Modeling exchange rate dynamics in Peru: A cointegration approach using the UIP and PPP

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April 2012

Online at https://mpra.ub.uni-muenchen.de/70772/
MPRA Paper No. 70772, posted 18 April 2016 04:33 UTC
Modeling exchange rate dynamics in Peru: A cointegration approach using the UIP and PPP

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Aprobado por Manuel Luy
Abril, 2012

Abstract

This paper investigates whether the Purchasing Power Parity (PPP) and the Uncovered Interest Rate Parity (UIP) hold for the Peruvian economy. We analyze Peruvian, international and trade-weighted foreign data for the period 1997 - 2011 using Johansen’s cointegration approach. The results of our analysis suggest the existence of two stationary relations obtained after imposing over-identifying restrictions, which represent a combination of the UIP with the PPP and an equation for an augmented risk premium. In terms of monetary policy, our findings provide evidence that the aim of the Central Bank’s intervention in the currency market is to smooth short-run volatility of the exchange rate, but does not have a long-run effect. After a successful identification of the stationary relations, we develop a permanent-transitory analysis following Gonzalo & Ng (2001) in order to analyze the impact of structural shocks to the error correction model variables and some relations of interest.

Keywords: PPP, UIP, cointegration, Johansen, exchange rate, impulse responses, permanent and transitory shocks.


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*The authors would like to thank Diego Winkelried for his excellent advice in this project. We also thank Juan Mendoza and Fabrizio Orrego for their useful comments. All errors are ours.

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I INTRODUCTION

During the past couple of decades, the Peruvian economy has implemented policies to increase its openness to the world. This higher exposure has resulted in a more dynamic trade balance and greater volatility of capital flows, which in turn leads to greater volatility in the nominal exchange rate. Peru—as a small economy—is strongly dependent on transactions with its trading partners and exposed to financial shocks such as the ones created by the current financial crisis.

Regarding the exchange rate, there are two theories that define its dynamics and that are associated to different markets. These theories are the Purchasing Power Parity (PPP) and the Uncovered Interest Rate Parity (UIP). They relate the exchange rate to the goods and capital markets, respectively. The first theory states that, in the absence of transaction costs and trade barriers, the nominal exchange rate must equal the ratio of price levels between two countries. If this condition suffered a departure from equilibrium, an arbitrage opportunity would arise in the commodities market, generating pressure on the nominal exchange rate to assure equilibrium.

The second theory states that expected returns on investment for any two currencies should be equal when measured in the same currency. Thus, if an arbitrage opportunity arises in the capital market, the nominal exchange rate will have to adjust as investors want to take advantage of any deviation from equilibrium.

This research analyzes financial and trade-weighted macroeconomic monthly data for Peru for the period 1997 - 2011\(^1\) to determine a model for the effective nominal exchange rate using the UIP and the PPP relations in Peru. Thus, we are interested to study if such relations hold in the short run or in the long run. A common feature found in the literature is the rejection of the PPP and UIP hypotheses as mechanisms for adjustment of the goods and capital markets. This typically occurs because these studies usually define the equilibrium exchange rate as a function of either the PPP or the UIP, but the commodity and capital markets may be related\(^2\). For this reason, we will follow Johansen & Juselius (1992) and Juselius (1995) to formulate a system approach with Johansen’s cointegration methodology. With this framework, we allow for different short-run and long-run effects in a Vector Error Correction Model (VECM) that ensures the stationarity of the PPP and the UIP. Furthermore, we will decompose the shocks into permanent and temporary shocks using Gonzalo & Ng (2001) approach to analyze the effect on the VECM’s variables.

The motivation for testing whether the UIP holds relies on its importance in the current global context and past events that have generated volatility in the exchange rate. In 2006, there was a favorable interest rate differential for the international assets. According to the UIP, depreciatory expectations should operate to compensate this gap in the short run, which in turn generates future appreciatory expectations. In 2011, following the 2008’s financial crisis, a large favorable differential towards domestic currency generated the opposite conditions in comparison to those occurred in 2006, so we would expect an exchange rate appreciation in the short run and a corresponding expected depreciation in the long run. These events raise the question of whether the Peruvian Central Bank should lower the interest rate to eliminate the differential or if this deviation from equilibrium is temporary and should return to its normal path. Moreover, the motivation for PPP exchange rate estimations is related to the importance of determining the degree of misalignment of the nominal exchange rate and to formulate appropriate policies (Sarno & Taylor, 1998).

Empirical tests have often yielded conflicting conclusions about the UIP and PPP relations. The explanations concerning the failure of the PPP include imperfect competition, pricing to market, composition of price indices, transport costs and trade barriers\(^3\). With regard to the UIP,

\(^1\)Data for 2011 includes information from January to August.
\(^2\)See Section 2 for a literature review.
failure explanations vary from a time-varying risk premium, expectation errors, etc. Moreover, invalid conditioning, nonlinear dynamics and low-power unit root tests have rendered mixed results, especially for the validity of the PPP hypothesis (Gokcan & Ozmen, 2002).

Regarding Peruvian evidence, Humala (2006) tests the UIP condition using different econometric approaches, including a linear model and a Markov Switching Model for regime changes. The author found poor results using the linear model with a small sample (2000 - 2005), but got favorable results with an increase in the sample to 1992 - 2005. Thus, this paper seeks to provide more evidence to determine whether the UIP or the PPP hold for the Peruvian economy.

The rest of the paper is organized as follows. Section 2 presents a brief literature review about PPP and UIP relations. Section 3 proposes theoretical relations of interest that we will test in the next sections. Section 4 presents the econometric framework. Section 5 estimates the empirical model using Johansen’s cointegration approach. Section 6 uses Gonzalo & Ng (2001) decomposition to separate permanent and transitory shocks and analyzes its impact to the system’s variables. Finally, Section 7 concludes.

II LITERATURE REVIEW

International finance and open macroeconomic models are usually based on two well-known arbitrage relations: the Purchasing Power Parity (PPP) and the Uncovered Interest Rate Parity (UIP). The first one establishes a relationship between the prices in one country to another measured in the same currency, while the last one indicates that movements in the exchange rate are positively related with the domestic-foreign interest rate differential.

Both international parities have been used as models of exchange rate determination, resulting in an overwhelming literature. Since the PPP proposes a stable long-run relation between the exchange rate and the domestic and foreign prices, its validity has been analyzed through cointegration studies. Moreover, PPP can also be interpreted in terms of the real exchange rate, in which case this last variable needs to be stationary in order for the PPP to hold empirically. However, in spite of this theoretical conclusion, the analysis of the real exchange rate through unit root tests has not lead to conclusive results (Roll (1979); Frenkel (1981); Adler & Lehmann (1983); Hakkio (1984); MacDonald (1985); Ng & Perron (2002); etc. found that the real exchange rate contains a unit root, whereas Huizinga (1987); Cumby & Obstfeld (1981); Abuaf & Jorion (1990); Diebold, Husted & Rush (1991); Whitt (1992); etc. have found that real exchange rates are stationary). Furthermore, cointegration studies trying to prove whether the PPP holds in the long run are also mixed, either using the Engle and Granger approach (Ardeni & Lubian (1983); Taylor (1988); McNown & Wallace (1990); Trozano (1992); and others) or Johansen’s methodology (Kim (1990); Fisher & Park (1991); Taylor (1992); Kugler & Lenz (1993); MacDonald (1993); MacDonald & Marsh (1994); MacDonald & Moore (1994); etc.). More recently, Rogoff (1996) surveyed the PPP puzzle and explained the empirical failure in validating the PPP.

Regarding UIP, most studies have analyzed whether or not the domestic-foreign interest rate differential is an optimal predictor of the depreciation rate. Results from Cumby & Obstfeld (1981), Davidson (1985) and Taylor (1987) suggest a strong rejection of the optimal predictor hypothesis; however, Melander (2009) found smaller deviations from the UIP for the case of a dollarized economy. On the other hand, more support has been found when using long-term in-
terest rates (Meredith & Chinn, 1998), and possible explanations for UIP failure are addressed by McCallum (1992) and Backus et al. (2010).

Johansen & Juselius (1992) suggest that one of the reasons why PPP and UIP do not hold is related to the fact that previous studies have not taken into consideration the links between the goods and capital markets. Since exports and imports depend on the real exchange rate, PPP affects directly the current account, while capitals respond to interest rate differentials so UIP has a strong effect on the financial account. Moreover, any current account deficit must be financed through the financial account, meaning that there is a strong link between PPP and UIP (MacDonald, 2000).

The first attempt to test a combined PPP-UIP relation was made by Johansen & Juselius (1992) in a study for the United Kingdom. Using the methodology for multivariate cointegration developed by Johansen (1991), they found that the PPP relation was nonstationary, but the results for the UIP were successful. However, after combining the PPP with the interest rate relations, they could not reject the stationarity hypothesis, showing that the interactions between these two parities in the long and short run are important and cannot be ignored.

Since the seminal paper of Johansen & Juselius (1992), many authors have used the same approach to test the validity of a joint estimation of the PPP-UIP relation. Juselius (1995) used the same framework to analyze the mechanisms explaining the inflationary effects transmitted from Germany to Denmark. She concluded that in the long run the exchange rates are determined from deviations from the PPP and that the link between the goods and asset markets, postulated by the PPP-UIP combined relation, is crucial for fully understanding the movements of the exchange rates, prices and interest rates. Furthermore, Hunter (1992) identifies two cointegration vectors, each corresponding to a long-run equation for the PPP and UIP. Camarero & Tamarit (1993) found empirical evidence on PPP and UIP for the Spanish economy vis a vis the European Community and their results support the importance of the interest differential as an explanatory variable for the short-term adjustment to the PPP. Caporale, Kalyvitis & Pittis (2001) found similar results for the German mark and the Japanese yen.

Gokcan & Ozmen (2002) investigated the long-run relations between Turkish and U.S. inflation rates, interest rates and exchange rate using Johansen’s cointegration approach. Their results supported the idea of two stationary relations concerning the long-run evolution of Turkish interest rates and the Turkish inflation rate. They also suggested that deviations from the PPP were explained by the interest rate differentials and, correspondingly, deviations from the UIP can be explained by inflation rate differentials. Using quarterly data for New Zealand, Stephens (2004) found that a strict PPP with a weak version of the UIP were the best identification scheme to describe the data. Sjoo (1995) and Choy (2000) found similar results for the same economy using different sample periods.

Rashid & Abbas (2008) used monthly information for four Asian countries (Bangladesh, India, Pakistan and Sri Lanka) vis a vis the U.S. Their results support the idea that the nominal exchange rate is determined by both relations (UIP and PPP). Moreover, they found evidence that domestic interest rates had more effect on nominal exchange rate than foreign interest rates, and by modifying the typical UIP and PPP relation to account for the effect of tradable and non-tradable goods and capital market imperfections they estimated more robust results (theoretically and statistically). In the case of emerging markets, Campbell et al. (2008) estimate models for the UIP and PPP relations using monthly data for 19 countries. Using the parity framework of PPP and UIP, they were able to assess whether a risk premium or expectation errors on exchange rate forecasts drove deviations from UIP and PPP. Similar conclusions regarding the validity of the

8The authors argue that if risk premium was the main force driving a deviation from UIP, then the basic UIP regression results in emerging markets would be worse than those found for developed economies. Instead, if...
PPP-UIP combined parity for a variety of countries can be found in Jore, Skjerpen & Swensen (1992), Helg & Serati (1996), MacDonald & Marsh (1997), MacDonald & Marsh (1999), Juselius & MacDonald (2000), Pesaran et al. (2000), Bjornland & Hunges (2002), and Bevilacqua (2006).

For the Peruvian economy, there are no studies that test whether the UIP and PPP relations hold in a joint framework, since most of the literature has analyzed each parity separately. For the PPP case, Arena & Tuesta (1998), Ferreyra & Herrada (2003), Ferreyra & Salas (2006) and Rodriguez & Winkelried (2011) have focused on analyzing the long-run fundamentals of the real exchange rate rather than taking a parity framework. Humala (2006) evaluates the UIP using several Peruvian financial instruments from a linear and nonlinear framework. He found that nonlinear models (Markov Switching) were able to distinguish between periods of UIP validity from those where it does not hold. On the other hand, Duncan (2000) evaluates the UIP formulating a micro-founded model of CAPM consumption, which accounts for the difference in favor of higher returns to foreign currency instruments. The author concluded that this differential can be explained through the higher covariance between returns in foreign currency with the private consumption rate than the covariance between returns in domestic currency with the same consumption rate (only if these covariances were equal the interest rate parity would hold).

In conclusion, the literature regarding the PPP and UIP has found mixed results when evaluating each of these international parities in isolation. However, as widely documented, the combination of both relations in a single model usually provides better results in comparison with treating them individually. However, this approach has not been done for a dollarized economy like Peru that could have an additional new long-run equation to correctly understand the dynamics of exchange rate determination.\(^9\)

### III THEORETICAL MODEL

The PPP concept is usually related to two type of relations: absolute PPP and relative PPP. The former is the exchange rate between two currencies which would equate two national price levels in a common currency, so that the purchasing power of a unit of one currency would be the same for both economies. The latter holds when the depreciation rate of one currency relative to another matches the difference in price inflation between both countries. Thus, when PPP holds, the real exchange rate should be constant, implying that changes in this variable represent deviations from PPP (Sarno & Taylor, 2002).

The basic building-block of the PPP is the law of one price. It relates common currency prices of similar goods. The law of one price in its absolute version can be written as:

\[
P_{i,t} = E_t P_{i,t}^* \quad i = 1, 2, 3, \ldots, n
\]

where \(P_{i,t}\) is the price of good \(i\) in terms of domestic currency at time \(t\), \(P_{i,t}^*\) is the price of good \(i\) in foreign currency at time \(t\), and \(E_t\) is the nominal exchange rate. This relation implies that the same good should have the same price across countries after expressing the prices in same currency with the exchange rate. Here, the idea of arbitrage is the main argument of why the law of one price should hold.

One of the problems with the law of one price is that it should hold only if goods produced domestically are perfect substitutes of those internationally produced. However, the presence of expectation errors were the driving force, then short-run deviations from both UIP and PPP would occur, especially for emerging markets.

\(^9\)Section 5 will provide an identification scheme that allows us to find a cointegration vector that relates the domestic interest rate in foreign currency with the foreign interest rate and other variables.
transport costs, product differentiation, etc. creates a gap between domestic and foreign prices. Following Sarno & Taylor (2002), by summing all the traded goods in each country, the absolute version of the PPP hypothesis requires that:

$$\sum_{i=1}^{n} \alpha_i p_{i,t} = E_t \sum_{i=1}^{n} \alpha_i p_{i,t}^*$$

(2)

where the $\alpha_i$ are weights that satisfy $\sum_{i=1}^{n} \alpha_i = 1$. Or alternatively:

$$\sum_{i=1}^{n} \gamma_i p_{i,t} = \alpha_t + \sum_{i=1}^{n} \gamma_i p_{i,t}^*$$

(3)

where $\gamma_i$ are weights satisfying the same condition as $\alpha_i$ and the lower case letters represent variables in logarithms. Using these relations, we can derive the absolute version of the PPP and an equation for the real exchange rate, respectively:

$$e_t = p_t - p_t^*$$

(4)

$$q_t = e_t - p_t + p_t^*$$

(5)

where $q_t$ is the log of real exchange rate and may be viewed as a measure of deviation from PPP. Thus, for the PPP to hold, we need the real exchange rate $q_t$ to be stationary; however, as Rogoff (1996) suggests, most empirical papers have found the opposite result.

As the PPP condition in (5) denote equilibrium in the goods market, implying that no arbitrage opportunities prevail, the uncovered interest parity (UIP) ensures equilibrium in the asset market. This theory suggests that the expected returns on deposits of any two currencies should be equal when measured in a common currency. Let $e_{t+m}$ be the exchange rate for period $t+m$, $i^m_t$ the domestic bond yield with maturity $m$ and $i^{m*}_t$ the yield of a foreign bond. The UIP is defined by:

$$E^e_t (\Delta_m e_{t+m})/m - (i^m_t - i^{m*}_t) = 0$$

(6)

UIP holds if the rate of return on foreign deposits equal the rate of return on domestic deposits when measured in the same currency, such that no type of deposit has an excess of demand or supply. If this equality does not hold, an arbitrage opportunity will exist in the asset market and the nominal exchange rate will adjust as investors take advantage. The empirical support for UIP, however, has been weak (Frydman & Goldberg, 2001). Thus, the literature often interpret this as evidence of the presence of a risk premium in (6):

$$E^e_t (\Delta_m e_{t+m})/m - (i^m_t - i^{m*}_t) = \epsilon_t$$

(7)

This lack of support for both relations encouraged many studies that combine the PPP and UIP\textsuperscript{10}. Theoretically, if we consider from the balance of payments constraint that any current account imbalance generated by persistent (nonstationary) movements has to be financed by the financial account, then we have a strong relation between the PPP and UIP conditions (Juselius, 2006).

Another relation of interest is the one that arises when comparing assets from different countries (like Peruvian assets vs. U.S. assets). We can fully exploit this relation with Peruvian data as there has been a high proportion of local assets in foreign currency for decades. This relation can be formulated as follows:

$$i^*_t = i^{**}_t + \zeta_t$$

(8)

where $i_t^*$ and $i_t^{**}$ are domestic and foreign rates in foreign currency, respectively, and $\zeta_t$ is a risk premium representing the underlying risk of emerging countries’ assets.

Finally, following a simple version of the sticky price model (Dornbusch, 1976), we can formulate a relation where the UIP and the PPP might hold together. First, recall (6), which can be rewritten as:

$$\Delta e_t = i_t - i_t^*$$

where expectations have been replaced with actual values\(^\text{11}\). The depreciation rate can be assumed to be a function of the gap between the log of the nominal exchange rate and the equilibrium rate $\bar{e}$:

$$\Delta e_t = v(e_t - \bar{e})$$

Furthermore, the equilibrium rate $\bar{e}$ is a function of the difference between the logarithms of the domestic equilibrium price $\bar{p}$ and the foreign equilibrium price $\bar{p}^*$, implied by the PPP hypothesis:

$$\bar{e} = \bar{p} - \bar{p}^*$$

If we combine (9) - (11), we can obtain a testable condition for exchange rate determination using a combination of PPP and UIP:

$$e_t - p_t + p_t^* = 1/v(i_t - i_t^*)$$

Equation (12) is an important relation in the Dornbusch model. This relation states that the nominal exchange rate is a function of the price level differential and the interest rate differential, whose speed of adjustment is given by the coefficient $1/v$. With this equation, we can formulate testable relations that we would expect to cointegrate:

$$e_t = p_t - p_t^* + 1/v(i_t - i_t^*) + \varepsilon_t$$

$$i_t^* = i_t^{**} + \zeta_t + \mu_t$$

where $\varepsilon_t$ and $\mu_t$ are white noises.

**IV THE ECONOMETRIC FRAMEWORK**

We now turn to a description of our econometric methodology, the statistical tests used to verify the formulated hypotheses and the permanent-transitory shock decomposition.

**IV.1 The Johansen’s approach for multivariate cointegration**

The general VAR formulation with $p$ lags for the $y_t$ vector, containing $n$ observable variables, can be expressed as:

$$y_t = \mu + \Pi_1 y_{t-1} + \Pi_2 y_{t-2} + \cdots + \Pi_p y_{t-p} + \Phi D_t + \varepsilon_t$$

where $\varepsilon_t$, a vector of residuals, is assumed to be i.i.d Gaussian with zero mean and positive definite covariance matrix $\Omega$ ($\varepsilon_t \sim N(0, \Omega)$); $y_0, \ldots, y_{-p+1}$ are assumed fixed and $D_t$ is a vector ($d \times 1$)

---

\(^{11}\)Expectations play a crucial role in the parity relations discussed. However, as Juselius (2006) suggests, replacing expectations with actual values should yield valid cointegration results, conditioned on two assumptions: (i) the difference between $E(y_{t+1})$ and $y_{t+1}$ is stationary (i.e. agents don’t make systematic errors in forecasting) and (ii) the differenced process $(y_{t+1} - y_t)/t$ is stationary.
of deterministic components (seasonal and intervention dummies).

An alternative way of expressing (15) is with a polynomial in the lag operator \( L \) (where \( L^jy_t = y_{t-j} \)):

\[
\Pi(L)y_t = \mu + \Phi D_t + \varepsilon_t
\]

where \( \Pi(L) \) is a \( n \times n \) matrix polynomial of order \( p \) and defined by \( \Pi(L) = I_n - \sum_{i=1}^{p} \Pi_i L^i \).

If \( y_t \) contains variables with a unit root (i.e. \( y_t \) is nonstationary) then some shocks affecting \( y_t \) will have permanent effects. However, if the vector of differences \( \Delta y_t \), defined as \( \Delta y_t = (1 - L)y_t \), is stationary and there are some linear combinations of the variables in \( y_t \) that are stationary, then it is said that \( y_t \) is cointegrated of order \((1,1)\) (CI\((1,1)\)). Under these conditions, (15) can be expressed as a vector error correction model (VECM).

\[
\Delta y_t = \mu + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \ldots + \Gamma_{p-1} \Delta y_{t-p+1} + \Pi y_{t-1} + \Phi D_t + \varepsilon_t
\]

where \( \Gamma_i = -\sum_{j=1+1}^{p} \Pi_j \). Alternatively, (17) may be written as:

\[
\Gamma(L)\Delta y_t = \mu + \Pi y_{t-1} + \Phi D_t + \varepsilon_t
\]

where \( \Gamma(L) = I_n - \sum_{i=1}^{p-1} \Gamma_i L^i \). Under the cointegration hypothesis, Johansen (1991) demonstrated that the presence of unit roots leads to a reduced rank condition on the long-run matrix \( \Pi \) such that \( \Pi = \alpha \beta' \) so (17) and (18) can be expressed as (19) and (20), respectively.

\[
\Delta y_t = \mu + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + \ldots + \Gamma_{p-1} \Delta y_{t-p+1} + \alpha(\beta'y_{t-1}) + \Phi D_t + \varepsilon_t
\]

\[
\Gamma(L)\Delta y_t = \mu + \alpha(\beta'y_{t-1}) + \Phi D_t + \varepsilon_t
\]

where \( \alpha \) and \( \beta \) are matrices of order \( n \times r \) and rank equal to \( r \) \((r \leq p)\). The columns of \( \beta \) are the cointegration vectors, which can be interpreted as long-run economic relations so that \( \beta'y_{t-1} \) is a \( r \times 1 \) vector of stationary relations, while the columns of \( \alpha \) correspond to the loading factors and denote how each variable in \( y_t \) “corrects”. Johansen (1988, 1991) suggested a maximum likelihood procedure to estimate the cointegration rank \( r \), and the matrices \( \beta \) and \( \alpha \).

**IV.2 Hypothesis testing**

After determining the number of cointegration vectors and estimating \( \alpha \) and \( \beta \), we can test different hypotheses using the procedures developed by Johansen and Juselius (1990, 1992).

A first group of tests is related to restrictions over \( \beta \). The constrained cointegration vector, \( \beta^c \), can be expressed in terms of \( s_i \) free parameters \( \varphi_i \):

\[
\beta^c = (\beta^c_1, \ldots, \beta^c_r) = (H_1\varphi_1, \ldots, H_r\varphi_r)
\]

where \( \varphi_i \) is a \((s_i \times 1)\) coefficient matrix and \( H_i \) is a \((p \times s_i)\) design matrix, where \( i = 1, \ldots, r \). Alternatively, we can impose the same restrictions by specifying the restriction matrices \( R_i \) \((p \times m_i)\) that define the \( m_i = p - s_i \) restrictions on \( \beta \):

\[
R_i'\beta_i = 0
\]

\[
R_r'\beta_r = 0
\]
Under this framework, we can test hypotheses regarding the same restriction on all the $\beta$ vectors, assume that some $\beta$ are known (to test the stationarity of a variable) or restrict only some $\beta$ coefficients.

The other hypothesis tests are related to the $\alpha$ matrix. For instance, we can impose $\alpha$ to have a zero row (i.e. weak exogeneity test) or to have a unit vector (i.e. variable with no permanent effects). The framework is similar to the $\beta$ case and also uses the $H$ or $R$ matrices.

**IV.3 A permanent and transitory shock decomposition**

In order to analyze and interpret the shocks, we are interested in expressing $\Delta y_t$ in terms of permanent and transitory shocks. From (16), we can make use of the Wold representation so that $\Delta y_t = C(L)\varepsilon_t$, where we have left out $\mu$ and $D_t$ to make the explanation simpler. As Gonzalo & Ng (2001) suggest, the shocks are defined as follows: let $y_t$ be a difference-stationary sequence whose VECM is given by (17). Denote $E_t$ as the expectation operator taken with respect the information set in period $t$. The $(n \times r) \times 1$ vector of shocks $\tilde{\eta}_t^y$ is said to have permanent effects on $y_t$ if $\lim_{h \to \infty} \partial(y_{t+h})/\partial \tilde{\eta}_t^y \neq 0$. Furthermore, the vector $\tilde{\eta}_t^y$ (of order $r \times 1$) is said to have transitory effects on $y_t$ if $\lim_{h \to \infty} \partial(y_{t+h})/\partial \tilde{\eta}_t^y = 0$.

To get these effects, Gonzalo & Ng (2001) suggest a transformation such that the data is expressed in terms of unorthogonalized permanent and transitory shocks. This is referred as a P-T decomposition. Let $G\varepsilon_t$ represent the $n \times n$ matrix of permanent and transitory shocks. The $G$ matrix is defined as:

$$G = \begin{bmatrix} \alpha'_1 \\ \beta' \end{bmatrix}$$

(23)

where $\alpha'_1$ is a $(n - r) \times n$ matrix such that $\alpha'_1 \alpha = 0$ and $\beta'$ is a $r \times n$ matrix. Thus, by making use of this $G$ matrix we can rotate $\varepsilon_t$ so that it can be decomposed into $u_t^P = \alpha'_1 \varepsilon_t$, defining the permanent shocks, and $u_t^T = \beta' \varepsilon_t$, defining the transitory ones.

However, these shocks are not mutually uncorrelated. For this reason, Gonzalo & Ng (2001) used a transformation to redefine $\Delta y_t = D(L)u_t$ to $\Delta y_t = D(L)\tilde{\eta}_t$, where $D(L) = C(L)G^{-1}$ and $\tilde{\eta}_t$ are mutually uncorrelated. To obtain this result, we follow the authors and construct a lower triangular matrix $W$ by applying a Choleski decomposition to $cov(G\varepsilon_t)$. With this, we can reformulate the rotated errors as orthogonalized permanent and transitory shocks such as:

$$\tilde{\eta}_t = W^{-1} G\varepsilon_t$$

(24)

Finally, we can reexpress the Wold representation with the orthogonalized errors:

$$\Delta y_t = C(L)G^{-1}WW^{-1}G\varepsilon_t = D(L)WW^{-1}u_t = \tilde{D}(L)\tilde{\eta}_t$$

(25)

**V EMPIRICAL MODEL**

We start this section with a six-variable system $y_t = [p_t - p_t^*, e_t, i_t, i_t^*, i_t^{**}, \zeta_t]$, which allows to test the validity of the PPP and the UIP hypotheses. $p_t - p_t^*$ represents the differential of Peruvian CPI and trade-weighted foreign CPI, $e_t$ is the effective nominal exchange rate, $i_t$ is the 90-days prime interest rate in domestic currency, $i_t^*$ is the 90-days prime interest rate in foreign currency, $i_t^{**}$ is the 90-days Libor and $\zeta_t$ is a risk premium proxy defined by the Emerging Markets Bond Index (EMBI)\textsuperscript{12}. The sample period is from January 1997 to August 2011 in a monthly frequency,
which includes the adoption of a fully-fledged inflation targeting scheme in 2002.

We use prime interest rates for two main reasons. First, the Peruvian capital market is not fully developed, especially in the first years of our sample. Furthermore, the number of companies that seek funding through issuing bonds is still very low and restricted mainly to large enterprises. In contrast, prime interest rates could provide useful information taking into account that these rates correspond to the funding costs that the banks charge to their most important corporate clients. Thus, these rates can be understood as the interest rates that these companies would get if they attempted to issue a bond in domestic or foreign currency.

Figure 1: Domestic interest rates comparison

![Graph showing domestic interest rates comparison between 1998 and 2010.](image)

Figure 2: Foreign interest rates comparison

![Graph showing foreign interest rates comparison between 1998 and 2010.](image)

The second reason is the lower volatility of prime rates compared to others, like the interbank rate. As Figures 1 and 2 show, the prime rates in domestic and foreign currency reflect the behavior of the interbank rates in domestic and foreign currency, respectively. However, the volatility of
the former rates is lower, especially in the case of the prime rate in foreign currency\textsuperscript{13}.

From section 4, the $6 \times 1$ system $y_t$ represented by the reparametrized VAR(p) process can be defined as:

$$
\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \ldots + \Gamma_{p-1} \Delta y_{t-p+1} + \Phi D_t + \varepsilon_t
$$

(26)

where $D_t$ is a dummy variable that accounts for the effects of the crisis. This variable is activated since 2008. Furthermore, given that $y_t$ is an I(1) set of variables, (26) is a vector error correction model (VECM) if the rank of matrix $\Pi$ is such that $0 < r < 6$, where $r$ is the number of cointegration vectors.

V.1 Empirical tests for determining the model

V.1.1 Cointegration and residuals diagnostics for model specification

In this section, we make several tests for a correct model specification following the approaches found in Johansen & Juselius (1992) and Juselius (1995). We start with a test of rank determination. It should be noted that a bootstrap was performed to simulate critical values of the trace test. This was done in order to account for the inclusion of a dummy variable in the model\textsuperscript{14}.

Table 1 presents the results that determine the number of cointegration vectors for the six-variable system $y_t = [p_t - p_*^t, e_t, i_t, i_*^t, i_*^{**t}, \zeta_t]$. Based on the trace statistic, we can conclude that there are two long-run relations and, therefore, four common stochastic trends in the model.

Table 1: Determination of Rank

<table>
<thead>
<tr>
<th>$H_0$: $r$</th>
<th>$p - r$</th>
<th>Trace statistic</th>
<th>Simulated critical values\textsuperscript{1}</th>
<th>$80%$</th>
<th>$90%$</th>
<th>$95%$</th>
<th>$99%$</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>75.45</td>
<td>82.93</td>
<td>85.01</td>
<td>89.55</td>
<td>102.05</td>
<td>0.041</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>60.23</td>
<td>52.14</td>
<td>56.42</td>
<td>59.82</td>
<td>66.80</td>
<td>0.046</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>36.09</td>
<td>33.78</td>
<td>37.12</td>
<td>39.99</td>
<td>45.63</td>
<td>0.124</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>16.11</td>
<td>19.47</td>
<td>21.97</td>
<td>24.13</td>
<td>28.27</td>
<td>0.430</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>4.78</td>
<td>8.94</td>
<td>10.69</td>
<td>12.18</td>
<td>15.22</td>
<td>0.678</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>0.10</td>
<td>1.46</td>
<td>2.23</td>
<td>3.02</td>
<td>4.75</td>
<td>0.741</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{1} Critical values have been corrected due to small sample bias as Harris & Judge (1998).

Note: The number of lags considered for the VAR model is four.

We also performed multivariate tests for a different number of lags in the VECM model with two stationary vectors. Table 2 reports various residual diagnostics to determine the correct number of lags in a system with two cointegration vectors. The null hypothesis for the normality and LM tests suggest that the errors are normally distributed and that there is no autocorrelation of order $p + 1$, respectively. The normality test rejects the null of normality; however, the cointegration results appear to be robust when it is caused by an excess of kurtosis (Gonzalo, 1994)\textsuperscript{15}. Furthermore, the LM test supports the idea of a four-lag specification and the LR test performed to a non-restricted VAR also suggests the presence of four lags in the cointegration model. According to these results, a VECM(4) seems to be a correct approximation of the data generation process.

\textsuperscript{13} Moreover, since the introduction of the monetary policy rate in January 2001, the interbank rate has closely followed the behavior of the former (the correlation coefficient between both rates is 0.99). However, the advantage of working with the prime rate in domestic currency stems from having information for years prior to 2001.

\textsuperscript{14} See Harris & Judge (1998) for more details.

\textsuperscript{15} Results for univariate kurtosis tests suggested that the first, second and sixth variables of the vector have an excess of kurtosis. Each test for those variables is distributed as $\chi^2(1)$.
Table 2: Multivariate tests for system evaluation and residual diagnostics

<table>
<thead>
<tr>
<th>Cointegration relations</th>
<th>Number of lags</th>
<th>Normality test</th>
<th>LM test</th>
<th>LR test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Statistic</td>
<td>p-value</td>
<td>Statistic</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>320.69</td>
<td>0.00</td>
<td>49.83</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>280.75</td>
<td>0.00</td>
<td>62.31</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>250.11</td>
<td>0.00</td>
<td>87.77</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>191.15</td>
<td>0.00</td>
<td>46.16</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>170.80</td>
<td>0.00</td>
<td>35.48</td>
</tr>
</tbody>
</table>

1 The normality test is distributed as $\chi^2(12)$.
2 The LM autocorrelation test is distributed as $\chi^2(36)$.
3 The LR test suggests the presence of five lags in a unrestricted VAR model. The LR statistic regarding the use of five lags in a VAR model is 66.70. This implies that the number of lags in the VECM is four.

V.1.2 Tests for cointegration vectors $\beta$ and the adjusting terms $\alpha$

Once we have determined the number of cointegration relations for the model, we proceed to impose a series of restrictions to the vectors and the adjustment coefficients. This procedure will reveal some interesting aspects of the variables, like stationarity using a multivariate approach and the significance of the vectors (i.e. if the variables are long-run excludable from the cointegration vectors). For the adjustment coefficients, we impose restrictions to determine the driving and pushing forces in the system (Juselius, 2006) and identify which of the variables have permanent or transitory effects due to different shocks. All the tests are performed at 5% of significance.

The test of a unit vector on $\beta$ is a multivariate approach to test for stationarity in the cointegration relations. The null hypothesis of this test implies that the variables are stationary $H(r) : \beta'y_t = 0$ (e.g. $\beta' = [1, 0, 0, 0, 0, 0]$). On the other hand, we perform a zero-row test in $\beta$. The aim of this test is to identify those variables that would not fit in the long-run equation. Thus, if the null hypothesis is not rejected, the variable should be removed from the system. Finally, the third test that we perform is a zero-row test in the $\alpha$ matrix which provides information about the pushing and pulling forces that drive the system. The null hypothesis of this test implies that any variable is a pushing force in the system or, equivalently, weakly exogenous (see Table 3).

Table 3: Unit vector and zero row tests for $\beta$ and $\alpha$

<table>
<thead>
<tr>
<th>$r$</th>
<th>$p_t$</th>
<th>$p_{t-1}$</th>
<th>$\iota_t$</th>
<th>$\iota_{t-1}^*$</th>
<th>$\iota_{t-2}^*$</th>
<th>$\iota_{t-3}^*$</th>
<th>$\iota_{t-4}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$\chi^2_{p-r}$</td>
<td>19.29</td>
<td>19.16</td>
<td>17.59</td>
<td>18.66</td>
<td>18.85</td>
<td>16.88</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Testing a unit vector in $\beta$ (stationarity)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\chi^2_{r}$</td>
<td>10.09</td>
<td>9.90</td>
<td>8.05</td>
<td>12.42</td>
<td>10.15</td>
<td>4.88</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
<td>0.00</td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td>Testing a zero row in $\beta$ (long-run exclusion)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>$\chi^2_{\alpha}$</td>
<td>6.77</td>
<td>7.25</td>
<td>5.97</td>
<td>3.26</td>
<td>2.78</td>
<td>6.07</td>
</tr>
<tr>
<td></td>
<td>p-value</td>
<td>0.03</td>
<td>0.02</td>
<td>0.04</td>
<td>0.02</td>
<td>0.25</td>
<td>0.04</td>
</tr>
<tr>
<td>Testing a zero row in $\alpha$ (weakly exogenous)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results presented in Table 3 suggest that the variables are non-stationary. Furthermore, all of the included variables in the system are significant at 5% of significance which means that they should be incorporated in the long-run equations. Finally, the zero-row test for $\alpha$ implies that only the foreign interest rate $i_{t-2}^*$ (the 90-days Libor) is weakly exogenous. According to this result, the $\alpha$ coefficient related to this variable will be set to zero, as a plausible interpretation that shocks in small economies like Peru do not affect foreign variables.
V.2 Identifying the long-run model

Before analyzing the long-run structure, from (4) and (7) we can hypothesize that (i) there is a stable relation between the price index differential and the nominal exchange rate and (ii) there is a positive correlation between the interest rate differential and the nominal exchange rate depreciation. Figure 3, Figure 4 and Figure 5 present these relations in the data.

Figure 3: Nominal Exchange Rate and Price Index Differential

Figure 3 shows that the price index differential follows a close pattern to the effective nominal exchange rate until the beginning of 2010. That period was characterized by variations in the nominal exchange rate that have been more pronounced\textsuperscript{16} than the domestic inflation relative to its main trading partners\textsuperscript{17}.

Figure 4: Interest rate differential and exchange rate depreciation

\textsuperscript{16}Probably because the bilateral exchange rate between Peru and its main trading partners have been more volatile.

\textsuperscript{17}We should take into consideration that Peruvian inflation has been one of the lowest in the region.
On the other hand, Figure 4 shows the behavior of the domestic-foreign interest rate differential in comparison to the nominal exchange rate depreciation. By the UIP, we expect a positive relation which can be seen for most of the sample. However, since 2006, there are some periods where a negative relation appears. This could be due to the higher risk characterizing the international markets today.

Finally, Figure 5 shows the domestic interest rate in foreign currency as well as the Libor. Both interest rates moved in the same way during the sample period. Furthermore, any difference between these two variables must reflect country risk, which we control by including the EMBI variable, as well as fundamentals, captured by the difference of price indices.

Taking into account the evidence presented above, we would expect two important relations in the formulated six-variable system. The first relation is related to a nested equation regarding the UIP and the PPP. The general long-run equation should take the form:

\[ \varepsilon_{1t} = (e_{1t} - p_{1t} + \pi_{1}) + \beta_{13}i_{1t} + \beta_{14}i_{1t}^* + \beta_{15}i_{1t}^{**} + \beta_{16}\zeta_{1t} \]  

(27)

This relation has been normalized to reflect any misalignment of the real exchange rate.

It may be convenient to analyze (27). For instance, assume that \( \beta_{15} = \beta_{16} = 0 \), which is a plausible assumption because if the UIP was tested using domestic interest rates, but denominated in different currencies, it would not be necessary to include a risk premium for investing in domestic (Peruvian) assets. Furthermore, we could also set \( \beta_{13} = \beta_{14} = 0 \), which would render a relation stating that the PPP is stationary by itself. However, this hypothesis would be difficult to prove empirically, due to imperfect substitution of domestic and foreign tradable goods\(^{18}\) and a Balassa-Samuelson effect (Arena & Tuesta, 1998) suggesting that deviations of the real exchange rate due to productivity differences prevent a fast technological and productive convergence.

For this reason, we formulate a joint hypothesis that the UIP and the PPP hold together in the long run. With this approach, we would expect the exchange rate to cointegrate with financial variables like the interest rates in order to control for discrepancies in the financial markets or the

\(^{18}\)Specially in Peru, where most of its exports are commodities while a large amount of its imports is related to industry inputs, capital goods and fuels.
cost of capital. Furthermore, a plausible hypothesis to test would be $\beta_{14} = -\beta_{13}$.

Acknowledging the fact that $\varepsilon_{1t}$ is an I(0) term, (27) can be written as:

$$e_{1t} = p_{1t} - p_{1t}^* + \beta_{13}(i_{1t} - i_{1t}^*) + \varepsilon_{1t}$$

where the hypotheses $\beta_{14} = -\beta_{13}$ and $\beta_{15} = \beta_{16} = 0$ have been imposed in (28).

The first part of (28) is the strong version of the PPP, while the second part relates the interest rate differential, or UIP, with the long-run nominal exchange rate. The parameter $\beta_{13}$ is anticipated to be positive. This because a long-run relation of the UIP and the nominal exchange rate would imply future depreciation conditional on an increase of the interest rate differential.

On the other hand, the second cointegration vector could be written as:

$$\varepsilon_{2t} = \beta_{21}(p_{2t} - p_{2t}^*) + \beta_{22}e_{2t} + \beta_{23}i_{2t}^* + \beta_{24}i_{2t}^* + \beta_{25}i_{2t}^{**} + \beta_{26}\zeta_{2t}$$

(29)

The second vector can be interpreted as a relation between a risk premium, an interest rate differential and some fundamentals. In this case, we have a huge advantage because one of the system’s variables is a domestic interest rate in foreign currency. Thus, this allows us to compare it to $i_{2t}^*$ (90-days Libor) and rule out exchange rate risk. However, as $i_{2t}^*$ is an emerging country’s interest rate, we add a risk premium to equal the foreign rate. Furthermore, to have a better control of the risk premium in our relation of interest, we include fundamentals in the form of a price differential 19.

We can impose restrictions to test our hypotheses by setting $\beta_{22} = \beta_{23} = 0$, $\beta_{24} = 1$ and $\beta_{25} = \beta_{26} = -1$. The coefficient regarding the price differential is left as a free parameter.

$$i_{2t}^* - i_{2t}^{**} = \zeta_{2t} + \beta_{21}(p_{2t} - p_{2t}^*)$$

(30)

Equation (30) can be understood as an augmented risk premium, where we model explicitly some fundamentals separately from $\zeta_{2t}$. In this particular case, we would expect $\beta_{21} > 0$ because a widening of the price index spread would imply a favorable productivity to the foreign economy relative to Peru.

Once we have examined the stationarity and relevance of the variables, we will analyze whether the PPP and UIP hold for the Peruvian economy. As stated in Section 4, Johansen & Juselius (1992) propose that for a p-dimensional system containing r cointegration relations, restrictions on the cointegration structure or the system can be tested by formulating

$$\beta^c = [\beta_1^c, \ldots, \beta_r^c] = [H_1\varphi_1, \ldots, H_r\varphi_r]$$

(31)

where $\beta^c$ is a constrained cointegration vector, $H_i$ are $(p \times s_i)$ design matrices and $\varphi_i$ are $(s_i \times 1)$ matrices for $s_i$ free parameters.

V.2.1 PPP and UIP with no interactions

First, we test whether the UIP and the PPP hold in the long run without being interacted. The first hypothesis postulates that the first vector only includes the PPP, while the second formulates that the UIP is the only relation in the second vector. In these cases, we are also constraining the foreign interest rate $i_{2t}^*$ and the risk premium $\zeta_{2t}$ to zero. The restricted vectors are:

$$\beta_1 = [-1, 1, 0, 0, 0, 0]$$
$$\beta_2 = [0, 0, 1, -1, 0, 0]$$

(32)

19 This variable can be thought of as a proxy of relative productivity.
with the corresponding $H$ matrices

$$H_1' = [-1, 1, 0, 0, 0, 0]$$
$$H_2' = [0, 0, 1, -1, 0, 0]$$ \hfill (33)

### Table 4: Cointegration analysis for the UIP and PPP

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\Delta(p_t - p_t^*)$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t - p_t^*$</td>
<td>-1</td>
<td>0</td>
<td>0.006</td>
<td>-0.007</td>
<td></td>
</tr>
<tr>
<td>$e_t$</td>
<td>1</td>
<td>0</td>
<td>$\Delta e_t$</td>
<td>-0.068*</td>
<td>0.065*</td>
</tr>
<tr>
<td>$i_t$</td>
<td>0</td>
<td>1</td>
<td>$\Delta i_t$</td>
<td>-0.003</td>
<td>-0.026</td>
</tr>
<tr>
<td>$i_t^*$</td>
<td>0</td>
<td>-1</td>
<td>$\Delta i_t^*$</td>
<td>-0.006</td>
<td>-0.004</td>
</tr>
<tr>
<td>$i_t^{**}$</td>
<td>0</td>
<td>0</td>
<td>$\Delta i_t^{**}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\zeta_t$</td>
<td>0</td>
<td>0</td>
<td>$\Delta \zeta_t$</td>
<td>0.086</td>
<td>-0.256</td>
</tr>
</tbody>
</table>

| LR test | 25.623 |
| p-value | 0.029 |

1 The LR test is distributed as $\chi^2(14)$.

Note: The values in brackets are the $t$-ratios. Values with * indicate significance at 5% level.

Note: The number of lags considered for the VAR model is four.

The p-value of the LR test in Table 4 is 0.029, suggesting that this structure, which treats both the PPP and the UIP separately, does not fit well in the data generating process. For instance, it is not surprising that the PPP or real exchange rate are not stationary processes. This can be explained by many factors, like imperfect substitution of imported and exported goods and a Balassa-Samuelson effect that implies differences in productivity preventing technological convergence between countries and adds up costs that the PPP does not take into account. Finally, the adjustment coefficients, with the exception of the nominal exchange rate in the first vector, are not significant at 5% level.

However, one may argue that omitting the existence of a risk premium in an emerging country like Peru would bias the results. For this reason, we include a risk premium with $\zeta$ in the second vector using the relation $i_t^* = i_t^{**} + \beta_2 \zeta_t$. With this, we obtain the vector $\beta_2 = [0, 0, 1, -1, -\beta_26]$ and the first vector remains as before. The design matrices for $\beta_1$ and $\beta_2$ would be, respectively:

$$H_3' = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$H_4' = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}$$ \hfill (34)

The results in Table 5 show that the restrictions are rejected at a level of significance of 5%. Regarding the restrictions imposed to $\alpha$ and $\beta$, they are also non-significant. Thus, evidence does not support the idea of treating UIP and PPP separately, even after including a risk premium. The next section tries to address this problem by allowing for interactions between the PPP and
the UIP in a same cointegration vector, as it is most likely that adjustments in both the capital and commodity markets could be interdependent in a small and open economy like Peru.

### V.2.2 A cointegration model with UIP and PPP interactions

A natural question is whether a joint estimation of both relations would render better results. Thus, it is important to explain the identification process related to both cointegration vectors. The first vector is a mixture between the PPP and the UIP. We formulate this vector in order to account for interactions between the commodity and capital markets.

The second vector is now restricted to represent the augmented risk premium established in (30). Furthermore, we explicitly model the second vector with the price differential as another proxy of fundamentals and as a way to have a better specification in the system.

\[
\beta_1' y_{t-1} = \begin{bmatrix} -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{t-1} - p_t^* \\ e_{t-1} \\ i_{t-1} \\ i^{*}_{t-1} \\ i^{**}_{t-1} \\ \zeta_{t-1} \end{bmatrix} \quad (35)
\]

\[
\beta_1' y_{t-1} = e_{t-1} - (p_{t-1} - p_t^* - i_{t-1} - i^{*}_{t-1}) \sim I(0) \quad (36)
\]

Note: The number of lags considered for the VAR model is four.

---

**Table 5: Cointegration analysis for the UIP and PPP with a risk premium**

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\Delta(p_t - p_t^*)$</th>
<th>$\Delta e_t$</th>
<th>$\Delta i_t$</th>
<th>$\Delta i_t^*$</th>
<th>$\Delta i_t^{**}$</th>
<th>$\Delta \zeta_t$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t - p_t^*$</td>
<td>-1</td>
<td>0</td>
<td>0.001</td>
<td>-0.079*</td>
<td>-0.003</td>
<td>-0.004</td>
<td>0</td>
<td>-0.013</td>
<td>[0.121]</td>
<td>[0.062]</td>
</tr>
<tr>
<td>$e_t$</td>
<td>1</td>
<td>0</td>
<td>[3.248]</td>
<td>[2.924]</td>
<td>[-1.43]</td>
<td>-0.462</td>
<td>0</td>
<td>[-1.419]</td>
<td>[NA]</td>
<td>[NA]</td>
</tr>
<tr>
<td>$i_t$</td>
<td>0</td>
<td>1</td>
<td>-0.013</td>
<td>-0.019</td>
<td>-0.005</td>
<td>-0.004</td>
<td>0</td>
<td>-0.027</td>
<td>[-0.085]</td>
<td>[-0.126]</td>
</tr>
<tr>
<td>$i_t^{*}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$i_t^{**}$</td>
<td>0</td>
<td>-1</td>
<td></td>
<td></td>
<td></td>
<td>-0.013</td>
<td>-0.004</td>
<td>-0.005</td>
<td>-0.013</td>
<td>-0.027</td>
</tr>
</tbody>
</table>

LR test$^1$ 23.957  

p-value 0.032

$^1$ The LR test is distributed as $\chi^2(13)$.

Note: The values in brackets are the t-ratios. Values with * indicate significance at 5% level.

Note: The number of lags considered for the VAR model is four.

---

20We refer it as “another proxy” of fundamentals because $\zeta_t$ is also subject to shocks to the fundamentals of the Peruvian economy.
\[ \beta'_{2y_{t-1}} = [\beta_{21} \ 0 \ 0 \ 1 \ -1] \begin{bmatrix} p_{t-1} - p^*_t \\ e_t \\ i_t \\ i^*_t \\ \zeta_t \end{bmatrix} \] (37)

\[ \beta'_{2y_{t-1}} = \beta_{21}(p_{t-1} - p^*_t) + i^*_t - i^*_t - \zeta_t \sim I(0) \] (38)

The design matrices are, respectively:

\[ H_5 = \begin{bmatrix} -1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad H_6 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix} \] (39)

Table 6 presents the estimation of the cointegration vectors and the adjustment coefficients for the model that considers these interactions. As we stated before for the identification process, we have imposed the first vector with the PPP and UIP, while the second vector is identified as a pricing equation for interest rates or a risk premium equation.

<table>
<thead>
<tr>
<th>Variable</th>
<th></th>
<th>Adjustment coefficients (\alpha)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Eq. (\alpha_1)</td>
<td></td>
<td>(\alpha_2)</td>
</tr>
<tr>
<td>(p_t - p^*_t)</td>
<td>-1</td>
<td>(\Delta(p_t - p^*_t))</td>
<td>0.003</td>
<td>0.001</td>
</tr>
<tr>
<td>(e_t)</td>
<td>1</td>
<td>(\Delta e_t)</td>
<td>-0.096*</td>
<td>0.003</td>
</tr>
<tr>
<td>(i_t)</td>
<td>-1.061*</td>
<td>(\Delta i_t)</td>
<td>-0.013</td>
<td>0.002</td>
</tr>
<tr>
<td>(i^*_t)</td>
<td>1.061*</td>
<td>(\Delta i^*_t)</td>
<td>-0.031*</td>
<td>0.003*</td>
</tr>
<tr>
<td>(i^*_t)</td>
<td>0</td>
<td>(\Delta i^*_t)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>(\zeta_t)</td>
<td>0</td>
<td>(\Delta \zeta_t)</td>
<td>-0.655</td>
<td>0.076*</td>
</tr>
</tbody>
</table>

| LR test\(^1\) | 11.977 |
| p-value | 0.287 |

\(^1\) The LR test is distributed as \(\chi^2(12)\).

Note: The values in brackets are the \(t\)-ratios. Values with * indicate significance at 5% level.

Note: The number of lags considered for the VAR model is four.

The previous table shows interesting results at first glance. The LR test accepts the imposed overidentifying restrictions with a p-value of 0.28 and the cointegration vectors and adjustment coefficients are highly significant. Also, the free parameters for the long-run equations have the expected sign and the adjustment coefficients for \(e_t, i^*_t\) and \(\zeta_t\) adjust to preserve the long term relations. For a better analysis, Table 6 can be written as:
This vector, we can formulate the equation for the dynamics of the exchange rate
\[ \gamma \]
regarding the effective nominal exchange rate. If we multiply some terms we can get the \( \Pi = \alpha \beta y \)
positive effect in the interest rates differential, and, therefore, the aggregate risk premium. Foreign productivity, which reflects higher relative prices in the domestic economy, would have a
trend of the nominal exchange rate. The aim of the Central Bank to smooth volatility in a short term, without changing the long-term
term dynamics dominated by the UIP hypothesis, in spite of the active intervention policy of the Peruvian Central Bank regarding the nominal exchange rate. This finding would also reflect the aim of the Central Bank to smooth volatility in a short term, without changing the long-term trend of the nominal exchange rate.

Regarding the second cointegration vector, we can formulate it as:
\[ i_t^* - i_t^{**} = \zeta_t + 2.2(p_t - p_t^*) \]  
(42)

The long-term relation in (42) has the expected coefficient (\( \beta_{21} = 2.2 > 0 \)) as an increase in
foreign productivity, which reflects higher relative prices in the domestic economy, would have a positive effect in the interest rates differential, and, therefore, the aggregate risk premium.

Furthermore, we are specially interested in the dynamics of the exchange rate. For this reason, if we multiply some terms we can get the \( \Pi = \alpha \beta y \) matrix:
\[ \alpha \beta y = \begin{bmatrix} 
0.003 & 0.001 \\
-0.096 & 0.003 \\
-0.013 & 0.002 \\
-0.031 & 0.003 \\
0.000 & 0.000 \\
-0.655 & 0.076 
\end{bmatrix} \begin{bmatrix} 
\alpha \beta y_{t-1} \\
\delta D08 
\end{bmatrix} \begin{bmatrix} 
p_{t-1} - p_{t-1}^* \\
e_t \\
i_t-1 \\
i_t^{**} \\
\zeta_t-1 
\end{bmatrix} \]  
(43)

If we now consider that \( \Delta y_t = \alpha \beta y_{t-1} + f(\Delta y_{t-4}) + \delta D08 \), then we can extract the equation regarding the effective nominal exchange rate.
\[ \Delta e_t = \gamma_2 \Pi y_{t-1} + f(\Delta y_{t-4}) + \delta D08 \]  
(44)

where \( \gamma_2 = [0, 1, 0, 0, 0, 0] \) is a selection vector that subtracts the second row of \( \Pi \) matrix. With this vector, we can formulate the equation for the dynamics of the exchange rate.
\[
\Delta e_t = 0.089(p_{t-1} - p^*_t) + -0.096\varepsilon_{t-1} \\
+ 0.102\zeta_{t-1} - 0.099i^*_t - 0.003\zeta^*_t - 0.003\zeta_{t-1} 
\] (45)

However, there are mixed effects of the coefficients due to the presence of some variables in both cointegration vectors. Using (40) we can isolate some effects and distribute them to each cointegration vectors. This can be done by analyzing the \( \alpha \) matrix.

\[
\Delta e_t = -0.096(e_{t-1} - p_{t-1} + p^*_t) + 0.102(i_t - i^*_t) \\
+ 0.003(i^*_t - i^*_t - \zeta_t) - 0.006(p_t - p^*_t) 
\] (46)

\[
\Delta e_t = -0.096(\text{RER}_{t-1}) + 0.102(i_t - i^*_t) \\
+ 0.003(i^*_t - i^*_t - \zeta_t) - 0.006(p_t - p^*_t) 
\] (47)

where \( \text{RER}_{t-1} \) is the real exchange rate.

It is convenient to analyze the expression in (47). In the long term, a positive deviation of 1 percentage point of the real exchange rate would require a negative adjustment of the nominal exchange rate of 0.096% to preserve the equilibrium. This result is consistent with the fact that the real exchange rate may deviate from equilibrium in the short run but the nominal exchange rate adjusts later to achieve the steady state again. The finding also supports the idea that the Central Bank intervenes in the market just to smooth exchange rate volatility in the short run, which in turn suggests that the real exchange rate would not deviate in the long run due to permanent deviations of the nominal exchange rate.

To analyze the other possible sources of real exchange rate misalignment, we can formulate the short-run equation of the price differential:

\[
\Delta(p_t - p^*_t) = 0.003(\text{RER}_{t-1}) - 0.003(i_t - i^*_t) \\
+ 0.001(i^*_t - i^*_t - \zeta_t) - 0.002(p_t - p^*_t) 
\] (48)

Regarding the real exchange rate in (48), even though its coefficient is not significant, the price differential would adjust in the short run to preserve the long-run relation. However, the speed of adjustment is considerably slower than the one seen in the nominal exchange rate, as a real depreciation of the exchange rate would imply a positive adjustment of the price spread of 0.003%. Thus, if we relate (48) to (47) we can anticipate that a real depreciation would cause the nominal exchange rate to appreciate to sustain equilibrium and, because of price stickyness reflected in a low rate of adjustment for prices, the coefficient related to the nominal exchange rate would have a higher reaction (Dornbusch-type effect). Furthermore, if we consider the following relation:

\[
\Delta i_t = -0.013(\text{RER}_{t-1}) + 0.012(i_t - i^*_t) \\
+ 0.002(i^*_t - i^*_t - \zeta_t) - 0.004(p_t - p^*_t) 
\] (49)

it is clear that a real exchange depreciation of 1 percentage point should cause the domestic interest rate to adjust faster than prices at a rate of -0.013% (at 5% of significance).
VI ANALYSIS OF PERMANENT AND TRANSITORY SHOCKS

Once the cointegration relations and the loading factors have been estimated through a VECM model, we are able to analyze what type of shocks are important to explain the dynamics of certain variables. For this part of the analysis, it is important to remember that, under a cointegration framework, it is known that if we have two cointegration vectors in a system of six variables there must be four common stochastic trends.

These stochastic trends correspond to the permanent shocks in the system since they affect the variables’ level in the long run, pushing off the system from its steady state. On the other hand, the two cointegration relations identify two shocks with transitory effects. Moreover, since we are working with domestic and foreign variables, we can also identify domestic shocks, which have effects only over domestic variables, and foreign shocks, which affect both domestic and foreign variables. Finally, the use of nominal variables in the system determines that we can only analyze nominal shocks.

Following the procedure developed by Gonzalo & Ng (2001) explained in Section 4, we estimate the $G$ matrix under the previously imposed restrictions on $\beta$ and $\alpha$. With this matrix, we are able to obtain the orthogonalized shocks which we classify as:

\[
\varepsilon_t = G^{-1}W\tilde{\eta}_t = G^{-1}W \begin{bmatrix} \tilde{\eta}^N_1 \\ \tilde{\eta}^N_2 \\ \tilde{\eta}^T_1 \\ \tilde{\eta}^T_2 \end{bmatrix} = G^{-1}W \begin{bmatrix} \tilde{\eta}^{N*}_1 \\ \tilde{\eta}^{N*}_2 \\ \tilde{\eta}^{T*}_1 \\ \tilde{\eta}^{T*}_2 \end{bmatrix}
\]

(50)

where $\tilde{\eta}^{N*}_i$ for $i = 1, 2$ are the nominal foreign permanent shocks, $\tilde{\eta}^{N}_i$ for $i = 1, 2$ are the nominal domestic permanent shocks and $\tilde{\eta}^{T*}_i$ for $i = 1, 2$ are the transitory shocks. The results are shown in the matrices below.

\[
\begin{bmatrix}
\varepsilon_{p_1-p_1^*} \\
\varepsilon_{\epsilon_t} \\
\varepsilon_{t_t} \\
\varepsilon_{t_t^*} \\
\varepsilon_{\epsilon_t^*}
\end{bmatrix} = \begin{bmatrix}
0.0000 & -0.0006 & -0.0031 & 0.0001 & -0.0008 & -0.0005 \\
-0.0037 & 0.0010 & -0.0018 & 0.0016 & 0.0089 & -0.0003 \\
-0.0004 & 0.0070 & -0.0005 & 0.0011 & -0.0011 & -0.0023 \\
0.0025 & 0.0004 & 0.0003 & -0.0006 & 0.0004 & -0.0025 \\
0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \\
-0.0693 & -0.0116 & 0.0138 & 0.0730 & -0.0155 & -0.0764
\end{bmatrix}
\]

(51)

We will analyze the permanent effect since it pushes the system off the long-run equilibrium. The first domestic nominal shock has a negative impact in the exchange rate, a negative effect in the domestic interest rate in domestic currency and a positive effect in the domestic interest rate denominated in foreign currency. The second shock corresponds to a external nominal shock which reduces the price differential, causes a depreciation, increases the domestic interest rates and negatively impacts the EMBI. The third shock is a foreign nominal shock that causes an appreciation and reduces the price differential. Also, it negatively impacts the domestic rate in local currency and causes a positive deviation in the domestic interest rate in foreign currency. The final shock is a domestic nominal shock that impacts positively almost all the variables, with the exception of the domestic interest rate in foreign currency.

\[
\begin{cases}
\varepsilon_{p_1-p_1^*} & \text{(1)} \\
\varepsilon_{\epsilon_t} & \text{(2)} \\
\varepsilon_{t_t} & \text{(3)} \\
\varepsilon_{t_t^*} & \text{(4)} \\
\varepsilon_{\epsilon_t^*}
\end{cases}
\]
VI.1 Impulse-response functions

The above results can be further complemented by analyzing how the variables of the VECM and definitions derived from them, respond to the structural shocks. Thus, in this section, we present the impulse-response functions of the permanent shocks.

Figure 6 explains the dynamic effects regarding the nominal exchange rate. The first shock is treated as a domestic interest rate shock in local currency (the TAMN). Its effect is straightforward from (41), as it causes an expected depreciation in the long run via the UIP hypothesis. The second permanent shock is labeled as a foreign price shock. This shock has a negative effect in the exchange rate at the beginning and it can be explained through the second cointegration vector in (42) as it generates a permanent long-run reduction in the domestic interest rate in foreign currency (the TAME). Thus, holding assets in foreign currency represents a higher relative cost compared to holding assets in domestic currency, which implies that investors will pressure the exchange rate to fall below its steady state level at the beginning. However, as the first cointegration vector in (41) suggests, the negative impact of the TAME will have a positive effect in the exchange rate through the UIP hypothesis that will raise it above its equilibrium level in the long run. This second order shock to the TAME is most likely to have a stronger effect than the original external price shock because the exchange rate remains positively deviated from its steady state in the long term.

Figure 6: Response of the nominal exchange rate to permanent shocks

The third permanent shock can be regarded as a foreign interest rate shock (LIBOR). The effects of this shock are first generated through the second cointegration relation, raising the domestic interest rate in foreign currency above its equilibrium level. This shocked variable, in turn,
generates a long-run appreciation due to the UIP element of (41), making it deviate permanently from its steady state. Finally, the last shock is labeled as a domestic price shock, which has a positive effect on the nominal exchange rate in (41). Furthermore, as the TAME is positively affected through the second cointegration relation, it exerts a negative long-run effect in the nominal exchange rate, defined in the first vector. However, as the first vector is also affected by the price shock, the net effect remains as a positive deviation from equilibrium.

The next impulse-response functions are related to aggregate variables that we created from the cointegration vectors. Thus, the analysis is focused in the impact of permanent shocks to the effective real exchange rate (RER), the interest rate differential (TAME - TAMN) and the EMBI. Figure 7 presents the response to the first permanent domestic shock, labeled now as a rise in the domestic price index. This increase produces a positive deviation of the TAME-TAMN differential implied by the second cointegration vector. Furthermore, by the joint PPP-UIP hypothesis, the real exchange rate decreases until the fourth period, were it exhibits a positive long run trend because of the reversion in the TAME-TAMN differential (which is related to the real exchange rate through the first cointegration vector). The EMBI variable, as expected, responds to fundamentals, as it shows a permanent negative level due to the permanent positive deviation of the real exchange rate above its steady state.

Figure 7: Response to the first permanent domestic nominal shock

[1] Confidence bands at 75% have been created with a bootstrap of 20,000 simulations.
[2] The shock implied by the $G^{-1}W$ matrix deviates the variables in the first period.
On the other hand, Figure 8 presents the response to the first permanent foreign nominal shock, which we interpret as a rise in the foreign price index. This shock generates an increase in the real exchange rate by the PPP hypothesis and a negative deviation of the TAME by the proposed second cointegration vector, which negatively relates it with the foreign price index. As a negative TAME-TAMN spread is given in the long run, an expected depreciation of the nominal exchange rate is generated, which also implies a real depreciation. Finally, the EMBI reacts accordingly to the fundamentals reflected in the real exchange rate as it is permanently below its steady state (showing a permanent increase in domestic competitiveness).

Figure 8: Response to the first permanent foreign nominal shock

---

[1] Confidence bands at 75% have been created with a bootstrap of 20,000 simulations.
[2] The shock implied by the $G^{-1}W$ matrix deviates the variables in the first period.

Figure 9 presents a response to the second permanent foreign nominal shock, which can be interpreted as a shock to the Libor rate. As implied by the second cointegration vector, there is a pronounced rise in the TAME-TAMN differential because of a positive adjustment of the TAME. While this effect could imply a short-run depreciation in the real exchange rate (due to a depreciation of the nominal rate), the long-run effect is an appreciation of the real exchange rate as suggested by the first formulated cointegration vector. Furthermore, the EMBI’s response is related to the shocked fundamentals, as it decreases due to the initial increase in the real exchange rate. In summary, the long-run equilibrium in this case is characterized by a permanent real appreciation, a permanent decrease of the EMBI (which seems to revert to a positive trend even though the initial shock was strong) and a permanent increase of the TAME-TAMN differential.
Finally, the response to the second permanent domestic nominal shock is presented in Figure 10. We characterize this shock as an increase in the domestic interest rate (TAMN). As can be seen, it generates a positive shock to the real exchange rate due to the joint PPP-UIP hypothesis (the first cointegration vector). Regarding the TAME-TAMN differential, it is permanently below its steady state because of the positive TAMN shock. Moreover, even though the EMBI is above its equilibrium it seems that it has a long-run negative trend reflecting the positive effect of the real exchange rate.
Figure 10: Response to the second permanent domestic nominal shock

[1] Confidence bands at 75% have been created with a bootstrap of 20,000 simulations.
[2] The shock implied by the $G^{-1}W$ matrix deviates the variables in the first period.
VII CONCLUSIONS

This study focuses on analyzing the nominal and real effective exchange rate dynamics with Peruvian data. For this, we used a parity view by linking the effective nominal exchange rate to two fundamental international parities, the Purchasing Power Parity (PPP) and the Uncovered Interest Rate Parity (UIP). These hypotheses are typically postulated as equilibrium conditions for commodity and capital markets but they are treated separately. However, as a disequilibrium in one of the markets can have effects on the other, treating them in isolation would yield misleading results.

One important advantage of Peruvian data is the existence of a domestic interest rate denominated in foreign currency, which has two important consequences. First, we are able to test and formulate an accurate UIP equation, since the difference in the interest rates corresponds solely to exchange rate movements and not country risk. Second, the availability of such rate determines the existence of a new cointegration equation that states a new arbitrage condition for a dollarized economy like Peru.

The results of Johansen’s cointegration approach suggest the existence of two stationary vectors. The first cointegration equation represents the joint PPP-UIP formulation while the second vector represents an interest rate equation with a risk premium and some fundamentals. Furthermore, consistent with the idea that Peru is a small economy, the $\alpha$ coefficient regarding the Libor is weakly exogenous.

Our estimations provide two important conclusions. First and foremost, it shows that neither the PPP nor the UIP hypotheses hold when modeled separately; however, a combination of both in a nested equation is a more suitable representation and better explains the dynamics of the nominal effective exchange rate in Peru. This suggests that, indeed, there are some spillover effects between commodities and capital markets. Secondly, it has important implications for monetary policy, as our findings provide evidence that the aim of the Central Bank’s intervention is to smooth short-run volatility of the exchange rate, but does not have a long-run effect.

Finally, the Gonzalo-Ng shock decomposition has allowed us to identify two nominal domestic and foreign structural shocks. In each case, they have been regarded as interest rate and price shocks. The results of permanent shocks, as expected, show that the analyzed variables (nominal exchange rate, real exchange rate, interest rate spread and EMBI) deviate from their equilibrium in the long run, generating a new steady state. Furthermore, the dynamics of the responses are accurately and correctly specified under our framework of two cointegration vectors. However, it is complicated to have a correct identification of structural shocks because the Gonzalo & Ng (2001) approach does not allow to recognize the specific shocks and just separates those permanent from the transitory ones.

Future research could seek to provide more evidence on the PPP and UIP relations working vis a vis with bilateral exchange rates from Peru’s most important trading partners in order to identify whether they hold. Finally, the inclusion of other variables such as credit could help to identify other channels in which the exchange rate influences the financial market.
References


Stephens, D. (2004). The equilibrium exchange rate according to PPP and UIP. Reserve Bank of New Zealand, DP2004/03.


