Risky Swaps

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Home

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Abstract.

In [10] we presented a reduced form of risky bond pricing. At default date, a bond seller fails to continue fulfilling his obligation and the price of the bond sharply drops. For no-default scenarios, if the face value of the defaulted bond is $1 then the bond price just after the default is its' recovery rate (RR). Rating agencies and theoretical models are trying to predict RR for companies or sovereign countries. The main theoretical problem with a risky bond or with the general debt problems is presenting the price, knowing the RR.

The problem of a credit default swap (CDS) pricing is somewhat an adjacent problem. Recall that the corporate bond price inversely depends on interest rate. In case of a default, the credit risk on a debt investment is related to the loss. There is a possibility for a risky bond buyer to reduce his credit risk. This can be achieved through buying a protection from a protection seller. The bondholder would pay a fixed premium up to maturity or default, which ever one comes first. If default comes before maturity, the protection buyer will receive the difference between the initial face value of the bond and RR. This difference is called 'loss given default'. This contract represents CDS. The counterparty that pays a fixed premium is called CDS buyer or protection buyer; the opposite party is the CDS seller. Note, that in contrast to corporate bond, CDS contract does not assume that the buyer of the CDS is the holder of underlying bond. Also note that underlying to the swap can be any asset. It is called a reference asset or a reference entity. Thus, CDS is a credit instrument that separates credit risk from corresponding underlying entity.

The formal type of the CDS can be described as follows. The buyer of the credit swap pays fixed rate or coupon until maturity or default in case it occurs before the maturity. If default does occur, protection buyer delivers cash or a default asset in exchange with the face value of the defaulted debt. These are known as cash or physical settlements.

Introduction.

The option valuation benchmark was developed by Black, Scholes, and Merton in 70s [2, 15]. It uses the present value neutralized reduction of the underlying security for the instrument pricing. It was highlighted in [5-8] that the underlying logic of the benchmark approach in many aspects contradicts the basic definition of pricing. Indeed, either the Black Scholes option pricing equation or a later developed binomial scheme does not depend on a real return of the underlying security. Therefore, these approaches suggest the price is the same for the options written on securities with an equal risk characteristic (volatility) and different expected rate of return, regardless whether it positive or negative. The benchmark price is the same for the options with the positive payoff at maturity having a probability arbitrary closed 1, and with probability as closed to 0 as we wish [5-7].

Recall that this incorrect pricing is constructed on the base of an idea known as 'self-financing' or 'no-arbitrage'. Briefly, the self-financing pricing scheme can be outlined as follows: The price of a financial instrument can be found by using standard transactions. First, one borrows funds from a bank at the risk free interest rate. Then he goes short in bonds and invests the funds in the instrument. The rule of pricing the instrument is that: the total price of the new instrument should be equal to 0 at any time in the future. Realization of this idea in stochastic setting can be achieved by putting the portfolio changes equal to 0 over the time. This represents 'no arbitrage' strategy. It should be clear that such no arbitrage approach based on assumption that market prices are expected values rather than a particular realization of a scenario.

This approach has its explicit drawbacks. If the instrument is risk free, then the self-financing strategy makes sense. In a stochastic setting self-financing scenario automatically implies deterministic interpretation of the market prices. Indeed, borrowing and investing funds in a risky market contains risk of losing an investment. In other words, the risk of investing in stochastic market could be described as getting a return bellow of what was initially planned.

In theory, one can assume that underlying security distribution and its' parameters are known. Fixing the distribution might lead to an opportunity to define a generalization of the classical arbitrage. The stochastic arbitrage is when an investor hopes to receive a statistical advantage. Nevertheless in finance it is difficult to exercise statistical arbitrage.

Theoretical distributions used for models in finance are implied and therefore statistical arbitrage for implied distributions is also implied. From mathematical point of view arbitrage is a necessary condition of a correct pricing.; that is if pricing is correct then arbitrage could not exist. Also, there is no arbitrage between stochastic stock price and deterministic bond price. Now let us express our remarks formally.

European call option price is a solution of the Black Scholes equation

$$\frac{\partial c}{\partial t} + rx\frac{\partial c}{\partial x} + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2 c}{\partial x^2} - rc = 0, \quad t \in [0, T), \ x > 0 \qquad \text{(BSE)}$$

$$c(T, x) = \max(x - K, 0)$$

The risk free interest rate (r), volatility coefficient (σ), and strike price (K) are assumed to be given constants. The unique solution to the problem (BSE) admits probabilistic representation in the form

$$c(t, x) = \exp - r(T - t) E \max(S_r(T) - K, 0)$$

where the random process $S_r(v) = S_r(v; t, x)$, $S_r(t) = x, t \le v \le T$ is a solution of the stochastic differential equation

$$d S_r(t) = r S_r(t) dt + \sigma S_r(t) dw(t)$$

on a complete probability space $\{ \Omega, F, P \}$ and $w(t)$ is a Wiener process on this space. Assume that option underlying security is governed by the equation

$$d S_\mu(t) = \mu S_\mu(t) dt + \sigma S_\mu(t) dw(t)$$

and $\mu \neq r$. The first remark to the Black Scholes pricing is that the solution of the problem (BSE) could not be called price. The notion of the price is commonly associated with the present value PV of the future cash flow. If each payment of the one cash flow strictly larger of the correspondent payment of the other cash flow then the spot price of the first flow should be greater than the spot price of the second. Otherwise, arbitrage exists. This observation can be formally expressed as the **Equal Investment Principle** (EIP).

Two investment opportunities are equal at the moment t, if they promise an equal instantaneous rate of return. Two investments are equal over the time interval $[t, T]$ if, they are ever equal at any moment of time of this interval.

Thus, equal investments are generated by the equal cash flows, and this implies equal spot prices. Assume that $0 < r < \mu$. Then as it follows from comparison theorem for one dimensional stochastic differential equations(SDE) $S_r(v; t, x) < S_\mu(v; t, x)$, $t \leq v \leq T$ with probability 1 and therefore the cash flow generated by the real μ-security is greater than the cash flow generated by the r-security. Hence, the present values of two cash flows are not equal. Remarkably, two investments can have equal present values, but according to EIP they could also be not equal. This remark shows that the benchmark present value principle is incomplete. The EIP was used earlier in [7, 9, 10] for fixed income and option valuations.

Looking at the equation (BSE), one can note that underlying securities having equal risk characteristic (σ) and promising arbitrary expected return $(-\infty, +\infty)$ are priced by the same amount. As **far as the future cash flows of two processes having opposite signs of the expected return satisfy** inequality $S_{-\mu}(v; t, x) < S_\mu(v; t, x)$, $t \leq v < +\infty$ then the present values of the correspondent cash flows over $[t, T]$ are different. That is

$$PV_t^T S_{-\mu}(\cdot; t, x) < PV_t^T S_\mu(\cdot; t, x)$$

That implies that the Black Scholes equation's solution could not be defined as a price based on the present value principle. Even though this function does not present an arbitrage, this function also can not be associated with the common notion of the price used in finance long before derivatives came into market. Therefore the benchmark option price is incorrect and cannot be used either by investors or by professors.

There is another aspect of the derivatives pricing we need to highlight. For the recent generation of the derivative instrument called credit derivatives, the risk neutral setting stems from the (BSE) interpretation. Note, that some researchers also used risk neutral setting for interest rate derivatives pricing. This problem relates to the probability theory and does not relate to finance. This is called risk neutral pricing. Let us briefly outline the essence of the problem. Usually researchers refer to the risk neutral world when they want to emphasize that in this world real μ-security would transform into neutralized r-security. Mathematical techniques behind this transformation are known as Girsanov theorem. Application of this theorem to the random processes S_μ, S_r states that measures m_μ, m_r corresponding these processes are absolutely continuous to each other and in particular

$$\frac{d m_\mu}{d m_r}(\omega) = \exp\{ \lambda [w(T) - w(t)] - \frac{1}{2}\lambda^2 (T - t) \}$$

where

$$\lambda = \frac{\mu - r}{\sigma}$$

Denote measure Q on the same measurable space $\{ \Omega , F \}$ with the help of the equality

$$Q(A) = \int_A \frac{d m_\mu}{d m_r}(\omega) P(d\omega)$$

Then the random process S_μ is the solution of the equation

$$d S(t) = r S(t) dt + \sigma S(t) d w_Q(t)$$

with the Wiener process $w_Q(t) = w(t) + \lambda t$ on the probability space $\{ \Omega, F, Q \}$. One could note that the measure Q depends on parameter λ and therefore it depends on parameter μ. Plus the calculation of the expectation of a functional given on risk neutral world $\{ \Omega, F, Q \}$ automatically converts it to the correspondent functional with respect to the measure P according to the formula

$$E_Q f(S_r (*; t, x)) = E f(S_\mu (*; t, x))$$

It is common to state that the solution of the problem (BSE) can be represented as an expected value of the functional over the risk neutral process S_r on the risk neutral world $\{ \Omega, F, Q \}$. Note the functional is a mathematical term that covers a variety of payoff classes used in derivative contracts. Taking into an account the change of variable represented in above equality one can see that risk neutral world does not perform the transformation of the real word security price S_μ into neutralized price S_r on risk neutral world. That means commonly stated the risk neutral setting fails to perform the task of the real world transformation of the parabolic equation. The real return to the parabolic equation has the risk free coefficient at the first order derivatives with respect to price variable. On the other hand, it is easy to apply the process S_r determined on the original probability space in order to present the solution of the parabolic equation (BSE) in the form represented above.

It is not a mistake to start on original probability space then to make Girsanov transformation and arrive at the risk neutral world. Following this way let us assume a certain number N of security instruments that composes a security market. Note, that this number N as well as a particular choice of these securities is a subjective heuristically notion. By choosing the market representation one could assume different stochastic dynamics on the risk neutral world. Nevertheless, the pricing and accurate calculation of the risk characteristics connected to these notions would display their dependence on security market parameters. That is irrelevant and incorrect. As well, it is difficult to reveal details of underlying calculations. It looks like the primary software simply omitted Girsanov density and used neutralized security which is mathematically incorrect, although can present close quantitative or qualitative results if the risk free and real returns are closed to each other.

CDS

A credit default swap is an over-the-counter (OTC) bilateral financial instrument used for hedging default risk of a risky debt instrument or instruments.
Default happens when one or two counterparties fail to make a scheduled payment.
CDS one of the most popular class of credit derivatives that allows trading of the counterparty risk from one to another without changing ownership of an underlying instrument.
A CDS is one of the most liquid instrument types where one party called protection buyer seeks for the credit protection on risky debt such as corporate bonds or loans. These are referred to as to reference entity. On the opposite side of the agreement is another party that agrees to pay in the event of default of the reference entity. This counterparty is called a protection seller. The protection buyer pays the protection seller a fee in the form of periodic fixed payments usually paid semiannually or quarterly. At the event of default the fixed payments stop and the protection seller pays to the protection buyer the predetermined amount. There are two common ways of the CDS settlement. With the physical settlement the protection buyer delivers the defaulted reference security or its equivalent to the protection seller and in exchange receives the par value of the reference security. Second way is through a cash settlement. In this case protection buyer receives the difference between par and recovery value of the security. This amount is called a loss given default. The physical settlement is a more common way of delivery. The most popular notional value is 5 or 10 million dollars with 5 years until expiration. Most of the CDS contracts are single name having a single corporate bond underlying. A portfolio names swap is written on a basket of bonds. For those contracts the first-to-default swap is most popular. It terminates when the first credit event happens or at the contract expiration. Other type of the swap written on a basket of the N bonds is n-th to default, $n \leq N$. This is a CDS contract that pays buyer of protection the difference between its face value and recovery rate at the n-th default event among the reference pool. Introduced CDS contract is more alike to the insurance contract.

There is a relationship between risky bond evaluation and CDS pricing. In a risky coupon bond valuation the face value and coupon payments can be interpreted as given parameters of the exposure for CDS seller. The variable risky bond price dynamics then becomes a subject to study. The CDS valuation is the problem of finding CDS spread with constant rates. Thus the CDS valuation is somewhat adjacent counterpart to the risky bond pricing.

Let us now outline CDS theoretical framework. We consider the two approaches to CDS valuation. The first valuation approach is standard based on comparison present values of the two counterparties involved in CDS. The main distinction of our approach is that we use stochastic setting in contrast to commonly accepted methods dealing with expected values of the cash flows to counterparties. Then we also consider another approach that uses the option pricing. Note that option-pricing method used in [5,7] in any respect does not relate to the Black Scholes benchmark derivative pricing [2,15].

Assume first for simplicity that default of the corporate coupon bond occurs only at a specified series dates t_k, $k = 1, 2 \ldots N$. These dates can or cannot be the coupon payment dates. Let $\tau(\omega)$ denote a random time of default and D_k denote the default event at the date t_k defined on a probability space $\{\Omega, F, P\}$. Thus $D_k = \{\omega : \tau(\omega) = t_k\}$, $k = 0, 1, ..., N$. Let (T) stand for the maturity of the bond. It is also possible to assume that $t_N < T$, for simplicity we put $t_N = T$. If a scenario $\omega \in D_k$ then a protection buyer pays a fixed periodic premium \$q at the dates $t_1, \ldots, t_{k-1}$. We assume that the premium (q) is not paid at the date of default. On the opposite side of the swap contract, the protection seller would compensate the losses of the protection buyer at the default date. This compensation value is an amount 'loss given default'. There are several reasonable possibilities to define the value of 'loss given default' (LGD). For instance it can be defined as

$$[F - \Delta_k]\,\chi\{\tau(\omega) = t_k\}$$
$$[F - R_q(t_k - 0, T;\omega)]\,\chi\{\tau(\omega) = t_k\}$$
$$[F\,B(t_k, T) - R_q(t_k, T;\omega)]\,\chi\{\tau(\omega) = t_k\}$$

Here, $R_q(t, T;\omega)$ is the value of the risky coupon bearing bond at the date t and maturity T. The first equality shows that the value of the risky bond at the default event is equal to its recovery rate. Therefore LGD is defined as the promised face value minus recovery. In the second equation, the LGD is the differential between bond's face value and the value just before default. The last equation corresponds to the case when default debt in the either form is delivered to the protection seller in exchange for the risk free bond with the equal face value and maturities. This case also covers the 'cheapest-to-delivery' delivery option as far as $B(t_k, T)$ could be issued at arbitrary moment in the past. Let $B(t_k, T; t)$ denote the risk free bond price at the date t_k issued at t with maturity T. Putting

$$B(t_k, T) = \min\{t, t \le t_k : B(t_k, T; t)\}$$

we note that the third equality can be interpreted as the 'cheapest-to-delivery' settlement.

The present value of future payment is considered by definition as the spot value of this payment. This reduction is usually presented in the form in which the future payment is multiplied by the zero coupon risk-free bond having maturity at the specified date in the future. Thus the discounted value is the seller's spot price. The series of the future payments received over a particular time period as coupon payments accumulated by the bond buyer at the maturity or at the default date, which ever one comes first. This other present value can be determined as a present value of the total balance at the date that is minimum between maturity or default dates. This present value relates to the bond buyer spot price and is by construction a random variable. On the other hand the bond buyer can buy the bond and go short on the same amount equal to discounted coupon future payments. In this case it is somewhat unrealistic; one needs to use the benchmark present value for the bond buyer's valuation. We consider such case as unrealistic because in a two party deal the security seller is a party looking for financing while the security buyer represents an investor. Therefore, in case when the investor buys a security and goes short getting cash is a mixed strategy combining investor's and financier's strategy. It might be a reasonable way but it should be more accurately specified. In this paper we consider market participants interested in getting maximum return on their investments.

The CDS value by definition is the value of a periodic payment q for which the balance of the cash flows to counterparties is equal to 0. This value q can be also interpreted as a coupon or premium but it is commonly called CDS spread. Hence the CDS spread is a fixed rate q during the lifetime of the CDS contract for which the cash flow to either counterparty is 0. The present value of the cash flow over the lifetime CDS to the protection seller is

$$\sum_{k=1}^{N} B(t, t_k)[q \sum_{j=0}^{k-1} B^{-1}(t_j, t_k) - (F - R(t_k, T; \omega))]\chi(\tau(\omega) = t_k) + \tag{1}$$

$$+ B(t, T) q [\sum_{k=1}^{N} B^{-1}(t_k, T)]\chi(\tau(\omega) > T) = 0$$

The solution of the equation (1) is a random variable $q = q_b(\omega)$ equal to

$$q_b = \frac{\sum_{k=1}^{N} B(t, t_k)[F - R(t_k, T; \omega)] \chi(\tau(\omega) = t_k)}{\sum_{k=1}^{N} [\chi(\tau(\omega) = t_k) B(t, t_k) \sum_{j=1}^{k-1} B^{-1}(t_j, t_k) + \chi(\tau(\omega) > T) B(t, T) B^{-1}(t_k, T)]} = \tag{2}$$

$$= \sum_{k=1}^{N} \frac{[F - R(t_k, T; \omega)]}{\sum_{j=1}^{k-1} B^{-1}(t_j, t_k)} \chi(\tau(\omega) = t_k)$$

The value q_b is the exact solution of the CDS pricing problem. From (2) in particular follows that for the no default scenario the value of the premium is 0. There is no need to buy protection for no default over the lifetime of the risky bond scenarios. Then for $\omega \in D_k =$ $= \{\tau(\omega) = t_k\}$ there is only one term in denominator and numerator that is not equal to 0 and in this case

$$q_b(\omega)\chi(\tau(\omega) = t_k) = \frac{[F - R(t_k, T; \omega)]}{\sum_{j=1}^{k-1} B^{-1}(t_j, t_k)}$$

and $q_b(\omega) = 0$ for $\omega \in \{\tau(\omega) > T\}$. Summing up these equalities we arrive at (2).
Remark. The benchmark CDS valuation models [4, 11] reduce the CDS spread notion to the break-even value that makes the expected value of the cash flows to counterparties equal. It is easy to check that the expected value of the exact solution of a linear algebraic equation with random coefficients does not coincide with the solution of the problem in which real random cash flows are replaced by the expectations of these cash flows. Besides that dealing with expectation of the cash flows one loss the market risk exposure implied by the random coefficients. Also the protection buyer and protection seller have asymmetric exposure to the

credit risk. Indeed, for no default scenarios protection buyer paid periodic premium over the lifetime of the CDS whereas protection seller does not pay anything to the protection buyer. The present value of these cash flows is presented below by the first term of the equality (3). If default occurred at the date t_k, then the protection buyer would receive from protection seller 'loss given default' amount equal to $(F - R (t_k , T ; \omega))$; in exchange for the fixed periodic payments q protection buyer has paid on dates t_j, $j < k$. For instance the present value for protection buyer is equal to 0 if:

$$\sum_{k=1}^{N} [B(t, t_k) (F - R(t_k , T; \omega)) - q_s \sum_{j=1}^{k-1} B(t , t_j)] \chi(\tau(\omega) = t_k) - \tag{3}$$

$$- q_s \sum_{k=1}^{N} B(t, t_k) \chi(\tau (\omega) > T) = 0$$

The periodic coupon implied by equation (3) is:

$$q_s (\omega) = \frac{\sum_{k=1}^{N} B(t, t_k) [F - R(t_k , T ; \omega)] \chi(\tau(\omega) = t_k)}{\sum_{k=1}^{N} \sum_{j=1}^{k-1} B(t, t_j) \chi(\tau(\omega) = t_k) + B(t, t_k) \chi(\tau(\omega) > T)} =$$

$$= \sum_{k=1}^{N} \frac{B(t, t_k) [F - R(t_k , T ; \omega)]}{\sum_{j=1}^{k-1} B(t, t_j)} \chi(\tau(\omega) = t_k) \tag{4}$$

The risk analysis of the CDS contract can be established as follows. Recall that the market risks of the protection seller and protection buyer are different and can be determined using formulas (2, 4). Counterparty risk depends on default time distribution. There are a variety of assumptions regarding a reasonable apply of default time distributions. In addition given D_k the protection buyer risk at t depends on future rates over periods $[t_j , t_k]$, $1 \leq j < k \leq N$ in formula (4). These formulas ignored protection seller's risk of default on claim amount at the date of default. If the seller's and the buyer's valuation formulas are different then it makes sense to present these risk formulas separately. When two counterparties come to the agreement regarding premium value then this value should be applied to estimate the market risk for either party of the contract. Thus, a settlement market price μ_q on CDS contract implies the risk for both counterparties. Let μ_q denote a market spread. Then market risk can be described by the following. The protection buyer's risk is a probability that the market spread value μ_q exceeds $q_b(\omega)$, i.e. $P \{ q_b (\omega) < \mu_q \}$. This probability represents the measure of a chance that the market price exceeds the exact contract price. Recall that for each scenario $\omega \in D_k$ the value $R (t_k , T ; \omega)$ is defined by its recovery rate [10]. This recovery rate can be found based on formula (4). If it is reasonable to use binomial distribution as an approximation of the time of default we can see that

$$P\{ \tau = t_j \} = (1 - p) p^{j-1} , \qquad P\{ \tau > t_j \} = 1 - p^j$$

Here the value 1 - p denotes the probability of default. Using these formulas any statistical characteristics of the swap can be calculated easily.

Some CDS variations.

CDS contract is the most popular credit derivative instrument. Nevertheless there are several important and popular variations of the standard CDS on the credit market. Let us first consider a **constant maturity default swap** (**CMDS**) contract. Goldman Sachs introduced this contract in 1997. A CMDS is almost identical to the standard CDS. The primary difference relates to the premium leg of the contract. For the standard CDS premium leg is a specified constant coupon paid periodically to the protection seller until the earliest between default and maturity. For a CMDS contract several parameters should be specified in advance. Those are the length of the constant maturity; a sequence of reset dates. These are dates when the previous market spread is replaced by a new CDS spread with a specified maturity, and a percentage factor that would be applied for. This percentage factor is also known as a participation rate. Given available information regarding CDS contracts the valuation of CMDS contract is the problem of calculation unknown percentage factor. First the protection leg payments are the same for the either type of the contract CDS or CMDS and equal to:

$$\sum_{k=1}^{N} [F - R(t_k , T; \omega)] \chi(\tau(\omega) = t_k)$$

A modification presented for CMDS contracts can be studied as follows. Let $L(s, t)$, $s \le t$ be the LIBOR rate at date s over $[s, t]$. It has been defined as the simple interest rate for Eurodollar deposit at s with maturity t. The cash flow from protection buyer to the protection seller for CDS and CMDS contracts are different and can be written in the forms

$$[\sum_{k=1}^{N} \chi(\tau(\omega) = t_k) \sum_{j=1}^{k-1} L^{-1}(t_j , t_k) + \chi(\tau(\omega) > T)] q_s$$

$$[\sum_{k=1}^{N} \chi(\tau(\omega) = t_k) \sum_{j=1}^{k-1} q_H(t_{j-1}) L^{-1}(t_j , t_k) + q_H(t_{N-1}) \chi(\tau(\omega) > T)] p$$

correspondingly. Here p is an unknown constant representing percentage factor and $q_H(t)$ is the CDS spread at the date t with maturity t + H. Protection seller receives payments on scheduled dates prior to default or maturity, which ever one comes first. The LIBOR rate is used here as a discount factor. Writing the equality in which the value of inflow is equal to the value of outflow from protection buyer to the protection seller we see that

$$p(\omega) = \sum_{k=1}^{N} \chi(\tau(\omega) = t_k) \frac{F - R(t_k, T; \omega)}{\sum_{j=1}^{k-1} q_H(t_{j-1}) L^{-1}(t_j, t_k)} \qquad (5)$$

Remark. It makes sense to highlight a calculation problem that relates to the derivatives valuation. The common approach in finance valuation practice to replace stochastic cash flow by its expectation. This reduction cannot be always accepted without critical remark. Sometimes for a particular problem it might make sense, but sometimes it might not. Let us consider an example. Let y be an unknown parameter and the present value of cash flows to and from an investor can be modeled as $[w^2(t) + 1] y$ and $5 w^2(t) + 3$. If we consider the equation generated by the expected cash flows we arrive at the solution

$$<< y >> = \frac{5t + 3}{t + 1}$$

On the other hand the exact solution of the problem is

$$y = \frac{5w^2(t) + 3}{w^2(t) + 1}$$

and its expectation does not coincide with the value <<y>>. This discrepancy highlights the fact that replacement of a stochastic flow by its expectation can lead to the crude estimate of the exact solution. In the case when a pricing problem admits an exact solution advanced reduction of the problem to the expected flows might be considered as mathematically incorrect. We also illustrate this point of view bellow.

The **equity default swap** (**EDS**) was launched by JP Morgan Chase in 2003 though the first equity swap was when Amoco Pension exchanged fixed rate on Japanese stock index excised in 1990. As a CDS benchmark contract EDS exchanges variable rate on a constant rate until maturity or a credit event which one comes first. A source of the variable rate could be a stock, a basket of stocks or price any of the market components. A stock basket can be referred to a traded index or a composed virtual index and rate and price can represent the gold price. Thus EDS links to equity market rather than to credit market.

Remark. First let us make a comment related to the equity swap (ES) contract pricing with zero chance of default. Such contracts first started to trade at the late 80's. The pricing models of the ES are well known [3]. The problem is: given stochastic equity price $S(t, \omega)$ to derive a fixed leg rate R of the swap contract. The benchmark formula that was developed using self-financing and no arbitrage general principles is quite simple. Following [3] the ES spread value is equal to

$$R = \frac{1 - B(0, t + n)}{\sum_{i=1}^{n} B(0, t + i)}$$

Here B(0 , T) is a Treasury bond price at the time 0 and maturity T, and B(T , T) = $1. Though in [3] it was remarked that "surprisingly the level of stock is irrelevant in determining the value of swap" the correct conclusion was not provided probably based on widely prevalent over financial community faith in perfection of the valuation methods. These methods might be reasonable when the market is constituted by securities which prices are subject to self-financing and no-arbitrage valuation. In stochastic setting security price S (t) is a given random process which price does not governed by this rule. Therefore it is impossible to expect that its derivatives in stochastic setting would be governed by the self-financing and no arbitrage principles. Though in [3] it was also noted that the stock price S (t , ω) does effect on swap pricing this effect relates to the market activity which does not formalized analytically. That is why the stock price effect does not appear in the theoretical formula. This is an example that illustrates when common sense follows behind the faith in correctness of the method.

From our point of view the formula provided above is incorrect. Indeed the introduced above formula suggests the same fixed rate spread for equity swap on different stocks for which expected return over a specified period is equal for example to 13%, 5%, 0% or -10%. One also can remark that the above formula does not depend on volatility of the stocks. The volatility of the stocks can be either equal or not. It is obvious that pricing method that leads to the above formula for the fixed rate R is incorrect regardless of its popularity or simplicity.

Let us briefly outline other framework of the ES pricing. In contrast to the self-financing method we will use equal investment principle in order to present equality two investment opportunities. This point of view assumes that an investor has sufficient funds available for investment. There are two market opportunities. These are investments in fixed or floating legs. Assume that the market is efficient, i.e. it provides equal information to the all market participants. This means in particular that an investor chooses long or short trade based on return analysis. A simple formula for the fixed rate R can be received if one equates the rates of return for both sides of the risk free contract over the time period [0 , T]. Note we could arrive at the same equation if one equates inflow and outflow to counterparty. If $1 invested in floating or fixed sides of 0 default equity swap at the time t then at the end of the first reset period $t_1 - 0$ these investments yield $\$1(t_0)[S(t_1)/(t_0)]$ and $\$1(t_0)(1 + R)$ correspondingly. Thus over the lifetime of the contract the equal investment principle leads to the equation

$$\$1(t_0) \prod_{j=1}^{N} \frac{S(t_{j+1})}{S(t_j)} = \$1(t_0)(1 + R)^N$$

Solving it for R we receive

$$R = [\prod_{j=1}^{N} \frac{S(t_{j+1})}{S(t_j)}]^{\frac{1}{N}} - 1 = [\frac{S(t_N)}{S(t_0)}]^{\frac{1}{N}} - 1$$

This rate is average over the time equity rate and presents approximate solution of the problem. Another approach that leads to the different pricing solution of the equity swap is based on the option pricing. This method in particular takes into account that only netted amount change hand at a transaction. Bellow we outline an application of the option pricing to the equity swaps pricing. In contrast to the above formula that is based on equality of the current date investments the option pricing reduce future payoff to its current date value.

Before writing general formulas let us consider a simple numeric example that displays typical equity swap transactions. Let us put the notional of the swap equal to 10. Then let the set of transactions of the equity swap can be specified by the table

Dates	t_0	t_1	t_2	$t_3 = T$
Floating rate: S (t)	2	6	3	12
Fixed rate: R	2	2	2	2
Transactions value		10[(6/2-1)-2]=0	10[(3/6-1)-2= -25	10[(12/3-1)-2]=10

The calculations in the last row show the cash transactions to the holder of the variable equity rate. The floating rate is exchanged for the fixed rate multiplied by the notional principal. At the date t_1 there is no cash changes hands. Then at t_2 amount 25 is gone from floating leg to the fixed rate holder. Then at the maturity T amount of 10 goes from fixed rate holder to the counterparty.

Now in general case let us assume that S (t) is a stochastic process. Denote 'A' a counterparty that receives a fixed rate and pays floating and therefore 'B' receives floating rate and pays fixed. The stochastic cash flows from counterparties can be represented in the form

$$I_{A \to B}(*, \omega) = \sum_{i=0}^{N-1} \chi_{i+1} [\frac{S(t_{i+1}, \omega) - S(t_i, \omega)}{S(t_i, \omega)} - R] \chi\{\frac{S(t_{i+1}, \omega) - S(t_i, \omega)}{S(t_i, \omega)} > R\}$$

where $\chi_{i+1} = \chi\{t = t_{i+1}\}$. The symbol '$*$' on the left hand side expresses a functional dependence of the cash flow on time. The flow to counterparty A is

$$I_{B \to A}(*, \omega) = \sum_{i=0}^{N-1} \chi_{i+1} [R - \frac{S(t_{i+1}, \omega) - S(t_i, \omega)}{S(t_i, \omega)}] \chi\{\frac{S(t_{i+1}, \omega) - S(t_i, \omega)}{S(t_i, \omega)} \le R\}$$

The netted cash flow to and from counterparty B is the difference between the first and the second functionals. The problem is to perform the date-t reduction of the netted cash flow. We use option pricing method introduced in [5]. This approach is consistent with the investment equality definition given above. Following [5] the call and put European option pricing equations are

$$\frac{S(T)}{S(t)}\,\chi\,\{S(T) > K\} \;=\; \frac{C(T, S(T))}{C(t, S(t))}$$

$$\frac{S(T)}{S(t)}\,\chi\,\{S(T) < K\} \;=\; \frac{P(T, S(T))}{P(t, S(t))}$$

Here the constant K is a known strike price and European call and put payoffs at expiration are defined as

$$C(T,\ S(T)) = \max\{S(T) - K,\ 0\}$$

$$P(T,\ S(T)) = \max\{K - S(T),\ 0\}$$

correspondingly. This approach represents the option price definition that does not coincide with Black Scholes pricing in two major issues. First, this approach does not advice the same derivative price for two instruments having the same volatility and different expected rates of return within $[-\mu, \mu]$, where $\mu > 0$ is an arbitrary constant. The second difference is that it does not relevant to self-financing pricing. Our point of view is that market participants make a decision to invest funds based on their expectation of the future return regardless a source where the funds have been received. A stochastic market could lead the investor either to a profit or to a loss. Self-financing pricing approach neglects profit - loss and binds pricing with risk free borrowing rate only. The solutions of the option pricing equations can be written in the form

$$C(t, S(t)) \;=\; \frac{S(t)}{S(T)}\, C(T, S(T))\,\chi\,\{S(T) > K\}$$

$$\text{(EO)}$$

$$P(t, S(t)) \;=\; \frac{S(t)}{S(T)}\, P(T, S(T))\,\chi\,\{S(T) < K\}$$

The payments $I_{B \to A}(*, \omega)$, $I_{A \to B}(*, \omega)$ are generated by the exchange of the floating equity rate $S(t_{j+1}) / S(t_j)$, $j = 0, 1, \ldots, N-1$ on the fixed rate R. Using the option pricing solution we enable to present the reduction of the cash flows at $t = t_0$. These values are equal to

$$I_{A\to B}(t, \omega) \;=\; \sum_{i=0}^{N-1} \frac{S(t_1, \omega)S(t_i, \omega)}{S(t_0, \omega)\,S(t_{i+1}, \omega)}\,\Big[\,\frac{S(t_{i+1}, \omega)}{S(t_i, \omega)} - 1 - R\,\Big]\,\chi\,\Big\{\,\frac{S(t_{i+1}, \omega)}{S(t_i, \omega)} > 1 + R\,\Big\}$$

$$I_{B\to A}(t, \omega) \;=\; \sum_{i=0}^{N-1} \frac{S(t_1, \omega)S(t_i, \omega)}{S(t_0, \omega)\,S(t_{i+1}, \omega)}\,\Big[\,1 + R - \frac{S(t_{i+1}, \omega)}{S(t_i, \omega)}\,\Big]\,\chi\,\Big\{\,\frac{S(t_{i+1}, \omega)}{S(t_i, \omega)} < 1 + R\,\Big\}$$

Scenarios in which indicator contains equality sign can be omitted as far as the corresponding terms in the sums are equal to 0. The value R for which the right hand sides of the above formulas are equal represents a solution of the equity swap pricing problem. This solution of the problem can be written in a simple compact form. Indeed the equality of the two cash flows at t results

$$I_{B \to A}(t, \omega) = I_{A \to B}(t, \omega)$$

Using equality $\chi \{ Q > x \} = 1 - \chi \{ Q \le x \}$ we see that the fixed rate is a random variable equal to

$$R(\omega) = \frac{\sum_{i=0}^{N-1} [1 - \frac{S(t_i, \omega)}{S(t_{i+1}, \omega)}]}{\sum_{i=0}^{N-1} \frac{S(t_i, \omega)}{S(t_{i+1}, \omega)}} = [\frac{1}{N} \sum_{i=0}^{N-1} \frac{S(t_i, \omega)}{S(t_{i+1}, \omega)}]^{-1} - 1 \tag{6}$$

This is the fixed rate of the equity swap with 0 chance of default. On the other hand when counterparties agree to accept a particular spread they are subject to risk. Let counterparties agreed to apply a value [R] as a contractual fixed rate of the swap. This value can be associated at a certain degree with the expectation of the random rate $R(\omega)$. Then the market risk is a chance that the real world payment to counterparty is bellow that implied by the exact value $R(\omega)$. It could be measured by the probability $P\{ R(\omega) < [R] \}$ for the fixed rate payer as far as the investor pays more than implied by the real world scenario. At the same time this is the value of the favorable chance for floating leg investor. The swap value by definition is equal to the difference

$$I_{A \to B}(t, \omega) - I_{B \to A}(t, \omega) = \frac{S(t_1, \omega)}{S(t_0, \omega)} \sum_{i=0}^{N-1} [1 - (1 + R) \frac{S(t_{i+1}, \omega)}{S(t_i, \omega)}] \tag{7}$$

Formulas (6) and (7) represent a solution of the equity swap pricing assuming 0 chance of default.

Now let us consider the risky equity swap. The risk of the swap is associated with a possibility of default. Assume that default could occur at the transactions date only and the date of default is specified for example by the event when the absolute value of the difference between the current equity rate and initial market spread becomes larger a particular barrier. With the other interpretation of the default time one could consider the difference between marked to market value. In the structural approach default is defined as the first moment of time when stochastic company's stock reaches a certain fraction q of the initial price. In this case default time would be defined as

$$\tau(\omega) = \min \{ t_i : S(t_i) / S(t_0) < q \}$$

Hence, if default occurred during the lifetime of the equity swap then the rate q is a threshold, which separates default from no default period. Given a distribution of the random process S (*) one can find an appropriate approximation of the default time distribution.

The cash flows to counterparties scheduled on the dates t_j, j = 1, 2,…, N can be represented in the form

$$I_{A\to B}(*, \tau(\omega)) = \sum_{i=1}^{N} \chi\{\tau(\omega) = t_i\} \sum_{j=0}^{i-2} \chi_j \left[\frac{S(t_{j+1}, \omega) - S(t_j, \omega)}{S(t_j, \omega)} - R_d \right] \times$$

$$\times\ \chi\left\{ \frac{S(t_{j+1}, \omega) - S(t_j, \omega)}{S(t_j, \omega)} > R_d \right\} + I_{A\to B}(*, \omega)\ \chi\{\tau(\omega) > T\}\ \chi_N ,$$

$$I_{B\to A}(*, \tau(\omega)) = \sum_{i=1}^{N} \chi\{\tau(\omega) = t_i\} \sum_{j=0}^{i-2} \chi_j \left[R_d - \frac{S(t_{j+1}, \omega) - S(t_j, \omega)}{S(t_j, \omega)} \right] \times$$

$$\times\ \chi\left\{ \frac{S(t_{j+1}, \omega) - S(t_j, \omega)}{S(t_j, \omega)} < R_d \right\} + I_{B\to A}(*, \omega)\ \chi_N ,$$

where $\chi_j = \chi\{t = t_j\}$ denotes indicator that specifies the transaction that taking place at the date t_j and $I_{B\to A}(*, \omega)$, $I_{A\to B}(*, \omega)$ are defined above. These formulas specify floating-fixed rates transactions. 'A' receives the fixed and pays the equity floating rate whereas 'B' receives floating and pays fixed rate at the given sequence of the reset dates. At the date of default we assumed that there are no transactions. It is possible to add to these formulas the term that covers recovery payments from defaulted side in exchange for full or a portion of the payment from the protection seller. In this case the contract in general might be interpreted as trilateral. Applying the option pricing method used above for every scenario $\omega \in \{\omega : \tau(\omega) = t_j\}$, j = 1, 2, …, N and summing up the corresponding terms we arrive at the formulas for the fixed swap rate and swap value

$$R_d(\omega) = \sum_{i=0}^{N-1} \chi\{\tau(\omega) = t_i\} \left\{ \left[\frac{1}{i-1} \sum_{j=0}^{i-2} \frac{S(t_j, \omega)}{S(t_{j+1}, \omega)} \right]^{-1} - 1 \right\} + \chi\{\tau(\omega) > T\} R(\omega) ,$$

$$I_{A\to B}(t, \tau(\omega) \wedge T) - I_{B\to A}(t, \tau(\omega) \wedge T) = \frac{S(t_1, \omega)}{S(t_0, \omega)} \sum_{i=0}^{N-1} \chi\{\tau(\omega) = t_i\} \times \qquad (8)$$

$$\times \sum_{j=0}^{i-2} \left[1 - (1 + R) \frac{S(t_{j+1}, \omega)}{S(t_j, \omega)} \right] + \left[I_{A\to B}(t, \omega) - I_{B\to A}(t, \omega) \right] \chi\{\tau(\omega) > T\}$$

The expression $R(\omega)$ in the first formula and the difference $I_{B \to A}(t, \omega) - I_{A \to B}(t, \omega)$ are defined in formulas (6), (7). They denote the spread and value of the risky equity swap. Thus the random spread $R - R_d$ could be interpreted as the possibility of default.

Now let us consider a credit protection problem. This is the problem of the premium valuation, which a protection buyer should pay to protection seller in order to receive a complete compensation in the case of default. The underlying to the equity default swap is the risky swap. Generally speaking the protection buyer can be either counterparty of the EDS. First we need to specify a reasonable value of the claim at the default event. If q is the default barrier then claim amount in the discrete time setting could be defined as following

$$F [Q - S(\tau(\omega)) / S(t_0)] \tag{9}$$

where F is a notional principal and Q, $Q > q$ is the initially specified rate that can be either a constant such a potion of R or a specified function depending on t. For example Q can be a variable index depending on t.

Credit default swap on a risky equity swap is a value of a fixed rate premium that should be paid periodically by protection buyer to protection seller until earliest date between default or maturity. Protection buyer can be an arbitrary market participant. The pricing problem is the premium value in exchange loss given default protection (9) delivered by protection seller to protection buyer at the default date. This problem is close to CDS valuation problem. Let R_B and R_A denote premium values from protection buyer or protection seller points of view correspondingly. When the credit event is coming up the protection seller pays amount (9) to the protection buyer. In the discrete time setting the time of default can be presented in the form

$$\tau(\omega) = \sum_{i=1}^{N} t_i \chi\{\tau(\omega) = t_i\} = \sum_{i=1}^{N} t_i \chi\{\frac{S(t_i)}{S(t_0)} \le q\} \prod_{j=1}^{i-1} \chi\{\frac{S(t_i)}{S(t_0)} > q\}$$

If $\omega \in \{\tau(\omega) = t_i\}$ then protection buyer makes $(i-1)$ regular payments '$\$FR_A$' on the scheduled reset dates. On the equity leg protection seller pays the amount (9) at the date $\tau(\omega)$ for any ω for which $\tau(\omega) \le T$. If there is no default over the lifetime of the EDS then protection seller accumulates all scheduled payments and does not pay the compensation on default. This reasoning leads to the equation

$$\sum_{i=1}^{N} \chi\{\tau(\omega) = t_i\} \sum_{j=1}^{i-1} FR_A B(t, t_j) + \chi\{\tau(\omega) > T\} \sum_{j=1}^{N} FR_A B(t, t_j) =$$

$$= \sum_{i=1}^{N} \chi\{\tau(\omega) = t_i\} F [Q - \frac{S(t_i)}{S(t_0)}] + 0 \chi\{\tau(\omega) > T\} \tag{10}$$

From which follows that

$$R_A(\omega) = \frac{\sum_{i=1}^{N} \chi\{\tau(\omega) = t_i\} \, [\, Q - \frac{S(t_i)}{S(t_0)} \,]}{\sum_{i=1}^{N} \chi\{\tau(\omega) = t_i\} \sum_{j=1}^{i-1} B(t, t_j) + \chi\{\tau(\omega) > T\} \sum_{j=1}^{N} B(t, t_j)} =$$

$$= \sum_{i=1}^{N} \frac{[\, Q - \frac{S(t_i)}{S(t_0)} \,]}{\sum_{j=1}^{i-1} B(t, t_j)} \chi\{\tau(\omega) = t_i\}$$

On the other hand the protection buyer prospective rate is

$$R_B(\omega) = \frac{\sum_{i=1}^{N} \chi\{\tau(\omega) = t_i\} \, [\, Q - \frac{S(t_i)}{S(t_0)} \,]}{\sum_{i=1}^{N} \chi\{\tau(\omega) = t_i\} \sum_{j=1}^{i-1} B^{-1}(t_j, t_i) + \chi\{\tau(\omega) > T\} \sum_{j=1}^{N} B^{-1}(t_j, t_i)} =$$

$$= \sum_{i=1}^{N} \frac{[\, Q - \frac{S(t_i)}{S(t_0)} \,]}{\sum_{j=1}^{i-1} B^{-1}(t_j, t_i)} \chi\{\tau(\omega) = t_i\}$$

The approach that is based on approximation of the risky equity rate by its mean over the time leads to the rate of the risky equity swap

$$R(\omega) = \sum_{i=1}^{N} \chi\{\tau(\omega) = t_i\} \{ [\frac{S(t_{i-1}, \omega)}{S(t_0, \omega)}]^{\frac{1}{i-1}} - 1 \}$$

Remark. In derivation of the premium payment one can use benchmark setting dealing with expected cash flows. In this case the last term on the left-hand side of the formula that corresponds to the formula (10) would not be equal be to zero. Denote $\langle\langle R_A \rangle\rangle$ a seller premium when stochastic cash flows are replaced by its expected values. Then the premium is

$$<< R_A >> = \frac{\sum_{i=1}^{N} E\chi\{\tau = t_i\}\,[\,Q - \frac{S(t_i)}{S(t_0)}\,]}{\sum_{i=1}^{N} P\{\tau = t_i\}\sum_{j=1}^{i-1} B(t, t_j) + P\{\tau > T\}\sum_{j=1}^{N} B(t, t_j)}$$

That of course does not coincide with the expectation of the rate $R_A(\omega)$. Thus the benchmark approximation derived by using expected cash flows is a bias estimate of the expectation of the precise rate.

An **asset swap** is a contract that transforms fixed coupon payments into floating rate coupon payments. Each payment of the asset swap represents periodically adjusted LIBOR plus a constant spread. The asset-swap market is an important segment of the Credit Derivatives Market. The LIBOR rate is usually interpreted as AA risky rate. The exchange a floating rate flow on a fixed rate coupon can be realized by entering into standard interest rate swap: paying floating and receiving fixed rate. An asset swap eliminates any changes of capital and left only coupon exchange of the floating rate on fixed. Two valuation methods can be applied for swap valuation. These are based on market and PAR bond values. An investor in the swap market buys risky bond at t paying its 'dirty' price, which includes the clean price and accrued interest. If there is no default the investor gets back the bond at the bond maturity T. In this case the investor would pay the difference of the PAR that is equal to the face value of the bond and the bond purchased price at t. Here we assumed that the bond is priced for discount. With the PAR valuation the investor pays PAR regardless of the bond price. In either case the investor receives the floating coupon. The difference between the market bond value and its PAR is known as the cash payment adjustment. It is paid up-front for a PAR asset swap or at maturity for a market value swap. Other cash flow is generated by the coupon payments. The risky bond would pay to the investor a fixed coupon 'c' until maturity or default, which one comes first. The fixed coupon coupon would then be exchanged for the LIBOR plus an asset swap spread. When the bond default, the IRS continues be active. The investor pays fixed and receives floating coupon until the maturity of the swap. The swap maturity is the same as the maturity of the underlying asset. The investor is also responsible for "loss given default" payment. At default the interest rate swap could be also closed out at the market price.

Let us consider a PAR asset swap in which an asset swap buyer X for up-front payment of PAR receives a risky coupon bond from an asset swap seller Y at date t. Then X enters into an interest rate swap paying the fixed rate c to the asset swap seller Y. This fixed payment has the same value as the coupon of the risky bond. In return, Y pays to X floating rate LIBOR plus spread s_X. Assume that a credit event might occur only when underlying bond defaults. This approximation might be realistic if default risk of the bond issued by the company is more significant than the 'X-Y' counterparty risk. Though it is possible to take into account counterparty risk we will study here the case when the counterparty risk can be ignored. If there is no default during the lifetime of the bond the company Y would exchanged for its PAR equal to the face value of the bond \$1 at the bond maturity T. Thus for a scenario ω for which $\tau(\omega) = t_i$ the cash flow from the asset swap buyer X to the asset swap seller Y is

$$\$1\,\chi\{\,t = t_0\,\} + \sum_{j=1}^{i-1} c\,\chi\{\,t = t_j\,\} + (\,1 - \Delta\,)\,\chi\{\,t = t_i\,\}$$

where $0 \leq \Delta \leq 1$. The cash flow to X is

$$< R_c\ (t, T; \omega) > \chi\{ t = t_0 \} + \sum_{j=1}^{N} (L_j + s_X)\ \chi\{ t = t_j \}$$

i = 1, 2, … , N. Here $< R_c (t , T ; \omega) >$ denotes the market price of the risky bond at t paying fixed coupon c at the same dates when default might occur. T denotes the bond maturity. The overall cash flow to X for a scenario ω for which $\tau (\omega) = t_i$ is

$$[< R_c\ (t, T; \omega) > - 1]\ \chi\{ t = t_0 \} + \sum_{j=1}^{N} (L_j + s_X - c)\ \chi\{ t = t_j \} - \\ - (1 - \Delta)\ \chi\{ t = t_i \} \tag{11}$$

For no default scenario ω for which $\{ \tau (\omega) > T \}$ the last term of the above expression is 0. Therefore the overall cash flow to X is 0 then

$$[< R_c\ (t, T; \omega) > - 1]\ \chi\{ t = t_0 \} + \sum_{j=1}^{N} (L_j + s_X - c)\ \chi\{ t = t_j \} = 0$$

Remark. If one admits default at intermediate date the term representing accrued interest must be added to the left-hand side of (11).

There are two ways to perform discount of the future payments. One is the US Treasury rate and other is the LIBOR rate. Let D (t , T) denote a discount factor over [t , T]. Then the spread value s_X from the counterparty Y perspective is equal to

$$s_X(\omega) = \begin{cases} \dfrac{1 - < R_c (t, T; \omega) > + \sum_{j=1}^{N} (c - L_j) D (t , t_j)}{\sum_{j=1}^{N} D (t , t_j)}\ \chi\{ \tau > T \} \\[2ex] \dfrac{1 - < R_c (t, T; \omega) > + \sum_{j=1}^{N} (c - L_j) D (t , t_j) - (1 - \Delta) D (t , t_i)}{\sum_{j=1}^{N} D (t , t_j)}\ \chi\{ \tau = t_i \} \end{cases}$$

i = 2, 3, … , N. Note that a particular value of spread implies risk for either counterparty. This risk can be measured by the probability of receiving return less than implied by the market spread value.

There exists another type of reduction that can be applied to derive the asset spread. This reduction includes calculation of the future value of the cash flow (11) at the date $\tau \wedge T = \min\{ \tau , T \}$. Then if we wish we can discount the future amount to its present value. Hence for a scenario $\omega \in \{ \tau(\omega) = t_i \}$ we have

$$[< R_c(t,T;\omega) > - 1]\, D^{-1}(t, T) + \sum_{j=1}^{N} D^{-1}(t_j , T)(L_j + s_X - c) - (1 - \Delta)\, D^{-1}(t_i , T) = 0$$

$$[< R_c(t,T;\omega) > - 1] + D(t, T) \sum_{j=1}^{N} D^{-1}(t_j , T)(L_j + s_X - c) - D(t, T) D^{-1}(t_i , T)(1 - \Delta) = 0$$

correspondingly. For a scenario $\{ \tau(\omega) > T \}$ the terms containing factor (1 - Δ) is equal to 0. Solving the first equation for spread we arrive at the formula

$$s_X^{(F)}(i) = \frac{[1 - < R_c(t,T;\omega) >]\, D^{-1}(t, T) + D^{-1}(t_i , T)(1 - \Delta) + \sum_{j=1}^{N} D^{-1}(t_j , T)(c - L_j)}{\sum_{j=1}^{N} D^{-1}(t_j , T)}$$

This is the par spread of the asset swap. The par spread given by the second equation is equal to

$$s_X^{(D)}(i) = \frac{[1 - < R_c(t,T;\omega) >] + D(t, T)[(1 - \Delta) + \sum_{j=1}^{N} D(t_j , T)(c - L_j)]}{D(t, T) \sum_{j=1}^{N} D^{-1}(t_j , T)}$$

for a scenario ω for which $\tau(\omega) = t_i$, $i \le N$. Therefore combining terms that do not depend on default and the term that does depend on default we arrive at the formula

$$s_X^{(F)}(\omega) = \frac{[1 - < R_c(t,T;\omega) >]\, D^{-1}(t, T) + \sum_{j=1}^{N} D^{-1}(t_j , T)[(c - L_j) + (1 - \Delta)\chi\{\tau = t_j\}]}{\sum_{j=1}^{N} D^{-1}(t_j , T)}$$

This is the spread formula based on the cash flows future reduction at the time $\min\{ \tau , T \}$. Discounted spread formula can be written analogously. There are several different ways LIBOR rate could be chosen. For example at the date t_j the rate $L_j = l(t_j ; H)$ could cover the next period $[t_j , t_{j+1}]$ where $H \ge T$ is a fixed maturity of the LIBOR rate. The rate could be assumed either stochastic or deterministic.

The second popular asset swap type is the market structure. This structure the investor buys the package of risky bond and IRS at the cash market price $< R_c (t , T ; \omega) >$ of the bond in contrast to PAR. At maturity if there is no default there is an exchange the original market price $< R_c (t , T ; \omega) >$ for PAR. Hence the cash flow to X for $\omega \in \{ \tau(\omega) = t_i \}$ is

$$\sum_{j=1}^{N} (L_j + s_X - c)\, \chi\{ t = t_j \} - (1 - \Delta)\, \chi\{ t = t_i \}, \quad i \le N$$

Then

$$[< R_c (t, T; \omega) > - 1]\, \chi\{ t = t_N \} + \sum_{j=1}^{N} (L_j + s_X - c)\, \chi\{ t = t_j \}$$

for a scenario $\tau(\omega) > T$. If one has belief that 0-cash flow represents the fair price then the spread over LIBOR is equal to

$$s_X(\omega) = \frac{[1 - < R_c (t, T; \omega) >]\, D(t, T)\, \chi\{ \tau > T \} + \sum_{j=1}^{N} (c - L_j) D(t, t_j)}{\sum_{j=1}^{N} D(t, t_j)} -$$

$$- \frac{(1 - \Delta) \sum_{i=1}^{N} \chi\{ \tau = t_i \} D(t, t_i)}{\sum_{j=1}^{N} D(t, t_j)}$$

With a **total return swap** (TRS) two counterparties X and Y exchange their cash flows. The maturity T of the TRS contract does not exceed the maturity T_Z of the underlying corporate bond issued by a company Z. Let us assume that bond might default only at the moments t_j, j = 1, 2, …N. It is common to call TRS counterparties total return TR receiver and payer. The framework mechanics can be outlined as following. The TR receiver 'X' obtains from counterparty Y the fixed coupons on corporate bond and changes in value of the bond at the reset dates t_j. The party X pays to TR payer 'Y' LIBOR plus spread.

If default occurred at t_i then X receives at the dates t_j, j = 0, 1, 2, …, i – 1

*) a specified coupon c ;

*) $[R (t_j , T_Z ; \omega) - R (t_{j-1} , T_Z ; \omega)]\, \chi\{ R (t_j , T_Z ; \omega) > R (t_{j-1} , T_Z ; \omega) \}$

*) $<R (T , T_Z)> - <R (t , T_Z)>$, if there is no default up to T,

In return if default occurred at t_i company X pays at the dates t_j, j = 1,2, …, i – 1

*) LIBOR rate L_{j-1} specified by the previous time period plus spread s

*) $[R (t_{j-1} , T_Z ; \omega) - R (t_j , T_Z ; \omega)]\, \chi\{ R (t_j , T_Z ; \omega) < R (t_{j-1} , T_Z ; \omega) \}$

*) the losses on default at the credit events at date t_i .

A reference obligation underlying of the TRS can be any risky asset or index.

The cash flow to the TRS receiver 'X' is then

$$\chi(\tau > T)\Big[< R(t, T_Z) > - < R(T, T_Z) > + \sum_{i=0}^{N} \chi(t = t_i)(c - L_i - s)\Big] +$$

$$+ \sum_{i=1}^{N} \chi(\tau = t_i)\Big\{ \chi(t = t_i)(\Delta_i - 1) + \sum_{j=0}^{i-1} \chi(t = t_j)[\, c - L_j - s +$$

$$+ R(t_j, T; \omega) - R(t_{j-1}, T; \omega)\,]\Big\} = 0$$

We can apply here the standard PV reduction usually used as the benchmark pricing approach in order to present corresponding approximation of the solution of the pricing problem. Then

$$D(t, T)\chi(\tau > T)\Big[< R(t, T_Z) > - < R(T, T_Z) > + \sum_{i=0}^{N} (c - L_i - s)\Big] +$$

$$+ \sum_{i=1}^{N} \chi(\tau = t_i)\Big\{ D(t, t_i)(\Delta_i - 1) + \sum_{j=0}^{i-1} D(t, t_j)[\, c - L_j - s +$$

$$+ R(t_j, T; \omega) - R(t_{j-1}, T; \omega)\,]\Big\} = 0$$

From the above equation it follows that TRS spread is equal to

$$s(\omega) = \chi(\tau > T)\,\frac{\Big[< R(t, T_Z) > - < R(T, T_Z) > + \sum_{j=0}^{N} D(t, t_j)(c - L_j)\Big]}{N + 1} + \tag{12}$$

$$+ \sum_{i=1}^{N} \chi(\tau = t_i)\Big\{\frac{1}{i} \sum_{j=0}^{i-1} D(t, t_j)[\, c - L_j + R(t_{j+1}, T; \omega) - R(t_j, T; \omega)] - (1 - \Delta_i) D(t, t_i)\Big\}$$

The cash flow reduction can be provided using another approach. The standard PV could be used for the reduction all fixed future payments that are known at initiation. On the other hand the payments that are depends on scenario could be reduced first to its future value at the date $\min\{\tau, T\}$ and then the total would be discounted to its value at t. Using this remark the adjustment of the formula (12) is quite simple. The constant terms will remain the same and the discount factor $D(t, t_j)$ of the terms that depend on t_j should be replaced by the
$D^{-1}(t_j, \tau \wedge T)\, D(t, \tau \wedge T)$.

Next credit instrument is a swap hybrid contract known as a **credit-linked note** (CLN). This is a funded type of credit derivatives, which combines two instruments: a corporate bond and a standard CDS. Lifetime of the CLN issued by a company Y is defined for simplicity by the corporate bond. This bond is assumed to be issued by a company Z. If there is no default of the bond then the company Y makes a fixed periodic coupon payments to the CLN buyers at payment dates t_j, j = 1, 2, …N and a principal CLN at the bond maturity T. If default occurs during lifetime of the CLN then the CLN contract is terminated. The next coupons are not paid and CLN buyers will receive value Δ on defaulted bond. If the bond defaults the CLN buyers deliver either the bond of the company Z or the amount representing the market value of the defaulted bond to the CLN issuer Y and in return Y pays loss given default. These are known as physical and cash settlements.

Along with CLN the company Y enters into CDS. Though it can be done before or later the CLN issue date we suppose for simplicity that the inceptions of the contracts CLN and CDS are the same. With CDS contract the company Y is the protection seller who will pay a protection 'loss given default' at the date of default in exchange for periodic premium until default and recovery at the default date. This recovery rate and premium are passed on to the CLN buyers to increase yield on the company Z bond.

Let $Q_{LN}(t, T; \omega)$ denotes the CLN price at the date t. Then using synthetic pricing one can split $Q_{LN}(t, T; \omega)$ price onto sum of two components. The one component is the cash flow stipulated by the bond. The PV of this cash flow is

$$\chi(\tau_Z > T)\,[\,B(t,T) - \sum_{i=1}^{N} B(t,t_i)\,c_{LN}\,] - \sum_{i=1}^{N} \chi(\tau_Z = t_i)\,[\sum_{j=1}^{i-1} B(t,t_j)\,c_{LN} + \Delta_Z B(t,t_i)]$$

Here c_{LN} denotes CLN coupon payment. Other component of the CLN is made conditional on CDS protection seller transactions. The CDS cash component promises the protection payment of $1 - \Delta_{CDS}$ paid by Y to the CDS protection buyers at the date of default. This is cash settlement setting of the CDS contract at the default. In physical settlement Y pays PAR in exchange of defaulted reference entity. Party Y also would receive CDS coupon payments up to default date. Thus the PV of the CDS transaction is

$$\sum_{i=1}^{N} \chi(\tau_{CDS} = t_i)\,\{\sum_{j=1}^{i-1} B(t,t_j)\,s_{CDS} - (1 - \Delta_{CDS})\,B(t,t_i)\} +$$
$$+\ \chi(\tau_{CDS} > T) \sum_{i=1}^{N} B(t,t_i)\,s_{CDS}$$

If there is no default the CLN issuer would receive CDS periodic payments of s_{CDS} and pays the CLN coupon c_{LN}. Applying the standard PV reduction for the outlined approximation of the CLN transactions and assuming that the PV of the sum of two cash flows to and from Y be equal to 0 we arrive at the equation

$$c_{LN}\Big[\sum_{i=1}^{N} B(t,t_i)\chi(\tau_Z > T) + \chi(\tau_Z = t_i)\sum_{j=1}^{i-1} B(t,t_j)\Big] =$$

$$= \sum_{i=1}^{N}\Big[B(t,t_i)\{\chi(\tau_{CDS} > T)s_{CDS} - \chi(\tau_{CDS} = t_i)(1-\Delta_{CDS})\} + \chi(\tau_{CDS} = t_i)\sum_{j=1}^{i-1} B(t,t_j)\, s_{CDS}\Big]$$

From which it follows that

$$c_{LN} = \frac{\sum_{i=1}^{N}\Big[B(t,t_i)\chi(\tau_{CDS} > T)s_{CDS} + \chi(\tau_{CDS} = t_i)\{\sum_{j=1}^{i-1} B(t,t_j)\, s_{CDS} - B(t,t_i)(1-\Delta_{CDS})\}\Big]}{\sum_{i=1}^{N} B(t,t_i)\chi(\tau_Z > T) + \chi(\tau_Z = t_i)\sum_{j=1}^{i-1} B(t,t_j)} \qquad (13)$$

This is the formula for the CLN spread. A typical CLN assumes that the bond is significantly higher rated than underlying CDS reference entity. This condition implies that risk of default of the bond significantly less than CDS reference entity and the probability $P\{\tau_{CDS} < \tau_Z\}$is closed to 1. With the first order approximation we could admit that the bond is default free. Consider the denominator of the first term

$$\frac{\sum_{i=1}^{N} B(t,t_i)\,\chi(\tau_{CDS} > T)}{\sum_{i=1}^{N} B(t,t_i)\,\chi(\tau_Z > T) + \chi(\tau_Z = t_i)\sum_{i=1}^{i-1} B(t,t_i)}$$

From identity $\chi(\tau_Z > T) = \chi(\tau_{CDS} > T, \tau_Z > T) + \chi(\tau_{CDS} \le T, \tau_Z > T)$ it follows that the expression that contains the second term equal to 0. Indeed when it is not 0 the numerator is 0. Then

$$\chi(\tau_{CDS} > T, \tau_Z > T) = \chi(\tau_{CDS} > T, \Omega \setminus \tau_Z \le T) = \chi(\tau_{CDS} > T) - \chi(\tau_{CDS} > T, \tau_Z \le T)$$

The probability of the last term is a small number by the assumption. Let us consider the term that contains indicator $\chi(\tau_Z = t_i)$.The event when it is not equal to 0 does not exceed the probability of event $\chi(\tau_{CDS} > T, \tau_Z \le T)$ that is small by the given assumption. Analogous arguments could be applied to the other terms of (13). Note that with this approximation one actually ignores the scenarios when low risk bond defaults prior CDS. In this case formula (13) can be simplified and

$$c_{LN} = s_{CDS} - \sum_{i=1}^{N} \chi(\tau_{CDS} = t_i)\frac{B(t,t_i)}{\sum_{k=1}^{i-1} B(t,t_k)}(1-\Delta_{CDS})$$

In order to produce higher order approximations of the statistical characteristics of the CLN coupon payments one needs reasonable assumption for the joint distribution of default of the bond and CDS reference entity. Admitting particular default time distributions it is possible to present calculations of the expected value of the CLN premium as well as its higher order risk characteristics. Taking into account discounting of type (2) we can perform other presentation of the CLN price

$$c_{LN}\Big[\sum_{i=1}^{N} B^{-1}(t_i, T)B(t, T)\chi(\tau_Z > T) + \chi(\tau_Z = t_i)\sum_{j=1}^{i-1} B^{-1}(t_j, t_i)B(t, t_i)\Big] =$$

$$= \sum_{i=1}^{N}\Big[B^{-1}(t_i, T)B(t, T)\{\chi(\tau_{CDS} > T)s_{CDS} -$$

$$- \chi(\tau_{CDS} = t_i)(1 - \Delta_{CDS})\} + \chi(\tau_{CDS} = t_i)\sum_{j=1}^{i-1} B^{-1}(t_j, t_i)B(t, t_i)\, s_{CDS}\Big]$$

In this adjustment the payments made by the counterparty Y are discounted by the standard PV after received payments are summing up at the earliest of maturity or default time using future rates. Ignoring possibility of default of the bond the correspondent formula for the CLN coupon can be presented in the form

$$c_{LN} = s_{CDS} - \sum_{i=1}^{N}\chi(\tau_{CDS} = t_i)\,\frac{B(t, t_i)}{\sum_{k=1}^{i-1} B^{-1}(t_k, T)B(t, T)}(1 - \Delta_{CDS})$$

Let us consider a CLN contract pricing bearing in mind counterparty risk. For simplicity we will assume that probability of default of the CDS floating leg asset is sufficiently small and it is possible to assume that the risk free bond generates the floating rate. Based on terms of the CDS and value of the protection it is clear that the protection seller's creditworthiness 'Y' should be taking into account for accurate pricing. Thus let us assume also that protection seller might also subject to default on protection transaction.

Denote $D_{ps}(\tau_{CDS}(\omega))$ a default event when the protection seller fails to deliver the protection of $F - R(\tau_{CDS}(\omega), T; \omega)$ to the protection buyer at the date of default. Subscript 'ps' here stands for 'protection seller'. Therefore if CDS underlying security defaults at t_k, i.e. $\omega \in \{\tau_{CDS}(\omega) = t_k\}$ there exist a chance $D_{ps}(t_k)$ that protection seller fails to deliver CDS obligation. Next for writing simplicity we will omit the subscript 'CDS'. Thus

$$(F - R(t_k, T; \omega))\chi\{\tau_{CDS}(\omega) = t_k\} = \Delta_{ps}(F - R(t_k, T; \omega))\,\chi\{\tau(\omega) = t_k\}\times$$

$$\times\chi(D_{ps}(t_k)) + (F - R(t_k, T; \omega))\,\chi\{\tau(\omega) = t_k\}[1 - \chi(D_{ps}(t_k))] =$$

$$= (F - R(t_k, T; \omega))\,\chi(\tau(\omega) = t_k)[1 - (1 - \Delta_{ps})\,\chi(D_{ps}(t_k))]$$

Here Δ_{ps} denotes recovery rate of the protection seller stipulated by the protection value 'loss given default' from Y to the protection buyer. Substitution of the right hand side of this equality in (2) and (4) leads to the refinement that takes into account the protection seller's creditworthiness. In this case for a scenario $\omega \in \{ \tau(\omega) = t_k \} \cap D_{ps}(t_k)$, k = 1, 2, .., N the protection buyer loss is

$$(1 - \Delta_{ps})(F - R(t_k, T; \omega))\chi\{\tau(\omega) = t_k\}\chi(D_{ps}(t_k))$$

The adjusted value of the spread can be written as follows

$$q_b^{(c)} = \sum_{k=1}^{N} \frac{[F - R(t_k, T; \omega)]}{\sum_{j=1}^{k-1} B^{-1}(t_j, t_k)} [1 - (1 - \Delta_{ps})\chi(D_{ps}(t_k))]\chi\{\tau(\omega) = t_k\} \qquad (2')$$

The spread value would be changed. This spread change reflects the chance that protection seller will not pay the full protection payment at the date of default. The correspondent seller's exposure will also be changed

$$q_s^{(c)} = \sum_{k=1}^{N} \frac{[F - R(t_k, T; \omega)]}{\sum_{j=1}^{k-1} B(t, t_j)} [1 - (1 - \Delta_{ps})\chi(D_{ps}(t_k))]\chi(\tau(\omega) = t_k) \qquad (4')$$

For calculations mean, variance, or other statistics of the random variables (2′) and (4′) we need to know the joint conditional distributions of the random vector $\{ R(t_k, T; \omega), \chi(D_{ps}(t_k)) \}$ conditioning on $\{ \tau(\omega) = t_k \}$. The event $D_{ps}(t_k)$ can be assumed to be independent on the event $\{ \tau(\omega) = t_k \}$ at least for the first order approximation. Nevertheless the distribution of the random variables $R(t_k, T; \omega)$ are depend on $\{ \tau(\omega) = t_k \}$. Therefore for realistic model one needs a realistic assumption regarding this conditional distribution.

Now let us consider the continuous time CDS contract. Assume that coupon is paid on fixed dates t_k, k = 1, 2,…N. Then the balance equation (3) should be adjusted taking into account continuous distribution of the default event. Bearing in mind accrual interest and a possibility of default of the protection seller on protection delivery at the dates $t_{k,i}$ the equality (3) can be rewritten in the form

$$\lim_{\varepsilon \downarrow 0} \sum_{k=1}^{N} \sum_{i=0}^{n} \chi\{\tau(\omega) \in [t_{k,i}, t_{k,i+1})\}\{B(t, t_{k,i+1})[F - R(t_{k,i+1}, T; \omega)] \times$$

$$\times [1 - (1 - \Delta_{ps})\chi(D_{ss}(t_{k,i}))] - q_s [\sum_{j=0}^{k-1} B(t, t_j) + [R(t_{k,i}, t_{k+1}; \omega) - R(t_k, t_{k+1}; \omega)]\} -$$

$$- \; q_s \sum_{j=0}^{N} B(t, t_j)\, \chi(\, \tau(\omega) > T\,) \;=\; 0$$

Here $t_k = t_{k,0} < t_{k,1} < t_{k,2} < \ldots < t_{k,n} = t_{k+1}$ is a sub-partition of the interval $[\, t_k , t_{k+1}\,]$, and $t_{k,i+1} - t_{k,i} = \varepsilon$. Denote $i_d(\, s , t ; \omega\,)$ the discount interest rate of the risky bond over the interval $[\, s , t\,]$. Then

$$q_s\,[\, R(\, t_{k,i}, t_{k+1} ; \omega\,) - R(\, t_k , t_{k+1} ; \omega\,)\,] =$$

$$= q_s [\, i_d(\, t_{k,i}, t_{k+1} ; \omega\,) - i_d(\, t_k , t_{k+1} ; \omega\,)\,] \times (\, t_{k,i} - t_k\,) / 360$$

is the unknown at the date t accrued future interest. If default occurs exactly at the moment t_k then from this formula follows that accrued interest will not be paid. If the chance that default occurs at the initiation date t does not equal to 0 then the term equal to

$$(\, F \;-\; R(\, t , T\,)\,)\, \chi\,\{\, \tau(\,\omega\,) = t\,\}$$

must be added to the left hand side of the above formula. In this case it looks reasonable that the first coupon payment of q be paid at t = 0. In the limit formula we assume that the protection delivery is occurred immediately after default. The spread value can be received then from the above equation. Thus

$$q_s = \lim_{\varepsilon \downarrow 0} \frac{\displaystyle\sum_{k=1}^{N}\sum_{i=0}^{n} B(t, t_{k,i+1})[\, F - R(t_{k,i+1}, T; \omega)\,]\,[\,1 - (\,1 - \Delta_{ps})\chi\,(D_{ps}(\, t_{k,i}))] \times}{\displaystyle\sum_{k=1}^{N}\sum_{i=0}^{n} \chi\{\tau(\omega) \in [t_{k,i}\;, t_{k,i+1})\,\{\sum_{j=0}^{k-1} B(t, t_j) + [\, R(t_{k,i}\,, t_{k+1}; \omega) -}$$

$$\frac{\times\; \chi\{\, \tau(\omega) \in (t_{k,i}\;, t_{k,i+1}]\,\}}{- R(t_k\,, t_{k+1}; \omega)\,]\,\} \;+\; \displaystyle\sum_{j=0}^{N} B(t, t_j)\, \chi\{\tau(\omega) > T\}} \;=$$

$$= \frac{B(t, \tau(\omega))\,[\, F - R(\tau(\omega), T; \omega)\,]\,[\,1 - (\,1 - \Delta_{ps})\chi\,(D_{ps}(\, t_{k,i}))]\, \chi\{\, \tau(\omega) \le T\,\}}{\displaystyle\sum_{k=1}^{N} \chi\{\tau(\omega) \in [t_k\,, t_{k+1})\,\}\,\{[\sum_{j=0}^{k-1} B(t, t_j)] + R(\,\tau(\omega), t_{k+1}; \omega\,) - R(\, t_k\,, t_{k+1}; \omega\,)\,\}}$$

Here in the denominator on the right hand side of the equality the term corresponding to no default can be omitted as far as the value of the numerator for such scenarios is equal to 0. Indeed for no default scenario there is any need to pay a protection.

Floating rate risky bond.

Let us briefly outline risk free floating rate bond. Besides the popularity of this contract it is also used for valuation of the interest rate swap. We introduce the valuation of the risk free floating rate bond following [9].

Let $t = t_0 < t_1 < \ldots < t_N = T$ be interest rate reset dates and assume that the tenor $\varepsilon = t_{j+1} - t_j$ does not depend on j. Let $i(t_j, t_{j+1})$ be the floating rate, which is applied at the date t_j over the period $[t_j, t_{j+1}]$. For simplicity assume that notional principal is \$1. Otherwise the values of transactions should be proportionally changed. The floating rate payments that would be applied for regular payments from the buyer of the contract to the contract seller are presented in the table bellow

Dates	t_0	t_1	t_2	…	$t_N = T$
Floating flow	-1	$i(t_0, t_0 + \varepsilon)$	$i(t_1, t_1 + \varepsilon)$	…	$1 + i(t_{N-1}, t_{N-1} + \varepsilon)$

Looking at this table one can see that one-dollar at date t_{N-1} is equal to

$$\$1(t_{N-1}) = \$[1 + i(t_{N-1}, T)](T)$$

at T. Hence

$$\$1(t_{N-1})\, i(t_{N-2}, t_{N-1}) + \$(T)[1 + i(t_{N-1}, T)] = \$1(t_{N-1})\,[1 + i(t_{N-2}, t_{N-1})]$$

Therefore the cumulative cash flow to the bond buyer over the time period [t , T] can be calculated backward in time starting from the date T to t. It yields

$$\$(t_1)\, i(t_0, t_1) + \$(t_2)\, i(t_1, t_2) + \ldots + \$(T)[1 + i(t_{N-1}, T)] =$$

$$= \$(t_1)\, i(t_0, t_1) + \$(t_2)\, i(t_1, t_2) + \ldots + \$(t_{N-1})[1 + i(t_{N-2}, t_{N-1})] = \ldots$$

$$\ldots = \$(t_1)[1 + i(t_0, t_1)] = \$1(t)$$

These calculations prove that \$1 invested in default free security at the date t generates the equivalent floating rate cash flow specified in the above table at the moments t_j, $j = 1, \ldots N$. This floating rate bond valuation has been used for the present value reduction in order to justify the pricing model. Thus floating bond seller receives \$1 at the date $t_0 = t$. Immediately investing it and paying next floating coupons of $i(t_{k-1}, t_k)$ at the dates t_k, $k = 1, 2, \ldots, N - 1$ and $1 + i(t_{N-1}, t_N)$ at the bond maturity T investor would exhausted the up-front funding of \$1. Note that this construction actually does not depend on T or ε and therefore can be applied for arbitrary T and variable ε. On the other hand bond buyer can estimate the future value of the annuity payments using following formula

$$Fl(T) = \sum_{j=1}^{N} i(t_{j-1}, t_j)\, B^{-1}(t_j, T) + [1 + i(t_{N-1}, t_N)]$$

This formula presents the date-T value of the floating payments. The ε-periodic compound interest rate formula we apply for the bond valuation. To avoid arbitrage over [t , T] one should expect that the floating bond and the 0-coupon bond issued by the same Government should provide the same rate of return. That implies that

$$Fl (T) / Fl (t) = 1 / B (t , T)$$

The solution of this equation is

$$Fl (t) = B(t, T) [\sum_{j=1}^{N} i (t_{j-1} , t_j) B^{-1} (t_j ,T) + 1 + i (t_{N-1} , t_N)]$$

Value of the rates $i (t_{j-1} , t_j) , j > 1$ are unknown at the date t therefore it makes sense to interpret them as a sequence of the random variables with the distributions consistence with historical data .

Now assume that underlying of the floating bond contract is a risky bond. Denote $FB_\lambda (t, T)$, $\lambda = \{ t_j ; j = 1, 2, \ldots, N)$ the cash flow generated by the sequence of payments $i (t_{j-1} , t_j)$ paid at t_j , j = 1, 2, …, N – 1 and the final payment of $1 + i (t_{N-1} , t_N)$ at T. These payments are made prior to default. A seller of the risky floating bond would pay λ-reset floating interest rate payments until default or maturity, which one comes first. Assume that the bond recovery is a known ratio $0 \le \Delta < 1$ and the floating bond investor pays the fixed coupon $s at initiation of the contract. Finding the value of the upfront premium $s given recovery rate Δ and a distribution of the default time is our problem. The bond buyer pays upfront $s in exchange of the cash flow would be received from the bond seller. This flow is

$$\sum_{i=1}^{N} \chi(\tau = t_i)[\sum_{j=1}^{i-1} i(t_{j-1} , t_j) + \Delta] + \chi(\tau > T) \sum_{i=1}^{N} i(t_{i-1} , t_i) = \sum_{j=1}^{N} \chi(\tau = t_j) \times$$

$$\times [FB_\lambda (t, t_j) - 1 + \Delta] + \chi(\tau > T)[FB_\lambda (t, T) - 1] = [FB_\lambda (t, \tau) - 1] - \Delta \chi(\tau < T)$$

Thus upfront premium value is

$$s(\omega) = [FB_\lambda (t , \tau) - 1] - \Delta \chi(\tau < T)$$

Taking into account theoretical assumption or historical default time data it is easy to calculate statistical characteristics of the spread s (ω). Any market price of the spread implies risk and therefore could not be interpreted as the perfect.

There is a possibility to study a CDS contract written on floating rate bonds. In this contract a protection buyer intention is to purchase a protection which would cover a loss of default. Let us assume for example that default might occur at the reset dates only. Then a protection seller should reimburse possible loss (1 - Δ) might occurred at one of the dates t_j .

On the other hand protection buyer would pay a fixed premium until the earliest between the date of default and maturity. Assume that the date of default is t_k. Then the loss of $L_k = L_k(t, \lambda)$ occurred at the dates $t = t_k$ is equal to the cash flow

$$L_k = \$[\, i(t_{k-1}, t_k) - \Delta \,]\,\chi(t = t_k) + \sum_{j=k+1}^{N} \$i(t_{j-1}, t_j)\chi(t = t_j) + \$1\chi(t = t_N) =$$

$$= (1 - \Delta)\,\$(t_{k-1})$$

Thus protection seller payment to protection buyer can be represented by the sum of the loss functions L_k. Hence

$$L(\lambda) = \sum_{k=1}^{N} L_k\,\chi(\tau = t_k) = \sum_{k=1}^{N} \chi(\tau = t_k)(1 - \Delta) = (1 - \Delta)\,\chi(\tau \le T))$$

The cash flow to the protection seller from the protection buyer is equal to

$$q\,[\sum_{k=1}^{N} (k - 1)\,\chi(\tau = t_k) + N\chi(\tau > T)\,]$$

By definition let $q(\omega) = 0$ for $\omega \in \{\omega: \tau(\omega) = t_1\}$. It conforms to the fact that if default occurs immediately after contract initiation i.e. at t_1 then the coupon payment will not be paid at this date. The equality of two cash flows to and from counterparty implies that

$$q(\omega) = \frac{(1 - \Delta)\,\chi(\tau \le T))}{\sum_{k=2}^{N} (k - 1)\,\chi(\tau = t_k) + N\chi(\tau > T)} = (1 - \Delta)\sum_{j=1}^{N-1} j^{-1}\chi(\tau = t_{j+1}) \qquad (14)$$

The term $N\chi(\tau > T)$ in denominator above can be omitted because for the scenario when it is positive the numerator is equal to 0. The moments of the random spread can be presented in a simple compact form

$$E q^n(\omega) = (1 - \Delta)^n \sum_{j=1}^{N-1} j^{-n} P(\tau = t_{j+1}) \qquad (15)$$

where n = 1, 2,… .

Remark. In contemporary credit derivatives studies it is common to replace random floating payments by its expectation. It seems important to note that for example the value of the spread received in such setting does not coincide with the first moment of the exact solution. Indeed the spread value calculated using expected cash flows to and from a counterparty of the CDS that is written on floating rate bond is

$$<< q >> = \frac{(1 - \Delta) P(\tau \leq T)}{\sum_{j=2}^{N-1} j P(\tau = t_{j+1}) + N P(\tau > T)}$$

The term $N P(\tau > T)$ does not exist in above formula and it can be either small or large. Therefore the reduction of the stochastic flows to their expectations can be sufficiently crude and biased.

Counterparty Risk of the Interest Rate Swap.

Following [9] let us recall a valuation model of interest rate swap (IRS) with 0 chance of default. A standard IRS is a two party contract. The counterparty A makes fixed semiannual or quarterly payments to counterparty B. The magnitude of the fixed payments are usually a pre-specified percent of the notional principal. In return, counterparty B pays a floating rate payments to A. All payments are made in the same currency and only netted amount is paid.

Let $t = t_0 < t_1 < \ldots < t_N = T$ be reset dates, q and $l(*, *)$ denote a fixed and a floating (LIBOR) interest rates correspondingly, and $1 is the notional principal. The fixed flow line in the table below represents the scheduled payments from A to B and the floating line is the payments from B to A.

Dates	t_0	t_1	t_2	…	$t_N = T$
Fixed flow	0	q	q	…	1 + q
Floating flow	0	$l(t_0, t_0 + \varepsilon)$	$l(t_1, t_1 + \varepsilon)$	…	$1 + l(t_{N-1}, t_{N-1} + \varepsilon)$

Recall that only netted payments are paid. Let us recall some important points of the swap valuation. The domestic risk free rate usually uses for calculations. If for a particular scenario $q > l(t_k, t_{k+1})$ then the payment of $q - l(t_k, t_{k+1})$ would made by A to B at t_{k+1}. This amount would be held until maturity is unknown at t. Thus the future value of the payments paid to counterparty B and A at the IRS maturity are

$$\sum_{k=0}^{N-1} [\, q - l(t_k, t_{k+1}) \,] \, \chi(q > l(t_k, t_{k+1})) \, B^{-1}(t_{k+1}, T)$$

$$\sum_{k=0}^{N-1} [\, l(t_k, t_{k+1}) - q \,] \, \chi(q < l(t_k, t_{k+1})) \, B^{-1}(t_{k+1}, T) \tag{16}$$

correspondingly. The PAR swap rate is by definition the rate q for which the present value of the all fixed side payments is equal to the present value of floating payments. In another words it is a value q for which the value of the swap at t is 0. Multiplying both expressions in (16) by the same factor $B(t, T)$ we arrive at that does not depend on this factor. That is the PAR IRS spread is equal to

$$q = \frac{\sum_{k=0}^{N-1} l(t_k, t_{k+1}) B^{-1}(t_k, T)}{\sum_{k=0}^{N-1} B^{-1}(t_k, T)} \qquad (17)$$

Definition. The value of the 0-risky swap $s = s(u, T)$ at the date $u \in [t, T]$ is the difference between fixed and floating legs at the date u.

Though the present value is still the benchmark concept in asset pricing we recall that in general it is an approximation of the Equal Investment Principal formulated above. Moreover the PV reduction along with the risk neutralization concept incorrectly present option pricing. The present value reduction can eliminate the real world risk. Let $t = t_0 < t_1 < t_2 = T$ be the dates of trade. Assume for instance that the risk free interest rate term structure is $i(t, t_1) = 4.1\%$, $i(t_1, T) = 4.2\%$ and $i(t, T) = 4.15\%$. In this case both counterparties are subjected to the market risk. The risk is that counterparty would receive a return less than it was expected. For the bond buyer it was occurred if the bond is exercised at t_1. This simple example suggests necessity of the stochastic modeling of the future interest rates. We discussed in more details it in [5-7].

Now we apply option-pricing approach for risk free IRS valuation. We will employ the option's valuation method [5-7] for the swap valuation. Recall that the cash flow to the counterparties A and B can be represented in the forms

$$C_A(\lambda, t) = \sum_{k=1}^{N} [\, l(t_k) - q \,] \, \chi\{ l(t_k) > q \} \, \chi\{ t = t_k \}$$

$$P_B(\lambda, t) = \sum_{k=1}^{N} [\, q - l(t_k) \,] \, \chi\{ l(t_k) < q \} \, \chi\{ t = t_k \} \qquad (18)$$

where $t = t_0$, $l(t_k) = l(t_{k-1}, t_k)$, $\lambda = \{ t_k, k = 1, 2, \ldots, N \}$. We interpret the value $l(t_k)$ as a variable portion of the dollar value at t_k. Recall the option price definition. The European call / put option prices $C(t, x)$, $P(t, x)$ at t written on underlying security $S = l(u)$, $u \geq t$, and $l(t) = x$ with strike price $K = q$ and maturities t_j are defined above as a solution of the equations (EO). This interpretation leads to the date-t reduction of the cash flows (18) in the form

$$C_A(t) = \sum_{j=1}^{N} \frac{l(t_1)}{l(t_j)} [\, l(t_j) - q \,] \, \chi\{ l(t_j) > q \}$$

$$P_A(t) = \sum_{j=1}^{N} \frac{l(t_1)}{l(t_j)} [\, q - l(t_j) \,] \, \chi\{ l(t_j) < q \} \qquad (19)$$

These are representation of the floorlets and caplets contracts pricing. Hence the swap value s by definition is the difference

$$s = C_A(t) - P_B(t) = \sum_{j=1}^{N} \frac{l(t_1)}{l(t_j)} [\, l(t_j) - q \,] \qquad (20)$$

The formula (17) can be now interpreted as an approximation of the swap spread value presented by the equation (19). Indeed we see that the left-hand side in formulas (16), (17) contains payments to and from counterparty A. The floating forward future rates $l(t_{k+1}) = l(t_k, t_{k+1})$ are unknown at t and could admit statistical interpretation. The hypothetical log-normal distribution is a benchmark used for theoretical modeling of the option valuations. As far as historical data is available one can provide statistical test for testing of the quantitative likelihood of the implied log-normal distribution.

On the other hand if the swap value s is given then the fixed leg rate q is equal to

$$q = \frac{N\, l(t_1) - s}{l(t_1) \sum_{j=1}^{N} l^{-1}(t_j)}$$

Putting in this formula s = 0 we obtain the value q that represents PAR value of the spread. The PAR value of a parameter is the value that is derived from the model in which PV of the fixed and floating leg are equal.

Now let us consider the case when one or both counterparties of the IRS are subject to credit risk. Assume first that floating rate payer B might default. Let us again assume that the only reset dates can be the dates of default. The party B does not bound with the bond and it might default before or after the date when the underlying bond defaults. The default of the counterparty B means that B is fail to deliver the floating coupon $[\, l(t_k) - q \,]$ to A. Assume that recovery rate of the party B implied by its credit rating is a constant $\Delta_B < 1$. If party B defaults at t_k assume that it pays to counterparty A the fraction of the amount due $\Delta_B [\, l(t_k) - q \,]$. Thus the cash flow from B to A given that B defaults prior to A is

$$C_{B \to A|B}(t, \lambda) = \sum_{k=1}^{N} \{ [\, \chi(\tau(\omega) = t_k)\, \chi(\tau_B(\omega) \geq t_k) + \chi(\tau_B(\omega) = t_k)\, \chi(\tau(\omega) > t_k) \,] \times$$

$$\times \sum_{j=1}^{k-1} [\, l(t_j) - q \,] \} + \chi\{\tau_B(\omega) = t_k\}\, \chi\{\tau(\omega) > t_k\} \Delta_B [\, l(t_k) - q \,]$$

Here $\tau(\omega)$ denotes the default time of the underlying security and $\tau_B(\omega)$ is the default time of the counterparty B. Note that random times $\tau(\omega)$ and $\tau_B(\omega)$ can be correlated. The cash flow from B to A given that B defaults prior to A has two components. One component covers the case when the default of underlying security is coming up before the default of B. The second component of the cash flow corresponds to the scenarios when B defaults before underlying security. We now present calculation of the fixed rate of the swap using the option valuation method. Let $\omega \in \{ \omega : \tau(\omega) = t_k \}$ then there are two mutually exclusive scenarios $\{ \omega : \tau_B(\omega) > t_k \}$ and $\{ \omega : \tau_B(\omega) = t_k \}$ for which we can apply formulas (19). Then

$$
\begin{aligned}
C_{B\to A|B}(t) = \sum_{k=1}^{N} \{ [\chi(\tau = t_k)\chi(\tau_B \ge t_k) + \chi(\tau_B = t_k)\chi(\tau > t_k)] \times \\
\times \sum_{j=1}^{k-1} \frac{l(t_1)}{l(t_j)} [l(t_j) - q]\chi(l(t_j) > q) + \qquad (21) \\
+ \chi(\tau_B = t_k)\chi(\tau > t_k)\Delta_B \frac{l(t_1)}{l(t_k)}[l(t_k) - q]\chi(l(t_k) > q)\} + \\
+ \chi(\tau > T)\chi(\tau_B > T) C_A^{(\lambda)}(t)
\end{aligned}
$$

where the value of $C_A^{(\lambda)}(t)$ is given by (19). Note that the cash flow from A to B at the date t given that B might default is

$$
\begin{aligned}
P_{A\to B|B}(t) = \sum_{k=1}^{N} \{ [\chi(\tau = t_k)\chi(\tau_B \ge t_k) + \chi(\tau_B = t_k)\chi(\tau > t_k)] \times \\
\times \sum_{j=1}^{k-1} \frac{l(t_1)}{l(t_j)} [q - l(t_j)]\chi(q > l(t_j)) + \\
+ \chi(\tau_B = t_k)\chi(\tau > t_k)\Delta_B \frac{l(t_1)}{l(t_k)}[q - l(t_k)]\chi(q > l(t_k))\}
\end{aligned}
$$

The swap value is a random process in time and depending on a scenario. This value is equal to

$$S_B(t) = C_{B\to A|B}(t) - P_{A\to B|B}(t)$$

The random function of $q = q(t, \omega)$ for which $S_B = 0$ is the default swap rate at t. The formula for the rate q can be presented in analytic form. Indeed taking into account equality

$$(l_j - q)\chi(l_j > q) - (q - l_j)\chi(l_j < q) = (l_j - q)$$

one can easy figure out that the solution of the problem $C_{B\to A|B}(t) = P_{A\to B|B}(t)$ can be written as

$$q_B = \frac{\sum_{k=1}^{N} \{ [\chi(\tau_B > \tau = t_k) + \chi(\tau > \tau_B = t_k)](k-1) + \chi(\tau > \tau_B = t_k)\Delta_B \}}{\sum_{k=1}^{N} \{ [\chi(\tau_B > \tau = t_k) + \chi(\tau > \tau_B = t_k)] \sum_{j=1}^{k-1} l^{-1}(t_j) + \chi(\tau > \tau_B = t_k) l^{-1}(t_k)\Delta_B \}}$$

Assume now that party A is also subject to default. In this case the only sign of the cash flow will be changed and all formulas remain the same as above when only company B is might default.

Now let us consider the case when both counterparties A and B can default simultaneously. The swap value then is defined as

$$S_{AB} = \sum_{k=1}^{N} \{ \chi[\tau_A(\omega) = \tau_B(\omega) = t_k] \chi(\tau(\omega) > t_k) \sum_{j=1}^{k-1} \{ \frac{1(t_0)}{1(t_j)} [1(t_j) - q] +$$

$$+ \frac{1(t_0)}{1(t_k)} [\Delta_A 1(t_k) - \Delta_B q] \} + \tag{22}$$

$$+ \sum_{k=1}^{N} \chi\{ \min[\tau_A(\omega), \tau_B(\omega)] > t_k \} \chi(\tau(\omega) = t_k) \sum_{j=1}^{k-1} \frac{1(t_0)}{1(t_j)} [1(t_j) - q] \}$$

The swap rate is the value of $q_{AB}(\omega)$ for which $S_{AB}(\omega) = 0$. From (22) it follows that

$$q_{AB} = \frac{\sum_{k=1}^{N} \{ [\chi(\tau_A = \tau_B = t_k)\chi(\tau > t_k)](k - 1 + \Delta_A) + \chi(\tau = t_k)\chi(\tau_A \wedge \tau_B > t_k)(k-1) \}}{\sum_{k=1}^{N} \{ [\chi(\tau_A = \tau_B = t_k)\chi(\tau > t_k)] \sum_{j=1}^{k-1} l^{-1}(t_j) + l^{-1}(t_k)\Delta_B \} + \chi(\tau = t_k)\chi(\tau_A \wedge \tau_B > t_k) \sum_{j=1}^{k-1} l^{-1}(t_j)}$$

where $a \wedge b = \min(a, b)$.

In general case when counterparties might default during the lifetime of the contract. In this case there are three mutually exclusive scenarios representing default

$$\alpha = \{ \tau_A(\omega) > \max[\tau(\omega), \tau_B(\omega)] \}, \quad \beta = \{ \tau_B(\omega) \geq \max[\tau(\omega), \tau_A(\omega)] \}$$

$$\gamma = \{ \tau(\omega) > \max[\tau_B(\omega), \tau_A(\omega)] \}.$$

For each of the possible scenarios one can apply a formula represented above and therefore in general case the value and the rate of the counterparty risky IRS is

$$S = S_A \chi(\alpha) + S_B \chi(\beta) + S_{AB} \chi(\gamma)$$

$$q = q_A \chi(\alpha) + q_B \chi(\beta) + q_{AB} \chi(\gamma)$$

Now all components of the counterparty risky interest rate swap value are presented.

Appendix. Remarks on Credit Swap Spread.

In this section we revise a basic notion related to dynamic valuation of the credit instruments. Recall that standard financial instruments such as bonds, stocks, or their plain vanilla derivatives in stochastic setting can be characterized by their stochastic rate of return. Higher price of a security corresponds to the higher rate of return. Structural instruments are valued differently. The CDS spread can be interpreted as insurance against possible default. Higher spread value reflects higher risk of default and lower spread corresponds to the less risky security on the floating side of the swap. The goal is to present a measure that dynamically characterizes changes of the CDS value over the time. The mark-to-market (MTM) value is a measure of change that can be applied for risky swap valuation [14].

Let us consider a CDS contract. Formula (4) represents the protection buyer spread $q(\omega) = q(t, T; \omega)$ at the date t. Time T denotes the maturity of the CDS contract. To define MTM value we need first to determine the way of selling CDS contract. Selling CDS or unwinding it can be realized by selling it to the current or assign it to a new counterparty as well as to enter into offsetting position [14]. Broadly speaking MTM value is the price of taking the offsetting position. We highlight details of the stochastic pricing approach in contrast to a commonly used benchmark in which historical market data interpreted as expected value of random spread.

Define (SMTM) stochastic mark-to-market CDS long protection at the date v, $v > t$ as the difference of the two future cash floors generated by the fixed spreads over [t , v]. The first cash floor is generated by the v-date spread until maturity T and the second is the cash payments over [t , T]. Thus

$$SMTM(t, v; T, \omega) = [\, q(v, T, \omega) - q(t, T, \omega) \,]\, FL(v, T, \omega) \tag{23}$$

where

$$FL(v, T, \omega) = \sum_{k : t_k > v}^{N} \Big\{ \sum_{j : t_j > v}^{k-1} B(t, t_j)\, \chi(\tau(\omega) = t_k) + B(t, t_k)\, \chi(\tau(\omega) > T) \Big\}$$

The value $FL(v, T, \omega)$ is the present value of the coupon payments of $1 by floating leg at the specified dates starting from date v until $\min\{\tau(\omega), T\}$. If $<q(v, T, \omega)>$ denotes the market spread at v then the unwinding value that estimates dynamic changes of the CDS contract is

$$UMTM(t, v; T) = [\, <q(v, T, \omega)> - <q(t, T, \omega)> \,] <FL(v, T, \omega)>$$

The value $<FL(v, T, \omega)>$ is a market estimate of the random function FL(*). It is clear that notation $< \xi >$ here does not coincide with the expectation of the random variable ξ.

Entering into CDS offsetting MTM position leads to the value

$$OMTM\,(t, v; T, \omega) = [\langle q(v, T, \omega)\rangle - \langle q(t, T, \omega)\rangle]\, FL\,(v, T, \omega)$$

The OMTM (*) is a random process where variable $v \in [t, \min\{\tau(\omega), T\}]$. These two MTM representations are different and UMTM can be considered as a statistics of the random OMTM. This observed statistics could be biased or not. That is the depending on market exposure expectation of the OMTM could be equal to UMTM or not. If <q> is measured in basic points then putting $RPV01\,(v, T; \omega) = FL\,(v, T; \omega)\ 0.01\%$ we arrive at the risky present value benchmark that is defined as payments of the 1bp until minimum between maturity and default. If H is the notional principal of the CDS contract then we have

$$UMTM\,(t, v; T) = H\,[\langle q(v, T, \omega)\rangle - \langle q(t, T, \omega)\rangle]\ 0.25 \langle RPV01\,(v, T; \omega)\rangle$$

$$OMTM\,(t, v; T, \omega) = H\,[\langle q(v, T, \omega)\rangle - \langle q(t, T, \omega)\rangle]\ 0.25 RPV01\,(v, T, \omega)$$

Here it was assumed that payments made quarterly. It is a standard in Credit Risk papers to call RPV the expected value of the random data. It might confuse to call 'risky' the expected value of the random variable. For example let Wiener process describes profit or losses of a portfolio over the time. If the portfolio's change is measured by its expectation then there is no change in the portfolio value and it looks no sense to associate the 0-value portfolio with the risk. This remark might advise to use mathematical notions in finance as they used in mathematics. Then the expected value is only a single characteristic of the risk.

Remark. According [1] RPV01 of a CDS is defined as the risky present value of one basic point of payments in the premium leg of the CDS. Then RPV01 is used to calculate MTM by entering into offsetting position. In this case the lifetime of the CDS is random equal to $\min\{\tau(\omega), T\}$. This definition corresponds to $RPV01\,(v, T, \omega)$ outlined above. Then it was remarked that RPV of a promised payment in the future is the PV of the future payment adjusted for the probability that the payment will be received. The RPV of a series of future payments is equal to the sum of the PV of the individual payments in the series. This refinement most likely corresponds to the expression

$$\sum_{j=1}^{N} P\{\tau = t_{j+1}\} C_j B(t, t_j)$$

This non-random expression does not correspond to the original definition.

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