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Abstract

This paper explores the shape of the Japanese money demand function in relation to the historical path of the Bank of Japan's policy rate by employing Saikkonen and Choi's (2004) cointegrating smooth transition model. The nonlinear model provides a unified econometric framework, not only for pursuing the time profile of interest elasticity, but also to test the linearity of the Japanese money demand function. The test results for the linearity of the Japanese money demand function provide evidence of nonlinearity with a semi-log model and linearity with a double-log model. Using a nonlinear semi-log model, the analysis also finds that Japanese money demand comprises three regimes and that the interest semi-elasticity began to increase in the early 1990s when the Bank of Japan set the policy rate below 3%.

JEL Classification Numbers: C22; E41; E58

Keywords: money demand, low interest rate policy, nonlinear cointegration, smooth transition model.

1 Introduction

This paper empirically explores the shape of the Japanese M1 demand function in relation to the historical path of the Bank of Japan's (BOJ) policy rate using the cointegrating smooth transition model in Saikkonen and Choi (2004).

From September 1995 onwards, the BOJ developed a unique low interest rate policy (see Figure 1). While the BOJ initially guided overnight call rates below 0.5%, in February 1999 it implemented the so-called zero interest rate policy, whereby the targeted overnight call rate was set at almost 0%. Accordingly, the relative amount of money in circulation, as represented by M1 relative to nominal GDP (the Marshallian k), rapidly increased towards 40% and higher from the mid-1990s, even though it had been hitherto stable between 25% and 30%. In March 2001, the BOJ adopted a new policy framework by expanding high-powered money. Although the quantity easing policy was lifted in March 2006, the targeted rate has since remained well below 0.5%.

The introduction of the low interest rate policy of the mid-1990s prompted studies that focused on the shape of the Japanese money demand function from the perspective of whether the Japanese economy had fallen into a liquidity trap. Investigating the shape of the Japanese money demand function, given the drastic increase in the Marshallian k under the small decrease in the call rate since 1995, can be classified into the following aspects.

The first aspect is involved with the issue of whether Japanese money demand became more interest elastic since the mid-1990s relative to other decades

(Miyao, 2004; Fujiki and Watanabe, 2004; Nakashima and Saito, 2006). Using cointegrating structural break models constructed by Hansen (1992) and Kuo (1998), these studies found that the absolute value of interest semi-elasticity substantially increased in 1995, whereas that of the interest elasticity was stable over time. [1] They confirmed that the estimates of interest elasticity ranged from -0.11 to -0.15 for the full-sample periods under consideration and that the estimates of interest semi-elasticity were -0.03 to -0.05 for sample periods prior to 1995, whereas they were -0.4 to -0.6 for sample periods after 1995.

The second aspect is to investigate whether the Japanese money market has a long-run equilibrium relationship. Previous studies, such as Miyao (2004) and Fujiki and Watanabe (2004), have provided significant evidence of cointegration in a double-log money demand model, but mixed evidence in a semi-log money demand model. [2]

The third aspect involves exploring a functional form to capture stable Japanese money demand in terms of goodness-of-fit. Bae, Kakkar, and Ogaki (2006) found that a double-log model outperformed a semi-log model in terms of out-of-sample prediction performance. [3]

The critical feature of previous studies on Japanese money demand is the assumption of linearity for both the semi-log and double-log money demand models. In the linear context, the test results for structural breaks show that a double-log model can capture Japanese money demand over time without considering the structural change of interest elasticity, whereas a semi-log model could not without considering the change of interest semi-elasticity in 1995. Further, the cointegration and goodness-of-fit test results commend use of the double-log model, not the semi-log model.

One objective of this paper is to reassess the performance of the semi-log and double-log money demand models. In particular, we evaluate the estimation and test results of previous work by considering the possibility of nonlinearity in both models. Both the semi-log and double-log models have their own theoretical backgrounds and policy implications. [4] Indeed, as pointed out by Lucas (2000), each model could derive a different policy implication if the level of nominal interest rates was drastically lower than rates thus far, similarly to the Japanese economy of the mid-1990s. [5] Hence, judging the advantage of one model over the other requires careful examination.

There is another motivation that is common with existing empirical studies on Japanese money demand: we investigate the shape of the Japanese money demand function. However, the approach adopted in this paper differs from previous studies in that we consider the time-varying interest elasticity (or semi-elasticity) as a function of the BOJ's policy rate. To fulfill our objectives, we employ the cointegrating smooth transition model in Saikkonen and Choi (2004).

Analyzing the history of interest elasticity (or semi-elasticity) in relation to a policy rate requires a state-dependent cointegrating model, that is, a model in which the coefficient parameter in question depends on the state of an explanatory variable. Saikkonen and Choi's (2004) nonlinear cointegrating model, being accompanied by their cointegration test (Choi and Saikkonen, 2005) and linearity test (Choi and Saikkonen, 2004), provides a unified econometric framework, not only to estimate the time-varying interest rate elasticity, but also to evaluate existing empirical studies on Japanese money demand under the extremely low interest rate regime.

Careful examination of the shape of money demand requires a transition value of a nominal interest rate, around which the interest elasticity is fluctuating. Another advantage of Saikkonen and Choi's (2004) model is that it can identify the transition value. Other nonlinear cointegrating models (e.g., Park and Phillips, 1999, 2001; Chang, Park, and Phillips, 2001; Bae and de Jong, 2007) and the cointegrating structural break models cannot identify this transition value. This paper attempts to uncover the shape of the Japanese money demand function by identifying the historical path of interest elasticity and the transition value of the BOJ's policy rate.

In this paper, the nonlinearity of the smooth transition model concerns modeling the long-run equilibrium of Japanese money demand, not the short-run error correction process to the equilibrium. Within a linear cointegration context, the theory of nonlinear error correction models including smooth transition error correction models has been developed by Saikkonen (2005, 2008) as an extension of Granger's representation theorem, which provides a link between linear cointegration and linear error correction models. [6] Within a nonlinear cointegration context in which a long-run equilibrium relation is allowed to be state dependent, however, an extension of Granger's representation theorem has not been developed. Consequently, this paper focuses on characterizing the long-run Japanese money demand by using the cointegrating smooth transition model in Saikkonen and Choi (2004). [7]

The paper is organized as follows. Section 2 introduces a cointegrating smooth transition model for Japanese money demand and discusses the estimation and test results. In this section, we reassess the performance of the semi-log and double-log models in terms of goodness-of-fit. Section 3 considers the shape of the Japanese money demand function based on estimates obtained in Section 2. Finally, Section 4 summarizes our empirical findings and discusses some issues for future research. The Appendix details the simulation methods and results.

2 Estimation and Test

In this section, we introduce a nonlinear model of Japanese money demand based on Saikkonen and Choi's (2004) cointegrating smooth transition model. In general, a smooth transition model, in identifying the transition value of an explanatory variable, deals with the dependence of a coefficient parameter on the state of the explanatory variable. [8]

In particular, Saikkonen and Choi's (2004) nonlinear cointegrating model has an econometric framework to conduct tests for the null hypotheses of cointegration with nonlinearity and linearity. As discussed, previous studies of Japanese money demand, with few reservations, assumed linearity for both the semi-log and double-log money demand models. However, if their assumption of linearity were invalid, their test results for cointegration and goodness-of-fit would be forced into a reexamination in a nonlinear context. Indeed, and according to Figure 2, a double-log model appears to be linear, whereas a semi-log model appears to be strongly nonlinear.

In this section, we undertake the following empirical steps. First, we find a cointegrating relationship with possible nonlinearity for each of the semi-log and double-log models. Next, we conduct the linearity test for the two models. If we find nonlinearity, we reassess the performance of the two models in terms of goodness-of-fit through modeling nonlinearity. Lastly, given the presence of nonlinearity, we estimate a nonlinear money demand model.

2.1 Nonlinear Model of Money Demand

We assume that the Japanese money demand function can be described by the following smooth transition model:

$$k_t = \text{constant} + \alpha i_t + \beta i_t \left(1 - \frac{1}{1 + \exp\left(-\gamma(i_t - i^*)\right)}\right) + \epsilon_t, \tag{1}$$

where k_t indicates the logarithm of the Marshallian k, that is, the ratio of nominal money stock to nominal GDP. Therefore, we adopt the velocity-based specification for the Japanese money demand function. [9] In the semi-log model, i_t merely indicates nominal interest as the opportunity cost of holding money, but in the double-log model, it includes the logarithm of nominal interest. The logistic function $\frac{1}{1+\exp(-\gamma(i_t-i^*))}$ makes the coefficient of i_t vary smoothly between α and $\alpha+\beta$. When the value of i_t sufficiently exceeds the transition value i^*, the coefficient for i_t takes a value close to α. When the value of i_t decreases and is far below the transition value, the coefficient for i_t changes and approaches $\alpha+\beta$. Furthermore, $\beta = 0$ reduces the nonlinear money demand model to a conventional linear model.

We build the set of quarterly data as follows. As nominal monetary aggregates we choose M1, compiled and seasonally adjusted by the BOJ, because M1 reflects to a greater extent the transaction demand for money and hence is frequently employed in empirical studies of the Japanese money demand function. Nominal GDP, as constructed by the Statistics Bureau, is used for nominal aggregate output. The overnight call rate, as reported by the BOJ, is used not only as the BOJ's policy rate but also as the opportunity cost of holding money. [10] Given the historical path of the call rate, regarded as the BOJ's policy rate, we should carefully choose the sample periods for inclusion. One strategy is to include sample periods prior to February 1999 when the BOJ began the zero interest rate policy. The inaction of the call rate at their lower bound (0%) for long-term periods (from 1999 to at least 2006) would cause serious econometric problems with our nonstationary and cointegration analysis. Existing nonlinear cointegration techniques including Choi and Saikkonen (2004) cannot deal with the prolonged inaction of the short-term nominal interest rates above the zero bound.

Another strategy is to include sample periods prior to March 2001, when the BOJ initiated the quantity easing policy. The departure from the policy regime of interest rate targeting prohibits us from assuming that the call rate reflects the BOJ's policy decisions. [11] In addition, for the sample period prior to August 2001, the short-term nominal interest rates stayed at low levels, but barely moved above zero rates over time. For the following, we employ a strategy of using sample periods up to the first quarters of 1999 and 2001; thereafter, we check the robustness of the estimation and test results.

We conducted unit root tests for each of the variables: the log of the Marshallian k, the level of the call rate, and the log of the call rate. We performed augmented Dickey–Fuller (1979) and Phillips–Perron (1988) tests using data from 1980 to 1999 or 2001. These test results confirm that each variable has a unit root.

2.2 Nonlinear Estimation Method and Cointegration Test

In this section, we find the presence of a cointegrating relationship with possible nonlinearity for the semi-log and double-log models using Choi and Saikkonen's (2005) test.

Choi and Saikkonen's (2005) cointegration test is based on the following two-step estimation procedure in Saikkonen and Choi (2004), which gives a consistent and efficient estimator of the five parameters (constant, α, β, γ, i^*). For simplicity, hereafter, we set $\theta =$ (constant, α, β, γ, $i^*)'$ and denote the logistic function $\frac{1}{1+\exp\left(-\gamma(i_t-i^*)\right)}$ as $g(i_t;\tau)$, where $\tau=(\gamma,\ i^*)'$.

The first step of the estimation involves obtaining a conventional nonlinear least squares (NLLS) estimator θ_T with respect to θ. Although the NLLS estimator is consistent, it is not efficient because of the regressor-error dependence. [12] Thereafter, to control the regressor-error dependence and thus obtain an efficient estimator, Saikkonen and Choi (2004) suggest considering the following auxiliary regression model by adding the short-run dynamics of i_t:

$$k_t = \text{constant} + \alpha i_t + \beta i_t\left(1-g(i_t;\tau)\right) + \sum_{s=-K}^{K} \pi_s \Delta i_{t-s} + \mu_t.$$

Plugging in the NLLS estimator θ_T, the second step of the estimation involves obtaining the following efficient estimator for θ and $\pi=(\pi_{-K}\ldots\pi_K)'$:

$$\begin{bmatrix} \theta_T^1 \\ \pi_T^1 \end{bmatrix} = \begin{bmatrix} \theta_T \\ 0 \end{bmatrix} + \left(\sum_{t=K+1}^{T-K} z_t z_t'\right)^{-1}\left(\sum_{t=K+1}^{T-K} z_t'\tilde{\epsilon}_t\right),$$

where $z_t=\left(\beta_T i_t\left(-\frac{\partial g(i_t;\tau_T)}{\partial\tau}\right)',\ 1,\ i_t,\ i_t\left(1-g(i_t;\tau_T)\right),\ \Delta i_{t-K},\ldots,\Delta i_{t+K}\right)'$ and $\tilde{\epsilon}_t$ indicate the fitted residuals in equation (1) using the NLLS estimates θ_T.

Saikkonen and Choi (2004) term the efficient estimator θ_T^1 as the "one-step Gauss–Newton estimator." They also propose plugging in the one-step Gauss–Newton estimator instead of the NLLS estimator. They term the estimator obtained in this manner the "two-step Gauss–Newton estimator."

To test for cointegration in a nonlinear context, Choi and Saikkonen (2005) propose a test for the null hypothesis of cointegration with possible nonlinearity that uses Kwiatkowski, Phillips, Schmidt, and Shin's (1992) (hereafter KPSS) test for the null of stationarity. They develop the cointegration test for the case of the NLLS estimator θ_T and the one-step Gauss–Newton estimator θ_T^1. According to Choi and Saikkonen (2005), the KPSS test using full-sample regression residuals has limiting distributions that depend on unknown nuisance parameters caused by the parameters of the models and regressor-error dependence. Accordingly, they propose the use of subsamples of the regression residuals with block size b and select the one that yields the maximum statistical values obtained by applying the KPSS test to each of the subsamples. The subresidual-based tests are not affected by the unidentified nuisance parameters. The selection of the block size b can be done by using the minimum volatility rule proposed by Romano and Wolf (2001). The rule comprises choosing b from $b = b_{small}$ to $b = b_{big}$ to minimize the standard deviations of $2m + 1$ statistical values that are calculated in the neighborhood of b, where m denotes an integer such that $m \geq 1$. [13] Choi and Saikkonen (2005) demonstrate that the calculated test statistics asymptotically converge to $\int_0^1 w^2(r)dr$, where $w(r)$ denotes a standard Brownian motion. [14]

Table 1 reports the test statistics and p-values of Choi and Saikkonen's test using the minimum volatility rule together with the selected block sizes. C_{θ_T} and $C_{\theta_T^1}$ denote cointegration tests based on the NLLS estimation and the one-step Gauss–Newton estimation with $K = 1, 2, 4$, respectively. To calculate the test statistics, we set $b_{min} = 10$ and $b_{big} = T - 4$, and choose $m = 2$ as in Choi and Saikkonen (2005). For the start points of the sample periods, we employ two cases, that is, sample periods from 1980/I and sample periods from 1985/I to check the robustness of the tests. The null hypothesis is cointegration with possible nonlinearity; we would be unable to reject cointegration for high p-values. The null hypothesis cannot be rejected at 5% or lower in the various periods, and consequently, we have strong evidence in favor of cointegration for both the semi-log and double-log models in equation (1).

Our test results for cointegration with possible nonlinearity are noteworthy because some empirical studies on Japanese M1 demand have pointed out the possibility of no-cointegration with the semi-log model in the linear context of $\beta = 0$ for equation (1). In contrast to previous work, our test results provide strong evidence that not only the double-log model but also the semi-log model has a cointegrating relationship with possible nonlinearity.

The small sample properties of the nonlinear cointegration test are reported in the Appendix. As detailed, our test results are not subject to serious small sample problems.

2.3 Linearity Test

As discussed earlier, previous research on the Japanese money demand function assumed linearity for both the semi-log and double-log money demand models. In this subsection, we evaluate the assumption of linearity.

We are interested in testing the null hypothesis that the money demand function (1) reduces to a linear money demand function. Accordingly, the null hypothesis of interest is $\beta = 0$, while the alternative is $\beta \neq 0$. However, conventional hypothesis testing is difficult because the nuisance parameters γ and i^* are not identified under the null hypothesis. Hence, for the linearity test of $\beta = 0$, we employ the first-order (T_1) and the third-order (T_2) tests suggested by Choi and Saikkonen (2004).

To calculate the T_1 test in the semi-log model of equation (1), the log of the Marshallian k is regressed on the call rate, the lead and lags of the differenced call rate, and the call rate to the second power using OLS techniques. For the T_2 test in the semi-log model, the call rate to the third power is additionally used as a regressor in the specification used for the T_1 test. As demonstrated by Choi and Saikkonen (2004), the linearity tests of $\beta = 0$ in T_1 and T_2 reduce to testing the significance of a parameter estimate for the second power of the call rate and the significance of parameter estimates for the second and third powers of the call rate, respectively. To calculate the two tests in the double-log model of equation (1), the logarithm value of the call rate should be used. The limiting null distributions of the T_1 and the T_2 test statistics are chi-square distributions with one and two degrees of freedom, respectively.

Table 2 illustrates the results of the linearity tests in the semi-log and double-log models. First, for the semi-log model, the null of linearity is rejected at a significance level of 5% in all cases of leads-lags and sample periods. For the double-log model, on the other hand, the null is not rejected in almost all cases. The test results clearly indicate that the use of the semi-log model requires the consideration of nonlinearity to capture Japanese M1 demand over time, whereas the double-log model does not.

Our test results for linearity establish the validity of assuming a linear specification, at least for the double-log model, but not for the semi-log model. [15]

The Appendix reports the small sample properties of the T_1 and T_2 tests for linearity. As demonstrated, the two tests are not subject to serious small sample problems.

2.4 Performance Comparison

In this subsection, we conduct a performance comparison of the four models in equation (1)—the linear semi-log model, the linear double-log model, the nonlinear semi-log model, and the nonlinear double-log model—in terms of goodness-of-fit.

Table 3 reports the sum of squared error (SSE) for the four models. The linear semi-log and double-log models are estimated with the fully modified OLS in Phillips and Hansen (1990). The SSE for the nonlinear semi-log and double-log models are based on estimates obtained using the two-step Gauss–Newton estimator with $K = 1$. The following results do not depend on the structure of the leads and lags. The SSE is calculated using the in-sample and out-of-sample prediction errors.

The linear semi-log and nonlinear double-log models are clearly inferior to the other models in terms of both in-sample and out-of-sample prediction performance. In particular, the result for the linear semi-log model is compatible with that of Bae, Kakkar, and Ogaki (2006) who have shown that the linear semi-log model is inferior to the linear double-log model in terms of out-of-sample prediction performance. On the other hand, the overall performance of the non-linear semi-log model appears to exceed that of the linear double-log model. For the in-sample prediction, the two models perform similarly. However, for the out-of-sample prediction, the nonlinear semi-log model partly exceeds the linear double-log model. We employed other sample periods for the performance comparisons, but the results did not qualitatively change.

Next, we conduct a comparative simulation study of the linear double-log and nonlinear semi-log models based on the test results for goodness-of-fit. In the simulation study, we examine how accurately the test results for goodness-of-fit are replicated when one is adopted as the true model to simulate draws and the other is used for the calculation of SSEs. If the empirical SSE presented in Table 3 is consistent with the SSEs obtained by simulating draws for the true model, we have a case for arguing that the true model is correct.

Table 4 reports the simulated mean square error (MSE) and bias (BIAS) of the SSEs of the in-sample and out-of-sample predictions. The MSE and the BIAS are defined as $(e - e^*)^2$ and $e - e^*$, where e and e^* indicate an empirical SSE as presented in Table 3 and the mean of the SSEs obtained using Monte Carlo simulation. When we adopt one as the true model, we simulate draws from the true model and calculate a simulated SSE of the other for each draw.

We conduct 1,000 Monte Carlo replications to obtain the mean of the simulated SSEs: e^*. The Monte Carlo simulation procedure is described in the Appendix.

Overall, the MSE and the BIAS of the nonlinear semi-log model appear to be smaller than those of the linear double-log model. The nonlinear semi-log model captures samples used for their estimation more accurately than the linear double-log model. For the in-sample prediction, the double-log model partly exceeds the nonlinear semi-log model, however, for the out-of-sample prediction the nonlinear semi-log model largely exceeds the double-log model for all of the prediction periods.

In sum, and in contrast to those of Bae et al. (2006), our results indicate that the linear double-log model does not always have the advantage over other functional forms in terms of goodness-of-fit. [16]

2.5 Estimation Results

Our results for goodness-of-fit indicate that the nonlinear semi-log model is as effective as the linear double-log model. In addition, and unlike the linear double-log model, the nonlinear semi-log model presents us with the opportunity to estimate the time profile of interest semi-elasticity in relation to the call rate as the BOJ's policy rate, thereby allowing us to investigate the shape of the Japanese M1 demand function in detail. In this subsection, we discuss the estimation results of the nonlinear semi-log model obtained using the two Gauss–Newton estimation methods. In the next section, we investigate the shape of the Japanese M1 demand function based on the estimation results.

Table 5 presents the estimation results for the two sample periods between 1985/I and 1999/I (hereafter period I) and between 1985/I to 2001/I (hereafter period II). [17] The absolute values of the estimated interest semi-elasticities for period I are smaller than those for period II. Indeed, for period I the estimates of interest semi-elasticity for the linear part (α) are in the range of -0.04 to -0.07, and for the nonlinear part (β) are approximately -0.2. For period II, the estimates of interest semi-elasticity for the linear part are in the range of -0.08 to -0.09, and for the nonlinear part, -0.2 to -0.35. The estimation results for the linear and nonlinear parts imply that the range of interest semi-elasticity ($\alpha + \beta$) is between -0.05 and -0.45. Estimated transition values (i^*) for the two periods range from 2% to 3%. The above estimation results do not depend on the structure of leads and lags.

Additional points of concern are as follows:

1. We estimate a nonlinear semi-log function without imposing unitary income elasticity by regressing the real money balance of M1 on real GDP and the call rate using Gauss–Newton methods. The results, being accompanied by estimated income elasticities close to one, are similar to those illustrated in Table 5.

2. We also use the monthly data set: the industrial index of production as a scale variable and the consumer price index as a price index. The results obtained by imposing unitary income elasticity are similar to those

illustrated in Table 5. Furthermore, the estimation without imposing the unitary income elasticity provides results like those illustrated in Table 5, bringing the estimated income elasticities close to one.

We calculate a bootstrap confidence interval at 95%. As illustrated in Table 5, the estimation results do not change substantially, even though we explicitly deal with small sample problems. The bootstrap procedure is described in the Appendix.

3 The Shape of the Japanese Money Demand Function

In this section, we investigate the shape of the Japanese money demand function using the nonlinear semi-log model.

Figure 3 illustrates the manner in which the interest semi-elasticity varies depending on the level of nominal interest rate between 0% and 14% by using semi-elasticity $= \alpha i_t + \beta i_t \left(1 - g(i_t;\tau)\right)$. The calculation of the values of interest semi-elasticity is based on the estimation results obtained by using the one-step and two-step Gauss–Newton estimators with $K = 1$ for period II. The figure shows that the interest semi-elasticity, taking a value of about -0.07, is quite stable above a level of the estimated transition value (i^*), which is about 3%. On the other hand, it is sharply increasing below the transition value and ends up at a value of about -0.45.

Next, we investigate the time profile of interest semi-elasticity using the actual path of the call rate. Figure 4 illustrates the actual path of the call rate and the path of the interest semi-elasticity. First, the interest semi-elasticity, taking a value of about -0.07, had been quite stable until the early 1990s, excluding the late 1980s, during which a low interest rate policy was temporarily conducted. Second, as the call rate gradually lowered from a level of about 3%, interest semi-elasticity gradually increased from the early 1990s to the mid-1990s. During the transition period, the interest semi-elasticity varies from -0.1 to -0.4. Third, in the period post-1995 when the BOJ guided the call rate below 0.5%, interest semi-elasticity took a larger value of about -0.45 and has been quite stable.

The novelty of our finding is that we identify a transition period of interest semi-elasticity from the early 1990s through to the mid-1990s. On the other hand, for the period up until the early 1990s and for the period post-1995, our estimates of interest semi-elasticity are compatible with existing empirical results on the Japanese M1 demand function (Miyao, 2004; Fujiki and Watanabe, 2004; Nakashima and Saito, 2006). Furthermore, compared with the existing empirical results on US M1 demand, Japanese money demand is remarkably interest elastic in the post-1995 period. [18]

The above-mentioned results indicate that Japanese M1 demand is composed of three regimes: Regimes I, II and III, as shown in Figure 5. In Regime I through the early 1990s, the policy rate was set above 3% and the Japanese money demand function had lower interest semi-elasticity. Regime II from the early 1990s to the mid-1990s is the transition period during which the interest semi-elasticity gradually changed. In Regime III from the mid-1990s, the policy rate was set below 0.5%, and the Japanese money demand function had higher interest semi-elasticity.

4 Conclusion

We have three substantive conclusions pertaining to the shape of the Japanese money demand function. First, the linearity test showed that there is statistically significant evidence of nonlinearity in the semi-log money demand model, whereas there is no comparable evidence pertaining to the double-log money demand model. This result indicates that if we examine a semi-log model for an extremely low interest rate economy similar to that in post-1995 Japan, we must specifically consider the nonlinearity of interest semi-elasticity. On the other hand, if we examine a double-log model, we can assume a conventional linear specification.

Second, we confirmed that a semi-log money demand model has a long-run equilibrium relationship with a nonlinear form. In addition, we provided evidence that a nonlinear semi-log model is as effective as a linear double-log model in terms of goodness-of-fit. Therefore, as long as we specifically introduce nonlinearity, we can capture stable Japanese money demand, even with a semi-log model. This suggestion is especially noteworthy because previous studies such as Fujiki and Watanabe (2004) and Bae, Kakkar, and Ogaki (2006) showed the possibility of no-cointegration with a semi-log model and the inferiority of a semi-log model to a double-log model without considering nonlinearity.

Third, we found that Japanese money demand comprises three regimes: the first, through to the early 1990s, was when the Japanese money demand function had lower interest semi-elasticity; the second, from the early 1990s to the mid-1990s, was when interest semi-elasticity gradually increased; and the third, after the mid-1990s, was when the Japanese money demand function had higher interest semi-elasticity.

In particular, with regard to the relationship between interest elasticity and the BOJ's policy rate, our estimation results indicate that it was when the BOJ set the policy rate below 3% in the early 1990s that interest semi-elasticity began to increase. As discussed by Lucas (2000), an interest rate of approximately 3% would arise in the US economy under a policy of zero inflation. The finding in the Japanese economy that even prior to 1999, when the BOJ started the zero interest rate policy and the liquidity trap phenomenon became self-evident, interest elasticity had already begun to increase would be beneficial in the actual implementation of monetary policy.

Two aspects remain yet unaddressed in our empirical investigation. First, we do not consider the relationship between the two competing models in the cointegration analysis, that is, between a nonlinear semi-log model and a linear double-log model and between a linear structural change model and a nonlinear smooth transition model. In particular, it is not yet understood whether each of the competing models is nested or reduces to a unified model, especially if the path of the short-term interest rate is monotonically falling as in Japan during the 1990s. [19] Second, we exclude the period after 1999 or 2001 from our sample because the inaction of nominal interest rates at their lower bound (0%) for long-term periods (from 1999 to at least 2006) would cause serious econometric problems. As pointed out by Elliott (1998), small amounts of mean reversion that cannot be consistently determined by unit root tests render standard cointegration inference highly misleading. If we include the sample period after 1999 to explicitly consider the zero bound, we must deal with this statistical problem using a more rigorous time series method. We would like to extend our research along these lines in the future.

Appendix: Simulation Methods and Results

A 1. Size and Power of the Linearity Test

To calculate the empirical power of the first-order (T_1) and the third-order (T_2) tests, we generate data using the following system:

$$k_t = \text{constant}_T^1 + \alpha_T^1 i_t + \beta_T^1 i_t \left(1 - g(i_t; \tau_T^1)\right) + \epsilon_t \tag{2}$$

$$i_t = i_{t-1} + v_t \tag{3}$$

$$\omega_t = \sum_{s=1}^{s=p} \Phi_s \omega_{t-s} + \xi_t \tag{4}$$

$$\xi_t \sim i.i.d.N\left(\bar{\xi},\ \sigma^2\right), \tag{5}$$

where (constant^1_T, α^1_T, β^1_T) and $\tau^1_T = (\gamma^1_T,\ i^{*1}_T)'$ indicate the parameter estimates obtained by the one-step Gauss–Newton estimator with a full sample of size T. ω_t is defined by $\omega_t = (\epsilon_t,\ v_t)'$. We calculate the empirical power through the following steps:

1. Obtain the fitted residuals $\{\epsilon_t : t = 1, \ldots, T\}$ in equation (2) using the estimates θ^1_T, and define $\{v_t : t = 1, \ldots, T\}$ in equation (3) by first differencing the observed value of the call rate, that is, $v_t = \Delta i_t$.

2. Suppose that the data generating process of $\left\{\omega_t = (\epsilon_t,\ v_t)' : t = 1, \ldots, T\right\}$ is given by the p-th order vector autoregression (VAR), as shown by equation (4), to capture the regressor-error dependence. In addition, estimate the VAR by OLS to obtain the estimates $\{\Phi_1, \Phi_2, \ldots, \Phi_p\}$ and the fitted VAR residuals $\{\xi_t : t = p+1, \ldots, T\}$. The order is chosen based on the Schwarz information criterion.

3. Draw a sample $\{\xi^*_t\}$ of size $T + p$ from the normal distribution (5) with a mean $\bar{\xi} = \frac{1}{T}\sum_{t=1}^{T} \xi_t$ and a variance $\sigma^2 = \frac{1}{T}\sum_{t=1}^{T} \left(\xi_t - \bar{\xi}\right)^2$.

4. Generate a sample $\left\{\omega^*_t = (\epsilon^*_t,\ v^*_t)' : t = 1, \ldots, T\right\}$ recursively, using the estimated VAR, and obtain a sample $\{i^*_t; t = 1, \ldots, T\}$ of the call rate by integrating v^*_t, that is, $i^*_t = i_0 + \sum_{j=1}^{t} v^*_j$, where i_0 indicates the initial value of i_t. In addition, generate a sample $\{k^*_t : t = 1, \ldots, T\}$ of the Marshallian k by substituting the residuals ϵ^*_t as well as the explanatory variable i^*_t into the money demand function (2) estimated by the one-step Gauss–Newton

estimator.

5. Apply the first-order and third-order tests for linearity based on the 5% asymptotic critical values to each set of the sample (k_t^*, i_t^*), and repeat this procedure 5,000 times.

Table A-1 reports rejection frequencies from Monte Carlo studies of the power of the linearity tests. As shown in each panel of Table A-1, the calculated power does not seriously deteriorate.

Next, we calculate the size of the linearity tests as follows. Assuming that $\beta_T^1 = 0$ under the null of linearity, we obtain the fitted residuals by using the estimates $(\text{constant}_T^1,\ \alpha_T^1)'$. Then, we follow the same procedure as in the calculation of power. Table A-1 reports the rejection frequencies under the null hypothesis as the size of the linearity tests. In the two sample periods, neither shows substantial over rejection.

The Monte Carlo studies demonstrate that the test results based on our sample do not suffer from serious small sample biases.

A 2. Size and Power of the Nonlinear Cointegration Test

To examine the small sample properties of Choi and Saikkonen's (2005) $C_{\theta_T^1}$ test for cointegration, we alter the Monte Carlo procedure for the linearity test as follows. First, we assume that a cointegrating error ϵ_t can be described as follows:

$$\epsilon_t \;=\; \rho\epsilon_{t-1} + w_t \tag{6}$$

$$w_t \sim i.i.d.N\left(\bar{w},\ \sigma_w^2\right). \tag{7}$$

Furthermore, we assume $\rho \neq 0$ under the null hypothesis and $\rho = 0$ under the alternative hypothesis. On calculation of size, we estimate AR(1) process (6) using the fitted residuals $\{\epsilon_t\}$, obtained in step 1 of the Monte Carlo procedure for the linearity test, whereas in the calculation of power, we first consider the difference of the fitted residuals. Then, we obtain a sample of $\{w_t : t = 1, \ldots, T\}$. Second, we assume that the sample $\{w_t\}$ follows an i.i.d. normal distribution (7) with a mean $\bar{w} = \frac{1}{T}\sum_{t=1}^{T} w_t$ and a variance $\sigma_w^2 = \frac{1}{T}\sum_{t=1}^{T}(w_t - \bar{w})^2$, and generate a sample $\{w_t^*\}$ of size $T+1$ from the normal distribution. Third, we obtain a sample $\{\epsilon_t^*; t = 1, \ldots, T\}$ by integrating w_t^*. Except for using the generated cointegrating errors $\{\epsilon_t^*\}$, we follow the same procedure as for the linearity test in calculating both size and power.

Table A-2 reports the calculated size and power of the $C_{\theta_T^1}$ test. Overall, the $C_{\theta_T^1}$ test shows good small sample performance. For the size, the test reveals no serious size distortion. The empirical power is also reasonably high.

A 3. Comparative Simulation Study of the Two Models

To conduct a comparative simulation study of the linear double-log and nonlinear semi-log models, we alter the Monte Carlo procedure for the linearity test as follows. First, when we adopt the linear double-log model as the true model, i_t indicates a logarithmic value of the overnight call rate in the system given by (2)–(5). We impose $\beta_T^1 = 0$ in equation (2) and estimate (constant$_T^1$, α_T^1) using the fully modified OLS in Phillips and Hansen (1990). Second, when we adopt the nonlinear semi-log model as the true model, we estimate (constant_T^1, α_T^1, β_T^1) and $\tau_T^1 = (\gamma_T^1,\ i_T^{*1})'$ using the two-step Gauss–Newton estimator with $K = 1$ and not using the one-step Gauss–Newton estimator. Third, to prepare for the out-of-sample prediction experiments for the periods up to 2001/I, and up to 2002/IV, we estimate the parameters of the true model using the sample from 1985/I to 2001/I, and from 1985/I to 2002/IV, respectively. Fourth, in step 5, we use the sample (k_t^*, i_t^*) obtained by simulating a draw for the true model, thereby calculating a simulated sum of squared error for the other. We repeat this procedure 1,000 times to obtain the mean of simulated sums of squared error: e^*. We calculate the mean square error (MSE) and bias (BIAS) defined as $(e - e^*)^2$ and $e - e^*$, where e indicates an empirical sum of squared error presented by Table 3.

According to Table 4, the MSE and the BIAS of the nonlinear semi-log model are smaller than those of the linear double-log model. For the in-sample prediction, the double-log model partly exceeds the nonlinear semi-log model, while for the out-of-sample prediction, the nonlinear semi-log model largely exceeds the double-log model in all of the prediction periods.

A 4. Bootstrap Confidence Interval

To obtain bootstrap confidence intervals, we alter the Monte Carlo procedure for the linearity test as follows. First, in step 1, we use the parameter estimates obtained by the one-step and two-step Gauss–Newton estimators with $K = 1, 2, 4$. Second, in step 3, we sample ξ_t^* of size T randomly with replacement from the centered VAR residuals $\{\xi_t - \bar{\xi} : t = 1, \ldots, T\}$, where $\bar{\xi} = \frac{1}{T}\sum_{t=1}^{T} \xi_t$. Third, in step 5, we calculate the confidence interval based on the one-step and the two-step Gauss–Newton estimators by using the bootstrap sample obtained in step 4. Except for these steps, we follow the same procedure as the linearity tests in calculating power.

According to Table 5, while the estimated confidence intervals are somewhat larger than those based on asymptotic distribution, the sign and significance of the estimated parameters do not change substantially.

Footnotes

1 As an exception, Hondroyiannis, Swamy, and Tavlas (2000) employed a random coefficient model and found that the absolute value of the interest elasticity estimate continuously decreased, even in the lower interest rate period. Their finding should, however, be reserved because it is not clear that the random coefficient model can be applied when dealing with the coefficients of integrated variables.

2 Miyao (2004) and Fujiki and Watanabe (2004) have shown that Gregory and Hansen's (1996) cointegration test, which allows for a possible structural break in a linear context, detected cointegration with a double-log model but not with a semi-log model. In contrast, Miyao (2004) showed that the Johansen (1988) and Johansen and Juselius (1990) cointegration tests detected cointegration with both models.

3 In addition to conventional linear cointegration techniques, such as fully modified

ordinary least squares (OLS) or dynamic OLS, Bae et al. (2006) used the nonlinear cointegration technique in Bae and de Jong (2007) to estimate a double-log money demand function, thereby dealing with the statistical issue of the nonlinear transformation of interest rates as the I(1) variable. Their finding for out-of-sample prediction performance did not depend on the techniques used for their estimation.

4 Cagan (1956) devised a semi-log money demand model to analyze hyperinflation. Nakashima and Saito (2006) developed a semi-log money demand model based on the classical Cagan model to examine deflation in Japan in the late 1990s. Lagos and Wright (2005) derived a semi-log model employing a search-theoretical approach. Conversely, the double-log money demand model mainly derives its theoretical background from new classical representative agent models, including the shopping-time model (see McCallum and Goodfriend (1989), and Lucas (1988, 2000)).

5 Lucas (2000) demonstrated that the estimated welfare cost of inflation varies depending on which of the two models is used. Miyao (2004) reviews recent developments in liquidity trap theories.

6 Applications of the smooth transition model for modeling the money demand equation include Wolters, Teräsvirta, and Lütkepohl (1998), Lütkepohl, Teräsvirta, and Wolters (1999), Sarno (1999), Huang, Lin, and Cheng (2001), and Teräsvirta and Eliasson (2001). Unlike the present paper, these studies all assume a linear long-run relation between money and other variables that is based on economic theories for money demand, thus using a smooth transition error correction model to capture nonlinear dynamics of a short-run correction process to a long-run equilibrium.

7 Christopoulos and León-Ledesma (2007) have applied Saikkonen and Choi's (2004) cointegrating smooth transition model to modeling the long-run Fisher equation.

They do not model the short-run error correction dynamics to the long-run equilibrium.

8 In contrast, a structural break model is a simple time-dependent parameter model. Hence, the structural break model is not appropriate for estimating the time profile of a parameter coefficient in relation to an explanatory variable of concern. Furthermore, the structural break model cannot identify the transition value, even though the model can identify the change point at which the set of coefficient parameters change.

9 Fujiki and Watanabe (2004) provide evidence that the estimates of income elasticity lie close to unity. Nakashima and Saito (2006) found that the income elasticity for Japanese money demand was stable over time with the partial structural change test of Kuo (1998). Miyao (2004) and Bae, Kakkar, and Ogaki (2006) adopted the velocity-based specification for modeling the Japanese money demand function. We follow these studies to form a base specification for the Japanese money demand function.

10 There is a long controversy over which short- or long-term rate should be included in the empirical specification of the money demand equations as the opportunity cost of holding money (e.g., Poole, 1988; Hoffman and Rasche, 1991). Hoffman, Rasche, and Tieslau (1995), however, argued that if two interest rates are cointegrated from the term structure, the interest rate used is irrelevant to the estimates of interest elasticity. Indeed, they employed the overnight call rate as the most representative short-term rate in Japan. Accordingly, we employ the call rate not only as the BOJ's policy indicator, but also as the opportunity cost of holding money.

11 Nakashima (2006) showed that the call rate represented the BOJ's policy decisions

up to February 2001 and that an equally weighted average of the call rate and reserves represented the BOJ's policy decisions from March 2001.

12 The NLLS estimator θ_T is consistently of the order $O_p(T^{-\frac{1}{2}})$, which differs from $O_p(T^{-1})$ obtained in the linear cointegrating cases.

13 More specifically, the algorithm of the minimum volatility rule is as follows. First, for each $b = b_{small}$ to $b = b_{big}$, we compute a statistical value $C(b)$. Next, for each b, we calculate the standard deviation of $C(b-m), \ldots, C(b+m)$, where m denotes an integer of $m \geq 1$. Lastly, we pick the value b^* with the smallest standard deviation and report $C(b^*)$ as the final statistical value.

14 Choi and Saikkonen (2005) give the cumulative distribution function of $\int_0^1 w^2(r)dr$ as

$$F(z) = \sqrt{2}\sum_{n=0}^{\infty} \frac{\Gamma(n+1/2)}{n!\Gamma(1/2)}(-1)^n \left(1 - f\left(\frac{u}{2\sqrt{z}}\right)\right), \; z \geq 0,$$

where $f(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-\beta)d\beta$ and $u = \frac{\sqrt{2}}{2} + 2n\sqrt{2}$. z indicates test statistics for the cointegration test. Choi and Saikkonen (2005) suggest truncating the series at $n = 10$.

15 We also conducted the linearity test for income elasticity without imposing unitary income elasticity. We have strong evidence pertaining to the linearity of income elasticity.

16 For the performance comparison of out-of-sample prediction, Bae et al. (2006) also used a nonlinear model implied by the money in the utility function with the constant elasticity of substitution. They showed that the nonlinear model displayed similar performance to the double-log model.

17 The estimation results for the sample periods from 1980/I are similar to those for sample period I.

18 Stock and Watson (1993) applied several estimation methods to the sample period 1946–1987 and found that the estimated interest semi-elasticity ranged from -0.02 to -0.09. By extending the postwar US data through to 1996, Ball (2001) demonstrated that the interest semi-elasticity was approximately -0.05. From existing empirical work on US M1 demand, we cannot find any point estimates of interest semi-elasticity around -0.4 or -0.5.

19 Bae and de Jong (2007) developed a nonlinear cointegration technique to estimate a double-log model, thereby rigorously modeling the nonlinearity of interest semi-elasticity using the double-log model. Carrasco (2002) studied the relationships between a structural change model, a threshold model, and a Markov-switching model. Her study, however, only concerns the modeling of stationary variables.

References

[1] Andrews, D., 1991, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix Estimator, Econometrica, 59, 817–858.

[2] Bae, Y., and De Jong, R., 2007, Money Demand Function Estimation by Nonlinear Cointegration, Journal of Applied Econometrics, 22, 767–793.

[3] Bae, Y., Kakkar, V., and Ogaki, M., 2006, Money Demand in Japan and Nonlinear Cointegration, Journal of Money, Credit and Banking, 38, 1659–1668.

[4] Ball, L., 2001, Another Look at Long-run Money Demand, Journal of Monetary Economics, 47, 31–44.

[5] Cagan, P., 1956, The Monetary Dynamics of Hyperinflation, in *Studies in the Quantity Theory of Money*, ed. Friedman, M., Chicago: University of Chicago Press.

[6] Carrasco, M., 2002, Misspecified Structural Change, Threshold, and Markov-switching Models, Journal of Econometrics, 109, 239–273.

[7] Dickey, D., and Fuller, W., 1979, Distribution of the Estimators for Autoregressive Time Series with a Unit Root, Journal of the American Statistical Association, 74, 427–431.

[8] Chang, Y., Park, J., and Phillips, P., 2001, Nonlinear Econometric Models with Cointegrated and Deterministically Trending Regressors, Econometrics Journal, 4, 1–36.

[9] Choi, I., and Saikkonen, P., 2004, Testing Linearity in Cointegrating Smooth Transition Regressions, Econometrics Journal, 7, 341–365.

[10] Choi, I., and Saikkonen, P., 2005, Test for Nonlinear Cointegration, mimeo.

[11] Christopoulos, D., and León-Ledesma, M., 2007, A Long-run Nonlinear Approach to the Fisher Effect, forthcoming in Journal of Money, Credit and Banking.

[12] Elliott, G., 1998, On the Robustness of Cointegration Methods when Regressors Almost Have Unit Roots, Econometrica, 66, 149–158.

[13] Fujiki, H., and Watanabe, K., 2004, Japanese Demand for Narrow Monetary Aggregate in the 90s: Time Series versus Cross-sectional Evidence from Japan, Monetary and Economic Studies, 22, 47–77.

[14] Gregory, A., and Hansen, B., 1996, Residual-based Tests for Cointegration in Models with Regime Shifts, Journal of Econometrics, 70, 99–126.

[15] Hansen, B., 1992, Tests for Parameter Instability in Regressions Analysis with I(1) Processes, Journal of Business and Economic Statistics, 10, 321–335.

[16] Hoffman, D., and Rasche, R., 1991, Long-run Income and Interest Elasticities of Money Demand in the United States, Review of Economics and Statistics, 78, 665–674.

[17] Hoffman, D., Rasche, R., and Tieslau, M., 1995, The Stability of Long-run Money Demand in Five Industrial Countries, Journal of Monetary Economics, 35, 317–339.

[18] Hondroyiannis, G., Swamy, P.A.V.B., and Tavlas, G., 2000, Is the Japanese Economy in a Liquidity Trap?, Economic Letters, 66, 17–23.

[19] Huang, H., Lin, J., and Cheng, J., 2001, Evidence on Nonlinear Error Correction in Money Demand: The Case of Taiwan, Applied Economics, 33, 1727–1736.

[20] Johansen, S., 1988, Statistical Analysis of Cointegrating Vectors, Journal of Economic Dynamic and Control, 12, 213–254.

[21] Johansen, S., and Juselius, K., 1990, Maximum Likelihood Estimation and Inference on Cointegration—With Application to the Demand for Money, Oxford Bulletin of Economics and Statistics, 52, 169–210.

[22] Kuo, B., 1998, Test for Partial Parameter Instability in Regressions with I(1) Processes, Journal of Econometrics, 86, 337–368.

[23] Kwiatkowski, D., Phillips, P., Schmidt, P., and Shin, Y., 1992, Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root: How Sure Are We That Economic Time Series Have a Unit Root?, Journal of Econometrics, 54, 159–178.

[24] Lagos, R., and Wright, R., 2005, A Unified Framework for Monetary Theory and Policy Analysis, Journal of Political Economy, 113, 463–484.

[25] Lucas, R., 1988, Money Demand in the United States: A Quantitative Review, Carnegie–Rochester Conference Series on Public Policy, 29, 137–168.

[26] Lucas, R., 2000, Inflation and Welfare, Econometrica, 68, 137–168.

[27] Lütkepohl, H., Teräsvirta, T., and Wolters, J., 1999, Investigating Stability of Linearity of a German M1 Demand function, Journal of Applied Econometrics, 14, 511–525.

[28] McCallum, B., and Goodfriend, M., 1989, Demand for Money: Theoretical Studies, in *The New Palgrave Dictionary of Economics: Money*, ed. Eatwell, J., Milgate, M., and Newman, P., New York: W.W. Norton.

[29] Miyao, R., 2004, Liquidity Traps and the Stability of Money Demand: Is Japan Really Trapped at the Zero Bound?, Mimeo, Kobe University.

[30] Nakashima, K., 2006, Ideal and Real Japanese Monetary Policy: A Comparative Analysis of Actual and Optimal Policy Measures, forthcoming in Japanese Economic Review.

[31] Nakashima, K., and Saito, M., 2006, Uncovering Interest-elastic Money Demand: Evidence from the Japanese Money Market with a Low Interest Rate Policy, Mimeo, Hitotsubashi University.

[32] Park, J., and Phillips, P., 1999, Asymptotics for Nonlinear Transformations of Time Series, Econometric Theory, 15, 269–298.

[33] Park, J., and Phillips, P., 2001, Nonlinear Regressions with Integrated Time Series, Econometrica, 69, 117–161.

[34] Phillips, P., and Hansen, B., 1990, Statistical Inference in Instrumental Variables Regression with I(1) Processes, Review of Economic Studies, 57, 99–125.

[35] Phillips, P., and Perron, P., 1988, Testing for a Unit Root in Time Series Regression, Biometrika, 75, 335–346.

[36] Poole, W., 1988, Monetary Policy Lessons of Recent Inflation and Deflation, Journal of Economic Perspectives, 2, 51–72.

[37] Romano, J., and Wolf, M., 2001, Subsampling Intervals in Autoregressive Models with Linear Time Trend, Econometrica, 69, 1283–1314.

[38] Saikkonen, P., 2005, Stability Results for Nonlinear Error Correction Models, Journal of Econometrics, 127, 69–81.

[39] Saikkonen, P., 2008, Stability of Regime Switching Error Correction Models under Linear Cointegration, Econometric Theory, 24, 294–318.

[40] Saikkonen, P., and Choi, I., 2004, Cointegrating Smooth Transition Regressions, Econometric Theory, 20, 301–340.

[41] Sarno, L., 1999, Adjustment Costs and Nonlinear Dynamics in the Demand for Money: Italy, 1861–1991, International Journal of Finance and Economics, 4, 155–177.

[42] Stock, J., and Watson, M., 1993, A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems, Econometrica, 61, 783-820.

[43] Teräsvirta, T., and Eliasson, A., 2001, Nonlinear Error Correction and the UK Demand for Broad Money, 1878–1993, Journal of Applied Econometrics, 16, 277–288.

[44] Wolters, J., Teräsvirta, T., and Lütkepohl, H., 1998, Modelling the Demand for M3 in the United Germany, Review of Economics and Statistics, 399–409.

Table 1: Test Results for Cointegration with Possible Nonlinearity

Double-log Model

Period		$C_{\theta_T^1}$			C_{θ_T}
		$K=1$	$K=2$	$K=4$	
	P-value	0.638	0.654	0.706	0.654
1980/I–2001/I		(0.440)	(0.464)	(0.553)	(0.465)
	Block Size	24	23	24	46
	P-value	0.598	0.772	0.801	0.764
1985/I–2001/I		(0.387)	(0.705)	(0.794)	(0.685)
	Block Size	35	48	21	16
	P-value	0.874	0.688	0.765	0.601
1980/I–1999/I		(1.586)	(0.520)	(0.687)	(0.390)
	Block Size	59	36	59	38
	P-value	0.628	0.822	0.801	0.103
1985/I–1999/I		(0.426)	(0.879)	(0.797)	(0.092)
	Block Size	16	37	35	39

Semi-log Model

Period		$C_{\theta_T^1}$			C_{θ_T}
		$K=1$	$K=2$	$K=4$	
	P-value	0.741	0.765	0.701	0.410
1980/I–2001/I		(0.628)	(0.686)	(0.544)	(0.214)
	Block Size	51	51	49	77
	P-value	0.727	0.758	0.667	0.224
1985/I–2001/I		(0.596)	(0.667)	(0.484)	(0.128)
	Block Size	29	52	27	52
	P-value	0.603	0.588	0.665	0.643
1980/I–1999/I		(0.392)	(0.374)	(0.482)	(0.447)
	Block Size	65	30	45	49
	P-value	0.541	0.641	0.725	0.185
1985/I–1999/I		(0.321)	(0.445)	(0.593)	(0.128)
	Block Size	46	41	11	47

1. C_{θ_T} and $C_{\theta_T^1}$ denote the cointegration tests based on the NLLS estimation and the one-step Gauss–Newton estimation, respectively.
2. K denotes the number of leads and lags in the regression model.
3. P-values are calculated using the cumulative distribution function in Choi and Saikkonen (2005).
4. Test statistics are reported in parentheses.
5. Block sizes are calculated using the minimum variance rule with $m = 2$.
6. The long-run variance is estimated using Andrews' (1991) method with an AR(4) approximation for the prefilter.

Table 2: Linearity Tests for the Japanese Money Demand Function

Specification	Period	T_1			T_2		
		$K = 1$	$K = 2$	$K = 4$	$K = 1$	$K = 2$	$K = 4$
Double-log	1980/I–2001/I	0.398	0.195	0.613	6.424**	2.850	2.282
	1985/I–2001/I	4.377**	0.335	1.110	8.255**	1.896	1.683
	1980/I–1999/I	3.646	3.065	1.668	3.646	3.103	1.746
	1985/I–1999/I	2.149	1.908	1.255	2.539	3.373	5.953
Semi-log	1980/I–2001/I	16.95**	12.21**	51.63**	22.68**	14.73**	61.62**
	1985/I–2001/I	5.024**	6.568**	4.43**	6.712**	9.637**	7.412**
	1980/I–1999/I	57.75**	9.235**	23.64**	74.86**	10.49**	25.81**
	1985/I–1999/I	14.56**	4.727**	4.999**	17.42**	6.126**	7.672**

1. K denotes the number of leads and lags in the regression model.
2. * and ** indicate the significance levels of 5% and 10%, respectively.
3. T_1 and T_2 have the asymptotic distributions with degrees of freedom one and two, respectively.
4. The long-run variance is estimated using Andrews' (1991) method with an AR(4) approximation for the prefilter.

Table 3: Test Results for Goodness-of-fit

Model	In-sample Prediction			Out-of-sample Prediction		
	1985/I-1999/I	1985/I-2001/I	1980/I-2001/I	1998/II-2001/I	1999/II-2001/I	1999/II-2002/IV
Linear Semi-log	0.381	1.029	2.987	0.867	0.475	1.225
Linear Double-log	0.081	0.367	0.431	0.345	0.403	0.611
Nonlinear Semi-log	0.094	0.230	0.356	0.449	0.025	0.295
Nonlinear Double-log	0.373	7.620	7.854	16.85	1.827	3.886

1. The table reports the sum of squares error (SSE) of the in-sample and out-of-sample prediction.
2. The SSEs of the out-of-sample prediction for the periods from 1998/II, and from 1999/II are based on estimates using the sample from 1985/I to 1998/I, and from 1985/I to 1999/I, respectively.
3. We calculate the SSEs for the linear semi-log and double-log models by performing the fully modified OLS estimation in Phillips and Hansen (1990).
4. We calculate the SSE for the nonlinear semi-log and double-log models through performing the two-step Gauss–Newton estimation with $K = 1$.

Table 4: Comparative Simulation Study of the Linear Double-log and Nonlinear Semi-log Models

In-sample Prediction

True Model		Prediction Period		
		1985/I-1999/I	1985/I-2001/I	1980/I-2001/I
Linear Double-log	MSE	1.138	0.317	0.913
	BIAS	-1.177	-0.563	-0.956
Nonlinear Semi-log	MSE	0.301	0.414	1.928
	BIAS	-0.548	-0.643	-1.388

Out-of-sample Prediction

True Model		Prediction Period		
		1998/II-2001/I	1999/II-2001/I	1999/II-2002/IV
Linear Double-log	MSE	0.881	0.783	10.86
	BIAS	-0.939	-0.884	-3.295
Nonlinear Semi-log	MSE	0.006	0.012	0.005
	BIAS	-0.079	0.110	0.068

1. The table reports the simulated mean square error (MSE) and bias (BIAS) of the SSEs presented in Table 3. The MSE and the BIAS are defined as $(e - e^*)^2$ and $e - e^*$, where e and e^* indicate an empirical SSE presented in Table 3 and the mean of the SSEs obtained using Monte Carlo simulation.
2. When the linear double-log model (the nonlinear semi-log model) is adopted as the true model, the MSE and the BIAS are obtained by simulating draws from the true model and estimating a SSE of the nonlinear semi-log model (the linear double-log model) for each draw.
3. The MSE and the BIAS are obtained with 1,000 Monte Carlo replications.
4. The procedure for the Monte Carlo simulation is described in the Appendix.

Table 5: Parameter Estimates of Nonlinear Money Demand Model

Period I (1985/I–1999/I)

Estimation	K	C.I.	α	β	γ	i^*
			-0.049	-0.219	0.756	3.246
	1	Asy	(-0.070, -0.027)	(-0.362, -0.076)	(0.369, 1.144)	(1.050, 5.442)
		Boot	(-0.055, -0.040)	(-0.319, -0.185)	(0.667, 0.822)	(2.778, 3.628)
			-0.048	-0.213	0.752	3.253
One-step	2	Asy	(-0.075, -0.021)	(-0.381, -0.044)	(0.329, 1.176)	(0.750, 5.757)
		Boot	(-0.055, -0.037)	(-0.329, -0.175)	(0.659, 0.823)	(2.732, 3.673)
			-0.043	-0.217	0.743	3.136
	4	Asy	(-0.080, -0.007)	(-0.437, 0.004)	(0.240, 1.247)	(0.106, 6.166)
		Boot	(-0.053, -0.029)	(-0.361, -0.168)	(0.638, 0.829)	(2.526, 3.641)
			-0.069	-0.204	0.872	3.144
	1	Asy	(-0.152, 0.014)	(-0.719, 0.312)	(-0.213, 1.957)	(-0.593, 6.880)
		Boot	(-0.105, -0.027)	(-0.601, -0.055)	(0.583, 1.137)	(2.201, 4.044)
			-0.070	-0.198	0.879	3.144
Two-step	2	Asy	(-0.156, 0.015)	(-0.719, 0.323)	(-0.234, 1.992)	(-0.750, 7.039)
		Boot	(-0.106, -0.028)	(-0.596, -0.034)	(0.565, 1.144)	(2.076, 4.037)
			-0.068	-0.202	0.887	2.989
	4	Asy	(-0.171, 0.353)	(-0.835, 0.432)	(-0.502, 2.276)	(-1.769, 7.747)
		Boot	(-0.117, -0.017)	(-0.661, -0.007)	(0.500, 1.242)	(1.713, 4.170)

Period II (1985/I–2001/I)

Estimation	K	C.I.	α	β	γ	i^*
			-0.081	-0.342	1.142	1.868
	1	Asy	(-0.099, -0.063)	(-0.520, -0.164)	(0.298, 1.985)	(0.242, 3.493)
		Boot	(-0.091, -0.069)	(-0.449, -0.225)	(0.873, 1.338)	(1.358, 2.333)
			-0.083	-0.303	1.049	2.109
One-step	2	Asy	(-0.103, -0.063)	(-0.501, -0.105)	(0.152, 1.945)	(0.338, 3.880)
		Boot	(-0.098, -0.078)	(-0.468, -0.261)	(0.886, 1.368)	(1.796, 2.716)
			-0.087	-0.206	0.754	2.727
	4	Asy	(-0.115, -0.059)	(-0.496, 0.085)	(-0.362, 1.869)	(0.396, 5.059)
		Boot	(-0.103, -0.082)	(-0.339, -0.176)	(0.602, 1.078)	(2.435, 3.400)
			-0.087	-0.397	1.458	1.566
	1	Asy	(-0.139, -0.035)	(-0.957, 0.163)	(-0.644, 3.560)	(-1.132, 4.265)
		Boot	(-0.132, -0.074)	(-0.901, -0.223)	(0.838, 2.142)	(0.791, 2.422)
			-0.089	-0.305	1.242	1.995
Two-step	2	Asy	(-0.131, -0.047)	(-0.688, 0.077)	(-0.012, 2.495)	(-0.052, 4.041)
		Boot	(-0.117, -0.075)	(-0.596, -0.204)	(0.917, 1.605)	(1.499, 2.574)
			-0.086	-0.118	0.743	3.383
	4	Asy	(-0.165, -0.007)	(-0.658, 0.422)	(-0.585, 2.071)	(-0.861, 7.627)
		Boot	(-0.117, -0.072)	(-0.403, -0.057)	(0.543, 0.943)	(2.780, 3.996)

1. One-step and Two-step denote the one-step and two-step Gauss–Newton estimations, respectively.
2. K denotes the number of leads and lags in the regression model.
3. The number in parentheses denotes the 95% confidence interval. To calculate the 95% confidence interval, we use the long-run variance estimated through Andrews' (1991) method with an AR(4) approximation for the prefilter.
4. Asy and Boot denote the asymptotic and bootstrap confidence intervals, respectively.

Table A-1: Empirical Size and Power of Linearity Tests

Period I (1985/I–1999/I)

Specification		T_1			T_2		
		$K = 1$	$K = 2$	$K = 4$	$K = 1$	$K = 2$	$K = 4$
Double-log	Size	0.100	0.120	0.156	0.082	0.092	0.130
	Power	0.781	0.832	0.997	0.744	0.803	0.927
Semi-log	Size	0.091	0.094	0.125	0.059	0.063	0.079
	Power	0.754	0.750	0.760	0.732	0.717	0.744

Period II (1985/I–2001/I)

Specification		T_1			T_2		
		$K = 1$	$K = 2$	$K = 4$	$K = 1$	$K = 2$	$K = 4$
Double-log	Size	0.103	0.154	0.230	0.087	0.122	0.293
	Power	0.547	0.698	0.851	0.464	0.583	0.868
Semi-log	Size	0.134	0.149	0.215	0.089	0.106	0.169
	Power	0.864	0.869	0.846	0.851	0.868	0.847

1. Data are generated as discussed in the Appendix.
2. K denotes the number of leads and lags in the regression model.
3. Empirical size is calculated under the null of linearity ($\beta = 0$), and empirical power is calculated under the alternative of nonlinearity ($\beta \neq 0$).
4. The number of iterations is 5,000 and the nominal size is 5%.
5. The long-run variance is estimated using Andrews' (1991) method with an AR(4) approximation for the prefilter.

Table A-2: Empirical Size and Power of the Nonlinear Cointegration Test

Period I (1985/I–1999/I)

Specification		θ_T^1		
		$K = 1$	$K = 2$	$K = 4$
Double-log	Size	0.055	0.061	0.054
	Power	0.437	0.406	0.401
Semi-log	Size	0.056	0.060	0.050
	Power	0.439	0.417	0.404

Period II (1985/I–2001/I)

Specification		θ_T^1		
		$K = 1$	$K = 2$	$K = 4$
Double-log	Size	0.058	0.048	0.035
	Power	0.591	0.539	0.530
Semi-log	Size	0.059	0.041	0.036
	Power	0.572	0.555	0.541

1. Data are generated as discussed in the Appendix.
2. K denotes the number of leads and lags in the regression model.
3. Empirical size is calculated under the null of cointegration and empirical power is calculated under the alternative of no-cointegration.
4. The number of iterations is 5,000 and the nominal size is 5%.
5. The long-run variance is estimated using Andrews' (1991) method with an AR(4) approximation for the prefilter.

Figure 1. Overnight Call Rates and Marshallian k

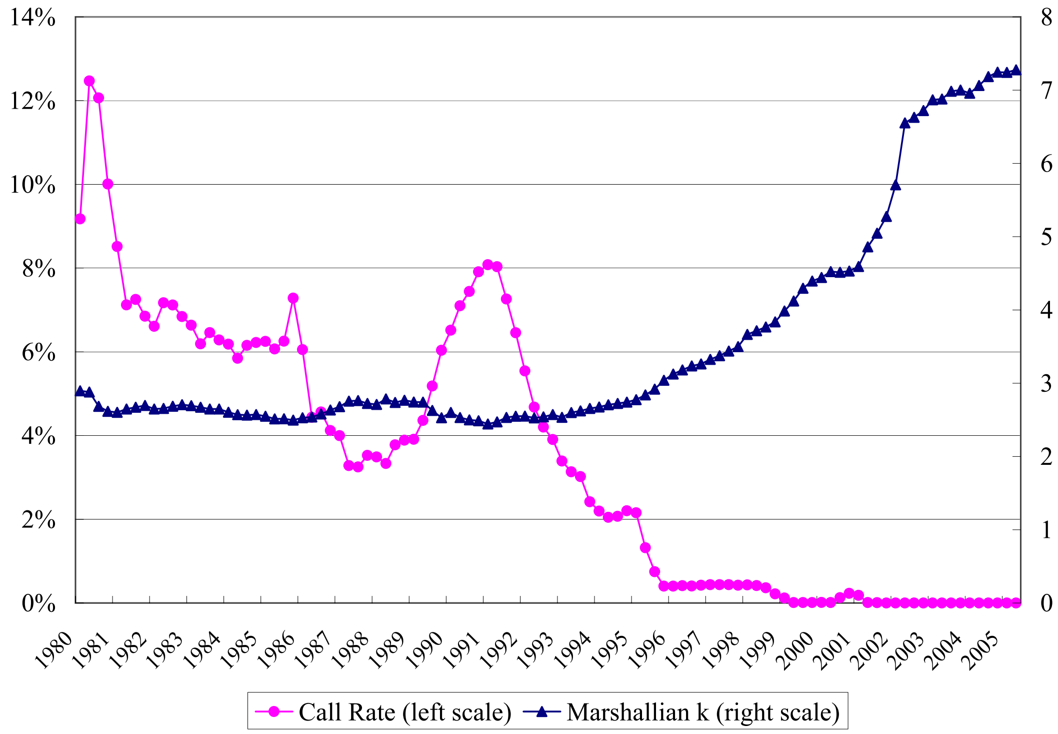
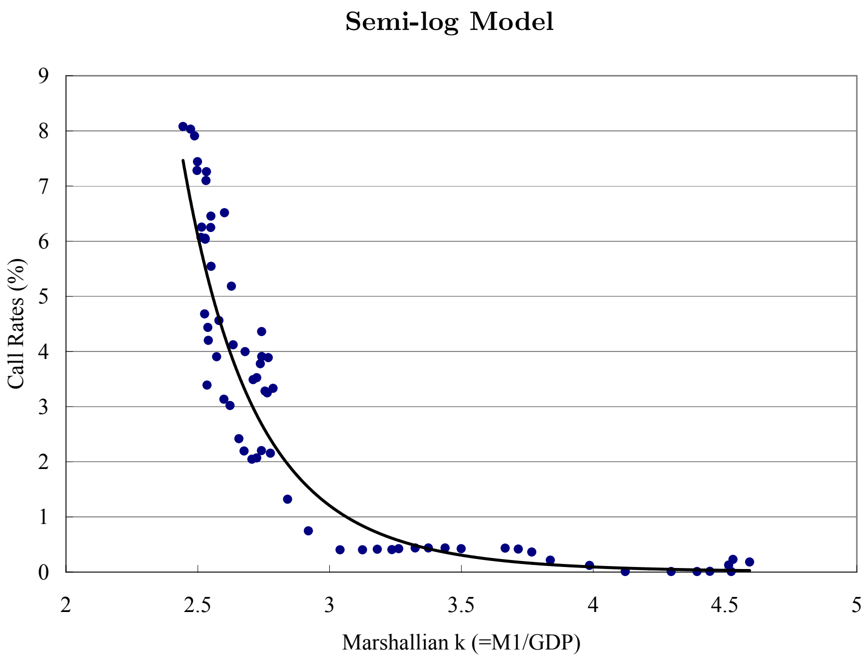


Figure 2: Semi-log Model and Double-log Model (1985/I-2001/I)



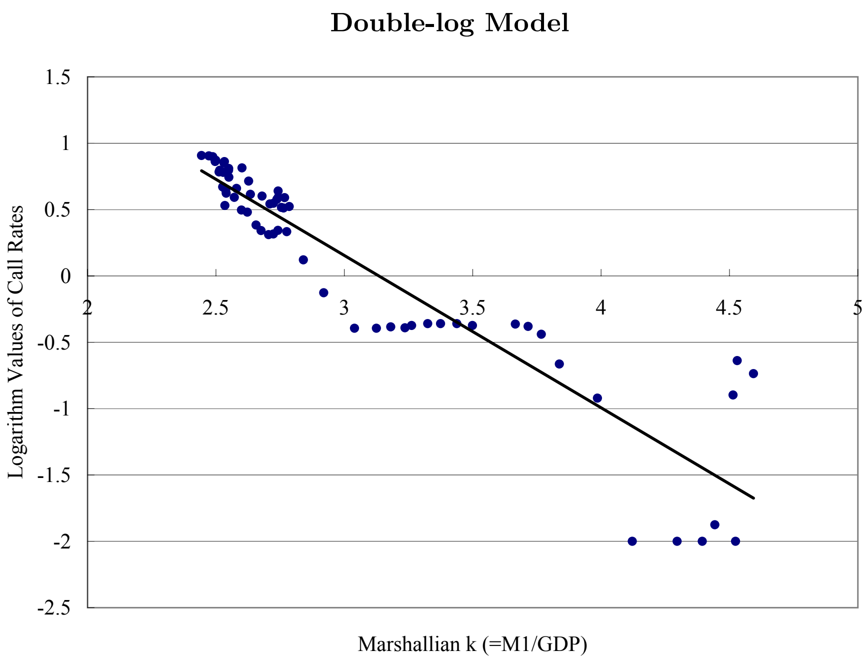


Figure 3. Level of Nominal Interest Rates and Interest Semi-elasticity

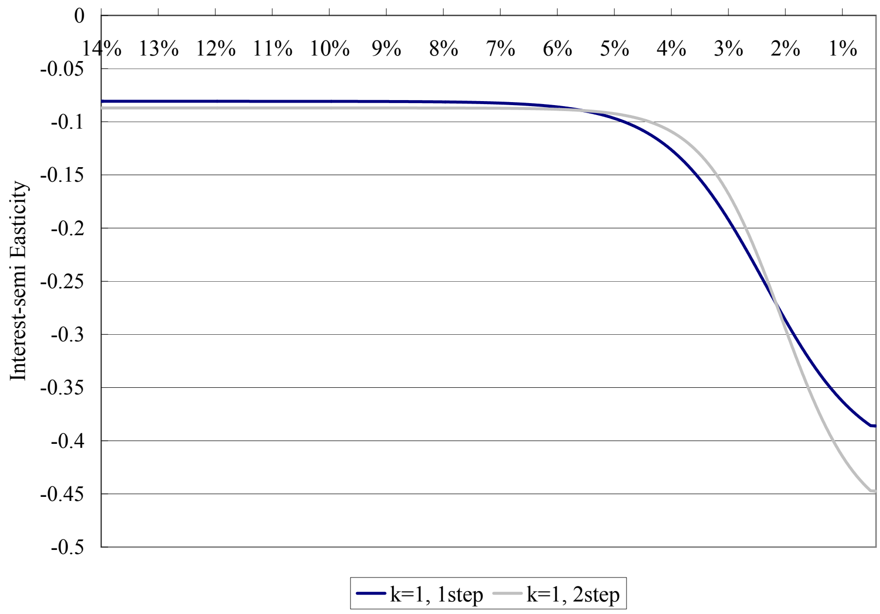


Figure 4. Time Profile of Interest Semi-elasticity

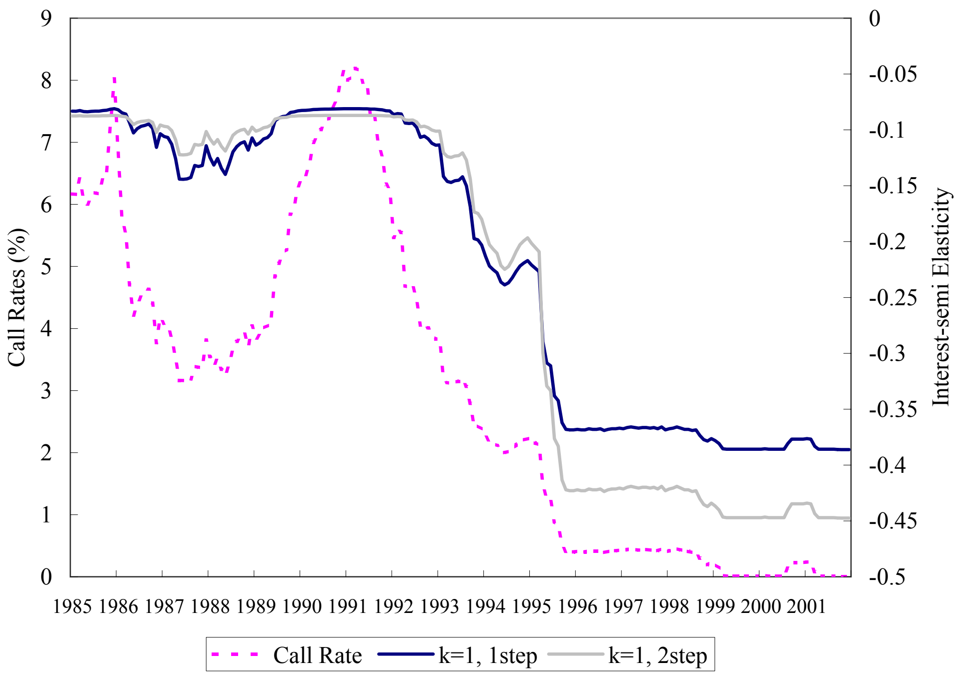


Figure 5. The Shape of the Japanese Money Demand Function