Analyzing Market Economies From the Perspective of Information Production, Policy, and Self-organized Equilibrium

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ANALYZING MARKET ECONOMIES FROM THE PERSPECTIVE OF INFORMATION PRODUCTION, POLICY, AND SELF-ORGANIZED EQUILIBRIUM

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SUMMARY: *A modern market economy is an exceedingly complex, infinite-dimensional, stochastic dynamical system. The failure of mainstream economists to characterize its dynamics may well be due to its intractability. This paper argues that the characterization of its dynamics becomes almost trivial when it is analyzed from the perspective of information production. Whether its Jacobian matrix is specifiable or not, a Lyapunov spectrum can be constructed from which the potential Kolmogorov-Sinai or Shannon entropy can be assessed. But, a self-organized equilibrium must first obtain, and for that a suitable policy must be operational.*

***KEYWORDS**: Complexity, Kolmogorov-Sinai entropy, Shannon entropy, Lyapunov spectrum, Lyapunov dimension, Efficient policy, Self-organized equilibrium.*

***JEL Classification**: B41, C52, C61*

1.0 INTRODUCTION

Market economies are often blamed for their bad outcomes such as thermodynamic entropy production, and unequal distribution of income or wealth which, in turn, is a source of conflicts. There is now sufficient evidence to argue factually that when market economies are unregulated or simply mildly regulated, they tend to become unstable, voracious, and predatory. For the proponents of unregulated markets, on the other hand, market economies are only sources of wealth creation.

In truth, modern market economies should be viewed as complex social constructs designed to facilitate exchanges, in which decisions regarding investment, production, and distribution are driven by supply and demand. Neoclassical economists model them as micro-founded-dynamic-stochastic-general-equilibrium constructs (DSGE) based on rational expectations, Walrasian market clearing, unique and stable equilibrium. Agents are infinitely-lived optimizing households with homothetic and identical preferences bent on maximizing outcomes. Despite the persistent reminders from people such as Dani Rodrick and Paul Krugman (Rosenberg, 2016), DSGE (the latest vintage of macro-models) is a poor guide to decision-making. If the modeling effort of the International Monetary Fund or Federal Reserve Bank of New York is an indication of DSGE's ability to explain and predict, one must conclude that it cannot fulfill these promises. For, in the absence of shocks and changes in model'structure, one could perhaps predict next year's outcome more accurately using a ruler. The reason is that market economies are infinite-dimensional webs of interrelationships with multiple feedbacks and feedforwards in which agents operate according to their own schema or local and public knowledge, while learning and adapting to emergent characteristics; that is, a process with many more affinities with biological rather than chemical or thermos-dynamical systems.

The 'deep parameters' of DSGE, namely elasticity of substitution, preferences, resource limitations, etc. are so-called, because they are supposed to be invariant to policy changes. But the only things "deep" in the capitalist economy (imagined by Professor Lucas in his "critique") are the inherent modes of action constituting the law of motion, such as monotonic increasing preferences, the attraction to incentives, and the quest of safety in domination. In reality, the structure of the economic model consists of exchange ratios, rates, identities, fractions of preference assigned to endowments, etc., that are constantly varying in response to changes in preferences, endowments, and policies, while the aggregate flows are noisy and sampled at large intervals. Furthermore, market economies share many attributes with biological systems in the sense that they can grow or decay, making their outcomes non-stationary and therefore non-ergodic. Otherwise put, market economies are complex-adaptive systems which are in addition subject to risks and uncertainties. Hence they are unable to throw-out fundamental statements. Conventional mainstream modelling of a large modern market economy appears almost an intractable problem. Nonetheless, model builders of such complexity could draw valuable lessons from both the logistic map (see below) and classical mechanics[1]. It is worth repeating that market economies are multi-dimensional, dissipative and heavily interconnected systems. For every event that occurs anywhere within them, small effects and uncertainties multiply over time, cascading into unpredictability (Petersen, 1983; Frigg, 2004). Being infinite-dimensional, they requires an infinite set of independent numbers to specify an initial condition. Similar to the well-known problem of classical mechanics, described in note 1 below, modelling them may be made trivial if viewed from the perspective of information production. In this respect at least, theorists are not powerless. For, Farmer (1982) has shown that such a system can be approximated by a finite-dimensional iterated system. And being dissipative, it almost surely possesses a chaotic attractor of finite-dimensions. Instead of attempting to tract elusive parameters and aggregate flows, the perspective of information simplifies the task, for it only requires that the focus be mainly on chaotic and predictable behaviors.

The main difference between predictable and chaotic behaviors is that predictable trajectories do not produce new information, whereas chaotic trajectories continuously do. That being the case, one can appeal to the notion of Kolmogorov-Sinai (or just metric) entropy as it provides a quantitative knowledge of how chaotic a dynamical system is. Moreover, chaotic attractors of finite dimensions have discrete spectra of Lyapunov characteristics exponents. These exponents provide a summary of local stability properties as well as the Lyapunov dimension of the attractor. Positive exponents measure the average exponential divergence of nearby trajectories, while negative exponents measure exponential convergence on the attractor; and together, they constitute the Lyapunov spectrum. We will make use of them, including the Kaplan-Yorke conjecture (1979a; 1979b) (which does not distinguish between infinite and finite-dimensional systems) to de-fang the infinite-dimensional dynamical market economy.

In Part II, we use the quadratic map to first establish a *spectrum of behaviors* within which we think this class of growth models lives, and in which we think a market economy belongs[2], and where it can easily be

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[1] Imagine a box filled with n particles. Putting together the space ($x \varepsilon \Re^3$) and momenta ($p \varepsilon \Re^3$) dimensions in one vector space called the phase space, X, which is a collection of all possible states $x \varepsilon X$, forming an abstract mathematical space in $\Re^{6n}$. With a sufficiently regular Hamiltonian function, one can find a unique solution of the position and momentum of each particle. However, one must first face a strongly coupled system of 10^{24} equations. The question that was subsequently raised was: If the system starts at a certain state (x_0, p_0), will it eventually return to a state close to that initial conditions? According to Sarig (2008), solving such a large system did appear intractable until Henri Poincaré made it trivial by viewing the problem from a different perspective. That new perspective led Poincaré to the Recurrence Theorem.

[2] Generally, the market economy is a dynamic pricing construct that may be analyzed as a pair of objects (X, T) consisting of a complete metric space X (i.e. the set of all possible states x of (X, T)) and a family T_t of continuous mappings of the space X into itself with the property $T_{t+\tau} = T_t \circ T_\tau$, where $t, \tau \varepsilon T_+$: $T_0 = I$. $T_+ \varepsilon \Re_+$ or $T_+ \varepsilon \mathbb{Z}_+ = \{0, 1, 2, \ldots\}$. X is called the phase space Γ, whereas the family T_+ is the evolutionary operator (a semi-group); the parameter t e ε T_+ plays the role of time. If $T_+ \varepsilon \mathbb{Z}_+$, the dynamic system is discrete. The law of motion indicates that if the system is at state x now, will it then evolve to state T(x) after Δt? $\{T^n(x)\}_{n \varepsilon \mathbb{Z}}$ is a record of the time evolution of the system, and understanding the behavior of $T^n(x)$ as $n \rightarrow \infty$ is the same as knowing

analyzed from the perspective of information production. Part III emphasizes the essential role of policy and stable equilibria. Part IV examines the role of self-organized equilibria in the assessment of the information produced by a chaotic system. Part V summarizes our findings.

2.0 THE SPECTRUM OF EQUILIBRIA

The quadratic map is one-dimensional and it is non-hyperbolic. It nevertheless offers a gamut of valuable lessons in the form of a spectrum of behavior of growth models. Using symbols such as $\llcorner$ to indicate "power on", $\rightarrow$ "imply", $\wedge$ for "and", $\vee$ for "or", and $\Leftrightarrow$ means 'equivalent to'. One can succinctly express equation (01) as:

$$X_{t+1} = f(x) = R\,X_t\,(1 - X_t); \qquad (1)$$

$$R\ \varepsilon\ \Re\ \varepsilon\ (0, 4]\ \llcorner\ X_t \wedge R \Leftrightarrow \omega$$

$$X_t(1 - X_t)\,\big|\,X_t > 0 \wedge R > 1 \Leftrightarrow \pi;$$

ω and π represent, respectively, the structure (or the Jacobian) and the policy set. Choosing the initial condition X_0 determines the outcome of n discrete steps in the following way: $X_1 = f(X_0) = f^1(X_0)$: $x_2 = f^2(X_0)$: …, $f^n(X_0) = f \circ f \circ \ldots \circ f(X_0)$ (n times), while $\Re$ stands for the real line.

One alternative is to use the Bernoulli shift map whose iterated dynamics can produce complicated motion as well. For example, let T: (0, 1) $\rightarrow$ (0, 1), $T_{(x)} := 2X$, mod 1, and the unit interval is divided in two segments at X = ½ . Assume now that the unit interval is filled with a uniform distribution of points. We can decompose the action of the shift map into 2 steps: i) the map stretches the distribution by 2 which in turn leads to divergence of nearby trajectories, and ii) cuts the line segment in the middle as per the modulo action mod 1, which leads to bounded motion on the unit interval. Thus the Bernoulli shift is an example of a nonlinear stretch-and-cut strategy to generate deterministic chaos in a closed dynamical system. Suppose now that points can leave the unit interval and escape to infinity, then the total number of points filling the unit interval is no longer conserved. We would have then an open system. We will deal mainly with open systems in what follows, but for now we return to the quadratic map, which in some respects is more suitable for the present purpose; furthermore, most of the concepts developed therein carry over to higher-dimensions.

Table 1 then displays the changes in the spectrum of equilibria as R in (1) is varied. As it can be seen, as the value of R is increased, the spectrum displays various modes that can be expected, depending of course on the structure of the process. For example, there is ω_L at ($1 < R < 3$) for which *all equilibria are stable sinks*; ω_{2c} ($3 < R \leq 3.57$) would produce *stable cycles-2*; ω_{Dc} ($R = 3.57$) would give some form of *deterministic chaos*; ω_{Hc} ($3.57 < R < 3.82$) is for *high-dimensional chaos*; ω_{3c} is for *cycles-3*; and ω_{Lc} is for *low-dimensional chaos.* In this study high and low dimensional chaos are distinguished only by the geometry of the attractor as measured by the Hausdorff dimension. With regard to modern markets, because of wild and irregular gyrations of output and feedbacks, one can safely rule out ω_L, ω_{2c}, and ω_{3c}. Another reason for discarding them is that they do not produce new information. We will assume that the structure of the market economy is either ω_{Hc} or ω_{Lc}.

Los (2000) has computed the third iterate of the quadratic map and he next ruled out negative and complex R values, but at R =3.832, in the middle of the so-called Li and Yorke interval, he found a cycle 3 x 2^k (k =

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the state of (X, T) in the future. The iterates of the map T are defined by induction. That is, $T^0 := id$, $T^n := T \circ T^{n-1}$, and the aim of the theory is to describe the behavior of $T^n(x)$ as $n \rightarrow \infty$. Since (X, T) is dissipative, it is not volume preserving and therefore does not preserve the Lebesgue measure.

1). Los does not say whether or not the unstable equilibria were about to bifurcate, but he noticed 3 distinct periods, and therefore all other periods become possible. However, for the present purpose, it is worth underlining that the Lyapunov exponents in that interval are negative in steady states. We ruled out ω_{3c} because no actual markets with these characteristics have been observed, but just before that interval i. e., at R = 3.82, Los found 2 stable equilibria at x* = 0.154 and x* =0.958 in the midst of high dimensional chaos. It goes without saying that if these equilibria were unstable instead, an appropriate policy (see next section) could elicit a phase change, which in turn could lead to a locally stable self-organized equilibrium. We will return to the concept of self-organized equilibrium in Part IV.

2.1 A MODEL OF THE MARKET ECONOMY

If 'complexity' implies diversity and arises from a multitude of connections between a wide variability of elements, it is then safe to say that the complexity of market economies is observable daily and is ubiquitous in aggregate data (Cohen and Stewart, 1994). Our contention is that being dissipative dynamical systems, their phase spaces (Γ) therefore contain invariant sets or attractors. Thus, valid or invalid assumptions notwithstanding, casual observations show that a market economy belongs to the class of models given by:

$$dX/dt = F_{(\pi)\,\omega}(x\,(t)); \tag{2}$$

$$\Lambda_{\Pi}\,\Lambda_{\omega}\,(\pi \llcorner (\omega \wedge x \quad (t_0) = x_0) \wedge \omega\ \varepsilon\ \Re^m \rightarrow \dim X \wedge \pi \neg \omega;$$

$$\varphi : \pi_i \llcorner\ \omega_{jk} \big|\, j, k\ \varepsilon\ q < m.$$

That is, for π and ω, where π has power over ω and $x_{0;}$ ω determines the dimension of X, and π is not ω. In other words, *ω is the structure of the model* or a vast networks of connections with nodes in the phase space Γ, and π is a policy space in which ω is embedded. φ is a reflexive "onto" map or a veto power either on ω, (i. e., capable of eliminating a few degrees of freedom) or capable of resetting x_0 in an attempt to put the system in a stable sub-space E^s (see below).

Value of R [1]	Equilibria x*	Lyapunov coefficient λ
$0 \leq R \geq 1$	no solution	Violation of π
$1 < R \leq 3$	linearity	< 0
$3 < R \leq 3.57$	Period 2^k, {k = 0, 1, 2 …	< 0
R = 3.57	Periodic & aperiodic cycles	= 0
$3.57 < R < 3.82$	Stability & instability	$\lesseqgtr 0$
$3.8284 \leq R$ 3.8414 [2]	P- 3^k {k = 0, 1, 2,…}	$\lesseqgtr 0$
$3.8414 < R \leq 4$	Low dim chaotic	> 0

Table 1: Equilibria and LCEs as a Function of R in the Quadratic map. (1Ac) counting for instrument noise. (2) Computed by Medio (1992).

First, suppose that *ω is known explicitly*, then its Jacobian is also known. That is,

$$J\,(t) = \partial\, T_i \,/\, \partial\, X_i\ , i\ \varepsilon\ m, \tag{3}$$

where T_i is the mapping in note 2, the ijth elements of the matrix $J_{ij} = \partial X_i (t)/\partial X_j$, where $X_i (t)$ is the ijth component of the state vector at time t, and J (t) is the observed square determinant (as it takes 2 for a connection) describing the overall contraction of the phase space volume, while its eigenvalues describe the divergence and convergence of trajectories. We first suppose that the square matrix J (t) has k distinct eigenvalues with negative real parts, h eigenvalues with positive real parts, and g = (m – k – h) eigenvalues with zero real parts.

The attractor Д of J (t) in this case is non-hyperbolic. However, a center manifold will not add anything to our discussion even though non-hyperbolic attractors are more common in the real world. For simplicity of exposition, we suppose that the attractor of (2) is hyperbolic,[3] and that all equilibria are translated to point **0** located at the origin.

If J (t) has pure imaginary eigenvalues in the form of $\sigma_j = a_j + ib_j$, then the generalized eigenvectors are $w_j = u_j + iv_j$. We will not spend much time on negative eigenvectors except to repeat that if the dominant eigenvector is negative, then no new information could be had since it would be known in advance that the flow would end up in the stable manifold ($Ш^s$); perhaps that is the reason why Kolmogorov initially thought that deterministic systems did not provide information (Sinai, 1959).

3.0 MEASURING THE METRIC ENTROPY AND THE LEVEL OF CHAOTICITY

The Kolmogorov-Sinai entropy (KS) notion is examined relative to another notion called partition (Kolmogorov, 1958). A partition $\gamma = \{\gamma_i \mid i = 1, 2, \ldots, n\}$ of X is a collection of non-empty, non-intersecting sets that can cover X. That is, $\gamma_i \cap \gamma_j = \emptyset, \forall\ i \neq j$ and $X = \cup_i^n \gamma_i$. Thus, if γ is a partition, so is $T_t^{-1} \gamma := \{T^{-1} \gamma_i \mid i\ \varepsilon\ n$.

Given the partition γ in a dynamical system, let

$$H_n (\gamma\ T) := (1/n)\ (\gamma \vee T^{-1} \gamma \vee, \ldots, T^{n+1} \gamma\ ). \qquad (4)$$

In the limit $H (\gamma\ T) := \lim_{n \to \infty} (H_n,T)$ exists. Then the KS entropy is defined (Frigg, 2004; Petersen, 1983; Kolmogorov, 1958; Shannon, 1949) as:

$$S_{KS} := \sup_\gamma \{H (\gamma, T)\} \qquad (5)$$

The KS entropy is linked to the Shannon entropy H (P). In the latter, it is assumed that there exists a source that is producing discrete messages and a receiver. Let a complete messages be $M = \{m_1, m_2, \ldots, m_n\}$ and its probability distribution be $P = \{p_1, p_2, \ldots p_n\}$, where $p_i > 0$ and $\sum p_i = 1$. Then the discrete Shannon entropy is:

$$H (P) := - \sum p_i \log_2 (p_i\ ). \qquad (6)$$

Thus $H_n (\gamma, T)$ measures the average amount of information produced by the system per step over the first n-steps relative to the coding γ. A positive KS entropy indicates that the system is unpredictable. To make this clearer, let us emphasize that the KS entropy measures the amount of information contained in an individual object (say a string) x by the size of the smallest program that generates it. It naturally characterizes a probability distribution over all possible binary strings Ӎ.

[3] In that case, there exist a stable subspace E^s of dim k $\vee$ unstable sub-subspace E^u of dim (m –k). E^s span $(v_1, v_2, \ldots, v_k) \wedge E^u$ span $(v_{k+1}, \ldots, v_m)$ such that $E^s \oplus E^u = \Re^m$. We then have a differentiable manifold $Ш^s$ tangent to E^s at $\mathbf{0} \wedge Ш^u$ tangent to E^u at **0**. Then $\forall t \geq 0$, the flow $\varphi_t (Ш^s) \subset Ш^s, \forall x_0\ \varepsilon\ Ш^s \mid \lim t \to \infty, x_0 = \mathbf{0}$. Similarly, $\forall t \leq 0, \varphi_t (Ш^u) \subset Ш^u, \forall x_0\ \varepsilon\ Ш^u \mid$ limit $t \to -\infty, \varphi_t (x_0) = \mathbf{0}$, where as before, x_0 stands for initial conditions. If J (t) has pure imaginary eigenvalues in the form of $\sigma_j = a_j + ib_j$, then the generalized eigenvectors are $w_j = u_j + iv_j$.

The Shannon entropy (H (P)) of a random variable X, on the other hand, is a measure of the average uncertainty. That is, the smallest number of bits required to describe x (the output of X) when the receiver comes to know the probability distribution. In the context of communication theory, it amounts to the minimum number of bits that is required to transmit x. Hence, it would seem that KS entropy and Shannon entropy H (P) are conceptually different concepts. The former is based on the length of programs, while the latter is based on probability distributions. Yet, for any distribution computable by a Turing machine, the total value of KS entropy is equal to H (P) up to a constant term Ɱ as shown below.

To recapitulate, we suppose a set of independent messages (M) and probability distributions (P). The receiver receives m_i and he gets $\log_2 (1 / p_i)$ of information. For Ɱ independent messages, he or she receives a total of information I, given by:

$$I = \sum_{i=1}^{m} (\text{Ɱ}\; p_i) \log_2 (1/p_i). \tag{7}$$

Then, the average information he gets per individual messages is:

$$\begin{aligned} \langle I \rangle &:= (1/\text{Ɱ}) \sum (\text{Ɱ}\; p_i) \log_2 (1/p_i) \\ &= \sum p_i \log_2 (1/p_i). \end{aligned} \tag{8}$$

According to Shannon, given a probability distribution P, its entropy is:

$$H (P) := \sum p_i \log_2 (1/p_i). \tag{9}$$

Therefore, H (P) = $\langle I \rangle$, implying that the entropy of P is just the expected value of the information given by P. If the Shannon entropy is equivalent to the potential information gained once the experimenter learns the outcome of the experiment, t*hen, the more entropy a system has, the more information one can potentially gained once one knows the outcome of that experiment or is able to apprehend its probability distribution.* Another way of seeing H (P) is that it is a way to quantify the potential reduction of one's uncertainty once one has learnt the outcome of a probabilistic process.

The KS entropy is also linked to the Lyapunov characteristic exponents (LCE) via the concept of exponential divergence. The LCEs measure the mean exponential divergence or convergence of solutions originating near x. Positive ones indicate that solutions diverge exponentially on the average and in some directions. One can then appeal to Persin's Theorem (1977) (see also Eckman and Ruelle (1985)) which asserts that under certain assumptions the sum of the positive LCEs is a measure of the KS entropy. If the system is chaotic then at least one of the LCE is positive. In addition, it may have dense orbits and sensitive dependence on initial conditions (SDIC), which is a critical hallmark of chaos. In fact, we consider the presence of SDIC as the main distinction between high and low-dimensional chaos in this study. For, whenever SDIC is present, the positivity of these exponents increases. Given their crucial role in the determination of chaotic behavior, a brief review their derivation might further increase understanding.

Suppose that initially we have two trajectories separated by a small distance d_0 on the unstable manifold. The trajectories will diverge at time t later by a distance d_t. The rate of separation of the two trajectories is measured by the Lyapunov exponents (λ) as $|d_t| \approx e^{\lambda t} |d_0|$. In statistical mechanics, one is mainly interested in limits as t goes to infinity. Here the final separation of the two trajectories depends on λ. We may then define the maximum λ as the normal exponent in the limit. The reason is that a chaotic trajectory will automatically follow its maximum expanding direction. That is,

$$\lambda := \lim_{t\to\infty} \lim_{d\to 0} (1/t) \ln |d_t / d_0| ; \tag{10}$$

there are m such exponents and whenever one is positive we know that we are in a chaotic regime.

3.1 THE LYAPUNOV SPECTRUM

Focusing on prediction errors observed in economic forecasts made by institutions such as the Federal Reserve Bank of New York, on can safely infer the complexity of economics. From the above discussion, it is supposed that the Jacobian of (2) is known and that its attractor is hyperbolic. Consequently, there are k negative LCEs and h positive ones, and the so-called *Lyapunov spectrum* can be set up as:

$$\{\lambda^{+}_{1h}, > \lambda^{+}_{2h} >, \ldots, > \lambda^{+}_{hh}, > \lambda^{-}_{1k}, > \lambda^{-}_{1k}, > \lambda^{-}_{2k} >, \ldots, > \lambda^{-}_{kk}\}.$$

Then from the Persin's Theorem, the *metric entropy* of the attractor is:

$$E_n\,(Д) := \Sigma_1{}^{h}\ \lambda^{+}_{hi}; \qquad (11)$$

that is, the metric or the KS entropy is just the sum of the positive LCEs or the average information generated by system (2).

The Kaplan-Yorke conjecture states that for an m-dimensional system, the index (D_{KY}) may be computed from the Lyapunov spectrum. In other words, *the information dimension* is the *Lyapunov dimension* as measured by the D_{KY} index:

$$D_{KY} := (\text{the order of } \lambda_{hh}{}^{+}) + (\Sigma_{ih}\ \lambda^{+}_{ih}) \,/\, |\lambda^{-}_{1k}|; \qquad (12)$$

where by 'the order of' it is meant the cardinal of the order of the least positive LCE in the spectrum.

It is worthwhile to recall here that it's all started with a conjecture by Kolmogorov to the effect that only stochastic systems produce information. However, it was also found later that several deterministic systems had positive Kolmogorov-Sinai entropy (KS). This is probably due to Yakov Sinai who, inspired by Kolmogorov and Shannon, was the first to come up with the mathematical foundation for quantifying the complexity of a dynamical system. Nowadays, it is widely accepted that the Kolmogorov-Sinai entropy is the basic tool used to capture the property of both stochastic and deterministic systems to produce information as the KS entropy measures the highest average information received from the present state of a dynamic system endowed with a coding, given its past states (that is, information that has already been received). Hence, the KS entropy measures the unpredictability of a dynamic system, a concept that is in harmony with the Shannon entropy where the next sequence is equivalent to new information

Suppose now that *ω is unknown*. According to Farmer, an infinite-dimensional system can be approximated by a finite-dimensional one. And simulation carried out by Farmer has effectively shown that the metric entropy does not vary much beyond a 20-D attractor. However, even a changing, finite-dimensional system may prove to be intractable. Economists can avoid such torment of trying to construct ω by appealing to Takens (1981; Mané, 1980; Liu, 2009; Medio, 1992) who have asserted that *in lieu* of an attempt to determine ω, a pseudo phase space can be constructed from observed data such as a time series. Obviously, a measured time series is only a scalar measurement from one variable which is not a trajectory. This difference is resolved by the delay coordinate embedding technique proposed by Takens. For if the dynamical system and the measured variable are generic, then the delay coordinate map from a smooth compact manifold of dimension, say, M to $\Re^{m}$ is a diffeomorphism on M. Therefore, under fairly general conditions, the unknown dynamical system can be reconstructed from the time series. After all, we have learned since Henri Poincaré that exact solutions are not necessary to understand and to analyze non-linear dynamical processes. Instead, the emphasis should be on describing the geometrical and topological structure of ensemble of solutions, and the structural elements of a non-linear process are attractors, subspaces, and the types of behavior.

If the structure of the market economy cannot be specified due to excessive complexity and high dimensionality, economists should focus on proven techniques used in other disciplines to recover information of an unknown model through the observations of one of its output. Thus, two theorems (see Takens, 1981;

Mané, 1980) provide the link between the true model and the dynamics of the model reconstructed from observed data. This is straightforward when the unknown model is dissipative, because one can be assured that the process converges on an attractor. Even though the true system might be infinite-dimensional, the resulting attractor may be low-dimensional. If the reconstructed attractor exhibits chaoticity, one can be sure that the unknown attractor is non-linearly deterministic and that its behavior is also unpredictable. If it is, then it produces information that can now be computed by following the procedure outlined in Part III.

The reconstruction process begins with a univariate time series such as:

$$Z(t), Z(t + \tau), Z(t + 2\tau), \ldots, Z(t + (n - 1)\tau),$$

where τ is the time delay. Medio observes that under mild conditions, it can be shown that the dynamics of the reconstructed phase space have the same asymptotic properties as those of the unknown attractor for almost any choice of τ, provided that the length of the univariate series is long enough and that the sampling period is short.

The Takens' method. Takens' Theorem asserts that if n is large enough compared with the dimension of the attractor, then the n-dimensional image of the reconstructed attractor provides a close topological picture of the unknown one. The question now is how large should n be? Both Takens and Mané suggest a condition on the size of n that is sufficient to produce a good projection; that is, if m is the dimension of the unknown attractor, then $n \geq 2m + 1$. Obviously, this is helpful if m is known and finite; anyhow, the reader is referred to these two sources for more details on that method.

Knowing that economic time series are seriously contaminated with noise, Medio recommends filtering before using the Takens' method in order to extract meaningful information. In sum, a good use of that method requires a long time series, short sampling period, proper window length, and filtering.

The Caterpillar-SSA method. According to Medio, Takens' method is very sensitive to noise. He recommends the *Caterpillar-singular spectrum analysis* (SSA hereafter) which gives a more accurate picture of the attractor, principally when the signal to noise ratio is low.

The SSA method is a powerful method of time series analysis developed independently in St-Petersburg (Russia) under the name 'Caterpillar' and in the US-UK under the name SSA. It is a model-free method that consists of the transformation of a one-dimensional series into a multi-dimensional series by one parameter translation procedure, singular value decomposition, and reconstruction of the series according to its principal components. It can analyze short and long series, stationary and nonstationary, almost deterministic and noisy series, and it can detect chaos. This is not the place for a detailed description of the method. The interested reader is referred to Golyandina and Zhigljavsky, (2005), (2013), Danilov and Zhigljavsky, (1984). It suffices here to emphasize the fact that the SSA method is widely and successfully used in many other disciplines. Once the attractor is reconstructed, formulae (11) and (12) can again be used; even though ω is not known explicitly[4].

4.0 SELF-ORGANIZED EQUILIBRIA

Neo-Keynesian economists such as Paul Krugman are firmly attached to the notion of equilibrium. For Krugman, there is no alternative to "maximization" and "equilibrium". We do not think that maximization is compelling, but there is no doubt that equilibrium is of central importance in the present context. It is well-known that flipping a fair coin once provides us with one bit of information per throw, but the information may not be accessible unless the outcome is actually observed. In the present set-up, the potential

[4] We would then have a situation similar to a case in cosmology. That is, by observing the angular velocity of visible masses around a galactic center, cosmologists can infer the presence of a black hole at the center of masses even though the black hole itself cannot be observed.

average information produced by system (2) is observable and accessible only if a self-organized equilibrium obtains. The law of physics says that information cannot be destroyed. Hence, on a chaotic attractor, information can only be dissipated on the unstable manifold (see note 3), while the true probability distribution remains unknown. Our central objective in this paper is to learn how to conserve the information produced by system (2). The only way to achieve this is to call on an appropriate policy that would induce a change in decision-making (the equivalent of a phase shift in physical systems). Put differently, a change in decision-making acting on ω would hopefully create a stable subspace of a self-organized equilibrium nearby.

The reason for this is motivated by the lesson of the quadratic map. Los' analysis reveals that a phase shift of 180 degrees always preceded a bifurcation. For example, as R increases from 3, a phase shift occurs at R = 3.44 followed by the first bifurcation at R = 3.50. Another phase shift occurs at R = 3.54 followed by the second bifurcation, and so on until R = 3.57. If the same phenomenon occurs in mathematical, chemical, cosmological, and biological systems, why not in social dynamical system? This lends support to the belief that in social dynamical system, a phase shift may well be the equivalent of a change in decision-making due to an efficient policy. It is worth repeating that we are assuming that a policy that inspires confidence will lead to changes in decision-making which in turn may lead to a change in the structure (ω). As shown above, at R =3.82, the process, on its own, alternates between stability and chaos. If a market were to show such intermittency, it would be safe to simply assume that economic agents would respond to a change in policy or a phase change leading to a self-organized equilibrium, where equation (11) can then be evaluated.

Self-organization is usually defined in various ways. One definition refers to the spontaneous order that arises out of local interactions between smaller parts. Another claims it arises out of random fluctuations that are subsequently amplified by positive feedbacks. In Prigogine and Stangers (1984), Nicholis and Prigogine, (1977), Hazy and Ashley, (2004), it is defined as some sort of order far from equilibrium. In chaos theory, self-organization is discussed in terms of islands of stability within a sea of chaos. In this paper we will adhere to the definition of von Foerster who defines it as the case where random fluctuations (but also a change in policy) increase the chance that a chaotic system may fall into the basin of a stable equilibrium arising out of a phase shift (Ashley, 1947; Mitchell *et al*., 1994).

In nature, self-organization is ubiquitous. It is regularly observed in physical, chemical, biological, and cognitive systems. It is also observed in ecology, neural networks as well as in social and mathematical systems. It obviously exists in cosmology judging by the apparent stability of our solar system; a stability that has lasted long enough to allow for the presence of conscious beings on planet earth.

It is of course legitimate to ask whether or not self-organization arises in market economies. It must be first recalled that economic agents can learn and adapt. The economy itself is a path-dependent system. Los' analysis among others clearly shows that following a phase shift of 180^{o}, previously stable equilibria become unstable while the resulting bifurcation reestablishes stability. We have argued that in a social dynamical system such a market economy, the equivalent of a phase shift is a change in decision-making (preceded by the observation of instability and followed by a change in policy that strengthens the confidence of agents in the immediate present and the near future). I do not have a definition of such a policy, but whatever it may be, it must be a policy that instills sufficient confidence to elicit a positive attitude on the part of economic agents.

The importance of striving for a stable outcome cannot be over emphasized. It is a *sine qua non* condition for the actual assessment of the average information produced by the market economy. Because, the average information rate, i. e., the entropy, enriches the collectivity through abundance and high productivity that in turn drives the growth of the economy.

5.0 CONCLUDING REMARKS

This paper argues that while market economies are often decried for their undesirable outcomes, besides producing goods and service, they have another beneficial side, i. e., they produce information. Modern market economies are very complex infinite-dimensional systems. Economists have built a plethora of models in an attempt to capture their dynamics. Yet the performance of these models leaves much to be desired. The alternative is to approach the problem from a different perspective. That is, from the perspective of information production.

To export our main argument we first draw on the quadratic map to establish a spectrum of equilibria of albeit dissipative dynamic systems as the set $\omega = \{\omega_L, \omega_{2c}, \omega_{Dc}, \omega_{Hc}, \omega_{3c}, \omega_{Lc}\}$. We next concluded that ω_L (that yields linear time invariant models), ω_{2c} and ω_{3c} (that produce period-doubling cascades) can be safely be ruled out either from observations of real markets or due to their inability to produce new information. The structure of a modern market economy most likely falls either within the intervals of high-dimensional or low-dimensional chaos. Therefore, they produce information.

After locating market economies in the spectrum, we next restrict ourselves to procedures for which there is a consensus such as the metric entropy and the Lyapunov spectrum in order to measure the level of chaoticity of a proposed model. We have also emphasized that the structure ω may not be specifiable. In such a case, one may proceed to reconstruct the unknown attractor from observed data such as a time series.

After observing the enormous waste that public institutions are capable of, neoclassical economist such as von Hayek, Friedman, Lucas, etc. tend to fall in the category of opponents of government policy. However, this is tantamount to go from one extreme to another even though extremes have no place in human affairs. There is plenty of evidence that in the absence of appropriate policies, market economies will soon become unstable. We have then argued for a more central role for policies that can induce confidence in economic agents. Efficient policies giving rise to change in decision-making are equivalent to a phase change in physical and mathematical systems. Such phase changes are necessary to bring about self-organized equilibria, where the entropy generated by the economy can be evaluated.

If new information produced by a chaotic economic process is not properly harnessed due to the presence of stop-gap policies or policies bought outright by powerful agents, one should observe wild gyrations of output and falling total factor productivity; that is, falling total factor productivity is evidence that the economy is trapped in unstable regimes. The US market presents a clear case in point. Since the later part of the 1990s, government policies have freed huge corporations from both their social responsibility and ethical market behavior. Moreover, the government has unwisely deregulated and subsidized the financial market thus allowing it to become ever since truly destructive and predatory. As a consequence, the market has moved on unstable trajectories. It could have been otherwise. When policies are conducive to stability, new information obtains, and it manifests itself through, say, the difference between, a Ford Model A and the Lincoln Continental, or between the slide rule and the computer, among many other examples. In sum, information + policy + innovation = growth. It suffices to think of the space program, internet, iPhone, solid-state memory, GPS, etc. Hence, any notion that associates market economies to linear time invariance or that claim that markets should be unfettered is untenable.

Thus, instead of going to the torment of building DSGE models, students of economics would do well to focus their attention on statistical methods, dynamic analyses, attractor reconstruction, and on the task of learning what constitutes policies conducing to self-organization. Because, in a self-organized equilibrium, competitive markets through-out new information which is the true modern asset of a society.

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