i.e., $\beta U(V_S - C_H) + (1 - \beta)U(V_m - C_H) < \bar{U}$ ⁶. Furthermore, we restrict any price list (P_L, P_H) to satisfy $P_d \geq C_d$ for all $d \in \{L, H\}$, which means that the expert can not cross-subsidise between the two treatment.

Following the literature, we define the following terms

DEFINITION 1. *Liability*: *The resolution of consumer's problem is verifiable costless ex post. **Verifiability**:* *The type of treatment the consumer receives is verifiable costless.*

By Liability, if the consumer's problem is not resolved after treatment, the expert could be heavily fined. Then, the expert could not providing minor treatment for the consumer with major problem, i.e., undertreatment could be excluded. By Verifiability, the expert can not provide a minor (major) treatment while charging a price for major

⁴This assumption can be easily extended to the situation that overtreatment only induce a friction of cost for major treatment. overtreatment is actually the situation that the expert privately learns that the consumer's problem is minor, So the expert only need fix the consumer's problem whilst make the consumer believe that he have performed a major treatment. Therefore, overtreatment in some cases may be less costly than actually performing a major treatment.

⁵We would like use the minor treatment as the base line to study the expert's incentive to overtreatment. Since there are only two cases, then it is a symmetric problem to focus on the expert's incentive to undertreatment, i.e., provide insufficiently minor treatment for major problem, which could only be possible when the assumption liability defined below is not held.

⁶Other cases will be discussed in later sections.

(minor) treatment, i.e., overcharging could be excluded. In our model we assume both the assumptions of liability and verifiability hold.

Timing of the Game. The game proceeds as follows:

1. The expert posts a price list (P_L, P_H) .
2. Nature draws the consumer's type. The consumer visits the expert. The expert performs a diagnosis and prescribes a treatment $d \in \{\emptyset, L, H\}$ for the consumer. \emptyset here means the expert could reject to treat the consumer.
3. The consumer decides whether to accept treatment d or not. If the consumer accepts, the treatment prescribed would be performed by the expert. If the problem is resolved, the expert receives her payment otherwise get nothing⁷. The game ends in other situations.

2.2 Equilibrium

The perfect Bayesian equilibrium is analyzed below. An equilibrium consists of the expert's strategy, the consumer's strategy, and the consumer's beliefs over the nature of problem, which satisfy the following:

1. The consumer's strategy maximizes her expected payoff given her beliefs.
2. The expert's strategy maximizes his expected utility given the consumer's strategy.
3. The consumer's beliefs are correct on the equilibrium path.

The consumer's strategy specifies the acceptance probability regarding to the expert's recommendation. Note that as long as the assumption liability holds, undertreatment would be excluded as mentioned above. The expert would only recommend minor treatment for the consumer with minor problem. And the consumer would believe that his problem is

⁷Here the payment for the expert is based on the resolution of the consumer's problem, so the liability holds. We think that it may be regarded as enough penalty for the expert's undertreatment behaviour.

indeed minor upon recommended minor treatment. Accepting the prescription at price P_L would bring the consumer with benefits of V_m , resulting in the consumer's utility being $U(V_m - P_L)$, while rejecting it retains the reserve utility \bar{U} .

Denote \bar{P}_L^* and \bar{P}_H^* that satisfy $U(V_m - \bar{P}_L^*) = \bar{U}$ and $U(V_S - \bar{P}_H^*) = \bar{U}$, respectively. Then \bar{P}_L^* and \bar{P}_H^* are the maximal prices at which the consumer would accept the treatment offer when he could make sure that his problem is minor and major, respectively. Any prices larger than the maximal prices would make the consumer reject and no transaction would be reached, which is suboptimal for the expert. So we restrict the prices with $P_L \leq \bar{P}_L^*$ and $P_H \leq \bar{P}_H^*$.

Then as long as $P_L \leq \bar{P}_L^*$ with $U(V_m - \bar{P}_L^*) = \bar{U}$, the consumer would accept the minor treatment prescription with probability one⁸. Denote λ as the probability of the consumer accepting major treatment.

The expert's strategy consists of posting a price list $\{P_L, P_H\}$ and a prescription strategy $d \in \{\emptyset, L, H\}$. For the consumer with serious problem, i.e. $r = S$, the expert has to prescribe major treatment ($d = M$) since the consumer's payment is based on the resolution of his problem and only major treatment could repair serious problem. Let ρ be the probability of prescribing major treatment for the consumer with minor problem, i.e. attempt to overtreat the consumer.

LEMMA 1. *There exists no equilibrium where the expert posts a single price $P_L = P_H = \bar{P}$ or a price list $\{P_L, P_H\}$ with $P_H \in [C_H, C_H + P_L - C_L)$.*

Proof. The proof can be divided into two parts: the first part shows that there exists no equilibrium with a single price $P_L = P_H = \bar{P}$ and the second part proves that a price list $\{P_L, P_H\}$ with $P_H \in [C_H, C_H + P_L - C_L)$ can not be optimal for the expert.

1. Assume that there is indeed an equilibrium where the expert posts a single price $P_L = P_H = \bar{P}$. If $\bar{P} < C_H$, then the expert would reject the consumer with serious problem. The consumer updates his belief that the expert would offer to treat him

⁸Assume that the consumer would accept the prescription when he is indifferent between accepting and rejecting the offer.

only when his problem is minor. Then for the expert's offer at such a single price, the consumer believe that his problem is minor with probability one and expected gain from the treatment is V_m . Therefore, the consumer would only accept the treatment at price $\bar{P} \leq \bar{P}^*$ and reject any treatment at price $\bar{P} > \bar{P}^*$. So the expert's maximal profits $\pi_m = (1 - \beta)(\bar{P}^* - C_L)$. However, the expert can always post a price list $\{P_L, P_H\}$ with $P_L = \bar{P}^*$ and $P_H = C_H + \xi$ ($\xi \rightarrow 0$). Overtreatment is not profitable for the expert under such a price list, then the consumer would accept both major and minor treatment prescriptions with probability one. The expert's profits are $\beta\xi + (1 - \beta)(\bar{P}^* - C_L) > \pi_m$. Therefore, the single price list $\bar{P} < C_H$ is suboptimal. If $\bar{P} \geq C_H$, although the expert would offer to treat the consumer in both states but no information about the consumer's problem could be updated from the price list and expert's recommendation, the consumer maintains his prior belief and the expected utility from treatment is $\beta U(V_M - P_H) + (1 - \beta)U(V_m - P_H) < \bar{U}$, then the consumer would reject the treatment with probability one. Then the market breaks down and the expert make no profits, which makes the price suboptimal.

2. For the price list $\{P_L, P_H\}$ with $P_H \in [C_H, C_H + P_L - C_L)$, it is strictly dominated strategy for the expert because the expert could strictly increase his profits by posting a price list $\{P_L, C_H + P_L - C_L\}$. This is because the consumer holds the belief that the expert would honestly treat him as long as $P_H \leq C_H + P_L - C_L$ and both liability and verifiability holds⁹, so the consumer would accept the expert's recommendation, either major or minor treatment, with probability one. Then the expert's profits are $\beta(P_H - C_H) + (1 - \beta)(P_L - C_L)$. The profits under any price list $\{P_L, P_H\}$ with $P_H \in [C_H, C_H + P_L - C_L)$ are $\beta(P_H - C_H) + (1 - \beta)(P_L - C_L) < P_L - C_L$, the profits under price list $\{P_L, P_H\}$ with $P_H = C_H + P_L - C_L$. Therefore, such a price list is also suboptimal.

□

⁹Assume that the expert would honestly treat the consumer when he is indifferent between honestly treating and overtreating.

Lemma 1 shows that some constraints can be put on the equilibrium price. Firstly, a single price is not feasible on the equilibrium. If the single price is too low to compensate the expert's cost of major treatment, the expert would reject treating the consumer with major problem at the price. Then the consumer learns that only minor problem would be repaired at the price based on this observation, so the consumer would only accepting a maximal price that leave him reserve utility. The expert can get all the rents from the consumer with minor problem but give up all the possible rents from the consumer with major problem, but such a price can not be optimal because the expert could always post a price list that get extra rents by treating the consumer with major problem. While if the single price can cover the cost of major treatment. The price is too high to make the consumer always reject the offer. Therefore, a single price is not optimal.

On the other hand, if the price list satisfies that the margin from major treatment is no bigger than that from minor treatment, then the expert would behave honestly and the consumer would accept the offer for certain. Given the consumer always accepting expert's offer, expert's profits is increasing in both prices. Then any price lists that makes the margin from major treatment smaller than that from minor treatment is dominated by that equals the price margin.

From the two points, we could focus our attention on the price list $\{P_L, P_H\}$ with $P_H \geq C_H + P_L - C_L$ in the equilibrium analysis below.

LEMMA 2 (Fully Overtreatment). *Given the price list $\{P_L, P_H\}$ with $P_L - C_L \leq P_H - C_H$, there exists no equilibrium where*

$$\rho = 1, \quad \lambda = 1. \quad (1)$$

Proof. We shows the result by contradiction. If $\rho = 1$, no information is updated and the consumer retains his prior belief that his problem is major with probability β . The expected utility from accepting the treatment is $\beta U(V_M - P_H) + (1 - \beta)U(V_m - P_H)$. Given the price $P_L - C_L \leq P_H - C_H$ and $\beta U(V_M - C_H) + (1 - \beta)U(V_m - C_H) < \bar{U}$, then $\beta U(V_M - P_H) + (1 - \beta)U(V_m - P_H) < \bar{U}$. Optimally, the consumer should reject the

treatment offer with probability one, i.e., $\lambda = 0$. \square

The lemma 2 establishes that the fully overtreatment could not be an equilibrium. Since fully overtreatment provides no information updating for the consumer about the nature of the problem, the consumer retains his prior belief and his acceptance for price of major treatment is constrained by such belief. As the price for major treatment is too high, the consumer should reject such an offer with probability one¹⁰.

We next direct our attention to mixed strategy equilibrium. Given the price list $\{P_L, P_H\}$ with $P_L - C_L \leq P_H - C_H$, we compute the consumer's optimal acceptance strategy λ at stage 3. Upon being recommended major treatment, the consumer would update his belief about the nature of his problem and weigh the price of the treatment and expected valuation from treatment. The expert would recommend major treatment in two situations: one where the consumer's problem is serious; the other where the expert overtreats the consumer with a minor problem, and the probability of the two situations is β and $(1 - \beta)\rho$, respectively. According to Bayes' rule, the consumer nurtures the belief that his problem is minor with probability $\frac{(1-\beta)\rho}{\beta+(1-\beta)\rho}$ and major with probability $\frac{\beta}{\beta+(1-\beta)\rho}$. If the consumer rejects the expert's recommendation, he would receive his reserve utility which is \bar{U} , while if he accepts, his expected utility is

$$EU = \frac{(1 - \beta)\rho U(V_m - P_H)}{\beta + (1 - \beta)\rho} + \frac{\beta U(V_S - P_H)}{\beta + (1 - \beta)\rho} \quad (2)$$

Assume that $\hat{\rho}$ be the probability which makes $EU = \bar{U}$, so $\hat{\rho}$ makes the consumer indifferent between accepting and rejecting the expert's recommendation. By simple calculations, $\hat{\rho} = \frac{\beta(U(V_S - P_H) - \bar{U})}{(1 - \beta)(\bar{U} - U(V_m - P_H))}$. Given the price list $\{P_L, P_H\}$ with $P_L - C_L \leq P_H - C_H$, the consumer should choose his acceptance probability $\lambda = 1$ if $\rho < \hat{\rho}$ and $\lambda = 0$ if $\rho > \hat{\rho}$. And if $\rho = \hat{\rho}$, any $\lambda \in [0, 1]$ is optimal for the consumer.

Backward, we consider the expert's recommendation strategy at stage 2. For the consumer with a minor problem, the expert could either prescribe minor or major treat-

¹⁰This argument holds because $\beta U(V_M - C_H) + (1 - \beta)U(V_m - C_H) < \bar{U}$ and $P_H \geq C_H$. $P_H \geq C_H$ because the expert lacks the commitment to provide major treatment by making a negative profit.

ment,i.e. overtreat or not. Given the price list $\{P_L, P_H\}$, if the expert treats the consumer honestly,i.e, prescribe a minor treatment, she could make a profit of $\pi(H) = P_L - C_L$ for sure, while if she overtreats the consumer,i.e, prescribe a unnecessary major treatment, her expected profit is $\pi(F) = \lambda(P_H - C_H)$. So the expert would always overtreats the consumer if $\pi(F) > \pi(H)$ while honestly repair the consumer's problem if $\pi(F) < \pi(H)$, and he is indifferent between overtreatment and honest treatment if $\pi(F) = \pi(H)$.

LEMMA 3 (Partial Overtreatment). *Given the price list $\{P_L, P_H\}$ with $P_L - C_L \leq P_H - C_H$, there exists a equilibrium where*

$$\lambda^* = \frac{P_L - C_L}{P_H - C_H}, \quad \rho^* = \frac{\beta(U(V_S - P_H) - \bar{U})}{(1 - \beta)(\bar{U} - U(V_m - P_H))} \quad (3)$$

and the expert's expected payoff $\pi = (P_L - C_L)$

Proof. 1. Suppose the posted price list satisfies $P_L - C_L \leq P_H - C_H$. Given $r = m$, the expert receives $\pi(H) = P_L - C_L$ by recommending $d = L$ and $\pi(F) = \lambda(P_H - C_H)$ by recommending $d = H$. Given the consumer's strategy, $\pi(H) = \pi(F)$, then $\rho \in [0, 1]$ is optimal.

Given the expert's strategy, the consumer who decline $d = H$ would retain his reserve utility \bar{U} and accept $d = H$ would receive a utility

$$\begin{aligned} U(d = H, \text{accept}) &= \left(\frac{(1 - \beta)\rho^*U(V_m - P_H)}{\beta + (1 - \beta)\rho^*} + \frac{\beta U(V_S - P_H)}{\beta + (1 - \beta)\rho^*} \right) \\ &= \bar{U} = U(d = H, \text{reject}). \end{aligned} \quad (4)$$

Then any $\lambda \in [0, 1]$ is an optimal strategy for the consumer. Therefore, for the given price list, the strategies of the expert and the consumer are mutually best responses.

2. Suppose the strategies in (3) form an equilibrium. For $\lambda \in [0, 1]$, it must hold that $P_H > C_H$ and $P_H - C_H \geq P_L - C_L$. For $\hat{\rho} \in [0, 1]$, it is necessary that $P_H \leq \bar{P}_H^*$.
3. Note that a prescription of $d = H$ is rejected with probability λ . The ex ante payoffs

of the expert are

$$\Pi = \beta(P_H - C_H) \frac{P_L - C_L}{P_H - C_H} + (1 - \beta)(P_L - C_L) = P_L - C_L \quad (5)$$

□

The lemma 3 shows that there indeed exists a mixed strategy equilibrium in which the consumer's strategy makes the expert indifferent between overtreating and honestly treating, whilst the expert's strategy makes the consumer indifferent between accepting and rejecting major treatment recommendation.

Lastly, we study the optimal pricing strategy for the expert given the optimal strategies of both players in continuation game and the following equilibria could be found.

PROPOSITION 1. *Under the assumption of liability and verifiability, there exists a continuum of equilibria where the expert posts the price list $\{\bar{P}_L^*, P_H^*\}$ with $P_H^* \in [\bar{P}_L^* + C_H - C_L, \bar{P}_H^*]$. The expert would honestly treat the consumer with major problem while attempt to overtreat the consumer with minor problem with probability $\frac{\beta(U(V_S - P_H^*) - \bar{U})}{(1-\beta)(\bar{U} - U(V_m - P_H^*))}$ and honestly treat the consumer with minor problem with probability $1 - \frac{\beta(U(V_S - P_H^*) - \bar{U})}{(1-\beta)(\bar{U} - U(V_m - P_H^*))}$. The consumer would accept minor treatment prescription with probability one and major one with $\frac{\bar{P}_L^* - C_L}{P_H^* - C_H}$.*

Proof. In lemma 3, the expert's expected payoff $\pi = (P_L - C_L)$. Obviously, the expert's expected profits are strictly increasing in P_L while constant at P_H . Then the expert should choose the price for minor treatment P_L as large as possible while decide the price for major treatment as long as the consumer would accept.

Since the consumer would accept any price $P_L \leq \bar{P}_L^*$ for minor treatment, thus the optimal price for minor treatment $P_L^* = \bar{P}_L^*$. While for the price of major treatment, $P_L - C_L \leq P_H - C_H$ gives that $P_H^* \geq C_H + P_L - C_L$. Moreover, the highest price that the consumer is willing to pay for his serious problem repaired is \bar{P}_H^* , then $P_H^* < \bar{P}_H^*$. In brief, $P_H^* \in [\bar{P}_L^* + C_H - C_L, \bar{P}_H^*]$. Therefore, the optimal price list is $\{\bar{P}_L^*, P_H^*\}$ with $P_H^* \in [\bar{P}_L^* + C_H - C_L, \bar{P}_H^*]$. □

In proposition 1, both the expert and the consumer play mixed strategy and their strategy is mutual best response. The expert's overtreatment probability makes the consumer indifferent between accepting or rejecting the treatment offer, whilst the consumer's acceptance probability equate the expert's profits from overtreatment and honest treatment. We can find that the expert would overtreat the consumer with strictly positive probability even when the price margin between two treatment is equal. And the social efficiency can not be achieved. Next, we could investigate how the degree of risk-aversion of the consumer affect the expert's overtreatment probability.

Corollary 1. *Given the price list $\{\bar{P}_L^*, P_H^*\}$ with $P_H^* \in [\bar{P}_L^* + C_H - C_L, \bar{P}_H^*]$ in the equilibrium¹¹, the optimal overtreatment probability ρ^* is decreasing with the coefficient of absolute risk aversion (CARA). Extremely, when CARA tends toward infinity, $\rho^* = 0$ ¹².*

Proof. Given certain price list $\{\bar{P}_L^*, P_H^*\}$ posted by the expert, the optimal probability of overtreatment for the expert

$$\begin{aligned} \rho^* &= \frac{\beta(U(V_S - P_H^*) - \bar{U})}{(1 - \beta)(\bar{U} - U(V_m - P_H^*))} = \frac{\beta U(V_S - P_H^*) + (1 - \beta)U(V_m - P_H^*) - \bar{U}}{(1 - \beta)(\bar{U} - U(V_m - P_H^*))} + 1 \\ &= \frac{U(\beta V_S + (1 - \beta)V_m - P_H^* - \gamma) - \bar{U}}{(1 - \beta)(\bar{U} - U(V_m - P_H^*))} + 1 \end{aligned} \quad (6)$$

where γ denote the certain equivalence that make $U(\beta V_S + (1 - \beta)V_m - P_H^* - \gamma) = \beta U(V_S - P_H^*) + (1 - \beta)U(V_m - P_H^*)$. It have been proved that a larger coefficient of absolute (relative) risk aversion (CARA or CRRA) means larger certainty equivalence¹³. Therefore, the more risk-averse the consumer is, γ is larger. It is clear to notice that ρ^* is decreasing in γ , then the result follows.

For the extreme case, when the coefficient of absolute risk aversion tends toward infinity, the consumer's expected utility function takes a form of maximin¹⁴. Then given the expert's recommendation for major treatment, the consumer's utility $U(P_H, \text{accept}) =$

¹¹Except the case with the price list $\{\bar{P}_L^*, \bar{P}_H^*\}$ because the expert would honestly treat the consumer regardless to the degree of consumer's risk aversion in that case.

¹²the result still holds when the coefficient of absolute risk aversion is replaced by the coefficient of relative risk aversion(CRRA).

¹³See THEROEM 1 in Pratt (1964) for the proof in details.

¹⁴See the Maximin Criterion in chapter 2(Laffont (1989)).

$\min\{U(V_m - P_H), U(V_S - P_H)\} = U(V_m - P_H)$ if the expert overtreats the consumer with any probability, while $U(P_H, \text{accept}) = U(V_S - P_H)$ if the expert honestly treats. Given our assumption $V_m < C_H$, then the consumer would always reject the expert's offer if the expert overtreats the consumer with any, even extremely small, probability because $U(V_m - P_H) < \bar{U}$ holds for any $P_H \geq C_H$. And the expert's expected profits from the strategy $\pi_1 = (1 - \beta)(1 - \rho^*)(\bar{P}^* - C_H)$.

On the other hand, if the expert honestly repairs the consumer's problem with $U(V_S - P_H) \leq \bar{U}$ and $P_H > C_H$, the consumer would accept the offer with probability one. Then the expert's expected profits $\pi_2 = (1 - \beta)(\bar{P}^* - C_H) + \beta(P_H^* - C_H)$. Obviously, $\pi_2 > \pi_1$ and honest treatment is optimal for the expert. \square

This result is quite interesting, but also intuitive. If the consumer become more risk-averse, he would obtain less utilities from the uncertain situation, in which the expert recommends major treatment and overtreats him with some probabilities. Then as long as the expert overtreats the consumer with any probabilities, the consumer who is more risk-averse would get less utility from accepting the major treatment recommendation and therefore become less tolerant with the expert's fraud behaviours, which in turn on the equilibrium would result in less overtreatments by the expert. Furthermore, if the consumer is extremely risk-averse and worries about the worst situation, then his utility function takes a form of max-min, and he would show zero tolerance for the expert's overtreatment behaviours, which would lead zero acceptance probability for major treatment if the expert overtreats the consumer with any, even very small, possibilities. Observing the consumer's strategy, Optimally, the expert would choose not to overtreat the consumer because the expert could always commit himself to honestly prescribing major treatment and make a strictly positive profits from major treatment while retain the same profits from minor treatment. Therefore, the expert would behave honestly if confronted with a extremely risk-averse consumers.

However, in our model we assume that the consumer's utility function is common knowledge, and then so is the degree consumer's risk-aversion. In some situations, this

information might be the consumer's private information and the expert is not informed about it. We have already shown that the expert's frauds decrease with the consumer's risk-aversion. Then the consumer of less risk-aversion may have incentive to pretend that he is much more risk-averse, which induce less frauds for him. If so, then two problems would arise. One is that how the consumer credibly signals out his degree of risk-averse, symmetrically the other is that how the expert optimally discriminates the consumer of different degree of risk-aversion. But, these two questions are out the range of this paper.

In the next sections, we extend the basic model by entitling the consumer with insurance option.

3 Analysis with Insurance Option

In the previous section, we have shown that the consumer's utility differs in different states on the equilibrium. If the problem is minor, he would obtain the reserve utility \bar{U} when he is not overtreated by the expert with probability $1 - \rho^*$, and he would a utility of $U(V_m - P_H) < \bar{U}$ when he is overtreated with probability ρ^* . If the problem turns out to be major, he could gain a utility of $U(V_M - P_H) > \bar{U}$. Thus, the consumer is not fully insured by the expert. Due to the expert's opportunistic behaviors, the consumer would obtain a expected utility $\rho^*U(V_m - P_H) + (1 - \rho^*)\bar{U} < \bar{U}$ in the state of minor problem while one above \bar{U} if the problem is major. Therefore, the consumer indeed have incentive to, if he could, purchase insurance from a third party.

As the consumer would obtain a utility below the reserve utility in states of minor problem while one above reserve utility with major problem, then the problem being minor is the "bad thing" for the consumer while a major problem is the "good thing". However, since both the insurer and the consumer can not learn exactly whether the consumer's problem is minor or major, the insurance could not be based on the states. But the price list posted by the expert is common knowledge for all, then the insurance can be based on the price list.

The consumer now is entitled with the insurance option, and we assume that he could

purchase some insurances provided by the insurance company based on the price list posted by the expert before he visits the expert. So it implicitly imply that both the consumer and the insurer observe the price list¹⁵. The insurer provide a insurance plan with (k, α) , k denote the price of the insurance or the amount of insurance with unit price and α means the compensation to the consumer if bad thing happens. After the expert posts the price list, the insurer determine the insurance scheme (k, α) and afterwards the consumer decides the optimal quantity of insurance to purchase. Once the treatments is implemented by the expert and the corresponding price paid by the consumer, if the consumer is charged with the price for minor treatment by the expert, the insurer would pay a compensation of α for the consumer, while pay nothing if the consumer being charged with the price for major treatment, no compensation is paid.

We assume that the insurance market is completely competitive, then the insurer would make zero expected profits and the insurance is actually fair. After the expert posts the price list and the insurer posts the insurance plan, the consumer chooses the amount of insurance to buy. The timing of new game is as follows.

Timing of New Game. The game proceeds as follows:

1. The expert posts a price list (P_L, P_H) .
2. The insurance company provide a actually fair insurance plan (k, α) . The consumer decide to buy k amount of insurance with $k \geq 0$.
3. Nature draws the consumer's type. The consumer visits the expert. The expert performs a diagnosis and prescribes a treatment $d \in \{\emptyset, L, H\}$ for the consumer.
4. The consumer decides whether accept treatment d or not. If the consumer accepts, the treatment prescribed would be performed by the expert. If the problem is

¹⁵Here we assume that both insurance company and the consumer observe the price list posted by the expert. The reality can justify the assumption very well. Besides, in our setting, we assume that the consumer must have a problem, minor or major. It could be easily extended to the case with healthy consumer but does not change the essence of the problem under concern.

resolved, the expert receives her payment, otherwise get nothing. The game ends in other situations.

The notation in the previous section will be retained in the following analysis. Before solving the game by backward induction, we proclaim that there is no equilibrium in which the expert posts a single price. Because if the expert indeed posts a single price, then no insurance scheme is feasible. The insurance option would alter nothing, then the same argument as that in Lemma 1 which establish no equilibrium with single price applies here also. Therefore, only price list should be under our concern. We subdivide the analysis into two cases, one with $P_L - C_L \leq P_H - C_H$ and the other with $P_L - C_L > P_H - C_H$

3.1 $P_L - C_L \leq P_H - C_H$

Firstly, we study the consumer's optimal acceptance strategy at stage 4. Given the price list $\{P_L, P_H\}$ with $P_L - C_L \leq P_H - C_H$ and \hat{k} amount of insurance purchased in previous stage, upon recommended minor treatment, accepting the offer would bring a utility

$$U(d = L, \text{accept}) = U(V_m - \hat{k} - P_L + \hat{\alpha}), \quad (7)$$

while rejecting will reserve \bar{U} .

On the other hand, the consumer would update his belief about the nature of his problem as before upon recommended a major treatment. The expert would recommend major treatment in two situations: one where the consumer's problem is serious; the other where the expert overtreats the consumer with minor problem, and the probability of the two situations is β and $(1-\beta)\rho$, respectively. According to Bayes' rule, the consumer forms the belief that his problem is minor with probability $\frac{(1-\beta)\rho}{\beta+(1-\beta)\rho}$ and major with probability $\frac{\beta}{\beta+(1-\beta)\rho}$. If the consumer accepts the offer, his expected utility is

$$U(d = H, \text{accept}) = \frac{(1-\beta)\rho U(V_m - \hat{k} - P_H)}{\beta + (1-\beta)\rho} + \frac{\beta U(V_S - \hat{k} - P_H)}{\beta + (1-\beta)\rho}, \quad (8)$$

and, again, rejecting will bring \bar{U} .

Denote $\hat{\rho}'$ that satisfy $U(d = H, \text{accept}) = U(d = H, \text{reject})$, i.e.,

$$\frac{(1 - \beta)\hat{\rho}'U(V_m - \hat{k} - P_H)}{\beta + (1 - \beta)\hat{\rho}'} + \frac{\beta U(V_S - \hat{k} - P_H)}{\beta + (1 - \beta)\hat{\rho}'} = \bar{U}, \quad (9)$$

then simple calculations lead to $\hat{\rho}' = \frac{\beta(V_S - \hat{k} - P_H) - \bar{U}}{(1 - \beta)(\bar{U} - U(V_m - \hat{k} - P_H))}$. Optimally, the consumer would choose his acceptance probability $\lambda = 1$ if $\rho < \hat{\rho}'$, $\lambda \in [0, 1]$ if $\rho = \hat{\rho}'$, and $\lambda = 0$ if $\rho > \hat{\rho}'$.

Backward, the expert's prescription strategy at stage 3 is the same as that in the analysis of previous section.

LEMMA 4 (Partial Overtreatment). *Given the price list $\{P_L, P_H\}$ with $P_L - C_L \leq P_H - C_H$ and $0 \leq \hat{k} \leq \bar{P}_H^* - P_H$, there exists a equilibrium where*

$$\lambda_1^* = \frac{P_L - C_L}{P_H - C_H}, \quad \rho_1^* = \frac{\beta(U(V_S - \hat{k} - P_H) - \bar{U})}{(1 - \beta)(\bar{U} - U(V_m - \hat{k} - P_H))} \quad (10)$$

and the expert's expected payoff $\pi_1 = (P_L - C_L)$

Proof. 1. Suppose the posted price list satisfies $P_L - C_L \leq P_H - C_H$. Given $r = m$, the expert receives $\pi(H) = P_L - C_L$ by recommending $d = L$ and $\pi(F) = \lambda_1(P_H - C_H)$ by recommending $d = H$. Given the consumer's strategy, $\pi(H) = \pi(F)$ because $\theta > 1$, then $\rho \in [0, 1]$ is optimal.

Given the strategies of the expert, declining $d = H$ gets reserve utility \bar{U} while accepting $d = H$ brings a utility

$$\begin{aligned} U(d = H, \text{accept}) &= \left(\frac{(1 - \beta)\rho_1^*U(V_m - \hat{k} - P_H)}{\beta + (1 - \beta)\rho_1^*} + \frac{\beta U(V_S - \hat{k} - P_H)}{\beta + (1 - \beta)\rho_1^*} \right) \\ &= \bar{U} = U(d = H, \text{reject}). \end{aligned}$$

Then any $\lambda \in [0, 1]$ is an optimal strategy for the consumer. Therefore, for the given price list, the expert's strategy and the consumer's strategy are mutual best responses.

2. Suppose the strategies in (10) form an equilibrium. For $\lambda \in [0, 1]$, it must hold that

$P_H > C_H$ and $P_H - C_H \geq P_L - C_L$. For $\rho_1^* \in [0, 1]$, it is necessary that $\hat{k} \leq \bar{P}_H^* - P_H$.

3. Note that a prescription of $d = H$ is rejected with probability λ . The expert's expected payoffs are

$$\Pi_1 = \beta(P_H - C_H) \frac{P_L - C_L}{P_H - C_H} + (1 - \beta)(P_L - C_L) = P_L - C_L \quad (11)$$

□

LEMMA 5 (Optimal Insurance Choice). *Given the price list $\{P_L, P_H\}$ with $P_L - C_L \leq P_H - C_H$ and players' strategy in (10), the consumer's optimal insurance choice*

$$k_1^* = \bar{P}_H^* - P_H \quad (12)$$

and then $\rho_1^* = 0$ under the consumer's optimal insurance choice.

Proof. In lemma 5, the consumer's expected utility is

$$\begin{aligned} EU(\hat{k}, P_L, P_H) &= \lambda_1^* \left(\beta U(V_S - \hat{k} - P_H) + (1 - \beta) \rho_1^* U(V_m - \hat{k} - P_H) \right) \\ &\quad + (1 - \lambda_1^*) (\beta + (1 - \beta) \rho_1^*) \bar{U} + (1 - \beta) (1 - \rho_1^*) U(V_m - \hat{k} - P_L + \hat{\alpha}) \\ &= (\beta + (1 - \beta) \rho_1^*) \bar{U} + (1 - \beta) (1 - \rho_1^*) U(V_m - \hat{k} - P_L + \hat{\alpha}) \end{aligned} \quad (13)$$

The second equation holds by using definitions of ρ_1^* . As we assume that the insurance market is perfectly competitive, free entry will ensure that insurance schemes bought in equilibrium make zero expected profits, so that if k is purchased,

$$k - (1 - \beta)(1 - \rho_1^*)\alpha = 0 \quad (14)$$

then $\alpha = \frac{k}{(1 - \beta)(1 - \rho_1^*)}$.

Therefore, the consumer's maximization program subject to the insurer's zero expected

profit is reduced to

$$\begin{aligned} & \arg \max \quad EU(\hat{k}, P_L, P_H) \\ \text{s.t.} \quad & \alpha = \frac{k}{(1-\beta)(1-\rho_1^*)}, \quad \rho_1^* = \frac{\beta(U(V_S - \hat{k} - P_H) - \bar{U})}{(1-\beta)(\bar{U} - U(V_m - \hat{k} - P_H))} \end{aligned} \quad (15)$$

Take both $\hat{\alpha}$ and ρ_1^* as a function of \hat{k} , denoted as $\hat{\alpha}(\hat{k})$ and $\rho_1^*(\hat{k})$, respectively. Note that $EU(\hat{k}, P_L, P_H)$ is differentiable. By taking partial derivative with respect to \hat{k} , we get

$$\frac{\partial EU}{\partial \hat{k}} = (1-\beta) \left(\rho_1^{*'} [\bar{U} - U(V_m - \hat{k} - P_L + \hat{\alpha})] + (1-\rho_1^*)(\hat{\alpha}' - 1)U'(V_m - \hat{k} - P_L + \hat{\alpha}) \right) \quad (16)$$

Due to $\bar{U} - U(V_m - \hat{k} - P_L + \hat{\alpha}) < 0$ and $\rho_1^{*'}(\hat{k}) < 0$, then the first term in equation 16 is positive. Moreover, $\hat{\alpha}' - 1 = \frac{1-\rho_1^*+k\rho_1^{*'}}{(1-\beta)(1-\rho_1^*)} - 1 > 0$, $\rho_1^*(\hat{k}) \in [0, 1]$, $\beta \in [0, 1]$ and $U'(\cdot) > 0$, the last term is also positive. Therefore, $\frac{\partial EU(\hat{k}, P_L, P_H)}{\partial \hat{k}} > 0$. So the consumer's expected utility is strictly increasing in the amount of insurance \hat{k} purchased. Since $0 \leq \hat{k} \leq \bar{P}_H^* - P_H$, the optimal amount of insurance for the consumer lies on the upper bound, i.e. $k_1^* = \bar{P}_H^* - P_H$. \square

LEMMA 6 (Pricing Strategy). *Given the consumer's optimal acceptance strategy and insurance choice, and the expert's optimal prescription strategy in lemma 4 and 5, the expert's optimal pricing strategy $\{P_L^*, P_H^*\}$ is*

$$P_L^* = \beta\bar{P}_H^* + (1-\beta)\bar{P}_L^* - \beta(C_H - C_L), \quad P_H^* = \beta\bar{P}_H^* + (1-\beta)\bar{P}_L^* + (1-\beta)(C_H - C_L) \quad (17)$$

and $\lambda_1^* = 1$ and the expert's maximal profit $\pi^* = \beta\bar{P}_H^* + (1-\beta)\bar{P}_L^* - \beta C_H - (1-\beta)C_L$ under the price list.

Proof. We study the optimal pricing strategy for the price list $\{P_L, P_H\}$ with $P_L - C_L \leq P_H - C_H$, the expert's profit $\pi = P_L - C_L$, which is strictly monotonically increasing in P_L while invariant to P_H .

To ensure that the consumer accepts minor treatment prescription, $U(d = m, \text{accept}) =$

$U(V_m - k^* + \alpha^* - P_L) \geq U(d = m, \text{reject}) = \bar{U}$. Then the optimal P_L should make that $U(V_m - k^* + \alpha^* - P_L^*) = \bar{U}$. Recall that $U(V_m - \bar{P}_L^*) = \bar{U}$, then $U(V_m - k^* + \alpha^* - P_L^*) = U(V_m - \bar{P}_L^*)$. It follows that $P_L^* + k^* - \alpha^* = \bar{P}_L^*$. Given k_1^* chosen by the consumer, the expert's optimal overtreatment strategy in lemma 4

$$\rho_1^* = \frac{\beta(U(V_S - k^* - P_H) - \bar{U})}{(1 - \beta)(\bar{U} - U(V_m - \hat{k} - P_H))} = \frac{\beta(U(V_S - \bar{P}_H^*) - \bar{U})}{(1 - \beta)(\bar{U} - U(V_m - \hat{k} - P_H))} = 0 \quad (18)$$

It follows that $\alpha_1^* = \frac{\bar{P}_H^* - P_H}{(1 - \beta)(1 - \rho_1^*)} = \frac{\bar{P}_H^* - P_H}{(1 - \beta)}$. Plugging $k^* = \bar{P}_H^* - P_H$ and $\alpha^* = \frac{\bar{P}_H^* - P_H}{(1 - \beta)}$,

$$P_L^* = \bar{P}_L^* + \frac{\beta}{1 - \beta}(\bar{P}_H^* - P_H). \quad (19)$$

Furthermore, $P_L^* - C_L \leq P_H^* - C_H$. To maximize P_L , P_H^* should take the lower bound, and $P_H^* = P_L^* + C_H - C_L$. Substituting P_H^* into equation 19, then $P_L^* = \beta\bar{P}_H^* + (1 - \beta)\bar{P}_L^* - \beta(C_H - C_L)$ and $P_H^* = \beta\bar{P}_H^* + (1 - \beta)\bar{P}_L^* + (1 - \beta)(C_H - C_L)$. The expert's maximal profits $\pi^* = P_L^* - C_L = \beta\bar{P}_H^* + (1 - \beta)\bar{P}_L^* - (\beta C_H + (1 - \beta)C_L)$.

Lastly, given the optimal price list, $\lambda^* = \frac{P_L^* - C_L}{P_H^* - C_H} = 1$. □

3.2 $P_L - C_L > P_H - C_H$

LEMMA 7 (Efficient Outcome). *Given the price list $\{P_L, P_H\}$ with $P_L - C_L > P_H - C_H$, there exists a equilibrium where*

$$\lambda_2^* = 1, \quad \rho_2^* = 0, \quad (20)$$

and the expert's expected payoff $\pi_2 = (1 - \beta)(P_L - C_L) + \beta(P_H - C_H)$.

Proof. 1. Suppose the posted price list satisfies $P_L - C_L > P_H - C_H$. since overtreatment is not profitable for the expert, then honest treatment is optimal for the expert, i.e, $\rho_2^* = 0$.

Given that the expert always honestly offer the treatment, the consumer would always accept the offer, i.e, $\lambda_2^* = 1$.

Therefore, under the given price list, the expert's strategy and the consumer's strategy are mutual best responses.

2. Suppose the strategies in (3) form an equilibrium. For $\lambda = 1$, it must hold that $P_L \leq \bar{P}_L^* + \hat{\alpha} - \hat{k}$ and $P_H \leq \bar{P}_H^*$. For $\rho = 0$, it is necessary that $P_L - C_L > P_H - C_H$.
3. Since the consumer would always accept the recommendation, the expert's expected payoffs are

$$\Pi_2 = \beta(P_H - C_H) + (1 - \beta)(P_L - C_L) \quad (21)$$

□

LEMMA 8 (Optimal Insurance Choice). *Given the price list $\{P_L, P_H\}$ with $P_L - C_L > P_H - C_H$ and players' strategy in (20), the optimal insurance choice for the consumer*

$$k_2^* = (1 - \beta)(V_S - V_m + P_L - P_H) \quad (22)$$

Proof. In lemma 8, the consumer's expected utility is

$$EU = \beta U(V_S - \hat{k} - P_H) + (1 - \beta)U(V_m - \hat{k} - P_L + \hat{\alpha}) \quad (23)$$

As the previous case, given the expert's optimal overtreatment probability ρ_2^* on the equilibrium path, zero expected profit in competitive equilibrium would ensure that

$$k = (1 - \beta)(1 - \rho_2^*)\alpha. \quad (24)$$

therefore $\alpha = \frac{k}{(1 - \beta)(1 - \rho_2^*)} = \frac{k}{1 - \beta}$. Optimally, the consumer maximize his expected utility subject to the insurer's zero expected profits constraint. As the insurance is actually fair, the consumer would fully insure for the risk. Then the consumer's optimal insurance choice is given by

$$V_S - k_2^* - P_H = V_m - k_2^* - P_L + \alpha_2^* \quad (25)$$

Solving k_2^* by combining this condition and equation (24), $k_2^* = (1 - \beta)(V_S - V_m + P_L - P_H)$

and $\alpha_2^* = V_S - V_m + P_L - P_H$. □

LEMMA 9 (Pricing Strategy). *Given the optimal strategy of both consumer and expert in lemma 7 and 8, the expert's optimal pricing strategy $\{P_L^*, P_H^*\}$ should satisfy*

$$\beta P_H^* + (1 - \beta)P_L^* = \beta \bar{P}_H^* + (1 - \beta)\bar{P}_L^*, \quad P_L^* - C_L > P_H^* - C_H \quad (26)$$

and the expert's maximal profit $\pi_2^* = \beta \bar{P}_H^* + (1 - \beta)\bar{P}_L^* - \beta C_H - (1 - \beta)C_L$.

Proof. The expert's profits $\pi_2 = \beta P_H + (1 - \beta)P_L - \beta C_H - (1 - \beta)C_L$, which are strictly increasing in both P_L and P_H . Then the pricing strategy should make the consumer indifferent between accepting and rejecting the recommendation and leave no rents for the consumer, which gives rise to

$$P_L = \alpha_2^* - k_2^* + \bar{P}_L^* \quad P_H = \bar{P}_H^* - k_2^* \quad (27)$$

Plugging k_2^* and α_2^* into the equations, we get $\beta P_H + (1 - \beta)P_L = \beta \bar{P}_H^* + (1 - \beta)\bar{P}_L^*$. Essentially, the two conditions in equation 27 are the same. Since the consumer would always purchase fully insurance, the consumer would retain the same utility in both states and the difference of utility between the two states under the price list would be cancelled out by insurance. Also, the optimal price list should meet the precondition, that is, $P_L^* - C_L > P_H^* - C_H$. The expert's profit follows. □

PROPOSITION 2. *Under the assumption of liability and verifiability, there exists two sets of equilibria where*

1. *the expert post a price list $\{P_L^*, P_H^*\}$ with $P_L^* = \beta \bar{P}_H^* + (1 - \beta)\bar{P}_L^* - \beta(C_H - C_L)$, $P_H^* = \beta \bar{P}_H^* + (1 - \beta)\bar{P}_L^* + (1 - \beta)(C_H - C_L)$, and the consumer purchase a mount of insurance $k_1^* = (1 - \beta)(\bar{P}_H^* - \bar{P}_L^* - C_H + C_L)$.*
2. *the expert posts the price list $\{P_L^*, P_H^*\}$ with $\beta P_H^* + (1 - \beta)P_L^* = \beta \bar{P}_H^* + (1 - \beta)\bar{P}_L^*$ and $P_L^* - C_L > P_H^* - C_H$, the consumer would purchase a mount of insurance $k_2^* = (1 - \beta)(V_S - V_m + P_L - P_H)$.*

In both sets of equilibria, the expert would honestly treat the consumer in both states, and the consumer would accept both minor and major treatment with probability one.

Proof. Comparing the expert's payoffs in lemma 6 and 9, the expert would achieve the same maximal amount of profits, which is $\beta\bar{P}_H^* + (1-\beta)\bar{P}_L^* - \beta C_H - (1-\beta)C_L$. Furthermore, the expert would always honestly treat the consumer and the consumer always accept the expert's offer, i.e, $\lambda_1^* = \lambda_2^* = 1$ and $\rho_1^* = \rho_2^* = 0$. Therefore, both optimal pricing strategies in lemma 6 and 9 with corresponding optimal strategy in the continuation game form an equilibrium of the game. And the social efficiency could be achieved, the expert would obtain all the social surplus from the transaction. \square

From the proposition 2, we show that the social efficiency could be achieved under both situations when the consumer could buy insurance in a competitive insurance market. The expert's profits is maximized and the consumer's problem is resolved with probability one. The logic behind the preferable result is that the consumer would fully insure himself in a competitive market, and then the consumer could always adjust his utility between the two treatment recommendation. Essentially, the consumer now have only one states and one utility under whatever the expert recommendation. Then anticipating that the consumer would buy full insurance, the expert would post the price list that make the consumer in the state indifferent between accepting or rejecting his recommendation, repair the consumer's problem honestly and grab all surplus at the same time. Therefore, the equilibrium ends up with the social efficiency outcome.

Without insurance option, the consumer could not cross-subsides himself under minor treatment and major treatment, then his decision under minor treatment and major treatment is separated, and he would only accept the recommendation with expected utility no lower than his reserve utility under each separate case. While with insurance option, the consumer could align the two choice into one and save in the good situation for case of the "bad thing" happening, which induce the consumer not restricted within acceptance strategy with respect to the expert's recommendation. the consumer could accept a treatment with expected utility lower than reserve utility and then receive compensation

from the insurance company.

When the price list gives rise to more profits from major treatment than minor treatment, corresponding to first case in the proposition, the expert's profits equal to the price margin from minor treatment due to the consumer's optimal acceptance probability on the equilibrium. Then, the expert would charge as much as possible for minor treatment. If the consumer purchase no insurance, the price for minor treatment maximizes at the price that leave the consumer with reserve utility after having minor problem repaired. While now the consumer could buy insurance and he would purchase as much amount insurance as he can because the insurance could not only alleviate the risk he face but also mitigate the expert's fraud problem. So the consumer now could receive compensation from the insurance company when he accept minor treatment, then the price for minor treatment could be bigger and maximize at the price which makes the consumer indifferent between accepting or rejecting the offer with minor problem resolved and receiving the compensation from the insurance company. Then the expert's optimal price for minor treatment is that price. But, the amount of insurance the consumer could purchase in insurance market is constrained by the residuals from having his problem repaired with major treatment and paying the price for major treatment. In other word, the money that the consumer could save for the "bad thing" is restricted by the money that he left when the "good thing" happens. Therefore, the expert could use the insurance that the consumer purchase as the tool that grab the surplus from major treatment by minor treatment. Since the consumer would fully insurance himself, then the consumer put aside all his surplus from having major problem repaired for minor treatment. As the compensation is based on the price paid, then the consumer could not get compensation even when he is overtreatment, but at same time he has already used all the surplus from having major problem repaired to purchase insurance, and then the consumer has no tolerance of overtreatment. The results follows.

As to the second case in the proposition, the argument is similar. The insurance acts as the tunnel by which the expert could transit the surplus from major treatment to

minor treatment, and then the expert captures all the surplus with only minor treatment. Since now all the surplus goes to the expert's pocket, the expert's incentive is aligned with social efficiency and he would post the price list and prescribe to ensure that the consumer always accepts the treatment. Therefore, social efficiency could be realised.

4 Discussions and Conclusions

In the previous sections, we assume that $\beta U(V_S - C_H) + (1 - \beta)U(V_m - C_H) < \bar{U}$, then fully overtreatment is not possible in the equilibrium. While if $\beta U(V_S - C_H) + (1 - \beta)U(V_m - C_H) \geq \bar{U}$, the expert could post a single price $\bar{P} > C_H$ to provide fully insurance for the consumer. Specifically, assume that the expert post a single price $\bar{P} > C_H$, the expert would repair consumer's problem, either minor or major, under the single price. The optimal single price could be set such that $\beta U(V_S - \bar{P}^*) + (1 - \beta)U(V_m - \bar{P}^*) = U(\beta V_S + (1 - \beta)V_m - \bar{P}^* - \gamma) = \bar{U}$, with γ being the certainty equivalence. Then $\bar{P}^* = \beta V_S + (1 - \beta)V_m - \gamma$. Under the optimal single price, the consumer would always accept the expert's recommendation and the expert's profits are maximized as the transaction could be reached with probability one and all surplus from the transaction is captured by the expert.

Therefore, cutting down the expert's cost of treatment or improving the expert's productivity of treatment to some threshold could avoid the social inefficiency induced by the risk-aversion of consumer when even both liability and verifiability assumption holds, the conditions that the credence goods market can achieve social efficiency in Dulleck and Kerschbamer (2006) where the consumer is homogeneous and risk-neutral. However, the expert's cost may differ from each other, some experts are efficient enough while others may be not¹⁶. Furthermore, there always exists some treatments that then the expert is not efficient enough and the expert could not provide fully insurance for the consumer on his own. Thus, the situation of our concern in this paper is pretty general.

¹⁶We study the impact of expert's heterogeneity of cost on the market performance in details in another paper.

Conclusion In this paper, we study the impact of consumer's risk-aversion on the expert's incentive for overtreatment and the efficiency of credence goods market when both liability and verifiability holds in the basic model. Then, we extend the model by introducing a competitive insurance company, and investigate the consumer's optimal insurance choices and its effect on the expert's fraud behaviour and social welfare on the equilibrium.

In the basic model, when both the type of treatment and resolution of consumer's problem are verifiable, i.e, both verifiability and liability hold, the consumer's risk aversion would lead to social inefficiency and social optimality¹⁷ can not be realised on the equilibrium. Besides, the degree of consumer's risk-aversion affects the expert's overtreatment behaviour. The result shows that the frequency of overtreatment of the expert decreases with the degree of consumer's risk-aversion or the coefficient of absolute risk aversion. And when the consumer is extremely risk-averse and the consumer's utility function takes a form of max-min, the expert would return to honestly treat the consumer and the market is socially efficient.

In the extensive model with insurance company, the consumer now could purchase any amount of actually fair insurance provided by a competitive insurance company. Social efficiency could always be achieved and the expert would obtain all the social surplus in the equilibrium. Two sets of equilibrium outcomes divided by expert's pricing strategy could be specified. When the expert posts a price list with price margin from major treatment being larger than that from minor treatment, the expert's equilibrium profits equal to the price margin from minor treatment and the consumer would purchase as much insurance as possible. Interestingly, the expert could grab all the social surplus facilitated by the insurance purchased by the consumer, acting as the tunnel that transports the consumer's surplus from major problem repaired to minor treatment. Optimally, the consumer's insurance choice would make the expert honestly treat his problem, and in turn the expert post a optimal price list to capture all the social surplus, and the social

¹⁷Here the social optimality means that all the consumer's problem is repaired at minimal cost, that is, the consumer with minor(major) problem is treated with minor(major) treatment. Then any rejection from consumer and any overtreatment is not efficient.

efficiency is implemented in the equilibrium.

The other equilibrium outcome relates to the situation where the expert posts a price list with profits from major treatment being smaller than that of minor treatment. Under such a price list, the expert credibly commits to honestly repair consumer's problem, and then the consumer would purchase full insurance and ends up with the same utility level no matter which type his problem may be. Again, the insurance is a surplus-grabbing device. Since the consumer would always purchase full insurance, then obtaining all the surplus in one situation (e.x. minor problem) means getting all the surplus in these two situations. Therefore, the expert only need post a price list that make the consumer with minor problem is indifferent between accepting and rejecting the treatment offer, and then obtains all possible surplus. More importantly, the expert indeed has the incentive to post such a price list as a honesty signal because all possible surplus could be captured under the price list.

Nevertheless, the results is based on some assumptions. In the model, we assume that the insurance market is completely competitive, and then the insurance is actually fair. We anticipate that the market may be not socially efficient any more if the insurance company have some monopoly power. Intuitively, the consumer would not purchase full insurance even in the second set of equilibrium outcome above and he would be under-insured. Although the insurance could still acts as tunnel for the experts, there exists loss of surplus in the transportation since the insurance company would obtain some consumer's surplus. Depending on the share captured by the insurance company, committing to honestly treatment may be not optimal for the expert and the equilibrium outcome is uncertain.

On the other hand, the consumer may be heterogeneous with respect to risk-aversion. Our basic model have show that the expert's probability of overtreatment decreases with the degree of consumer's risk-aversion. With consumer's risk-heterogeneity, the less risk-averse consumer would have the incentive to pretend to be more risk-averse. This may give rise to screening problem for the expert. While if the insurance company is in position,

then risk-heterogeneity may also bring about selection problem for the insurance company. And it might be interesting to study the fraud problem of expert mingled with the selection problem. All these problems are interesting while remains unanswered in the literature, which point out the direction for future researches.

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