Earnings, Coalitions and the Stability of the Firm

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Abstract. This paper presents an economy in which workers hired by a firm receive without cost a firm-specific training that enables them to potentially become independent producers. Thus, this specific training changes a worker's outside option according to the firm in which he works. Under such circumstances, by modelling explicitly the workers' decision to stay or to leave the firm, the paper determines a *stable arning profile* of the economy. Two main results are obtained by this approach. Firstly, that such a stable earning profile can allow for a vector of wages higher than the basic neoclassical wage and for wages differentials across industries even for initially homogenous workers; secondly, that an industry equilibrium wage depends upon the relative degree of competition existing therein. Both the results seem to match labour markets empirical evidence. Furthermore, a game-theoretic framework is introduced to characterize a stable earning profile as a particular case of *core of an economy with coalitions of players* behaving *à la* Nash in the product market.

Keywords: *Wages Negotiations, Oligopoly, Coalitions.*

1. Introduction

It is empirically and theoretically well accepted that companies enjoying larger market shares in the product market are used to pay higher wages to their workers.[1] One of the most common explanations of such phenomenon is that firms with higher surpluses, usually associated with larger market shares, allow unions to grasp higher rents over workers' reservation wages. Using for instance a simple Nash bargaining model, the resulting equilibrium wage - given each group of players' bargaining power - turns out to be positively related to the level of firm's expected profit.

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[1] The number of empirical studies on wage differentials and unionization taking into account market shares differences across firms and industries is very high. For an account of some of them, see, for instance, Booth (1995) or, more recently, Lamo et al. (2010).

A fact usually not accounted for by this type of analysis, however, is that a high market profitability usually increases workers' reservation wages. This may be the case if the value of a trained worker's outside option when she or he decides to leave the firm (for instance after a break of the wage negotiation) either setting up a new venture as entrepreneur or establishing a partnership with other colleagues or, finally, becoming self-employed, becomes greater the higher is the industry expected profitability. As a consequence, it is not entirely clear which cause is responsible of the phenomenon described above.

The aim of this paper is to explicitly model the workers' decision to stay or to leave the firm in which they are employed, and to observe which firm's *stable earning profile* raises as a result.[2] The main assumption of the model is that at the beginning two different groups of individuals exist in the economy: the *entrepreneurs,* endowed with a specific knowledge of a given industry production process and the *workers,* endowed with just their workforce and without any knowledge about how to produce a commodity. Once an entrepreneur decides to set up a firm in a given industry, however, she or he hires a certain number of workers and these workers acquire a firm-specific training. When this happens every worker can bargain with the firm his compensation, threating to leave whenever, given his available outside options, the wage proposed by the entrepreneur is not satisfactory. Since the model assumes oligopolistic competition whithin each industry, in such a strategic environment every entrepreneur is sensitive to the possibility that employees leave their workplace setting up new competing production units. This simple ingredient of the model permits to obtain some interesting results. On the one hand, an economy stable earning profile usually comprehends a vector of wages higher than the basic neoclassical (reservation) wage, also giving rise to wage differentials across industries for initially homogeneous workers; on the other hand, this vector of wages depends upon the relative degree of competition of the industry in which firms operate. Moreover, the framework characterizes a stable earning profile as a particular case of *core of an economy with coalitions of players* behaving *à la* Nash in the product market. The equilibrium earning profile of the economy can in fact be proved to belong to such a solution set.

Among the several bargaining models existing in the literature at least two frameworks include the option for the employees to leave a firm becoming potential competitors. One, by Feinstein and Stein (1988), considers the behaviour of a firm that is aware of the potential danger of its employees' know-how. In this model, the main answer of the firm is to hire more workers yielding a sort of internal employees' redundancy. Hence, the firm is able to lower the workers' outside option and their threat

[2] Throughout all the paper this expression indicates a vector of payments received both by the employees and by the owners of a firm that respects certain stability requirements.

point in the wage negotiation. The reduction of the employees' competing firms value is thus used as a device to moderate the employees wage demand.

Another model, by Mailath and Postlewaite (1990), shows that when each worker's reservation wage is private information it might be the case that for leaving employees, even when convenient, finding an agreement on how to distribute their new firm's value can be impossible. The reason of this result is the difficulty, in a multi-agent adverse selection setup, to implement a satisfactory agreement on a collective matter.

In two somehow related papers Stole and Zwibel (1996a, 1996b) apply an intra-firm multilateral bargaining framework with non binding agreements between a firm and its worker to yield an equilibrium level of wages and employment. By assuming complete irreplaceability of each single worker, a stable (i.e. non renegotiable) earnings profile for the firm and the workers is characterized and proved to be equal to the Shapley value of the corresponding cooperative game. Two main assumptions seem to be responsible for their results. The first is the adoption of a decreasing returns of scale production function that modifies the usual split-the-pie bargaining solution in what gives the firm an incentive to hire more workers in order to reduce their marginal contractual power. The second is that the behaviour of the firm is substantially parametric when facing each employee's departure. This feature basically implies that the workers' reservation wage is unaffected by every action subsequent their departure. More recently, Baccara and Razin (2007) model the sequence of information spillover among agents as a sequential bargaining process occurring between informed and uninformed agents. Similarly, in Marini (2006) it is modelled explicitly the process of sequential entry of informed workers increasing market competition in a simple multi-stage setup. None of the above mentioned papers considers the possibility that workers leave the firm in *coalitions*.

The paper presented here, albeit through a different framework, takes a further step toward the direction of explicitly modelling each worker's outside option and investigating its consequences. The main assumption responsible of the paper results is basically one: after being hired by a firm as employees the workers dispose of the necessary know-how to potentially set up a new production unit. As a consequence, although the entrepreneurs of the initial firms can substitute without cost the departing workers with other unemployed people, there is an indirect cost to be paid in terms of product market increased competition. This simple fact permits to calculate a stable earning profile, that is, a not improvable payoff vector for all individuals of the economy.

In its basic structure the model can be considered as mainly heuristic. The idea that all employees of a firm can immediately acquire a specific ability to become entrepreneurs or stockholder of a new firm, without being credit-constrained when bearing the necessary setup costs, is a very extreme assumption. Once some basic results are obtained, however, it would not be difficult to introduce specific types of transaction costs that, constraining each individual's behaviour, could make the picture definitely more realistic.

The next section outlines the basic structure of the model. Section 3 introduces an application of the model describing the main results of the paper. Section 4 is devote to presenting in greater detail the game-theoretic nature of the solution concept adopted. Section 5 extends some of the results of the paper. Section 6 concludes the paper.

2. The Structure of the Model

2.1. Basic Assumptions. We present here a very simple oligopolistic economy in which the most relevant feature is that every worker cannot autonomously produce without having first been trained as employee in a firm. Once one individual is recruited as employee by an entrepreneur she or he acquires a specific training and then can decide whether to stay or to leave the firm. Whenever the compensation is not convenient the worker can choose either to set up a new firm as entrepreneur, or to become self-employed or to participate as a member to a partnership with other workers. By imposing some *stability* requirements, this simple framework can thus be used to determine an equilibrium earning profile of the economy. The main assumptions of the model are listed below.

i) In the economy there are two types of agents, *entrepreneurs* and *workers*. The entrepreneurs can be distinguished into m different types, according to the specific know-how they possess. Each worker is endowed with a unit of labour, without any particular knowledge of production processes;

ii) Each entrepreneur needs to hire a certain number of workers to apply his knowledge and activate the specific production process;

iii) the employees that decide to leave an *entrepreneurial firm* can be substituted without direct costs by the entrepreneur (as long as sufficient unemployed workers are available);

iv) a leaving employee can set up a new firm alone as self-employed, as entrepreneur (using the old firm's employees), or as a member of a egalitarian partnership (with the other employees);

v) there are no explicit credit constraints for leaving workers;

vi) members of the unemployed group cannot set up a firm or be self-employed before having been trained as employees or members of a firm;

vii) the workers' training is specific to the firm and is immediately obtainable (for simplicity) once a worker become employee or member of a firm of a particular industry. This means that any deviation of a coalition of workers cannot involve workers belonging to different firms;[3]

viii) every firm acts non cooperatively within a given industry, behaving as a separate coalition;

[3] The model assumes homogeneous firms within each industry, so that the training could be considered, from this point of view, industry-specific.

ix) before being trained within a firm the workers of the economy are homogeneous;

x) the market is oligopolistic. The competition is modeled in quantities.

xi) in general, only symmetric equilibria are considered.

2.2. A Simple Oligopolistic Multi-sector Economy. This section describes a simple economy in which a finite set of individuals $N = \{1, 2, ..., n\}$ is initially distributed among two subsets, such that $N = (\{I_K\} \cup \{I_L\})$, where I_K represents the subset of entrepreneurs of the economy, while I_L is the subset of workers. As anticipated above, a distinctive feature of the economy is that, at the beginning, each *entrepreneur* owns a specific knowledge of a given industry production process, while *workers* do not.[4] For simplicity, it is assumed that every entrepreneur $i \in I_K$ sets up just one firm by hiring a certain number of workers from the group I_L. As a consequence of their lack of know-how, the members of I_L, at least initially, cannot set up a firm without first being recruited by at least one member of I_K, or by someone that worked with him before, and so on. Let us assume m different specific know-how $(l = 1, 2, ..., m)$ and, for each one, k^l entrepreneurs disposing from the beginning of this particular knowledge.[5] It turns out, thus, that the number of firms in the economy will be equal to $\underline{\mathbf{v}}^{\mathrm{T}}\underline{\mathbf{k}}^l$, where $\underline{\mathbf{v}}^{\mathrm{T}}$ is the transposed l -dimensional unitary vector and $\underline{\mathbf{k}}^l$ is the vector representing the number of entrepreneurs (and then of firms) having a knowledge of each l -th sector of production $(l = 1, 2, ..., m)$.

Since not necessarily all the workers in I_L will be hired by one of the existing entrepreneurs, the potential number of coalitions in every initial coalition structure of this economy comprehends $(\underline{\mathbf{v}}^{\mathrm{T}}\underline{\mathbf{k}}^l + 1)$ coalitions (firms) denoted S^{jl}, where $j = 1, 2, ...k^l$. That is, in the economy there are m sectors, each one with a certain number of firms $(j = 1, 2, ...k^l)$ devoted to producing a homogeneous commodity y^l. Furthermore, in general there is a coalition (that can also be empty) including all unemployed people that do not belong to any firm S^{jl}. Denoting the set of all the firms of a certain industry l as $\sum^l = \bigcup\limits_{j=1,..k^l} S^{jl}$ the set of all the unemployed people can be represented as $U = N \diagdown \bigcup\limits_{l=1,..,m} \sum^l$.

Let the production for self-consumption be excluded, by requiring every commodities to be sold in the market. After bearing a given fixed cost F, each firm S^{jl} has a production function specific for the industry $g_l : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ represented by:

$$y_{jl} = g_l \left(\mid I_L \cap S^{jl} \mid \right) \tag{1}$$

where $\mid I_L \cap S^{jl} \mid$ indicates the number of workers hired by each firm operating in that particular industry. This means that the number of workers in each firm S^{jl} will

[4] This assumption is very strong but undoubtedly useful to have an initial condition for a model whose focus is not specifically who detains firms' property right but which stable earning profiles can be achieved starting from an istitutionally very simple social situation.

[5] This means that the number of entrepreneurs can be different in each l-th sector.

be decided by every entrepreneur according to the inverse function $I^N\left(g^{-1}\left(y_{jl}\right)\right) = \mid I_L \cap S^{jl} \mid$, where the function $I^N(.) : \mathbf{R}_+ \rightarrow \mathbf{I}_+$ transforms every real number into the neareast natural number.[6]

Given the existing production function, every firm is assumed to compete *à la* Cournot in the l -th homogeneous good market, with a firm's payoff function $\pi_{jl} : \mathbf{R}_+^2 \rightarrow \mathbf{R}$ given by:

$$\pi_{jl}\left(y_{jl}, y_l\right) = p_l\left(y_l\right) y_{jl} \underset{\substack{\forall j=\left(1,..k^l\right) \\ \forall l=(1,..m)}}{} \tag{2}$$

where $p_l : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ indicates the inverse demand function of the l-th sector and $y_l = \sum_{j=1}^{k^l} y_{jl}$. Note that, in order to ensure existence and uniqueness of a Cournot equilibrium in every l-th sector $l = (1, ..m)$, the following assumptions will be considered to hold in the subsequent analysis:

A.1 the payoff of each firm of a given sector is a function of its own strategy and of the sum of strategies of all existing firms in that sector;

A.2 strategy sets Y^j are, for every firm, compact and convex and, in particular, $Y^j \subset R_+ = \left\{y_j : y_j = \left[0, \overline{y}_j\right]\right\}$, where $\overline{y}_j$ represent every j-th firm's production boundary given its capacity constraint;

A.3 every firm's payoff function, $\pi_j : Y^j \times \Re_+ \rightarrow \Re_+$, is twice continuously differentiable;

A.4 $\frac{\partial^2 p_l(y_l)}{\partial y_l^2} y_j + \frac{\partial p_l(y_l)}{\partial y_l} < 0$;

A.5 $\frac{\partial p_l(y_l)}{\partial y_l} - \frac{\partial^2 C_j(y_j)}{\partial y_j^2} < 0$;

A.6 $\pi_{jl}(.,.)$ is, for every j-th firm, strictly decreasing on y_l.

Definition 1. *A Cournot-Nash equilibrium of the multi-sector oligopolistic economy represented here is a vector of quantity y_l^* for each l-th sector such that, for every $j = 1, ..., k^l$ in a given sector, the following condition holds:*

$$\pi_j\left(y_j^*, y_{-j}^*\right) \geq \pi_j\left(y_j, y_{-j}^*\right) \ \forall y_j \in Y^j.$$

where y_{-j} is the sum of all firms' output in a given sector minus j.

It is well known that from assumptions A.1-A.5, the existence of a unique Cournot-Nash equilibrium can be proved (see, for instance, Dubey, Mas-Colell, Shubik (1980)). Consequently, also the multi-sector oligopolistic economy presented here has a unique Nash equilibrium.

In what follows, a further simplification will be made regarding the compensation system adopted by every firm. The basic reservation wage of unemployed workers,

[6] We assume that, when the real number is exactly in between two integers, the lowest one is selected.

i.e., all $i \in (\{I_L\} \cap \{U\})$, is equal to zero (because they cannot produce without first being recruited by a firm). Moreover, the entrepreneurs are assumed to pay each worker of their firm a share α_i^l of the firm's surplus (2), where $\alpha_i^l \in [0,1]$ is such that $\sum_{i \in S^{jl}} \alpha_i^l = 1$.[7]

Given these assumptions, we have that for each coalition S^{jl} the surplus is given by (2) and it is distributed according to the vector $\underline{\alpha_i}^l$. Assuming linear utility for every player, the earnings can be expressed as:

$$\left\{ \begin{array}{c} u^i = \alpha_i^l \pi_{jl}\left(y_{jl}, y_l\right), \ \forall i \in \left\{I_L \cap S^{jl}\right\}, \\ u^i = \left(1 - \sum\limits_{i \in I_L \cap S^{jl}} \alpha_i^l\right) \pi_{jl}\left(y_{jl}, y_l\right), \forall i \in \left\{I_K \cap S^{jl}\right\}. \end{array} \right. \tag{3}$$

2.3. The stability of an earning profile. Given the oligopolistic economy described above, a *stable earning profile* of the economy can be defined as a feasible vector of payments such that, given certain specific assumptions concerning the behaviour and the constraints of the individuals of the economy, anyone cannot improve upon. In our framework, given the Nash equilibrium quantities, a stable earning profile is an income distribution within each firm such that none, individually or as a group, wants to leave the firm by establishing a new production unit.

In the economy, under the assumptions made before, the Nash equilibrium surplus of every firm in an industry l is given by:

$$\pi_{jl}^* = \left\{\pi_{jl}\left(y_{jl}^*, y_l^*\right)\right\}_{\forall j=1,2,\ldots,k^l} \tag{4}$$

where, as said before, y_l^* is the Nash equilibrium quantity vector in every l-th industry. This vector also determines the partition of the workers belonging to I_L into two different groups: one group, denoted by D^*, is the set of all workers employed by all k^l entrepreneurs in each industry at the Nash equilibrium, i.e., $D^* = \sum_{l=1}^{m} \sum_{j=1}^{k^l} I^N \left(g^{l^{-1}}\left(y_{jl}^*\right)\right)$; the other, conversely, is made of all unemployed workers $U = I_L \setminus D^*$. Assuming the existence at the Nash equilibrium of a sufficiently large and non empty set U, in the economy there are many different roles to be potentially undertaken by each individual active in a firm. For each employee the choice is between:

(1a) staying in the firm as employee and negotiating with the entrepreneur a share α_i^l of the firm's equilibrium profit (4);

(2a) leaving the firm (after being trained) alone (or with other workers), becoming entrepreneur(s) of a new firm in the industry, by recruiting workers either from the

[7] Basically this feature of the model simplifies the problem when compared to the adoption of a fixed wage, without any particular loss of generality. Note also that, since firms are assumed symmetric in each industry l, the share to be paid is the same in each j firm of a particular industry and then is indicated as α^l.

same or from another firm (in the same industry), knowing that the entrepreneur in S^{jl} will recruit other workers from the set U;

(3a) leaving individually the firm and becoming self-employed, producing a certain amount of the product (with only one unit of work), given that the existing k^l entrepreneurial firms will continue to produce according to their best-reply;

(4a) leaving the firm (after being trained) with other firm's workers and becoming member of a partnership, given that the existing k^l entrepreneurial firms will continue to produce according to their best-reply;

(5a) entering the unemployed set, obtaining the basic reservation wage of the economy equal to zero.

For each entrepreneur, apart from the choice to stay in the firm as entrepreneur, choices (1a)-(5a) described above are similarly feasible. Therefore, in equilibrium, the entrepreneur will never earn less than her or his workers, otherwise she or he would prefer to deviate setting up another firm.

Thus, an earning profile of the economy (a vector $\underline{\mathbf{z}}$ of remunerations for all the players) can be viewed as stable if, given the partition of the N players into $(\underline{\mathbf{v}}^{\mathrm{T}}\underline{\mathbf{k}}^{l}+1)$ coalitions (k^l firms S^{jl} in each industry plus the unemployment group U) according to the Nash equilibrium quantity vector of every l-th industry y_l^*, no individual or group of individuals within each firm S^{jl} can improve upon $z^i \in \underline{z}^{S^{jl}}$ by deviating, where $\underline{z}^{S^{jl}}$ describes the vector of remunerations obtained in equilibrium within every S^{jl}.[8] Note that the allowed deviations are as in (1a)-(5a) above. Moreover, the behaviour of individuals within the complementary coalitions, i.e. all the coalitions $S^{jl}\diagdown T$, (where T indicates a deviating coalition) is supposed to be that of continuing to produce as before the deviation occured, according to their best-reply. This is possible through the recruitment of a certain number of workers from the unemployment set U.[9] Moreover, note that only consistent deviations are considered, that is, deviations that cannot, in turn, be objected.

It is now time to describe the formal conditions required to an earning profile $\underline{\mathbf{z}}$

[8] By assumption the unemployed people cannot autonomously setup firms and therefore cannot deviate from $\underline{z}$. We will describe more in detail the formal setup behind the analysis in the next section.

[9] This behaviour may appear myopic but can actually be justified. In fact, suppose as a benchmark a constant returns to scale production function like $y^j = \ell^j$ for each j firm, where ℓ^j is the number of workers hired by each entrepreneur of a certain sector. In this particular case the equilibrium price of each market $p\left(L^*\right)$, where $L^* = \sum_{j=1}^{k^l} \ell^{j*}$, is not affected by transferring trained workers from a firm to another. So it easy to see that the equilibrium condition for each firm to be indifferent whether or not to hire other firms' workers would be in equilibrium to fix a wage $w = p\left(L^*\right)$. This is the wage such that makes the firm that steals a worker to another firm sure to not be stolen the same worker. Since for this equilibrium wage every entrepreneur obtains zero profits, it seems totally reasonable to assume that he will react to any of her or his workers'deviation by hiring non trained workers, rather than keep the number of trained workers fixed. Thus, firms' Cournot behaviour after a deviation appears reasonable at least in the case of constant returns to scale.

to be stable.

The Nash equilibrium quantity (and consequently the number of I_L employed in each S^{jl}) is, by the symmetry of the equilibrium considered, identical for all existing firms in an industry l. Thus, the set of conditions making each firm's earning profile $z^{S^{jl}}$ stable can be characterized through a level of share α^l of the profits (equal by symmetry for all firms of an industry so the superscript j can be dropped) paid to *all employees* of a firm of that given industry, i.e., $\alpha^l = \sum_{i \in \left\{I_L \cap S^{jl}\right\}} \alpha_i^l$. The level of α^l can be determined through the respect of the following constraints:

a) No employee of every firm has to find convenient to become entrepreneur and setting up a *new* firm by hiring an optimal number of workers and paying them a share of the profit sufficient for them to stay. This condition holds when:

$$\frac{\alpha^l\left(k^l\right)\cdot\pi_{jl}^*\left(k^l\right)}{\ell_{jl}^*\left(k^l\right)} \geq \left(1-\alpha^l\left(k^l+s'+1\right)\right)\pi_{jl}^*\left(k^l+s'+1\right) \tag{5}$$

where $s' = k^l \cdot \left(\triangle\ell_{jl}^*\left(k^l\right)\right) = k^l\left(\ell_{jl}^*\left(k^l\right)-\ell_{jl}^*\left(k^l+1\right)\right)$ is the number of firms (positive by assumption A.4 and A.5) created by all workers dismissed from the k^l firms as a result of the entry of the new firm; $\alpha^l\left(k^l+s+1\right)$ indicates the share of the new firm's profit that the worker become entrepreneur earns in the new firm; $\pi_{jl}^*\left(k^l+s+1\right)$ represents the equilibrium profit of the new firm (now the market includes $\left(k^l+s+1\right)$ firms); $\pi_{jl}^*\left(k^l\right)$ represents each initial firm's equilibrium payoff and $\ell_{jl}^*\left(k^l\right) = \left\{I^N\left(g^{l^{-1}}\left(y_{jl}^*\left(k^l\right)\right)\right)\right\}_{\forall j\in k^l}$ is the number of employees hired by every firm given the initial number of firms in the market. Note that, by symmetry, within each firm, $\alpha_i^l\left(k\right) = \frac{\alpha^l\left(k^l\right)}{\ell_{jl}\left(k^l\right)}$. Moreover, note that when condition (5) holds, leaving the firm and becoming entrepreneurs it is not even profitable for a group of workers (that thus need to pay the other reruited unemployed workers their equilibrium share);

b) No employee of a firm has to find convenient to become a member of a newly created partnership with some other firm's employees. This condition holds when:

$$\frac{\alpha^l\left(k^l\right)\cdot\pi_{jl}^*\left(k^l\right)}{\ell_{jl}^*\left(k^l\right)} \geq \frac{\pi_{jl}^*\left(k^l+s''+1\right)}{\ell_{jl}^*\left(k^l+s''+1\right)} \tag{6}$$

where the RHS represents each partnership member's equilibrium payoff when a new firm of this type (besides the s'' induced new entries) enters the market;

c) No employee of a firm has to find convenient to become self-employed:

$$\frac{\alpha^l\left(k^l\right)\cdot\pi_{jl}^*\left(k^l\right)}{\ell_{jl}^*\left(k^l\right)} \geq E^*\left(k^l+s'''+1\right) \tag{7}$$

where E^* is a self-employed's equilibrium payoff when a new firm of this type (besides the s'''induced new entries) enters the market;

d) None of k^l entrepreneurs in each market has to earn less than an employee. This corresponds to the condition:

$$\left(1-\alpha^l\left(k^l\right)\right)\pi^*_{jl}\left(k^l\right) \geq \frac{\alpha^l\left(k^l\right)\cdot\pi^*_{jl}\left(k^l\right)}{\ell^*_{jl}\left(k^l\right)} \tag{8}$$

e) *internal consistency*:

$$\left(1-\alpha^l\left(k^l\right)\right)\pi^*_{jl}\left(k^l\right) \geq \left(1-\alpha^l\left(k^l+s+1\right)\right)\pi^*_{jl}\left(k^l+s+1\right), \tag{9}$$

that expresses the fact that each entrepreneur has to find convenient to pay her or his employees the equilibrium wage (characterized by the share $\alpha^{k^l,l}\left(k^l\right)$) rather than let one (or more than one) employee leaving, establishing a new firm and then paying workers the new equilibrium wage. Note that when condition (5) holds with equality, expression (9) implies (8). Therefore, constraint (8) is required just when condition (6) or (7) imply shares $\alpha^l\left(k^l\right)$ higher than that respecting condition (5). Moreover, due to the recursive nature of condition (5), a further assumption is needed. In fact, the solution of condition (5) implies that, in each round of entry, the most profitable deviation for employees is always that expressed by the choice (2a). In order to avoid that a choice that is unprofitable in a given round becomes profitable in successive rounds of entry, thus making impossible to solve expression (5), the following condition is imposed in what follows.

A.7 (*No-crossing condition*) *Let α_1, α_2 and α_3 be the share $\alpha^l\left(k^l\right)$ that respect condition (5), (6) and (7), respectively, with equal sign. When, for a given number of existing firms k^l, $\alpha^l\left(k^l\right)=\max\left\{\alpha_1,\alpha_2,\alpha_3\right\}$ then the same condition holds for every $t>k^l$.*

Hence, when the vector $\alpha^l\left(k^l\right)=\left(\alpha^1,\alpha^2,...\alpha^m\right)$ respect all conditions listed above, the corresponding earning profile of the economy $\underline{\mathbf{z}}\left(\underline{\alpha}^l\left(k^l\right)\right)$ can be considered to possess some properties of stability. Using (7)-(11) and given an arbitrary initial number k^l of firms existing in each industry, the proposition that follows characterizes the vector $\underline{\alpha}^l$ associated to the stable earning profile $\underline{\mathbf{z}}\left(\underline{\alpha}^l\right)$ of the economy.Let us, for ease of notation, denote $I^N\left(\ell_{jl}\left(k^l\right)\right)$ as $\widehat{\ell}_{jl}\left(k^l\right)$.

Proposition 1. *Under no-crossing condition and for a given number of entrepreneurs (and firms) existing in every industry, a stable earnings profile of the economy is characterized by the following share α^{l*} of profit in every S^{jl} of a given industry: (i)*

$$\alpha^{l*}=\frac{\sum_{t=1}^{T}(-1)^{t+1}\pi^*_{jl}\left(k^l+\left(k^l+t\right)\cdot\sum_{h=1}^{t}\left(\triangle^h\widehat{\ell}^*_{jl}(k+h-1)+t\right)\right)\prod_{h=0}^{t-1}\widehat{\ell}^*_{jl}\left(k^l+\left(k^l+t\right)\cdot\sum_{h=0}^{t-1}\left(\triangle^h_{jl}\widehat{\ell}^*_{jl}(k+h-1)+h-1\right)\right)}{\pi^*_{jl}\left(k^l\right)}$$

if $\alpha_1 = \max\ \{\alpha_1, \alpha_2, \alpha_3\}$ *and* $\alpha^{l*} \leq \frac{\pi^*_{jl}(k^l) - E^*_{j,l}(k^l)}{\pi^*_{jl}(k^l)}$; *(ii)*

$$\min\left\{\frac{\widehat{\ell}^*_{jl}}{(1+\widehat{\ell}^*_{jl})}, \frac{\pi^*_{jl}\left(k^l\right) - E^*_{j,l}\left(k^l\right)}{\pi^*_{jl}\left(k^l\right)}\right\} \geq \alpha^{l*} = \frac{\pi^*_{jl}(k^l+\triangle\widehat{\ell}^*_{jl}(k^l)+1)\cdot\widehat{\ell}^*_{jl}(k^l)}{\pi^*_{jl}(k^l)\cdot\widehat{\ell}^*_{jl}(k^l+\triangle\widehat{\ell}^*_{jl}(k^l)+1)}$$

if $\alpha_2 = \max\ \left\{\alpha_1^{'}, \alpha_2, \alpha_3\right\}$, *and*

$$\alpha_1^{'} = \frac{\pi^*_{jl}(k^l+\triangle\widehat{\ell}^*_{jl}(k^l)+1)\cdot\widehat{\ell}^*_{jl}(k^l)}{\pi^*_{jl}(k^l)} - \frac{\pi^*_{jl}(k^l+\triangle\widehat{\ell}^*_{jl}(k^l)+\triangle^2\widehat{\ell}^*_{jl}(k^l+1)+2)\widehat{\ell}^*_{jl}(k^l)\widehat{\ell}^*_{jl}(k^l+\triangle\widehat{\ell}^*_{jl}(k^l)+1)}{\pi^*_{jl}(k^l)\widehat{\ell}^*_{jl}(k^l+\triangle\widehat{\ell}^*_{jl}(k^l)+\triangle^2\widehat{\ell}^*_{jl}(k^l+1)+2)};$$

(iii)

$$\frac{\ell^{k^l,l*}}{\left(1+\ell^{k^l,l*}\right)} \leq \alpha^{l*} = \frac{E^{l*}(k^l+\triangle\ell^*_{jl}(k^l)+1)\cdot\ell^*_{jl}(k^l)}{\pi^*_{jl}(k^l)}$$

if $\alpha_2 = \max\ \left\{\alpha_1^{''}, \alpha_2, \alpha_3\right\}$ *and*

$$\alpha_1^{''} = \frac{\pi^*_{jl}(k^l+\triangle\widehat{\ell}^*_{jl}(k^l)+1)\cdot\widehat{\ell}^*_{jl}(k^l)}{\pi^*_{jl}(k^l)} - \frac{E^{l*}(k^l+\triangle\widehat{\ell}^*_{jl}(k^l)+\triangle^2\widehat{\ell}^*_{jl}(k^l+1)+2)\widehat{\ell}^*_{jl}(k^l)\widehat{\ell}^*_{jl}(k^l+\triangle\widehat{\ell}^*_{jl}(k^l)+1)}{\pi^*_{jl}(k^l)}.$$

Proof. When every α^{l*} is equal to the maximum value amongst the three expressions above (i), (ii) and (iii) , the constraints (5), (6) and (7) are respected for each coalition S^{jl} of a given industry. When $\alpha_1 = \max\ \{\alpha_1, \alpha_2, \alpha_3\}$, the first expression for $\alpha^{l*}$is obtained by iteratively solving the differential equation in (5) by assuming a finite number of potential entrants $t = T$. This is the maximum number of available entrants, given that the set of unemployed people is finite. Note that, for $t = (1, ...T-1)$, we indicate with $\triangle^t\widehat{\ell}^*_{jl}\left(k^l+t+1\right) = \widehat{\ell}^*_{jl}\left(k^l+t\right) - \widehat{\ell}^*_{jl}\left(k^l+t+1\right)$ the number of workers dismissed by every firm at each round of entry as a consequence of the reduction of output. However, the expression can still be solved assuming an infinite number of potential entrants t. We will consider these different possibilities in the model application. It has to be noticed that when the constraint (5) is satisfied, no coalition of workers would like to become entrepreneur of a new firm. The share $\left(1-\alpha^l\left(k^l+1\right)\right)$ of the new firm profit should be in fact divided among all deviators ensuring, consequently, a lower earning. When the best workers' outside option is characterized by α_2 (corresponding to the option to create a partnership), the entire problem must be modified to account for the fact that, in the iterative solution of (5), the best outside option for the employees working in the new entrepreneurial firm created at the first round of entry is now represented by the share α_2. Provided this, α_1 must be modified accordingly, thus yielding α_1' as expressed above. The share α_2 is thus nothing but the solution of constraint (6). A similar reasoning must be done when the best workers' outside option is to be self-employed (corresponding to the share α_3), which is then used to derive α_1''. We already said above that the condition is always respected when α^{l*} respects all constraints (5)-(9). Thus the initial entrepreneurs will always find convenient to pay their employees the equilibrium share of the profit $\alpha^{l*}\left(k^{l}\right)$ rather than risk their departure and an increase in the existing market competition. Finally, the LHS of the expressions above represents the solution of constraints (8) and (9), ensuring that every entrepreneur does not have incentive to deviate or give rise to a self-employed equilibrium. Since α^{l} is equal to the maximum among the three expressions presented above, by satisfying the tightest of the constraints, the entrepreneur will automatically respect the others. Furthermore, straightforward calculations show that the solution of difference equation (9) is simply given by $\alpha^{k^{l},l} \leq 1$, that is always respected by definition. Thus for the entrepreneurs is always convenient to pay their employees the equilibrium share of the profit $\alpha^{l}\left(k^{l}\right)$ rather than risk their departure and an increasing in the existing market competition. When these inequality hold, provided that under the equilibrium share α^{l} the constraints (2.5)-(2.7) are satisfied, the entrepreneurs will never find convenient neither to set up a new firm as entrepreneur, partner or self-employed, nor to be self-employed from the beginning. In general then, given the partition of the economy determined by the Nash equilibrium, when the vector $\underline{\alpha}^{*}=\left(\alpha^{1*},.,..\alpha^{m*}\right)$ respects all the expression represented above, the corresponding earning profile of the economy:

$$\underline{\mathbf{z}}\left(\underline{\alpha}^{*}\right)=\left\{\left(1-\alpha^{l*}\right)\pi^{1,l*},...,\left(1-\alpha^{l*}\right)\pi^{k^{l},l*}\right\}_{\forall i\in\left(I_{K}\cap S^{jl}\right)\text{ and } l=1,..m},$$

$$\left\{\frac{\alpha^{l*}\pi^{1,l*}}{\ell^{1,l*}}\right\}_{\forall i\in\left\{I_{L}\cap S^{1,l}\right\}},...,\left\{\frac{\alpha^{l*}\pi^{k^{l},l*}}{\ell^{k^{l},l*}}\right\}_{\forall i\in\left\{I_{L}\cap S^{k^{l},l}\right\}\text{and } l=1,..m},\ \{0\}_{\forall i\in U},$$

is stable. Existence and uniqueness of such a solution will be considered in detail in the next section. ■

The proposition just indicates the way to find the stable earning profile $\underline{z}(\underline{\alpha}^{*})$ of the oligopolistic economy described above. What it is basically stated is that, under simple stability conditions, each firm has to offer a wage sufficiently high to keep its workers inside the firm. This wage - expressed as a share of the firm's profit - has to be high enough to prevent that even the finest subcoalition of the firm, represented by each single worker, can setup a new firm in the form of an entrepreneurial firm, a partnership or a self-employed unit by using the workforce available in the economy.

3. A LINEAR APPLICATION OF THE MODEL

3.1. A One-industry Economy. In order to show that the workers' equilibrium share of the profits (and then their remuneration) possess the features described in the introduction, let us introduce a simple single-industry example. For simplicity, let the production function of each firm of the industry be a linear function of the number of workers within the firm, that is, $y^{j} = \mid S^{j}\cap I_{L}\mid$. In order to make calculations even simpler, let $p\left(Y\right)=a-Y$, be the linear inverse demand for the homogeneous good, where $Y = \sum_{j=1}^{k} y^j$ is the total quantity of the good delivered in the market, with $a > Y$.

Straightforward manipulation of the best reply functions obtained from the maximization of (4), (5) and (6) yield the Nash equilibrium values of the main variables and which of every firm's payoff, as shown in the table below.

Tab.1 - Relevant results of the oligopolistic market

VARIABLES	EQUILIBRIUM VALUES
y^{j*}, ℓ^{j*}	$\left(\frac{a}{(k+1)}\right)$
Y^*	$\frac{a \cdot k}{(k+1)}$
$p(Y^*)$	$\left(\frac{a}{(k+1)}\right)$
π^{k*}	$\left(\frac{a^2}{(k+1)^2} - F\right)$
π^{k+1*}	$\left(\frac{a^2}{(k+2)^2} - F\right)$
V^{k+1*}	$\left(\frac{a-(k+2)\sqrt{F}}{(k+1)}\right)$
E^{k+1*}	$\left(\frac{a+1}{k+1} - F\right)$

By proposition 1, using the results of tab 1, the value of α^* that makes z core-stable is given by the maximum between:

$$\alpha_1 = \frac{\pi^{k+1*} . \ell^{k*}}{\pi^{k*} \cdot \left(1+\ell^{k+1*}\right)} = \frac{\left(\frac{a^2}{(k+2)^2} - F\right) \cdot \left(\frac{a}{(k+1)}\right)}{\left(\frac{a^2}{(k+1)^2} - F\right) \cdot \left(1 + \frac{a}{(k+2)}\right)},$$

$$\alpha_2 = \frac{V^{k+1*} . \ell^{k*}}{\pi^{k*}} = \frac{\left(\frac{a-(k+2)\sqrt{F}}{(k+1)}\right) \cdot \left(\frac{a}{(k+1)}\right)}{\left(\frac{(a)^2}{(k+1)^2} - F\right)}$$

and

$$\alpha_3 = \frac{E^{k+1*} \ell^{k*}}{\pi^{k*}} = \frac{\left(\frac{a+1}{k+1} - F\right) \cdot \left(\frac{a}{(k+1)}\right)}{\left(\frac{(a)^2}{(k+1)^2} - F\right)}$$

subject to the fact that:

$$\max \ \{\alpha_1, \alpha_2, \alpha_3\} \leq \alpha_4 = \frac{\ell^{k*}}{(1+\ell^{k*})} = \frac{\left(\frac{a}{(k+1)}\right)}{(1 + \frac{a}{(k+1)})}. \tag{10}$$

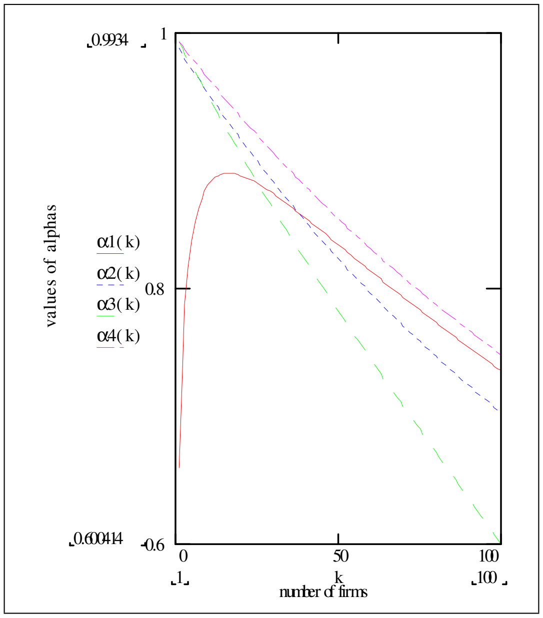

Fig..1 - Values of $\alpha_1, \alpha_2, \alpha_3$ and α_4
$(a = 1000, F = 15$ and $k = 1, ...100.)$

The picture above plots the values of α_1, α_2, α_3 and α_4 (for given values of parameters) over a variable number of initial existing firms in the economy ($k = 1, ...100$) .It is immediately visible that, for the values of the parameters selected, α_4 is greater than the other alphas over all the range of k. This implies that the LHS of the condition (13) is respected and every entrepreneur earns more than his employees. Moreover, for a low number of firms in the economy, α_2 is the maximum among $\alpha_1, \alpha_2, \alpha_3$ and then it represents the equilibrium workers' share of the profits in every firm. For a larger number of firms, however, α_1 becomes the equilibrium share. The values of α_1 are positively related to k for a relatively low number of firms. This depends on the fact that the leaving employee has to pay a lower share β to convince the other employees to follow him (in order to keep their remuneration unaffected), the higher the number of firms in the market. However, for a high number of firms k, the negative effect of the number of firms on the equilibrium α (needed to keep each employee within the firm) prevails on the effect of k on β. Note that when every firm's profit tends to zero the equilibrium earning share tends to zero as well. This feature ensures that, since the workers' outside option varies with the given partition of the economy, the economy equilibrium earning profile changes according to the monetary value of every firm's market share. When either the dimension of the market (represented by the parameter a) increases, or the fixed cost F necessary to set up a firm decreases, or the number of firms in the economy rises, the equilibrium workers compensation decreases as well. This appears to be a basic feature of the model presented.

3.2. A Two-sector Economy. It is straightforward to extend the results obtained above to an economy in which there is more than one industry. An interesting feature of this extension is that, since the workers' training is specific to every firm belonging to a certain industry, each worker can negotiate a share of his firm's profit by using the threat of setting up a new firm *only* in that particular industry. This feature makes possible the existence of an economy earning profile characterized by different levels of compensation for individuals that, although initially homogeneous, operate in different industries. The Figure 2 below depicts the equilibrium α^{l*} in two different industries designed to represent, for instance, a heavy industry ($l = 1$) characterized by a large market size and high fixed costs and a light industry ($l = 2$), with a small market size and low fixed costs. In this example, $\alpha^{1*} = \alpha_3$, that means that the light industry workers' compensation depends on a self-employment leaving threat, while the heavy industry equilibrium employees' share is $\alpha^{2*} = \alpha_2$, that indicates that the partnership-type of deviation is the active threat for the workers of this industry. A result of this extreme specificity of the workers' training across industries is the very high discrepancy between workers' equilibrium compensations between the two industries, even when the number of firms is the same in each industry. In Figure 3 the equilibrium earnings (equal to $z^i\left(\alpha^{l*}\right) = \alpha^{1*} \cdot \frac{\pi^{k^l,l*}}{\ell^{k^l,l*}}$, $l = 1, 2$) for both industry workers are plotted over the number of firms existing in those industries.

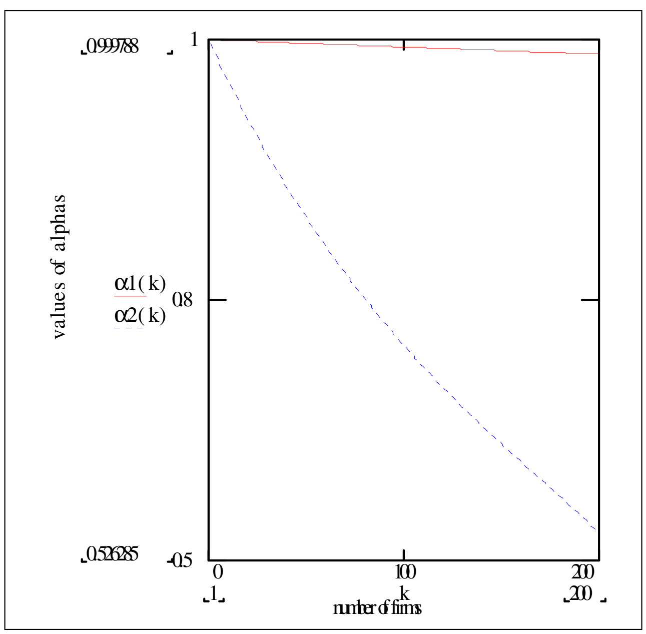

Fig. 2 - Values of α^{1*} (for $a^1 = 100000$, $F^1 = 50$) and α^{2*} (for $a^2 = 250$, $F^2 = 1.03$) for $k^l = 1, .., 100$.

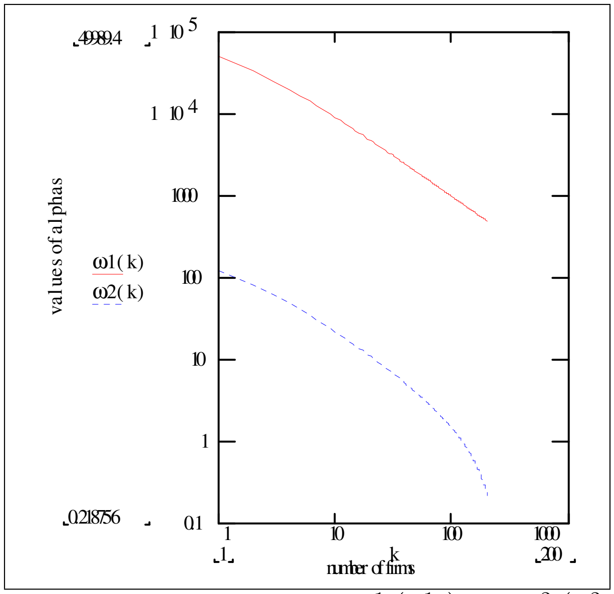

Fig. 3 - Equilibrium remuneration $\omega^1\left(\alpha^{1*}\right)$ and $\omega^2\left(\alpha^{2*}\right)$ for the employees working in the two industries ($k = 1, ...200$).

It is evident that the larger surplus obtained by every firm in the heavy industry togheter with the more aggressive employees' threat operative in that industry determines a large wage differential over the equilibrium remunerations across the two sectors of the economy.

Many other cases are possible in the framework presented. The economy vector of workers' remuneration can vary across industries due to the different number of firms operating in those industries even with the same size of market and equal fixed costs. A certain industry workers' remuneration can be for instance be lower (due to the large number of firms), even in presence of a level of fixed investment higher than that of the other sectors of the economy, and so on.

4. Game-theoretic Nature of a Stable Economy Earning Profile

This section briefly outlines the game-theoretic features underlining the model presented. The purpose is mainly to show that the stable earning profile described in previous sections is equivalent to the core of a game in which only certain deviations are allowed and coalitions act as separate entities behaving *à la* Nash in any event. Firstly, some basic notions are introduced in order to illustrate the main result obtained in this section.

4.1. Game and Equilibrium Concept. The framework described above can be represented as a particular type of *n-players transferable utility game in normal form* given by:[10]

[10] Given a n-players game in coalitional (or characteristic function) form (N, v) where N is the finite set of players $N = \{1, 2, ..., n\}$ and for each coalition $S \subseteq N$, $v(S)$ is a compact non empty subset

$$\Gamma = \left\{N, Y^{jl}, v\left(S^{jl}\left(y^{l}\right)\right), z^{i}\right\} \tag{11}$$

where N is a finite set of players, Y^{jl} is the set of strategies (quantities) available to each coalition (firm), $v\left(S^{jl}\left(y^{l}\right)\right)$ is the worth of each coalition S^{jl}, and z^{i} is the payment received by each player within a coalition. It can be noticed that $v\left(S^{jl}\left(y^{l}\right)\right)$ depends upon the strategy profile $y^{l} \in Y^{l} = \Pi_{j\in\Sigma^{l}} Y^{jl}$ selected by the coalitions opearating in each l -th sector. This for two reasons: firstly, y^{l} determines the number of each firm's workers through the function $\left|I_{L} \cap S^{jl}\right| = g^{l^{-1}}\left(y^{jl}\right)$ defined for each firm of a sector; secondly it defines the surplus available to each coalition S^{jl} through the function (4), (5) and (6) defined above. The assumption of the previous sections was that the strategy of a coalition is decided by the player (or players) that detains the property rights over the fixed asset of a coalition and that then pays F. Thus $y^{jl} \in Y^{jl}$ is decided respectively by the entrepreneur (or new entrepreneur), in the event that any deviation occurs or a deviation occurs both to set up a new entrepreneurial firm (like in (2a) and (2b)) and to switch the role whithin the firm (like in (6b)), by a single self-employed (case (3a) or (3b)) and, finally, by the leaving workers (case (4a) or (4b)). In all these cases for each firm $j = 1, ..., k^{l}$, y^{jl} is chosen in a Cournot fashion within the l -th industry. In the model presented above, chosing a strategy means to select the number of workers offering a unit of labour to be included in the coalition S^{jl}. Furthermore, it is assumed that $Y^{U} = \{\emptyset\}$, that is, unemployed people do not have any strategy available.

It has been already pointed out as the possible coalitional deviations are constrained to the cases (1a)-(5a) and (1b)-(6b) illustrated above. In any of these events the assumed reaction of complementary coalitions is to continue to behave as before by using the available workers in U and adopting a Nash strategy. Under these assumptions, an equilibrium of the game (14) can be defined as a couple $(\underline{y}^{*}, \underline{\alpha}^{*})$ such that $\underline{y} \in \prod_{l=1,..m} Y^{l}$ is a Nash Equilibrium profile of strategies for all the firms of all industries, i.e.:

$$\left\{\underline{y}^{*} \mid \forall l \text{ and } \forall j \nexists\, y^{jl} \in Y^{jl} \text{ s.t. } \left(1-\alpha^{l}\right)\pi^{jl}\left(y^{jl}, y^{l*}_{-j}\right) \geq \left(1-\alpha^{l}\right)\pi^{jl}\left(y^{jl*}, y^{l*}_{-j}\right)\right\}$$

and $\underline{\alpha}^{*} = \left(\alpha^{1}, \alpha^{2}, ..., \alpha^{l}\right)$ is such that:

$$\left\{\underline{\alpha}^{*} \mid \forall l \text{ , } \underline{z}^{S^{jl}}\left(\alpha^{l*}\right) \in C\left[S^{jl}, v\left(S^{jl}\left(y^{l*}\right)\right)\right] \right\}$$

of the Euclidean space $\mathbf{R}^{S}$, the mapping v is called characteristic function. Moreover, the game is a transferable utility (TU) game if, for every $S \subseteq N$, there exists a nonnegative scalar $k(S)$ such that $v(S) = \left\{x \in \mathbf{R}^{S}_{+} \mid \sum_{i\in S} x_{i} \leq k(S)\right\}$. This nonnegative scalar is thus the range of the mapping $v : S \rightarrow \mathbf{R}$ and it is called the worth of the coalition S. This game can be reduced in normal form by assigning to each coalition a given set of strategies.

for $y^{l*} \in Y^l = \Pi_{j\in\Sigma^l} Y^{jl}$ and the core is defined as:

$$C\left(\left(S^{jl}\right), v\left(S^{jl}\right)\right) = \left\{z \in v\left(S^{jl}\right) \mid \nexists\ T \subseteq S^{jl}\left(y^{l*}\right) \text{ and } \mathbf{e}_t \in v\left(T\right) \text{ s.t. } \mathbf{e}_t \geq z^{S^{jl}\left(y^{l*}\right)}\right\}$$

where $z_t^{S^{jl}\left(y^{l*}\right)}$ is the projection of z on $\mathbf{R}^{S^{jl}\left(y^{l*}\right)}$ and, for every $T \subseteq S^{jl}\left(y^{l*}\right)$, $v\left(T\right) \in \mathbf{R}$ (a non negative scalar), such that, $\mathbf{e} \in v\left(T\right)$ if and only if $e^i \geq 0$ for all $i \in T$ and $\sum_{i\in T} e^i \leq v\left(T\right)$. Thus, it turns out that the concept of core used above is a variant of the usual core, for two main reasons:

- firstly, each coalition S^{jl} acts non cooperatively with respect to all other coalitions it can interacts with;

- secondly, in the core solution concept enclosed within the specific non cooperative game it is assumed that deviating coalitions formed with players belonging to different coalitions S^{jl} do not have sufficient blocking power and then only intra-coalitions $T \subseteq S^{jl}$ can profitably deviate from a given imputation $\mathbf{z}^{S^{jl}}$.[11] Now, it becomes possible to present the main result of this section.

Proposition 2. *Given that the set N of economy players distribute themselves between $(\underline{\mathbf{v}}^{\mathrm{T}}\underline{\mathbf{k}}^l + 1)$ coalitions,such that, for each $\left(\underline{\mathbf{v}}^{\mathrm{T}}\underline{\mathbf{k}}^l\right)$-th coalition S^{jl}, $\left|S^{jl} \cap I_L\right| = g^{l^{-1}}\left(y^{jl*}\right)$ and the last coalition is $U = I_L \diagdown D^*$, where y^{jl*} is the Nash equilibrium strategy for each S^{jl} and $D^* = \sum\limits_{l=1}^{m}\sum\limits_{j=1}^{k^l} g^{l^{-1}}\left(y^{jl*}\right)$, when the earning profile $\underline{\mathbf{z}}\left(\underline{\alpha}^*\right)$ is characterized by the equilibrium vector of shares $\underline{\alpha}^* = \left(\alpha^{1*}, \alpha^{2*}, ..., \alpha^{m*}\right)$ defined by Proposition (1), and when the allowed deviations are as in (1a)-(5a) and (1b)-(6b), each $z^{S^{jl}} \in \underline{\mathbf{z}}\left(\alpha^{l*}\right)$ is such that $z^{S^{jl}} \in C\left[S^{jl}, v\left(S^{jl}\left(y^{l*}\right)\right)\right]$.*

Proof. From the proposition 1 we know that when $\underline{\alpha}^* = \left(\alpha^{1*}, .., \alpha^{m*}\right)$ respects the equilibrium condition, any single individual cannot, given the Nash equilibrium, improve his payoff by deviating alone or with a group of the firms' workers . Since the allowed deviations by every firm's subcoalition $T \subset S^{jl}$ are those assumed, it follows that, given that the other coalitions in each industry behave à la Nash in the event of any deviation (adopting a strategy $y^{S^{jl}*}$ given the other firms' equilibrium

[11] In this respect both these assumptions make our equilibrium concept similar to the *hybrid equilibrium* considered in Zhao (1992), that is a refinement of Aumann's α-core for a *given* coalition structure of the set of players. There are, however, two main differences: firstly, our equilibrium assumes that complementary coalitions $S^{jl} \diagdown T$ continue to behave *à la* Nash; secondly, the coalition structure is not fixed since every deviation implies a finer coalition structure of the N players of the economy.

strategy $y_{-S^{jl}}{}^{*}$), in each firm of an industry, given the vector of quantity $\left\{\underline{y}^{l*}\right\}_{l=1,..m}$, the earning profile $\underline{\mathbf{z}}\left(\alpha^{l*}\right)$ coincides with the core of every firm of the industry:

$$\underline{\mathbf{z}}\left(\alpha^{l*}\right)=\left\{\underline{\mathbf{z}}\in v\left(S^{jl}\left(y^{l*}\right)\right)\,\middle|\,\nexists T\subset S^{jl}\left(y^{l*}\right)\text{ and }\mathbf{e}\in v(T(y^{l*}))\text{ s.t. }\mathbf{e}\geq z^{S^{jl}\left(y^{l*}\right)}\right\}$$

where $z^{S^{jl}\left(y^{l*}\right)}$indicates the equilibrium allocation within each j-th firm belonging to the l-th sector of the economy when the Nash equilibrium vector y^{l*} has been implemented. Since this is true for every industry and no deviations are allowed across industries, the result follows. ■

5. SOME EXTENSIONS OF THE MODEL

5.1. The Case of Full (or Near Full) Employment. When in section 2 we described all the possible deviations available to the economy coalitions, the assumption was that in the economy a non empty set of unemployed workers $U=I_L\diagdown D^*$ exists in equilibrium. When, conversely, this set does not include sufficient people to replace leaving workers in the event of a deviation, the stable earning profile described above turns out to be different. In particular the following distinct cases can be described:

i) $\ell_{h=1,...r}^{k^l+h,l^*}\leq|U|<\ell^{k^l+1,l^*}$, that is, in the economy there are not enough people to replace the leaving workers (from one of the k^l -th firms) when they decide to setup a new entrepreneurial firm, while there are enough to replace them when they decide to setup a partnership. In this case the stable earning profile $\underline{z}^*\left(\underline{\alpha}^*\right)$ is characterized by a vector $\underline{\alpha}^*=\left(\alpha^{1*},\alpha^{2*},...,\alpha^{m*}\right)$ such that, for each $l=(1,..,m)$:

$$\frac{\ell^{k^l,l*}}{\left(1+\ell^{k^l,l*}\right)}\geq\alpha^{l*}=\max\left\{\frac{\pi^{(k^l+1,l)^{S.O.}*}.\ell^{k^l,l*}}{\pi^{k^l,l*}.\left(1+\ell^{(k^l+1,l)^{S.O.}*}\right)},\frac{V^{k^l+1,l*}.\ell^{k^l,l*}}{\pi^{k^l,l*}},\frac{E^{k^l+1,l*}\ell^{k^l,l*}}{\pi^{k^l,l*}}\right\}\tag{12}$$

where $\pi^{(k^l+1,l)^{S.O.}*}$ and $\ell^{(k^l+1,l)^{S.O.}*}$ represent each firm's equilibrium profit and workforce under a deviation of type (1a) when it cannot recruit the optimal number of workers to replace the leaving workers and then selects a sub-optimal strategy (S.O.). Since in this event not only $\pi^{(k^l+1,l)^{S.O.}*}$, but also β^l, the share that has to be paid to workers in the new firm increases, the result of shortage of members in U has an uncertain effect on every α^{l*}. Using the figure 2.1, when $\alpha^{1*,l}=\frac{\pi^{(k^l+1,l)^{S.O.}*}.\ell^{k^l,l*}}{\pi^{k^l,l*}.\left(1+\ell^{(k^l+1,l)^{S.O.}*}\right)}$ turns out to be the maximum between the RHS expressions in brackets of (15), the effect of near-full employment on the workers' pay will be negative (respectively positive) if the equilibrium lies in the increasing (decreasing) part of the function $\alpha^{1*,l}$;

ii) $1\leq|U|<\ell_{h=1,...r}^{k^l+h,l^*}$, that is, there are not enough unemployed people to replace workers in the event of a partnership-type of deviation. When $\alpha^{2*,l}=$

$\frac{V^{(k^l+1,l*)^{S.O.}}.\ell^{k^l,l*}}{\pi^{k^l,l*}}$ is the maximum amongst expressions $\alpha^{1*,l}$, $\alpha^{2*,l}$ and $\alpha^{3*,l}$ (under the sub-optimality of the first two), in this case the effect is clear-cut: since $\frac{V^{k^l+1,l*}.\ell^{k^l,l*}}{\pi^{k^l,l*}} > \frac{V^{(k^l+1,l*)^{S.O.}}.\ell^{k^l,l*}}{\pi^{k^l,l*}}$, every stable workers' share of profit α^{l*} will rise;

iii) When $|U| = 0$ and there is complete full employment, except for extreme cases, the maximum value of expressions in brackets will be $\alpha^{1*,l}$ and this value will be such that α^{4*} holds with an equal sign. In fact, the only possible equilibrium implies that workers and entrepreneurs earn the same pay, otherwise the workers would continue deviating to become entrepreneurs of new firms. This means that, under ful employment, the stable workers' compensation will tend to rise with respect to a situation in which the set U is non empty and contains a sufficient quantity of available workers.

5.2. The Case of Fixed Wage. In the model described in section 2 it was assumed a profit-sharing type of remuneration for the workers. This feature of the model has the advantage to simplify the analysis since the vector of Nash equilibrium quantities in the economy is virtually independent of the stable share of profits α^{l*}assigned to the active workers of each sector. When this assumption is removed and a fixed remuneration is assumed for workers, the equilibrium can be described by a triple $\left(\underline{y}^*\left(\underline{w}^*\right), \underline{w}^*, \underline{\pi}\left(\underline{w}^*\right)\right)$, that gives respectively the economy equilibrium quantities, wages and profits. It is now obvious that the equilibrium quantities selected within each industry depend on the equilibrium wage expected to be paid by the entrepreneurs in that industry. It is easy to see that, when the profit function of each firm in every sector is given by:

$$\pi^{jl}\left(y^{jl}, \sum_{h \neq j} y^{hl}\right) = \left\{ R^{jl}\left(y^{jl}, \sum_{h \neq j} y^{hl}\right) - w^l g^{l^{-1}}\left(y^{jl}\right) - F^l \right\}_{\substack{\forall j=\left(1,..k^l\right) \\ \forall l=(1,..m)}} \tag{13}$$

the stable wage is determined, in each sector, as:

$$\frac{p^{k^l,l*}\left(w^{l*}\right) \cdot y^{k^l,l*}\left(w^{l*}\right) - F^l}{\left(1+y^{k^l,l*}\left(w^{l*}\right)\right)} \geq w^{l*} =$$
$$\max \left\{ \frac{p^{k^l+1,l*}\left(w^{l*}\right) \cdot y^{k^l+1,l*}\left(w^{l*}\right) - F^l}{\left(1+y^{k^l+1,l*}\left(w^{l*}\right)\right)}, \frac{p^{k^l+1,l*}\left(w^{l*}\right) \cdot y^{k^l+1,l*}\left(w^{l*}\right) - F^l}{\ell^{k^l+1,l*}\left(w^{l*}\right)}, \ p^{k^l+1,l*\left(w^{l*}\right)} y^{k^l+1,l*\left(w^{l*}\right)} \right\}$$

that is calculated by a procedure analogous to that followed for proposition 1. Note also that in this case the internal consistency is respected for $p^{k^l,l*}\left(w^{l*}\right) \geq w^{l*}$, that should always holds for each k^l -th market to be profitable.

Two main differences appear, however, in the case of fixed wage. The first is that, in each sector, under the stable wage, the following condition must hold:

$$\pi^{k^l+1,l*}\left(w^{l*}\right) \le 0 \tag{14}$$

Since from the equilibrium condition it must be true that:

$$w^{l*} \cdot \ell^{k^l+1,l*}\left(w^{l*}\right) \ge p^{k^l+1,l*}\left(w^{l*}\right) \cdot y^{k^l+1,l*}\left(w^{l*}\right) - F^l \tag{15}$$

this implies expression (17). The meaning is that in each industry the stable wage must be so high to prevent every $\left(k^l+1\right)$ -th firm (entrepreneurial or of a partnership-type) to be profitable at the current wage. This is a condition stronger than what contained in proposition 1.

The second observation comes from the dependence of firms' quantities on stable wages. Since every firm's equilibrium quantity can be expressed as $y^{k^l,l*}\left(w^{l*}\left(k^l\right)\right)$, it can happen in a sector that, for a given initial number of firms, the total quantity produced by all firms coincides with the collusive solution, i.e. that achieved by the whole industry joint profit maximization. In other words, it might be the case that in an industry the high stable wages paid to employees turn the firms into *hiddenly collusive firms*. One peculiar feature of this collusion is that is stable against the temptation for every oligopolistic firm to deviate from the given equilibrium. The figure below, given a certain fixed labour supply, shows how in the usual one-sector linear example exists a certain initial number of firms such that the quantity selected by every firm under the non cooperative stable wage equilibrium coincides with the quantity selected under collusive agreement.

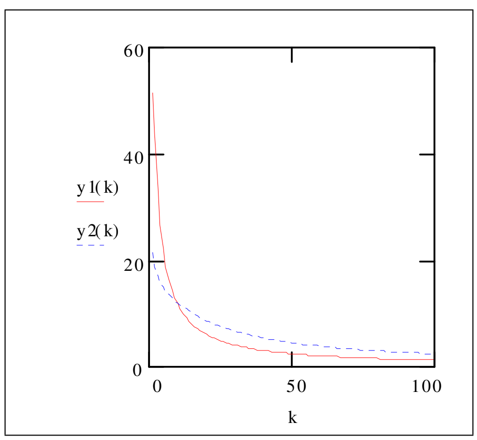

Fig. 3 - Firm Equilibrium quantities under collusive agreement $y^1(k)$ and under stable wage $y^2(w(k))$, for $k = 1, ., 100$, $a = 250, F = 1.3, U = 147$.

6. Discussion

The paper has described an oligopolistic economy in which each worker employed by a firm receives a firm-specific training. This training enables him to potentially leave the firm in which is employed whenever - given the structure of the sector he belongs to and given his specific competence - he finds the compensation received not satisfactory. By carefully defining the conditions needed to ensure that anyone in the economy, given his payoff, does not want to change his situation through a feasible deviation, a stable earning profile for the economy has been characterized. The wages associated to this earning profile present two features: they are higher than neoclassical reservation wages (that in the model are equal to zero); they turn out to be sensitive to the size of firms' market shares and then in turn to the dimension of fixed costs, that of the market and to the number of firms operating in each industry. Moreover, the paper proved that the stable earning profile belongs to the core of the economy when the players are constrained to certain deviations and each coalition is independent and behaves *à la* Nash. Finally, the case of fixed wages has revealed the possibility of the firms' stable collusive behaviour.

Two extensions would be worthwhile of further exploration. One could be to let the type of firms existing at the beginning in the economy be endogenously determined. This extension would imply to build an entrepreneurial theory of firm formation jointly with the theory of stable earning profiles described above. A second route of analysis might be instead that of extending the allowed deviations to coalitions formed by agents belonging to different firms and/or introduce different individuals' constraints to describe different bargaining processes and more realistic institutional set up of the economy.

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