The Dilemma Facing Guests Enjoying a Party

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17 April 2016

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Abstract. A partially ordered set formalizes and generalizes the intuitive notion of ordering, sequencing, or arrangement of the elements in the set. In the present paper under Monotone (or Monotonic) System we understand a totality of sets of guests charity positions arranging guests utilities possessing monotone (monotonic) property, which reflects the dynamic nature of utilities. Utilities are increasing or decreasing along with the partial order induced by subsets of some general set. The theory, was initiated by the author in 1971, [1a], and published in Russian periodical of MAIK in 1976. In English, it was originally distributed by Plenum Publishing corporation [1b]. The theory produces Greedy type algorithms, which guarantee the optimal solution. Further development and application of the theory was first held in Tallinn [2], and then at some Universities in Israel [3], Moscow [4], USA [5], London [6], and Georgia, Tbilisi [7].

Suppose we observe a set W, $|W| = n$, n guests, $j = \overline{1,n}$, participating in a party. Let, in particular, a group of guests, denoted by H, are all those who could enjoy the party in companions with their sole mates. Consider a totality of sets $\{H\}$ of all 2^n such groups $H \subseteq W$ of companions, where $W - H = W \setminus H$; $\overline{H} = W - H$ signifies guests enjoying the party alone.

Let $\pi(j, H)$ estimates the utility of guests $j \in H$ who are in companions and who will stand by in their companions. In our nomenclature the utility $\pi(i, \{i\})$ estimates thus the utility for those $i \in \overline{H}$ enjoying the party alone.

In highlighting our pedagogical scenario when a guest $j \in H$ has decided to enjoy the party alone, we suppose that for all others $i \in H - \{j\}$ remaining in companions as a group indicated by $H - \{j\}$, *i.e.*, those still deciding to stand by in any companions, the utility to stand by in companions decreases:

$$\pi(i, H - \{j\}) \le \pi(i, H) \text{ for all } i \in H - \{j\}.$$ [1]

[1] In his work "Cores of Convex Games" Shapley investigated a class of n-person's games with special convex (supermodular) property, International Journal of Game Theory, Vol. 1, 1971, pp. 11-26. The author was not familiar with this work and could not predict the close connection between this basic monotonicity property and the above definition of a monotone system.

Given a utility threshold u, we say that a group H_u, as a whole, for those enjoying the party in companions, obeys u-stable condition if $\pi(j, H_u) \geq u$, *i.e.*, $\forall j \in H_u | \pi(j, H_u) \geq u$, even in the worst case when all guests $i \in W - H_u$ within $W - H_u = \overline{H}_u$ as an opposing guests to the group H_u in companions, have eventually, or incidentally, left their companions and become a standalone gests $\{i\}$.

The u-stable group H_u is called u-critical, *i.e.*, say an u-critical group H_u^c, when for all sub-groups $X \subseteq H_u^c$, the condition $\pi(j, X - \{i\}) < u$ is fulfilled for some $i \in X - \{j\}$ to leave their companions, *i.e.*, $\forall X \subseteq H_u^c \to \exists i \in X - \{j\} | \pi(i, X - \{j\}) < u$.

Consider now the situation when one of the standalone guests $j \notin X$ wishes to join guests X with a certain utility $\pi(j, X + \{j\})$ [2] depending on guests X already enjoying the party in companions. It is clear that in this way a function $\pi(j, X)$ is extended, and now the utilities are defined for all guests $j \in W$, as well as for those $\overline{X} = W - X$ standing alone. At the same time, we understandably assume that the smaller is the L group of guests accompanied by their sole mates, the lower are the utilities $\pi(j, L)$ for the guests in L, $L \subseteq G$, to remain with their accompanies, and the less likely that anyone will join L (to become a member of L); contrary, it is more likely to join G (to become a member of G). Formally, the following inequalities must be true

$$\forall i \in W | \pi(i, L) \leq \pi(i, G) \text{ for all pairs } L, G \text{ such that } L \subseteq G.$$

Given a utility threshold u, consider a mapping $V_u(X) = \{i \in W | \pi(i, X) \geq u\}$ of the group X in the set theoretical sense. We can rigorously prove that a group S, as a fixed point $S = V_u(S)$ represents a stable group $H_u = S$.

The problem. Given threshold u what can we say about the set "structure" $\{H_u\}$ of all u-stable groups H_u, including u-critical, while u increases? How to find a stable group or groups $H_u | u \leftarrow \max$? Is this maximization problem well defined?

Example. Let some numbers p_i, $i = \overline{1, n}$, represent guests charity positions. Assume that some guests denoted by X are enjoying the party in companions; $\overline{X}$ are those self-esteem as being alone. To determine the utilities π for the guests in a group X, let the utilities for all guests $j \in X$ enjoying the party in companions equal $\pi(j, X) = \left(|X|/n\right) \cdot p_j$. Obviously, if none of the guests could find a sole mate, the utility $\pi(i, \{i\}) = {}^{|i|=1}/n \cdot p_i$ is n times smaller than ${}^{|W|=n}/n \cdot p_i$ in contrast to the case when each of the n guests is a sole mates for someone in W, *i.e.*, all are enjoying the party in companions. Now, as a player $j \in X$

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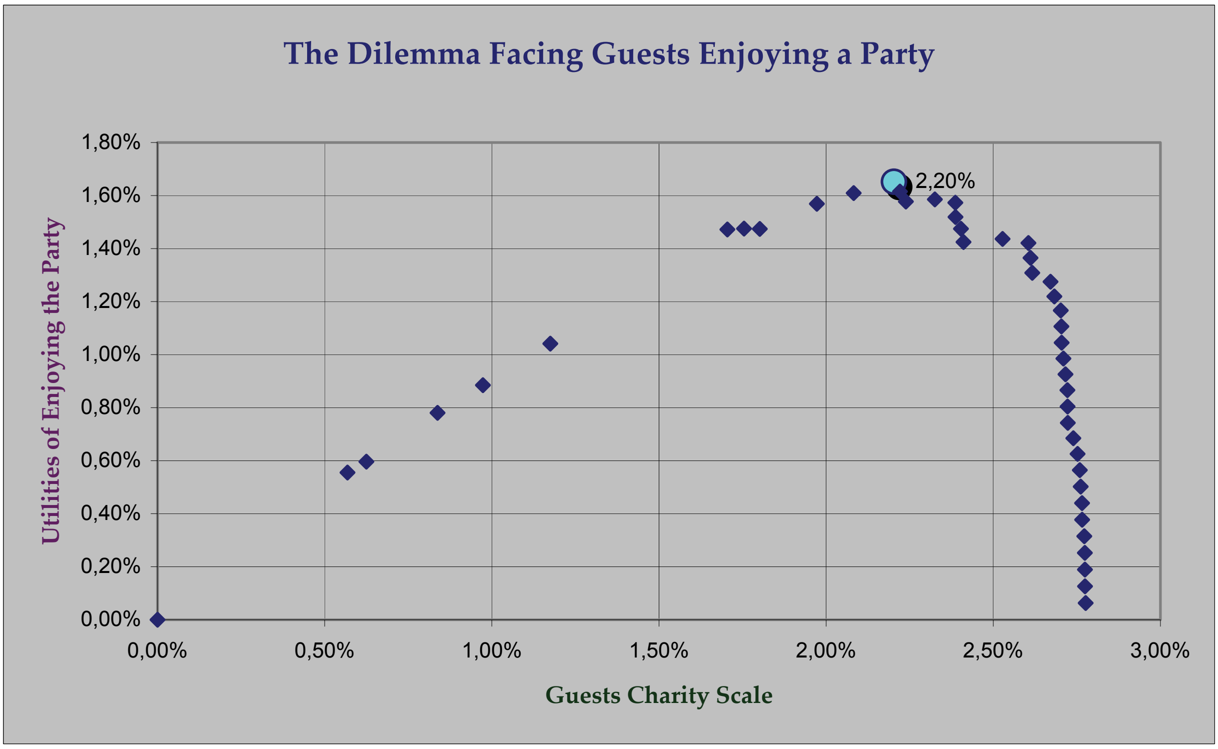
[2] Sometimes to extend π values to all elements in W we do not need this extension: π values on the whole set W appear in a natural way, see the example.

decides to enjoy the party alone, the utilities for all, including those still enjoying in companions, the utilities decrease, or increase when someone standalone guest $j \in \overline{X}$ wishes to join X and become a member of $X + \{j\}$. Typical graph below shows guests charity positions on x-axis, against utilities on y-axis.



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