How accounting accuracy affects DSGE models

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Abstract

This paper explores how accounting consistency affects DSGE models. As many DSGE models descended from real business cycle models, I explore a simple labor-only RBC model with an exogenous external sector introduced. The conclusion reached in this paper is that once an external sector is introduced, DSGE models may suffer from accounting inconsistency, unless disequilibrium or some non-orthodox theory of price level, real monetary supply or bonds is accepted.

1 Accounting consistency of a simple labor-only RBC model with exogenous government and without money

The model is the infinite-life representative agent framework. The household obtains utility $u(C_t, N_t)$ at time t, where C_t is consumption and N_t is labor. Total utility of the household is given by

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, N_t) \tag{1}$$

where β is time preference. In this economy, nominal factor can be ignored, and thus every variable will be a real variable.

$$u(C_t, N_t) = \frac{{C_t}^{1-\sigma}}{1-\sigma} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \tag{2}$$

The household has budget constraint as follows:

$$C_t + {R_t}^{-1} B_t \leq B_{t-1} + W_t N_t + \Pi_t \tag{3}$$

where B_t is bond, R_t is real interest rate, Π_t is dividend received from the firm. One can immediately stop here and notice that for the fixed income in the right-hand side, there is no reason why the household would buy B_t, unless it affects future consumptions. The rest of this section is developed to demonstrate in

the economy specified that buying more B_t does not increase or decrease future consumption and does not increase or decrease future labor quantity. Future consumption and labor quantity are affected only by expected technology A_{t+k} and expected government deficit spending G_{t+k} that are assumed to be money-financed solely (in other words, finance deficit by printing money), instead of being debt-financed. I will assume that g_t is exogenous, but that the government announced the full path of g_t from present to the infinite future.
Let the lower-case z of upper-case variables Z represent $z = \log(Z)$. The optimality conditions in the log form are:

$$w_t = \sigma c_t + \varphi n_t \tag{4}$$

$$E_t\left[c_{t+1}\right] = c_t + \frac{1}{\sigma}(r_t - \rho) \tag{5}$$

where $\rho = -\log\beta$.
Let the firm maximize profit:

$$\Pi_t = Y_t - W_t N_t \tag{6}$$

with

$$Y_t = C_t + G_t = A_t {N_t}^{1-\alpha} \tag{7}$$

where G_t is government deficit spending, financed through money. I will not consider inflation as price level P_t is assumed to be uniform across sectors. The optimality condition is

$$w_t = a_t - \alpha n_t + \log(1-\alpha) \tag{8}$$

By log-linearization assumption, assume:

$$y_t = cc_t + gg_t = a_t + (1-\alpha)n_t \tag{9}$$

$$c_t = \frac{a_t + (1-\alpha)n_t - gg_t}{c} \tag{10}$$

where c and g are defined around steady-state values.
Labor-market clearing requires:

$$\sigma\left[\frac{a_t + (1-\alpha)n_t - gg_t}{c}\right] + \varphi n_t = a_t - \alpha n_t + \log(1-\alpha) \tag{11}$$

$$\left[\frac{\sigma}{c}(1-\alpha) + \varphi + \alpha\right] n_t = \left[1 - \frac{\sigma}{c}\right] a_t + \frac{\sigma g}{c} g_t + \log(1-\alpha) \tag{12}$$

$$n_t = \frac{\left[1 - \frac{\sigma}{c}\right] a_t + \frac{\sigma g}{c} g_t + \log(1-\alpha)}{\frac{\sigma}{c}(1-\alpha) + \varphi + \alpha} \tag{13}$$

If technology a_t is assumed to be exogenous, but with known future expected values, then y_t is uniquely specified. Since g_t is already known, c_t is already known. Thus, just from knowledge of g_t, present and expected future c_t can

be calculated. Thus it is now established that B_t does not affect real economy. This result is not affected by whether one takes linearization approximation or not.
The problem, then is the following. For accounting consistency,

$$Y_t = C_t + S_t = C_t + G_t \tag{14}$$

is required (which means $Y_t > C_t$ whenever $G_t > 0$), assuming there is no foreign sector and there is no investment (because this economy is labor-only economy). S_t refers to savings in national accounting. Thus $G_t = S_t$. But notice Equation 3, replicated below:

$$C_t + {R_t}^{-1} B_t \leq B_{t-1} + W_t N_t + \Pi_t$$

We know that

$$Y_t = W_t N_t + \Pi_t \tag{15}$$

This is true by definition. Thus the budget constraint can be re-written as

$$C_t + {R_t}^{-1} B_t \leq Y_t + B_{t-1} \tag{16}$$

$Y_t + B_{t-1}$ can be considered as available budget. For the fixed budget $Y_t + B_{t-1}$, there is simply no reason why the household would buy B_t, as this would decrease the household's utility. Furthermore, according to the calculation above, C_{t+1} is unaffected by the quantity of B_t. Thus, $B_t = 0$ in equilibrium for all time t. But this runs in contradiction to Equation 14, as now

$$C_t = Y_t + B_{t-1} \geq Y_t \tag{17}$$

Whenever $G_t > 0$, this causes contradiction.
The inevitable conclusion is that in this basic economy, unless government deficit spending is zero ($G_t = 0$), disequilibrium is unavoidable, unless the idea of forced savings is adopted.

1.1 Interpreting government deficit spending as exports

It can easily be seen that G can be replaced with X, exports. Assume that X is exogenously given and there is zero import. (I will save M for representing money quantity.) One can assume that the foreign sector shares the same currency as the domestic sector, and all central banks have money-printing rights, and that the representative agent of each country cannot change its citizenship. Then it is clear that one faces the exactly same accounting problem.

2 Gali (2014)'s review of money-financed deficit spending

The discussion above is important, as this problem is not properly recognized when dealing with money-financed government spending problems. Gali (2014)

[2] does the exactly same analysis as in the above analysis in the classical monetary economy section, with some utility simplification and additions and some further analysis. Mainly, money is introduced into utility, so utility now looks as:

$$u(C_t, N_t) = \frac{{C_t}^{1-\sigma}}{1-\sigma} + \frac{{M_t}^{1-\nu}}{1-\nu} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \tag{18}$$

where M_t is "real" value of money (in Gali (2014), it is M_t/P_t), with budget constraint:

$$C_t + {R_t}^{-1}B_t + M_t \leq B_{t-1} + W_tN_t + \Pi_t + M_{t-1} \tag{19}$$

But even with this modification, the only extra optimality condition one obtains is:

$$M_t = \left(\frac{{C_t}^{\sigma}}{1-{R_t}^{-1}}\right)^{1/\nu} \tag{20}$$

By given knowledge and market clearing, C_t and E_tC_{t+1} are known. Thus, R_t is also known. This means M_t is also known. $B_t = 0$ also in "equilibrium." Let us re-write the budget constraint into equality (as the household does best to maximize its utility):

$$C_t + M_t - M_{t-1} = C_t + S_t = C_t + G_t = Y_t \tag{21}$$

Thus, $G_t = M_t - M_{t-1}$ must be satisfied. But notice again that C_t and R_t are determined independently of M_t. Suppose that it was found that $G_t = M_t - M_{t-1}$. Then one can adjust ν to make this equality to be untrue, given that the path of G_t remains the same as before.

2.0.1 Fiscal theory of real money supply?

In some ways, these results suggest that some form of fiscal theory of real money supply (here, M_t) is needed to properly form a equilibrium - that the current money-financed deficit spending defines the change in real money supply ($M_t - M_{t-1}$). If this were true, then central banks, by setting nominal money supply M_tP_t defines price level P_t. In a way, this is similar to fiscal theory of price level.

Intuitively, the theory does make sense. After all, G_t is assumed to be all money-financed and this all adds up to real money supply. The problem rather here is why it is the only change possible in net aggregate. Though explaining this constraint may reveal how price level is affected by government spending as equilibrium adjustments.

Also, if one replaces G with X, then the theory converts to current account (CA) surplus/deficit theory of real money supply. One can try to combine two as external surplus/deficit theory of real money supply. But whether this theory is plausible would be left as a question. Notice that the form of a theory can change depending on how budget constraint/utility are specified, so $G_t = M_t - M_{t-1}$ does not always come out as a constraint.

3 Debt-financed deficit spending via bonds, with interest money-financed

So far, in equilibrium $B_t = 0$. Suppose that the government finances its deficit spending G_t by bonds, so $G_t = {R_t}^{-1} B_t$, if there are equivalent demands, and finance interest by printing money. Again, however, the household has zero demand on B_t. Thus to form an equilibrium properly without G_t constrained to zero or to adopt a non-orthodox theory of money supply, let us introduce B_t into utility.

$$u(C_t, N_t) = \frac{{C_t}^{1-\sigma}}{1-\sigma} + \frac{{B_t}^{1-\nu}}{1-\nu} - \frac{{N_t}^{1+\varphi}}{1+\varphi} \tag{22}$$

with the previous budget constraint:

$$C_t + {R_t}^{-1} B_t \leq B_{t-1} + W_t N_t + \Pi_t$$

Here, I drop M_t from utility. But the optimality conditions of the household do change significantly as follows:

$${B_t}^{-\nu} - {C_t}^{-\sigma} {R_t}^{-1} + \beta E_t \left[{C_{t+1}}^{-\sigma} \right] = 0 \tag{23}$$

Other optimality conditions remain the same. Notice that C_t and C_{t+1} are unaffected by the changed optimality condition. The affected is R_t, and the below is the log-linearizied approximation of r_t:

$$r_t = \frac{\sigma(E_t[c_{t+1}] - c_t) + \rho + \nu g_t}{1-\nu} \tag{24}$$

If g_t is replaced with x_t, then the foreign sector is buying the goods in the domestic sector and selling B_t that the domestic sector willingly takes. Without further restriction, it is certainly possible that the domestic sector continuously buys B_t at all time t that the foreign sector wishes to sell to finance x_t (for the foreign sector this is import). Thus, CA deficits go without the problem in this economy, though this certainly is only theoretical.
The inclusion of B_t in utility results in a different conclusion of welfare effects of fiscal deficit, but I will not explore this question.
However, notice also here that the accounting problem re-appears. To satisfy both budget constraint and accounting consistency,

$${R_t}^{-1} B_t - B_{t-1} = G_t \tag{25}$$

needs to be satisfied. This implies that the government needs to issue bonds more than it really needs in order to avoid disequilibrium. To the dominant effects, Equation 24 can be used for qualitative analysis.

4 Hot potato effects

The budget constraint/accounting problem underlined in this paper also shows how hot potato effects, in Monetarist jargon, may be understood in representa-

tive agent classical models. Inherently, hot potato effects arise only in disequilibrium conditions.
Let us start with a classical equilibrium of the first model presented in this paper where $G_t = 0$, $B_t = 0$. If G_t is forced into an economy, this generates extra savings S_t that the household did not want. Thus, the household wants to get rid of it - but the household would fail in doing so because the accounting identity cannot be violated. In short words, the household wants $S_t = 0$, but $S_t > 0$ always if $G_t > 0$ by accounting equivalence of $S_t = G_t$ (assuming investment does not exist, taxes do not exist and so on). Thus, this generates extra Y_t that ends only when N_t reaches its maximum value, defined either by physical limitation or legal constraints.
With this maximum labor limitation constraint, and with $G_t > 0$, the economy gravitates toward the maximum labor economy, instead of the pseudo-equilibrium that was obtained without $B_t = 0$ demand constraint.
These hot potato effects may show how the government deficit might be effective when the economy is demand-deficient, either because of self-fulfilling belief problems associated with many multiple equilibria models. In a way, this section did present a simple multiple equilibria model, if we consider the two pseudo-equilibria that are not truly equilibria as equilibria.

5 Including investment

Let us introduce investment into the first model. Without discussing full optimality conditions, first look at the household budget constraint:

$$C_t + I_t + {R_t}^{-1} B_t \leq B_{t-1} + W_t N_t + r_{K,t} K_t + \Pi_t \tag{26}$$

One can assume $E_t\left[r_{K,t+1}\right] = R_t - 1 + \sigma$ and $Y_t = C_t + I_t + G_t = W_t N_t + r_{K,t} K_t + \Pi_t$.

$$C_t + I_t + {R_t}^{-1} B_t \leq Y_t + B_{t-1} \tag{27}$$

Assume that $I_t = K_{t+1} - (1-\sigma)K_t$ where σ is depreciation rate with K_t representing capital.
When $G_t = 0$, infinite number of equilibria are possible, as any time path of B_t that satisfies ${R_t}^{-1} B_t = B_{t-1}$ is an equilibrium consistent with the accounting identity $C_t + I_t = Y_t$ with equal C_t at all cases. Usually one eliminates explosive solutions by one more constraint and obtain a unique equilibrium $B_t = 0$.
But if $G_t > 0$, it is no longer possible to assume that bond demand would obviously be zero, because even if one fixes I_t as it was before, increasing C_t may affect C_{t+1} and all other variables as a result. Though if the household only thinks about the current time, given the current income and bond payment $Y_t + B_{t-1}$, it is optimal to increase consumption C_t and set $B_t = 0$. Also the choice does affect $r_{K,t}$, if equilibrium conditions are followed.
However, notice that in the specification of the household problem, what the household does is take $r_{K,t}, W_t, K_0, B_{-1}, R_t$ (with B_{-1} and K_0 determined already outside the equilibrium process and with current time being $t = 0$) and

maximize total expected utility by varying C_t, N_t, B_t. (here, $r_{K,t}, W_t$ refer to the path, not just the variable at $t = 0$.) And if we fix N_t given by some income value determined from the calculation that I show does not yield proper equilibria, we are left with varying C_t, B_t. And B_t does not offer utility. Thus, it is better for the agent to maximize utility by not buying a bond $B_t = 0$.
If one refuses to consider the correct interpretation of the household utility maximization problem, then this saves an ordinary real business cycle model from being accounting-inconsistent even when the government is included in - allowing analysis like the Ricardian equivalence principle [1].
However, this apparent consistency becomes mere coincidence, once one extends RBC models to include some plausible frictions. Consider the household budget constraint the simplified Smets-Wouters economy [3] made somehow classical:

$$C_t + I_t + {R_t}^{-1} B_t \leq W_t N_t + r_{K,t} K_t + B_{t-1} + \Pi_t - T_t - \textbf{utilization costs} \tag{28}$$

with the government budget constraint (so far this paper was dominantly about money-financed fiscal deficit, so a government budget constraint did not exist):

$$G_t + B_{t-1} = T_t + {R_t}^{-1} B_t \tag{29}$$

where T_t refers to taxes.
Substituting Equation 29, one obtains:

$$C_t + I_t + G_t \leq W_t N_t + r_{K,t} K_t + \Pi_t - \textbf{utilization costs} \tag{30}$$

But by the logic of the model $Y_t = W_t N_t + r_{K,t} K_t + \Pi_t$. Thus,

$$C_t + I_t + G_t + \textbf{utilization costs} \leq Y_t \tag{31}$$

But by the accounting identity, $C_t + I_t + G_t = Y_t$.

$$Y_t + \textbf{utilization costs} \leq Y_t \tag{32}$$

which of course makes no sense unless utilization costs are zero.
While the Smets-Wouters model discusses utilization costs, one can replace utilization costs with any psychological cost - in form of "money is there, but you cannot use it." This may be the tribute paid by some nation to another nation that never gets used/consumed.
The above case is much more problematic - for if a rational agent deviates from perfect rationality slightly, then equilibrium simply disappears - but modern business cycle models have been founded on deviations still achieving equilibrium.
Also, this example shows how rational expectation models place strict restriction on the behaviours that can be modelled. Once one introduces slightly irrational behaviours, one increases the chance of models producing only disequilibrium.
Thus, this demands an addition procedure when creating a rational expectation model - one first starts from a simple RBC model without government, introduce variables one by one and check whether the final consolidated budget constraint satisfies accounting identities. But this itself does not give a solution to why DSGE models should fail on modelling some economies.

5.1 What if B_t does not exist?

If G_t is financed by money all the time, then there is no need for the government to issue bonds B_t. Assume thus that it does not. Let us return to the standard RBC model without capital utilization costs. The household budget constraint is:

$$C_t + I_t \leq W_t N_t + r_{K,t} K_t + \Pi_t \tag{33}$$

If $G_t > 0$, then $C_t + I_t < W_t N_t + r_{K,t} K_t + \Pi_t = Y_t$ must hold. The main role $R_t - 1$ played is allowing one to compute expected future consumption. But even without R_t, one can simply substitute in $E_t\left[r_{K,t+1}\right] - \sigma = R_t - 1$ and obtain expected future consumption.

Now the agent faces forced savings $S_t = G_t$ that gives zero interest. And the problems mentioned in this section get much worse.

6 Can relaxation of equilibrium conditions save models?

The answer is no. The previous section derives an inconsistency only by using budget constraints. The idea may be that by relaxing equilibrium conditions, one obtains multiple equilibria, and hopefully only one may turn out to be consistent with accounting identities. But recall the household budget constraint of the first model:

$$C_t + \frac{B_t}{R_t} \leq W_t N_t + \Pi_t + B_{t-1}$$

Unless one can derive a way to get the demand function for the bond (and even then the trouble appears, as shown in previous sections), any relaxation will simply give inconsistency - $C_t = Y_t = C_t + G_t$ when $G_t > 0$.

7 Conclusion

In short words, this paper demonstrates that when $G_t > 0$ where G_t is government deficit spending financed by money printing, a simple RBC model extended with G_t can only result in disequilibrium if we properly enforce the bond demand that has to be zero. One can bake the $C_t + G_t = A_t {N_t}^{1-\alpha} = Y_t$ constraint into the household optimization problem, but doing so is avoiding the problem without dealing with it, because what the equilibrium resulted says is "the household somehow is forced to buy bond quantity given by the government deficit spending," which is not at all a characteristic of a free market economy. The household is supposed to be free in choosing its income spending proportion, when income is given.

With this in mind, I described what can be done to restore equilibria, and what analysis can be done. I hope these discussions lead to fruitful economics developments.

References

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