Large-cap versus small-cap, a downside risk comparison

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In this paper we estimated for the period 1990 - 2015, Sortino Ratio and Return Level using a Generalized Pareto Distribution to evaluate downside risk of large-cap companies, approach through S&P 500 Index, and small-cap companies, approach through Russell 2000 Index. Small-cap depicted higher downside risk than large-cap.
I. Market Capitalization

Market capitalization is the market value of outstanding shares of a traded company, it’s calculated multiplying total number of outstanding shares by the company’s stock price per share.

Traded companies are usually grouped in different market capitalization categories. Threshold values of each categories is a relative and subjective amount that differ from source, have to be adjusted over time, and change by country, too.

According to Investopedia.com US Large-Cap have a market value higher than US$10 billion, and US Small-Cap have a market value less than US$2 billion.¹

The S&P 500 Index is an appropriate benchmark for large-cap portfolios, and the Russell 2000 Index is an appropriate benchmark for small-cap ones (Hayes, 2015).

¹ Extended categories include mega cap (≥ US$200 billion), mid-cap (from $2 billion to $10 billion), micro-cap (between $50 million to $300 million), and nano-cap (≤US$50 million)
II. S&P 500 Index and Russell 2000 Index

We estimated monthly returns of S&P 500 Index and Russell 2000 Index from January 2\textsuperscript{nd}, 1990 to December 1\textsuperscript{st}, 2015, downloaded from Yahoo Finance, for a total 332 observations per index.

Figure I presents S&P 500 Index and Russell 2000 Index monthly returns from 1990 to 2015.

![Fig. I. S&P 500 Index and Russell 2000 Index Monthly Returns](image_url)

Next table summarized main statistical data:

<table>
<thead>
<tr>
<th></th>
<th>Russell 2000</th>
<th></th>
<th>S&amp;P 500</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Period</td>
<td>Obs.</td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
</tr>
<tr>
<td>1990-2015</td>
<td>312</td>
<td>0.6120%</td>
<td>0.0556</td>
<td>0.5625%</td>
</tr>
</tbody>
</table>
During the time period analyzed, the average monthly returns and the standard deviation of Russell 2000 Index was higher than the average monthly returns and the standard deviation of S&P 500 Index.

Generally, small-cap companies have registered higher returns and higher volatility than large-cap companies.

For a robust analysis of small-cap outperformance large-cap see Davenport & Meissner, (2014).

Small-cap stocks are potentially riskier than large-cap stocks due to lower liquidity, less access to capital, a lack of operational history, and fewer public information available (Maverick, 2015).

III. Downside Risk

Downside risk is the possibility of losses, a decline in value, of an investment.

In this section, we evaluated the Sortino Ratio and the Return Level at the Russell 2000 Index monthly returns and the S&P 500 Index monthly returns for the time period 1990 - 2015.
III.1. Sortino Ratio

The Sortino Ratio, SR is defined as:

\[
SR = \frac{\bar{X} - T}{\sqrt{\frac{1}{N} \sum_{i=1}^{N} (\text{Min}(0, X_i - T))^2}}
\]

where \(\bar{X}\) is the average of monthly returns, \(T\) is the target monthly return defined at 0% for our case, \(X_i\) is the monthly return, and \(N\) is the number of observations.

The higher the impact of the presence of negatives monthly returns \((X_i<0)\), the lower the Sortino Ratio.

In addition, see Rollinger and Hoffamn (2013) for a basic example calculation of Sortino Ratio.

Next table summarized Sortino Ratio calculated:

<table>
<thead>
<tr>
<th>Period</th>
<th>Russell 2000 SR</th>
<th>S&amp;P 500 SR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1990-2015</td>
<td>0.15349</td>
<td>0.1878</td>
</tr>
</tbody>
</table>

During the time period analyzed, the Sortino Ratio value of S&P 500 Index monthly return was higher than the Sortino Ratio value of Russell 2000 Index monthly returns.
III.2. Return Level

The return level $R_k^n$ is the level expected, on average, to be exceeded in one out of $k$ periods of length $n$.

The return period is the amount of time expected to wait for particular return level to be exceed; return period is the inverse of the probability of an event (e.g. a called “100 years event” has a 1% (1/100) probability of exceed the record level in a given year).

For a Generalized Pareto Distribution, the $k$ year return level is defined:

$$ R_k \approx \mu + \frac{\bar{\sigma}}{\bar{\xi}} ([k^n y \cdot Pr(X>\mu)]^{\bar{\xi}} - 1) \text{ for } \bar{\xi} \neq 0 $$

where $\mu$ is the defined threshold, $\bar{\sigma}$, and $\bar{\xi}$ are the parameters of the Generalized Pareto Distribution, $n_y$ is the number of observations per year, and $Pr(X>\mu)$ is equal to number of exceedances of threshold ($Nu$) divided by total number of observations ($N$).

Using the in2extRemes Toolkit developed by Eric Gilleland and Richard Katz, within statistical software R, we conducted the estimation of the Generalized Pareto Distributions.
A threshold of a negative returns of 8.33\% was selected for the estimation of the Generalized Pareto Distributions.

Figure II depicts Russell 2000 and S&P 500 Index negatives monthly returns from 1990 to 2015.

![Fig. II. S&P 500 Index and Russell 2000 Index Monthly Negative Returns](image)

For the period 1990 – 2015, the Russell 2000 Index recorded a total of 121 cases of negatives monthly returns, a maximum negative monthly return of 23.5\%, and 17 cases of negative returns over -8.33\%.

\[2\] Usually, average log monthly returns are transformed to annually returns multiplying it by 12, therefore a -8.33\% monthly returns was selected to represent a 100\% annual potential loss.
Russell 2000 Index monthly returns for that period give a Generalized Pareto Distribution with parameters \((\mu): 8.33, (\sigma): 4.23190\) and \((\xi): 0.01535\)

For the period 1990 – 2015, the S&P 500 Index recorded a total of 115 cases of negatives monthly returns, a maximum negative monthly return of 18.6%, and 12 cases of negative returns over -8.33%.

S&P 500 Index monthly returns for that period give a Generalized Pareto Distribution with parameters \((\mu): 8.33, (\sigma): 1.53381\) and \((\xi): 0.44631\)

Figure III illustrates S&P 500 Index and Russell 2000 Index negative monthly returns at different returns levels.

![Chart](image_url)

Fig. II. S&P 500 Index and Russell 2000 Index Return Levels
Given any return level, the Russell 2000 Index monthly negative return is higher than the S&P 500 Index monthly negative return, for example at the 25 years return level, the $R_{25}$ Russell 2000 Index is -20.41% against $R_{25}$ S&P 500 Index of -15.13%. Or, in other words, given a negative monthly return value, the probability of exceed it in any given year is higher in the Russell 2000 Index than the S&P 500 Index, for example at -20.0%, there is a 4.34% chance of exceeded ($R_{23}$) in the Russell 2000 Index compared to a 1.66% chance of exceed ($R_{60}$) in the S&P 500 Index.

IV. CONCLUSION

Through the analysis of the Sortino Ratio and the Return Level values, we can concluded that for the period 1990-2015, small-cap showed higher downside risk than large-cap.

Future lines of research could apply estimation of return level using extreme value theory to others time periods, stocks, portfolio analysis, or markets like commodities and currencies, to evaluate downside risk.
REFERENCES


