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Abstract

This study explores the growth and welfare effects of monetary policy in a scale-invariant Schumpeterian growth model with endogenous human capital accumulation. We model money demand via a cash-in-advance (CIA) constraint on R&D investment. Our results can be summarized as follows. We find that an increase in the nominal interest rate leads to a decrease in R&D and human capital investment, which in turn reduces the long-run growth rates of technology and output. This result stands in stark contrast to the case of exogenous human capital accumulation in which the long-run growth rates of technology and output are independent of the nominal interest rate. Simulating the transitional dynamics, we find that the additional long-run growth effect under endogenous human capital accumulation amplifies the welfare effect of monetary policy. Decreasing the nominal interest rate from 10% to 0% leads to a welfare gain that is equivalent to a permanent increase in consumption of 2.82% (2.38%) under endogenous (exogenous) human capital accumulation.

JEL classification: O30, O40, E41
Keywords: monetary policy, economic growth, R&D, human capital

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1 Introduction

How does monetary policy affect economic growth and social welfare? To explore this question, this study develops a scale-invariant monetary Schumpeterian growth model with human capital. The novelty of our analysis is that we allow for endogenous human capital accumulation and show that the interaction between endogenous technological progress and human capital accumulation gives rise to important implications on the effects of monetary policy. Following previous studies, such as Chu and Cozzi (2014) and Chu *et al.* (2015), we model money demand via a cash-in-advance (CIA) constraint on R&D investment.[1] In this growth-theoretic framework, we find that an increase in the nominal interest rate leads to a decrease in R&D and human capital investment, which in turn reduces the long-run growth rates of technology and output. This result stands in stark contrast to the case of exogenous human capital accumulation in which the long-run growth rates of technology and output are independent of the nominal interest rate.

The intuition of the above results can be explained as follows. We follow the setup in the seminal Romer (1990) model in which human capital (or skilled labor) is allocated between production and R&D. An increase in the nominal interest rate raises the cost of R&D via the CIA constraint and leads to a reallocation of human capital from R&D to production, which in turn improves the marginal product of raw labor (or unskilled labor) in the production sector. As a result, more labor is allocated to production crowding out the amount of labor available for education, which in turn reduces the growth rate of human capital. Given the increasing-complexity effect of technology on the productivity of R&D in our scale-invariant Schumpeterian growth model, the long-run growth rate of technology is determined by the growth rate of human capital. Therefore, the negative effect of the nominal interest rate on education also leads to a negative effect on the growth rates of technology and output in the long run. However, in the case of exogenous human capital accumulation, the growth rate of human capital is exogenous and independent of monetary policy.

We also calibrate the model to simulate the transitional dynamics of the economy from a change in the nominal interest rate. We find that under *exogenous* human capital accumulation, decreasing the nominal interest rate from 10% to 0% leads to a welfare gain that is equivalent to 2.38% of consumption. Allowing for *endogenous* human capital accumulation amplifies the welfare gain to 2.82%. In other words, the additional long-run growth effect under endogenous human capital accumulation raises the welfare effect of monetary policy.

This study relates to the literature on inflation and economic growth; see Stockman (1981) and Abel (1985) for seminal studies of the CIA constraint on capital investment in the Neoclassical growth model. Instead of analyzing the effects of monetary policy in the Neoclassical growth model, we consider an R&D-based growth model in which economic growth in the long run is driven by innovation and endogenous technological progress. The seminal study in this literature on inflation and innovation-driven growth is Marquis and Reffett (1994), who analyze the effects of a CIA constraint on consumption in the Romer variety-expanding model. In contrast, we consider a Schumpeterian quality-ladder model and explore the effects of monetary policy via a CIA constraint on R&D investment as in

[1] See Berentsen *et al.* (2012), Chu and Cozzi (2014) and Chu *et al.* (2015) for a discussion of empirical evidence for the presence of CIA constraints on R&D.

Chu and Cozzi (2014), Chu *et al.* (2015), Chen (2015) and Huang *et al.* (2015).[2] However, this study differs from previous studies by allowing for human capital accumulation. To our knowledge, this is the first study that analyzes the effects of monetary policy in a growth-theoretic framework featuring both R&D-driven innovation and human capital accumulation as dual engines of economic growth. Furthermore, we find that allowing for endogenous human capital accumulation amplifies the welfare effect of monetary policy.

This study also relates to the literature on innovation and human capital. Early studies, such as Romer (1990), Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), on innovation-driven economic growth do not consider human capital accumulation. More recent studies, such as Eicher (1996), Zeng (1997, 2003), Strulik *et al.* (2013) and Hashimoto and Tabata (2016), explore human capital accumulation and its interaction with endogenous technological progress. Our study complements these studies by introducing money into an R&D-based growth model with human capital to explore the effects of monetary policy on the interaction between endogenous technological progress and human capital accumulation.

The rest of this paper is organized as follows. Section 2 sets up the monetary Schumpeterian growth model. Section 3 analyzes the growth and welfare effects of monetary policy. The final section concludes.

2 A monetary Schumpeterian growth model

In this section, we consider a monetary version of the quality-ladder growth model in Grossman and Helpman (1991).[3] Following previous studies, we model money demand via a CIA constraint on R&D investment and also a more conventional CIA constraint on consumption. We also allow for human capital accumulation and remove the scale effect through an increasing-complexity effect of technology similar to Segerstrom (1998).[4] Given that the quality-ladder model has been well-studied, we will describe the familiar features briefly to conserve space and discuss the new features in details.

2.1 Household

There is a representative household which has the following lifetime utility function:

$$U = \int_0^{\infty} e^{-\rho t} \ln c_t dt. \tag{1}$$

[2]For other approaches of modeling money demand in the Schumpeterian growth model, see Funk and Kromen (2010) who consider sticky prices, Chu and Lai (2013) who consider the money-in-utility approach, and also Chu and Ji (2016) who consider the CIA constraint on consumption in a scale-invariant Schumpeterian model with endogenous market structure. However, these studies do not feature human capital.

[3]See also Aghion and Howitt (1992) and Segerstrom *et al.* (1990) for other seminal studies of the quality-ladder growth model.

[4]See Jones (1999) for a discussion of the scale effect in R&D-based growth models.

The variable c_t denotes the consumption of final goods (numeraire) at time t. The parameter $\rho > 0$ is the subjective discount rate. The asset-accumulation equation is given by

$$\dot{a}_t + \dot{m}_t = r_t a_t + w_{l,t} l_t + w_{h,t} h_t + \tau_t - c_t - \pi_t m_t + i_t b_t. \tag{2}$$

a_t is the real value of financial assets (in the form of equity shares in monopolistic intermediate goods firms), and r_t is the real interest rate. l_t is raw labor supplied to production, and $w_{l,t}$ is the real wage rate of raw labor. h_t is human capital supplied to production and R&D. $w_{h,t}$ is the real wage rate of human capital. The household also receives a real lump-sum transfer τ_t from the government (or pays a lump-sum tax if $\tau_t < 0$). π_t is the inflation rate that determines the cost of holding money, and m_t is the real money balance held by the household partly to facilitate purchases of consumption goods. The CIA constraint is given by $\gamma c_t \leq m_t - b_t$, where the parameter $\gamma > 0$ determines the strength of the CIA constraint on consumption. b_t is the amount of money borrowed by entrepreneurs to finance R&D investment, and the rate of return on b_t is i_t.

At any time t, the household has one unit of raw labor that is allocated between work l_t and education e_t subject to

$$l_t + e_t = 1. \tag{3}$$

The accumulation equation of human capital is given by[5]

$$\dot{h}_t = \xi h_t e_t, \tag{4}$$

where ξ is a productivity parameter for human capital investment.

From standard dynamic optimization,[6] we derive a no-arbitrage condition given by $i_t = r_t + \pi_t$; therefore, i_t is also the nominal interest rate. The optimality condition for consumption is

$$c_t = \frac{1}{\eta_t (1 + \gamma i_t)}, \tag{5}$$

where η_t is the Hamiltonian co-state variable on (2). The intertemporal optimality condition is

$$-\frac{\dot{\eta}_t}{\eta_t} = r_t - \rho. \tag{6}$$

In the case of a constant nominal interest rate i, (6) becomes the familiar Euler equation $\dot{c}_t / c_t = r_t - \rho$. Finally, we also have the following no-arbitrage condition that equates the return to financial assets given by r_t and the return to human capital:

$$r_t = \xi h_t \frac{w_{h,t}}{w_{l,t}} + \frac{\dot{w}_{l,t}}{w_{l,t}}. \tag{7}$$

We will show that this condition determines the equilibrium growth rate of human capital.

[5] We do not impose a CIA constraint on human capital investment for the following reason. Although human capital investment may be subject to credit constraints that are influenced by the real interest rate, there is no evidence that human capital investment is subject to CIA constraints that are influenced by the nominal interest rate.

[6] We provide the derivations in an unpublished appendix (see Appendix B).

2.2 Final goods

Final goods are produced by competitive firms that aggregate a unit continuum of differentiated intermediate goods using a standard Cobb-Douglas aggregator given by

$$y_t = \exp\left(\int_0^1 \ln x_t(j)dj\right). \tag{8}$$

The variable $x_t(j)$ denotes intermediate good $j \in [0,1]$. From profit maximization, the conditional demand function for $x_t(j)$ is

$$x_t(j) = y_t/p_t(j), \tag{9}$$

where $p_t(j)$ is the price of $x_t(j)$ denominated in units of final goods.

2.3 Intermediate goods

There is a unit continuum of industries producing differentiated intermediate goods. Each industry is temporarily dominated by an industry leader until the arrival of the next innovation, and the owner of the new innovation becomes the next industry leader.[7] The production function for the leader in industry j is

$$x_t(j) = z^{q_t(j)}[h_{x,t}(j)]^{\alpha}[l_t(j)]^{1-\alpha}. \tag{10}$$

The parameter $z > 1$ is the step size of productivity improvement, and $q_t(j)$ is the number of productivity improvements that have occurred in industry j as of time t. $l_t(j)$ is raw labor employed for production in industry j. $h_{x,t}(j)$ is human capital employed for production in industry j. From cost minimization, the marginal cost of production for the industry leader in industry j is

$$mc_t(j) = \frac{1}{z^{q_t(j)}}\left(\frac{w_{h,t}}{\alpha}\right)^{\alpha}\left(\frac{w_{l,t}}{1-\alpha}\right)^{1-\alpha}.$$

It is useful to note that we here adopt a cost-reducing view of vertical innovation as in Peretto (1998).

Standard Bertrand price competition leads to a profit-maximizing price given by $p_t(j)$ determined by a markup $\mu = p_t(j)/mc_t(j)$ over the marginal cost. In the original Grossman-Helpman model, the markup μ is assumed to equal the step size z of innovation. Here we consider patent breadth similar to Li (2001) and Iwaisako and Futagami (2013) by assuming that the markup $\mu \in (1, z]$ is a policy instrument determined by the patent authority.[8] This

[7] This is known as the Arrow replacement effect in the literature. See Cozzi (2007) for a discussion of the Arrow effect.

[8] Intuitively, the presence of monopolistic profits attracts potential imitation; therefore, stronger patent protection allows monopolistic producers to charge a higher markup without losing their markets to potential imitators. This formulation of patent breadth captures Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".

formulation provides as a simple way to separate the markup μ from the step size z. The amount of monopolistic profit in industry j is

$$\Pi_t(j) = \left(\frac{\mu - 1}{\mu}\right) p_t(j) x_t(j) = \left(\frac{\mu - 1}{\mu}\right) y_t, \tag{11}$$

where the second equality follows from (9). Finally, wage income for $h_{x,t}(j)$ and $l_t(j)$ is

$$w_{h,t} h_{x,t}(j) = \left(\frac{\alpha}{\mu}\right) y_t;\ w_{l,t} l_t(j) = \left(\frac{1-\alpha}{\mu}\right) y_t. \tag{12}$$

2.4 R&D

Denote $v_t(j)$ as the real value of the monopolistic firm in industry j. Given that $\Pi_t(j) = \Pi_t$ for $j \in [0,1]$ from (11), $v_t(j) = v_t$ in a symmetric equilibrium that features an equal arrival rate of innovation across industries.[9] The familiar no-arbitrage condition for v_t is

$$r_t = \frac{\Pi_t + \dot{v}_t - \lambda_t v_t}{v_t}. \tag{13}$$

This condition equates the real interest rate to the asset return per unit of asset. The asset return is the sum of (a) monopolistic profit Π_t, (b) potential capital gain $\dot{v}_t$, and (c) expected capital loss $\lambda_t v_t$ from creative destruction for which λ_t is the arrival rate of the next innovation.

There is a unit continuum of R&D firms indexed by $k \in [0,1]$. They employ human capital $h_{r,t}(k)$ for innovation. The wage payment is $w_{h,t} h_{r,t}(k)$; however, to facilitate this wage payment, the entrepreneur needs to borrow money from the household. Each entrepreneur borrows the amount $b_t(k)$ of money from the household. Following Chu and Cozzi (2014), we impose a CIA constraint on R&D investment, and the cost of borrowing per unit time is $b_t(k) i_t$. To parameterize the strength of this CIA constraint, we assume that a fraction $\beta \in [0,1]$ of R&D investment requires the borrowing of money from the household such that $b_t(k) = \beta w_{h,t} h_{r,t}(k)$. Therefore, the total cost of R&D per unit time is $w_{h,t} h_{r,t}(k)(1 + \beta i_t)$.

The CIA constraint on R&D gives the monetary authority an ability to influence the equilibrium allocation of human capital across sectors through the nominal interest rate. The zero-expected-profit condition of firm k is

$$v_t \lambda_t(k) = (1 + \beta i_t) w_{h,t} h_{r,t}(k). \tag{14}$$

The *firm-level* innovation arrival rate per unit time is $\lambda_t(k) = \overline{\varphi}_t h_{r,t}(k)$, where $\overline{\varphi}_t = \varphi / Z_t$ captures an increasing-complexity effect of technology.[10] This formulation of increasing R&D

[9] We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the quality-ladder growth model.

[10] See Venturini (2012) for empirical evidence based on industry-level data that supports the presence of increasing R&D difficulty.

difficulty serves to remove a scale effect of human capital[11] in the innovation process as in Segerstrom (1998).[12] Finally, the *aggregate* arrival rate of innovation is

$$\lambda_t = \int_0^1 \lambda_t(k)dk = \frac{\varphi h_{r,t}}{Z_t} = \frac{\varphi h_t}{Z_t} s_{r,t}, \tag{15}$$

where we have defined $s_{r,t} \equiv h_{r,t}/h_t$ as the R&D share of human capital. Similarly, we will define $s_{x,t} \equiv h_{x,t}/h_t$ as the production share of human capital. Finally, we will also define a transformed variable $\Omega_t \equiv \varphi h_t/Z_t$.

2.5 Monetary authority

The nominal money supply is denoted by M_t, and its growth rate is $\dot{M}_t/M_t$. By definition, the aggregate real money balance is $m_t = M_t/P_t$, where P_t denotes the price of final goods. The monetary policy instrument that we consider is i_t. Given an exogenously chosen i_t by the monetary authority, the inflation rate is endogenously determined according to $\pi_t = i_t - r_t$. Then, given π_t, the growth rate of the nominal money supply is endogenously determined according to $\dot{M}_t/M_t = \dot{m}_t/m_t + \pi_t$. Finally, the monetary authority returns the seigniorage revenue as a lump transfer $\tau_t = \dot{M}_t/P_t = \dot{m}_t + \pi_t m_t$ to the household.

2.6 Decentralized equilibrium

The equilibrium is a time path of allocations $\{c_t, m_t, h_t, l_t, e_t, y_t, x_t(j), l_t(j), h_{x,t}(j), h_{r,t}(k)\}$, a time path of prices $\{p_t(j), w_{l,t}, w_{h,t}, r_t, v_t\}$, and a time path of monetary policy $\{i_t\}$. Also, at each instance of time, the following conditions hold:

- the household maximizes utility taking $\{i_t, r_t, w_{l,t}, w_{h,t}\}$ as given;
- competitive final-goods firms produce $\{y_t\}$ to maximize profit taking $\{p_t(j)\}$ as given;
- each monopolistic intermediate-goods firm j produces $\{x_t(j)\}$ and chooses $\{l_t(j), h_{x,t}(j), p_t(j)\}$ to maximize profit taking $\{w_{l,t}, w_{h,t}\}$ as given;
- R&D firms choose $\{h_{r,t}(k)\}$ to maximize expected profit taking $\{i_t, w_{h,t}, v_t\}$ as given;
- the market-clearing condition for raw labor holds such that $l_t + e_t = 1$;
- the market-clearing condition for human capital holds such that $h_{x,t} + h_{r,t} = h_t$;
- the market-clearing condition for final goods holds such that $y_t = c_t$;

[11] The level of education has been increasing in many developed countries. However, this increase in the level of human capital is not accompanied by a rise in the growth rate of total factor productivity; see for example Jones (1995).

[12] Segerstrom (1998) considers an industry-specific index of R&D difficulty. Here we consider an aggregate index of R&D difficulty to simplify notation without altering the aggregate results of our analysis.

- the share value of monopolistic firms adds up to the total value of the household's assets such that $v_t = a_t$; and
- the real money balance borrowed by R&D entrepreneurs from the household is $b_t = \beta w_{h,t} h_{r,t}$.

Substituting (10) into (8) yields the aggregate production function given by

$$y_t = Z_t (h_{x,t})^{\alpha} (l_t)^{1-\alpha} = Z_t (h_t s_{x,t})^{\alpha} (l_t)^{1-\alpha}, \tag{16}$$

where $s_{x,t} \equiv h_{x,t}/h_t$ and aggregate technology Z_t is defined as

$$Z_t = \exp\left(\int_0^1 q_t(j) dj \ln z\right) = \exp\left(\int_0^t \lambda_v dv \ln z\right). \tag{17}$$

The second equality of (17) applies the law of large numbers. Differentiating the log of (17) with respect to t yields the growth rate of aggregate technology given by

$$g_{z,t} \equiv \frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z = s_{r,t} \Omega_t \ln z, \tag{18}$$

where $s_{r,t} \equiv h_{r,t}/h_t$ and $\Omega_t \equiv \varphi h_t / Z_t$.

2.7 Balanced growth path

We consider the balanced growth path in this section. We first derive the steady-state equilibrium growth rates of technology and human capital. On the balanced growth path, the R&D share of human capital s_r is constant and the arrival rate of innovation is also constant. Therefore, h_t and Z_t must grow at the same rate as implied by (18). In other words, the steady-state growth rate of technology g_z is equal to the steady-state growth rate of human capital g_h.

$$g_z = g_h \Leftrightarrow s_r \Omega \ln z = \xi e. \tag{19}$$

We now manipulate the R&D free-entry condition in (14) to determine the steady-state equilibrium allocation s_r. Combining (12) and (14), we derive the first condition for solving the steady-state equilibrium as follows.

$$\frac{v_t \lambda_t}{(1+\beta i) h_{r,t}} = w_{h,t} = \frac{\alpha y_t}{\mu h_{x,t}} \Leftrightarrow \frac{s_r}{1-s_r} = \frac{1}{1+\beta i}\left(\frac{\mu-1}{\alpha}\right)\frac{\lambda}{\rho+\lambda}, \tag{20}$$

where we have used $s_x = 1 - s_r$, $v_t = \Pi_t/(\rho+\lambda)$ and (11). The steady-state equilibrium innovation-arrival rate λ is given by

$$\lambda = g_z / \ln z = \xi e / \ln z, \tag{21}$$

where we have used (18) and (19). Given that education is endogenous, we need a second condition to determine the steady-state equilibrium allocation e. Substituting (12) into (7) yields

$$\rho = \frac{\xi\alpha}{1-\alpha}\left(\frac{1-e}{1-s_r}\right), \tag{22}$$

where we have used the Euler equation $\dot{c}_t/c_t = r_t - \rho$ and the steady-state condition $\dot{c}_t/c_t = \dot{w}_{l,t}/w_{l,t}$.

We are now ready to solve for the steady-state equilibrium $\{s_r, e\}$. Substituting (21) into (20) yields the following R&D free-entry condition, which we refer to as the R curve:

$$\frac{s_r}{1-s_r} = \frac{1}{1+\beta i}\left(\frac{\mu-1}{\alpha}\right)\frac{\xi e}{\rho \ln z + \xi e}. \tag{23}$$

Re-expressing (22) yields the household's optimality condition for education, which we refer to as the E curve:

$$\frac{s_r}{1-s_r} = \frac{\rho}{\xi}\frac{1-\alpha}{\alpha}\frac{1}{1-e} - 1. \tag{24}$$

We impose the following parameter restriction to ensure the existence of a unique equilibrium:

$$\rho/\xi < \alpha/(1-\alpha). \tag{P1}$$

Figure 1 plots (23) and (24) in terms of $s_r/(1-s_r)$ against e and shows that a unique equilibrium must exist given (P1).

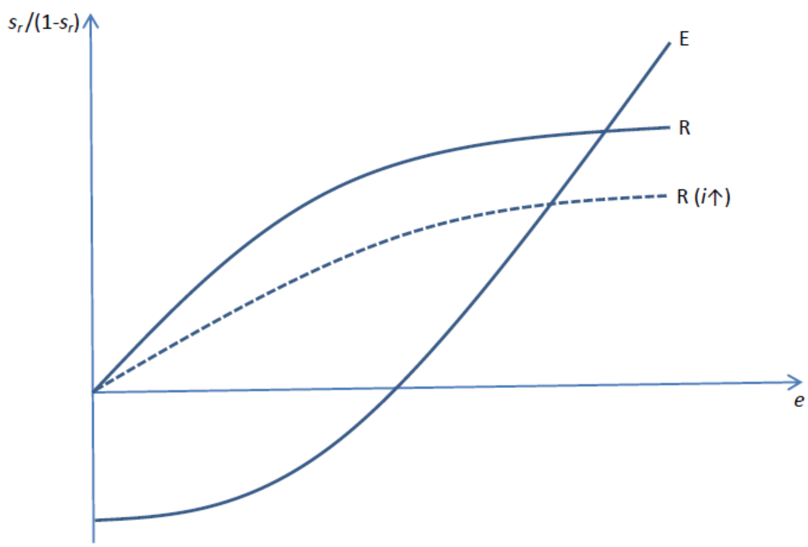

Figure 1: Steady-state equilibrium

3 Growth and welfare effects of monetary policy

In this section, we analyze the growth and welfare effects of monetary policy. In Section 3.1, we analyze the effects of the nominal interest rate on economic growth. In Section 3.2, we calibrate the model and simulate the transitional dynamics to provide a quantitative analysis on the effects of the nominal interest rate on economic growth and social welfare. In Section 3.3, we present the results from a simplified version of our model with exogenous human capital accumulation.

3.1 Growth analysis

Figure 1 shows that an increase in the nominal interest rate i rotates the R curve downwards and leads to a decrease in both R&D share s_r and education e. First, the effect of the nominal interest rate i on R&D share s_r operates through the CIA constraint on R&D captured by β rather than the CIA constraint on consumption captured by γ due to the absence of leisure in utility. Then, from (19) we know that the long-run growth rate of technology is given by $g_z = g_h = \xi e$. Therefore, the decrease in education e reduces both the long-run growth rates of human capital g_h and technology g_z. Intuitively, the higher nominal interest rate raises the cost of R&D via the CIA constraint on R&D and leads to a reallocation of human capital from R&D to production, which in turn improves the marginal product of labor l and its wage rate in the production sector. As a result, the increase in l crowds out education e, which in turn reduces the growth rate of human capital and also the growth rate of technology given that the long-run growth rate of technology is determined by the growth rate of human capital in the model. As for the effect of i on the growth rate of output, (16) implies the following steady-state equilibrium growth rate of output:

$$g_y = g_z + \alpha g_h = (1 + \alpha) g_h. \tag{25}$$

Therefore, the long-run growth rate of output is also decreasing in the nominal interest rate. We summarize these results in Proposition 1.

Proposition 1 *An increase in the nominal interest rate reduces the growth rates of human capital, technology and output.*

Proof. Proven in text. ■

Using the Fisher identity $i = r + \pi$ and the Euler equation $g_c = r - \rho$, we can write down an expression for the equilibrium inflation rate given by

$$\pi = i - g_c - \rho, \tag{26}$$

where $g_c = g_y$ is decreasing in the nominal interest rate i. Differentiating (26) with respect to i yields the following positive *long-run* relationship between the inflation rate and the nominal interest rate:[13]

$$\frac{\partial \pi}{\partial i} = 1 - \frac{\partial g_c}{\partial i} > 0. \tag{27}$$

Therefore, we have the following empirical implications from (27) and Proposition 1. First, an increase in the nominal interest rate is associated with a decrease in innovation and an increase in the inflation rate. This finding is consistent with the empirical evidence in Chu and Lai (2013) and Chu *et al.* (2015), who provide empirical evidence for a negative relationship between inflation and R&D. Second, an increase in the nominal interest rate is associated with a decrease in the growth rate of output and an increase in the inflation rate. This negative relationship between inflation and economic growth is supported by the empirical results in recent studies, such as Vaona (2012) and Chu *et al.* (2014).

3.2 Quantitative analysis

In this section, we calibrate the model and simulate the transitional dynamics to provide a quantitative analysis on the growth and welfare effects of monetary policy. Proposition 2 provides the three differential equations that summarize the dynamics of the economy.

Proposition 2 *The dynamics of the economy is given by the following differential equations:*

$$\dot{s}_{r,t} = (1 - s_{r,t}) \left\{ \rho + \left[s_{r,t} - \frac{\mu - 1}{\alpha(1 + \beta i)}(1 - s_{r,t}) - s_{r,t} \ln z \right] \Omega_t + \xi e_t \right\}, \tag{28}$$

$$\dot{e}_t = (1 - e_t) \left[\rho - \frac{\xi \alpha (1 - e_t)}{(1 - \alpha)(1 - s_{r,t})} \right], \tag{29}$$

$$\dot{\Omega}_t = \Omega_t \left[\xi e_t - s_{r,t} \Omega_t \ln z \right]. \tag{30}$$

Proof. See Appendix A. ∎

The model features the following set of parameters $\{\rho, \mu, z, \alpha, \xi, \beta\}$. For the discount rate ρ, we set it to a conventional value of 0.04. We follow Acemoglu and Akcigit (2012) to calibrate the innovation step size z by targeting an innovation-arrival rate λ of 1/3, which implies an average duration of 3 years between the arrival of innovations. We calibrate the markup μ by targeting an R&D share of GDP of 0.03, which is in line with recent US data. As for the human-capital intensity α in production, we consider a conventional value of 1/3; see for example Mankiw *et al.* (1992). As for the education productivity parameter ξ, we will use the human-capital growth rate g_h to calibrate its value. We consider a long-run GDP per capita growth rate g_y of 2%, and we pin down the value of $g_h = g_y/(1 + \alpha)$ from (25). Given $\alpha = 1/3$, we have $g_h = g_z = 1.5\%$, which is in line with the the long-run total factor productivity growth rate reported in Jones and Williams (2000). Then, we find a value of ξ such that $g_h = \xi e(\xi) = 0.015$, where $e(\xi)$ is the steady-state equilibrium value determined

[13] Empirical studies, such as Mishkin (1992) and Booth and Ciner (2001), provide evidence for this Fisher effect of a positive long-run relationship between inflation and the nominal interest rate.

by (23) and (24). Finally, we set the parameter β in the CIA constraint on R&D to 1.[14] In summary, the parameter values are $\{\rho, \mu, z, \alpha, \xi, \beta\} = \{0.04, 1.04, 1.05, 0.33, 0.09, 1\}$. We consider a policy experiment of decreasing the nominal interest rate from 10% to 0% and use the relaxation algorithm developed by Trimborn *et al.* (2008) to simulate the transitional dynamics of the economy.

Figure 2 shows the original balanced growth path and the transition path of the human-capital growth rate $g_{h,t}$. The decrease in the nominal interest rate increases the amount of human capital allocated to R&D, which in turn leads to a decrease in human capital and raw labor allocated to production. As a result, there is more labor allocated to education, and hence, the growth rate of human capital jumps up and gradually converges to the new steady-state growth rate that is higher than the initial steady-state growth rate.

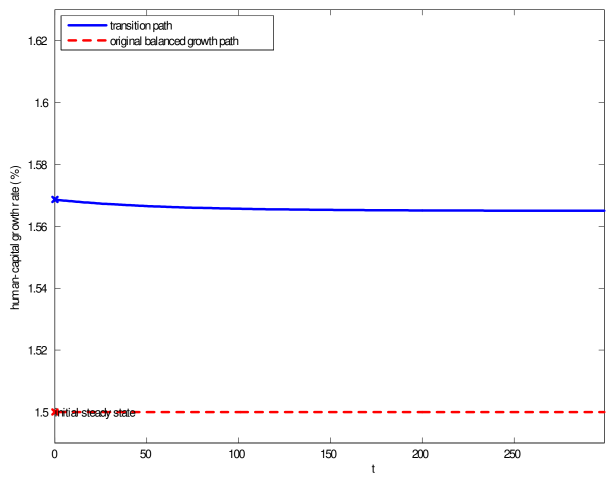

Figure 2: The transition path of $g_{h,t}$

Figure 3 shows the original balanced growth path and the transition path of the technology growth rate $g_{z,t}$. When the nominal interest rate decreases, the growth rate of technology jumps up on impact and gradually converges to the new steady-state growth rate that is higher than the initial steady-state growth rate. The steady-state growth rate of technology increases because of the higher steady-state growth rate of human capital.

[14]The growth and welfare effects of the nominal interest rate is roughly proportional to the value of β. Due to the lack of an empirical value, we consider $\beta = 1$ as an illustrative benchmark. However, it is useful to note that our focus is to compare the welfare effects of monetary policy under endogenous human capital accumulation and under exogenous human capital accumulation. Our finding of a larger welfare effect under endogenous human capital accumulation is robust to different values of β.

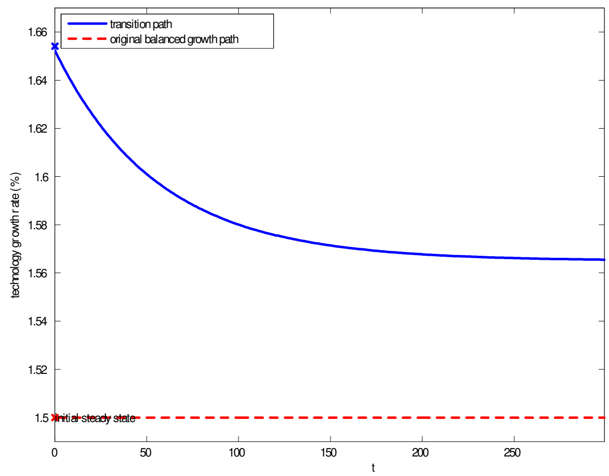

Figure 3: The transition path of $g_{z,t}$

Figure 4 shows the original balanced growth path and the transition path of the (log) level of consumption $\ln c_t$. The decrease in the nominal interest rate leads to a decrease in production human capital $h_{x,t}$ and production labor l_t, which in turn reduces output y_t and consumption c_t initially. Then, the higher growth rates of technology and human capital give rise to a higher growth rate of output and consumption. Gradually, the level of consumption converges to the new balanced growth path that features a higher growth rate than the initial balanced growth path.

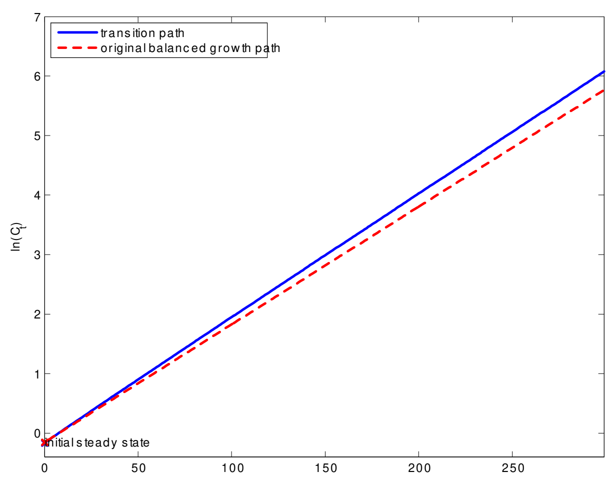

Figure 4: The transition path of $\ln c_t$

Table 1 summarizes the initial and new steady-state growth rates of technology, human capital and output. It also reports the effect of the decrease in the nominal interest rate on the household's lifetime utility. In summary, the welfare gain is equivalent to a permanent increase in consumption of 2.82%.

Table 1: Endogenous human capital				
i	g_z	g_h	g_y	ΔU
10%	1.50%	1.50%	2.00%	n/a
0%	1.57%	1.57%	2.09%	2.82%

3.3 Exogenous human capital accumulation

To highlight the importance of endogenous human capital accumulation, we also consider the case in which human capital accumulation is exogenous. In this case, the steady-state growth rate of technology is determined by the exogenous growth rate of human capital as $g_z = g_h = \xi \bar{e}$, and the R&D free-entry condition in (23) becomes

$$\frac{s_r}{1-s_r} = \frac{1}{1+\beta i}\left(\frac{\mu-1}{\alpha}\right)\frac{\xi\bar{e}}{\rho \ln z + \xi \bar{e}}, \tag{31}$$

where $\bar{e}$ is an exogenous parameter. We consider the same parameter values as before. Therefore, we can calibrate the value of $\bar{e}$ using $\bar{e} = g_h/\xi = 0.17$. The resource constraint on labor becomes $l = 1 - \bar{e} = 0.83$. The dynamics of the economy is now determined by the following two differential equations:

$$\dot{s}_{r,t} = (1-s_{r,t})\left\{\rho + \left[s_{r,t} - \frac{\mu-1}{\alpha(1+\beta i)}(1-s_{r,t}) - s_{r,t}\ln z\right]\Omega_t + \xi\bar{e}\right\}, \tag{32}$$

$$\dot{\Omega}_t = \Omega_t\left[\xi\bar{e} - s_{r,t}\Omega_t \ln z\right]. \tag{33}$$

In this case, the growth rate of human capital is exogenous and constant at $g_h = \xi\bar{e}$. Figure 5 shows the original balanced growth path and the transition path of the technology growth rate $g_{z,t}$. When the nominal interest rate decreases, the growth rate of technology jumps up on impact and gradually converges back to the initial steady-state growth rate. The steady-state growth rate of technology does not change because of the constant and exogenous growth rate of human capital.

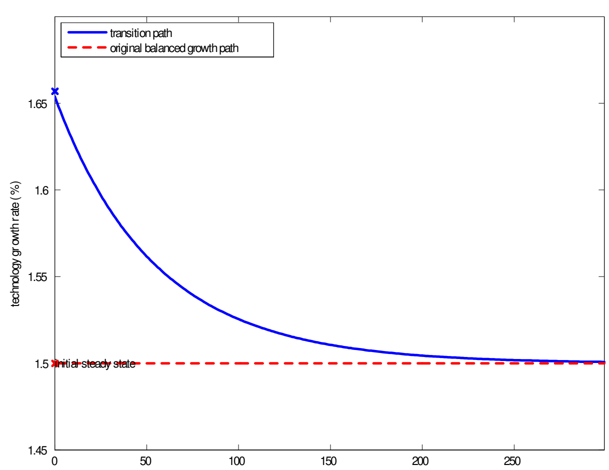

Figure 5: The transition path of $g_{z,t}$

Figure 6 shows the original balanced growth path and the transition path of the (log) level of consumption $\ln c_t$. The decrease in the nominal interest rate leads to an increase in R&D human capital $h_{r,t}$ and a decrease in production human capital $h_{x,t}$, which in turn reduces output y_t and consumption c_t initially. Then, the higher transitional growth rate of technology gives rise to a higher transitional growth rate of output and consumption. Gradually, the level of consumption converges to the new balanced growth path that is higher than the initial balanced growth path but features the same growth rate in the long run.

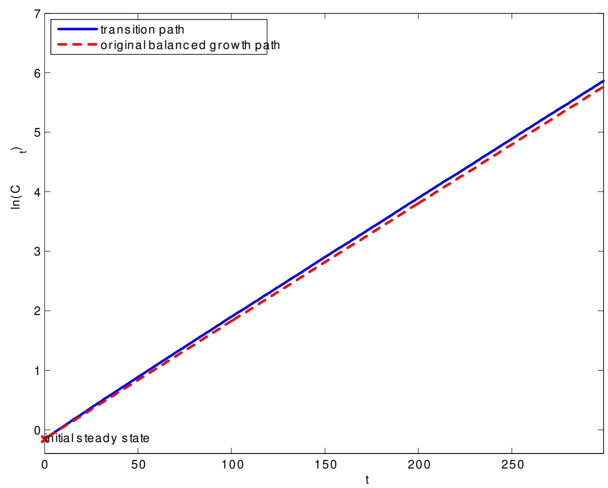

Figure 6: The transition path of $\ln c_t$

Table 2 summarizes the initial and new steady-state growth rates of technology, human capital and output. The steady-state growth rates of technology and output do not change because the growth rate of human capital is exogenous and constant. Table 2 also reports the effect of the decrease in the nominal interest rate on the household's lifetime utility. In summary, the welfare gain is equivalent to a permanent increase in consumption of 2.38%, which is smaller than the welfare gain under endogenous human capital accumulation.

Table 2: Exogenous human capital

i	g_z	g_h	g_y	ΔU
10%	1.50%	1.50%	2.00%	n/a
0%	1.50%	1.50%	2.00%	2.38%

4 Conclusion

In this study, we have analyzed the effects of monetary policy in a scale-invariant Schumpeterian growth model. The novel element in our analysis is endogenous human capital accumulation, which gives rise to some interesting results. In the case of exogenous human capital accumulation, an increase in the nominal interest rate has no effect on the long-run growth rates of technology and output despite the CIA constraint on R&D. However, in the case of endogenous human capital accumulation, an increase in the nominal interest rate reduces the long-run growth rates of technology, human capital and output. Due to this additional long-run growth effect, endogenous human capital accumulation amplifies the welfare effect of monetary policy. Therefore, we argue that when evaluating the effects of monetary policy on economic growth and social welfare, it is important to take into consideration this interaction between endogenous technological progress and human capital accumulation that has been neglected in the literature.

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Appendix A

Proof of Proposition 2. Substituting the Euler equation $r_t = \rho + \dot{c}_t/c_t$ and (12) into (7) yields

$$\rho + \frac{\dot{c}_t}{c_t} = \frac{\xi\alpha}{1-\alpha}\frac{l_t}{s_{x,t}} + \frac{\dot{w}_{l,t}}{w_{l,t}}, \tag{A1}$$

where we have used $s_{x,t} = h_{x,t}/h_t$. Differentiating the log of (12) with respect to time and substituting the resulting expression into (A1) yields

$$\rho = \frac{\xi\alpha}{1-\alpha}\frac{l_t}{s_{x,t}} - \frac{\dot{l}_t}{l_t}, \tag{A2}$$

where we have used $\dot{c}_t/c_t = \dot{y}_t/y_t$. Applying $s_{x,t} = 1 - s_{r,t}$, $l_t = 1 - e_t$ and $\dot{l}_t = -\dot{e}_t$ to (A2) yields (29).

Substituting $\overline{\varphi}_t = \varphi/Z_t$ into (14) and differentiating the resulting expression with respect to time yields

$$\frac{\dot{v}_t}{v_t} - \frac{\dot{Z}_t}{Z_t} = \frac{\dot{w}_{h,t}}{w_{h,t}}. \tag{A3}$$

Substituting (13) and then (14) into (A3) yields

$$r_t + \lambda_t - \frac{\Pi_t\lambda_t}{(1+\beta i)w_{h,t}h_{r,t}} - \frac{\dot{Z}_t}{Z_t} = \frac{\dot{w}_{h,t}}{w_{h,t}}. \tag{A4}$$

Substituting the Euler equation $r_t = \rho + \dot{c}_t/c_t$, (11) and (12) into (A4) yields

$$\rho + \frac{\dot{c}_t}{c_t} + \lambda_t - \frac{(\mu-1)\lambda_t}{\alpha(1+\beta i)}\frac{s_{x,t}}{s_{r,t}} - \frac{\dot{Z}_t}{Z_t} = \frac{\dot{w}_{h,t}}{w_{h,t}}, \tag{A5}$$

where we have used $s_{x,t} = h_{x,t}/h_t$ and $s_{r,t} = h_{r,t}/h_t$. Differentiating the log of (12) and substituting the resulting expression into (A5) yields

$$\rho + \lambda_t - \frac{(\mu-1)\lambda_t}{\alpha(1+\beta i)}\frac{s_{x,t}}{s_{r,t}} - \frac{\dot{Z}_t}{Z_t} = -\frac{\dot{h}_{x,t}}{h_{x,t}}, \tag{A6}$$

where we have used $\dot{c}_t/c_t = \dot{y}_t/y_t$. Adding $\dot{h}_t/h_t = \xi e_t$ to both sides of (A6) yields

$$\rho + \lambda_t - \frac{(\mu-1)\lambda_t}{\alpha(1+\beta i)}\frac{s_{x,t}}{s_{r,t}} - \frac{\dot{Z}_t}{Z_t} + \xi e_t = -\frac{\dot{s}_{x,t}}{s_{x,t}}, \tag{A7}$$

where we have used $\dot{s}_{x,t}/s_{x,t} = \dot{h}_{x,t}/h_{x,t} - \dot{h}_t/h_t$. Substituting (15) and (18) into (A7) yields

$$\rho + \left[s_{r,t} - \frac{\mu-1}{\alpha(1+\beta i)}s_{x,t} - s_{r,t}\ln z\right]\Omega_t + \xi e_t = -\frac{\dot{s}_{x,t}}{s_{x,t}}. \tag{A8}$$

Applying $s_{x,t} = 1 - s_{r,t}$ and $\dot{s}_{x,t} = -\dot{s}_{r,t}$ to (A8) yields (28).

As for (30), we differentiate the log of $\Omega_t \equiv \varphi h_t/Z_t$ with respect to time to obtain

$$\frac{\dot{\Omega}_t}{\Omega_t} = \frac{\dot{h}_t}{h_t} - \frac{\dot{Z}_t}{Z_t} = \xi e_t - s_{r,t}\Omega_t \ln z, \tag{A9}$$

where the second equality comes from (4) and (18). ■

Appendix B: Not for publication

Household's dynamic optimization: In this appendix, we solve the household's dynamic optimization problem. The Hamiltonian function is

$$H_t = \ln c_t + \eta_t[r_t a_t + w_{l,t}(1-e_t) + w_{h,t}h_t + \tau_t - c_t - \pi_t m_t + i_t b_t] + \zeta_t \xi h_t e_t + \vartheta_t(m_t - b_t - \gamma c_t).$$

The first-order conditions include

$$\frac{\partial H_t}{\partial c_t} = \frac{1}{c_t} - \eta_t - \gamma\vartheta_t = 0, \tag{B1}$$

$$\frac{\partial H_t}{\partial e_t} = -\eta_t w_{l,t} + \zeta_t \xi h_t = 0, \tag{B2}$$

$$\frac{\partial H_t}{\partial b_t} = \eta_t i_t - \vartheta_t = 0, \tag{B3}$$

$$\frac{\partial H_t}{\partial a_t} = \eta_t r_t = \rho\eta_t - \dot{\eta}_t, \tag{B4}$$

$$\frac{\partial H_t}{\partial m_t} = -\eta_t \pi_t + \vartheta_t = \rho\eta_t - \dot{\eta}_t, \tag{B5}$$

$$\frac{\partial H_t}{\partial h_t} = \eta_t w_{h,t} + \zeta_t \xi e_t = \rho\zeta_t - \dot{\zeta}_t. \tag{B6}$$

Combining (B1) and (B3) yields

$$\frac{1}{c_t} = \eta_t(1 + \gamma i_t). \tag{B7}$$

Substituting (B3) into (B5) and equating it to (B4) yield $i_t = r_t + \pi_t$, which is the nominal interest rate. Taking the log of (B2) and differentiating it with respect to t yield

$$\frac{\dot{\zeta}_t}{\zeta_t} + \frac{\dot{h}_t}{h_t} = \frac{\dot{\eta}_t}{\eta_t} + \frac{\dot{w}_{l,t}}{w_{l,t}}. \tag{B8}$$

Substituting (B2), (B4) and (B6) into (B8) yields

$$r_t = \xi h_t \frac{w_{h,t}}{w_{l,t}} + \frac{\dot{w}_{l,t}}{w_{l,t}}. \tag{B9}$$