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Johnson, Christopher

University of California, Davis

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Differences of Opinion, Liquidity, and Monetary Policy

Christopher P. Johnson

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Abstract

Liquidity considerations are important in understanding the relationship between asset prices and monetary policy. Differences of opinion regarding the future value of an asset can affect liquidity of not only the underlying asset, but also of competing media of exchange, such as money. I consider a monetary search framework in which money and risky assets can facilitate trade, but where the asset is an opinion-sensitive medium of exchange in that traders may disagree on its future price. A pecking-order theory of payments is established between money and risky assets, which can go in either direction depending on the respective beliefs of both agents in a bilateral trade. In short, optimists prefer to use money over assets, whereas pessimists prefer to use assets over money. In contrast to a majority of the differences of opinion literature, not only do pessimists actively participate in the purchasing of assets, but in some cases their demand coincides with that of optimists. Additionally, in support of Bernanke and Gertler (2000), I find that monetary policy aimed at reducing asset price volatility need not be welfare-maximizing. Instead, the Friedman rule is welfare-maximizing.

Contents

1 Introduction

Stories of liquidity are capable of explaining various classical macroeconomic puzzles. Lagos (2010) uses an asset pricing model in which the liquidity channel of financial assets is used to explain the equity-premium puzzle and the risk-free rate puzzle. Lagos (2013) rationalizes the rate-of-return dominance puzzle in equilibria that exhibit "liquidity effects" of open-market operations. Liquidity differences among competing media of exchange lie at the heart of these analyses. Informational asymmetries may play a role in the liquidity structure of asset yields, as in Rocheteau (2011). This may be most relevant in complex derivatives markets, especially during the time surrounding the Great Recession. However, informational asymmetries may not be adequate in explaining liquidity differences among information-rich financial markets such as bond, money, or stock markets. In such markets, traders may be exposed to the same level of information, but may interpret the information heterogeneously. I explore the extent to which differences of opinion under the same level of information can explain differences of liquidity among assets and money. Using this liquidity channel, I consider monetary policy implications on welfare and asset price volatility, the latter of which is often used as a proxy for financial market instability.

Models of disagreement have been used in the context of financial markets in order to explain various empirical regularities involving asset prices and trading volume. While the literature pioneered by Harrison and Kreps (1978) was not concerned with liquidity, disagreement among traders regarding future asset prices introduce a speculative component to liquidity. I consider a model of disagreement similar to Eyster and Piccione (2013), where agents hold heterogeneous theories about how the current-period dividend of a risky asset affects next period's asset price. These differences of opinion are persistent over an infinite horizon and the underlying theories are statistically correct, which prevents deviations from one's theory. These theories can be seen as the limit point of a statistical learning process. The asset studied in my model is an infinitely-living Lucas tree which pays a risky dividend according to a Markov process. While agents in my model disagree regarding future asset prices, they share homogeneous beliefs regarding money, whose supply is controlled by a monetary authority.

I first consider an economy without money, where the only non-perishable object available to traders is an opinion-sensitive risky Lucas tree. Agents can obtain assets in a centralized Walrasian market and use them as a medium of exchange in a decentralized market, where buyers and sellers meet bilaterally with possibly different opinions regarding the asset. Terms of trade are determined through bargaining, where buyers trade assets for a specialized good that only sellers can produce. Surplus in a decentralized market meeting is not only a function of output produced, but also a function of assets exchanged. Specifically, if the buyer is more pessimistic than the seller, then surplus increases when more assets are traded in exchange for output. This occurs because the asset is more valuable in the hands of the seller due to her optimism. Contrastingly, if the buyer is more optimistic than the seller, then the asset is valued more by the buyer. In both cases, the surplus-maximizing level of output is achieved when the marginal change in surplus due to increased output is equal to the marginal change in surplus due to the exchange of the asset. The surplus-maximizing level of output in matches where the buyer is more optimistic than the seller is less than the surplus-maximizing level of output that prevails in Lagos and Wright (2005), where there are no differences of opinion. The surplus-maximizing level of output in matches where the buyer is more pessimistic than the seller is greater than the Lagos and Wright (2005) surplus-maximizing level of output. In a steady state equilibrium, it is impossible for the surplus-maximizing level of output to be achieved in every possible match. This is in contrast to Geromichalos, Licari, and Suárez-Lledó (2007), where a sufficiently high supply of the asset sets the asset price equal to its fundamental value, which will guarantee that the surplus-maximizing level of output is achieved in every match. My result supports a role for an opinion-insensitive medium of exchange, such as fiat money, in order to improve welfare.

Various theoretical approaches to asset pricing under differences of opinion conclude that the asset price reflects the most optimistic trader's valuation of the asset, whom which is the sole participant in the market. Specifically, Miller (1977) finds that disagreement can lead to higher asset prices when short-sale constraints are present, which occurs due to pessimists sitting out of the market, implying that the asset price only reflects the valuation of the optimists. Such a conclusion can be attributed to an asset's lack of liquidity in such frameworks. In contrast, my monetary search approach to asset pricing under differences of opinion gives assets a role as a medium of exchange. Hence, assets matter not only for equity, but also for liquidity. I find results on the contrary to Miller (1977), by which pessimists not only actively participate in the asset market, but in some cases mirror the asset demand of optimists. This result can be attributed to the fact that optimists value the asset more as an investment than pessimists due to their higher subjective fundamental value, but value the asset less as a form of liquidity since pessimists receive better terms of trade in the decentralized market from their own perspective. The asset price reflects the valuation of both optimists and pessimists. Specifically, for any trader, the asset price is equal to a trader's subjective fundamental value plus a speculative liquidity premium. The speculative liquidity premium is higher for pessimists, whereas the subjective fundamental value is higher for optimists. As the price of the asset increases, its value as an investment decreases, whereas its value as a form of liquidity increases. If participation in the decentralized market is guaranteed, then at sufficiently high prices, optimists and pessimists demand the same amount of assets. At lower prices, optimists demand more assets than pessimists because the asset's value as an investment is greater. If there is uncertainty regarding participation in the decentralized market, then an optimist's asset demand dominates the asset demand of a pessimist at any equilibrium price. However, pessimists still demand some positive amount of assets.

Money is introduced in my model to study liquidity under competing media of exchange, as well as the monetary policy implications on welfare. Due to differences of opinion regarding the value of the asset, a pecking-order theory of payments is established. If the buyer is more optimistic than the seller, then her preferred method of payment is with money. Specifically, if the level of output that the buyer can afford is below a specific threshold, then she first spends all of her money holdings, then funds the remainder with her assets. If she can afford more than this threshold level of output with money alone, then she only spends her money and holds onto her assets. At this level of output, using any assets results in a lower level of surplus, even though a greater level of output is produced. If the buyer is more pessimistic than the seller, then her preferred method of payment is with the asset. Once again, if the level of output that the buyer can afford is below a specific threshold, then she will spend all of her assets to fund the production of output, with the remainder funded by money holdings. If she can afford more than this threshold level of output, then she will only use her assets to fund output and hold onto her money. At this level of output, spending money lowers surplus, even though more output is produced. My result is similar in spirit to Rocheteau (2011), who finds that under informational asymmetries regarding the future value of assets, agents use their risk-free bonds (money) first in order to finance the counterparty's output, and use the asset only if their holdings of bonds (money) are depleted. However, while Rocheteau (2011) relies on informational asymmetries to support a pecking order, I assume that all agents are equally informed about the asset, but have differences in opinion in which they agree to disagree during the bargaining process. Additionally, my model yields pecking orders in two directions depending on which trader is an optimist and which trader is a pessimist, whereas the pecking order in Rocheteau (2011) is always in one direction.

I find that the optimal monetary policy that maximizes expected aggregate surplus coincides with the Friedman rule, i.e., a zero nominal interest rate. This result is consistent with the benchmark monetary model of Lagos and Wright (2005), even with a richer behavioral environment and the inclusion of a risky asset. Under the Friedman rule, the asset price is equal to the fundamental value of the most optimistic trader. Hence, the liquidity premium of assets under the Friedman rule is nonexistent in equilibrium. Under such a price, only the most optimistic trader participates in the asset market. It follows that lowering the nominal interest rate not only reduces the price of the asset, but it also changes the composition of aggregate asset demand among the set of opinions in the economy. As the nominal interest rate decreases, each trader's subjective liquidity premium decreases and the asset is valued more for its return than its liquidity.

Financial instability has been an increasingly important topic to policymakers around the world, especially in the aftermath of the Great Recession. Such concerns regarding instability in financial markets are warranted, as Jordà, Schularick, and Taylor (2013) document that financial crisis recessions are costlier relative to typical recessions. One particular metric of financial market instability is asset price volatility. Due to the link between asset prices and interest rates, policymakers may be interested in monetary policy aimed at reducing asset price volatility. On this front, Bernanke and Gertler (2000) conclude that monetary policy should be aimed at combating underlying inflationary pressures. They find that the relevance of asset prices for a monetary authority is their ability to signal potential inflationary or deflationary forces. In my model, a monetary authority who seeks to maximize social welfare need not focus on asset price volatility, as traders are concerned with the first moment of asset prices rather than second moments. Thus, asset price volatility should not be a concern in a steady state equilibrium if improving welfare is the policy objective of a central bank.

The remainder of this paper proceeds as follows. First, I provide brief literature reviews of differences of opinion in financial markets and of the third generation of monetary search models. In Section 3, I then describe the way in which differences of opinion are modeled in my framework and I describe a steady state equilibrium in an benchmark economy without a decentralized market. In Section 4, I explore an economy in which the asset can be used as a method of payment if a spending shock is observed to participate in the decentralized market. Terms of trade in a bilateral match, equilibrium asset prices, and differences in demand among heterogeneous market participants are all explored. In Section 5, I consider an economy similar to that in Section 4, except money is introduced as a method of payment. Monetary policy implications on welfare and asset price volatility are described as well. Section 6 concludes the paper.

2 Related Literature

2.1 Differences of Opinion in Financial Markets

The assumption of rational expectations lies at the heart of neoclassical economic thought. Many economic stylized facts can be explained using models that utilize the rational expectations hypothesis. However, in the realm of financial markets, models assuming rational expectations fall short in explaining asset price and trading volume anomalies. As a result, a literature focusing on differences of opinion among market participants emerged in an attempt to explain many of the empirical regularities in financial markets, where rational expectations does not suffice as an assumption. For example, Scheinkman and Xiong (2003) utilize differences of opinion among investors in their model to focus on asset price bubbles and the coexistence of high prices and high trade volume, as seen in the Internet stock boom prior to their paper's publication. Also, Hong and Stein (2003) utilize differences of opinion in order to explain the negative skewness in stock returns, large asset price movements occurring without dramatic news events, and crashes involving a degree of cross-stock contagion. Models of differences of opinion in financial markets arguably started with Harrison and Kreps (1978). They find that in an environment with heterogeneous expectations within the community of potential investors, stock prices contain a speculative component in addition to reflecting the stocks fundamental value. The heterogeneity among potential investors in their model is expressed via non-common priors for each investor class about an assets dividend process. An alternate modeling of differences of opinion can be found in Varian (1985), who studies differences of opinion and asset pricing in an Arrow-Debreu model, where agents have heterogeneous subjective probabilities of each state of nature. Harris and Raviv (1993) introduce differences of opinion in the way traders interpret announcements of public information in the context of a risky asset. Morris

(1996) builds on Harrison and Kreps (1978) by considering agents with non-common priors who trade in a financial market as learning occurs.

A more modern approach to a model of asset pricing under heterogeneous beliefs culminates in Eyster and Piccione (2013), who seeks out to describe the relationship between investor sophistication and returns. The key ingredient of their model is that agents hold heterogeneous incomplete theories of how the next-period price of a long-term asset depends on the current state of the world. Unlike models of non-common priors, all agents in the model have statistically correct beliefs, which can be seen as the limit point of a statistical learning process. Informational asymmetries are not a factor in this model, which is critical if a theorist is interested in isolating disagreement, rather than including disagreement as a byproduct of asymmetric information.

2.2 A Monetary Approach to Asset Pricing and Liquidity

In macroeconomics, the canonical model of asset pricing can be attributed to Lucas (1978), which features a frictionless exchange economy. While asset prices reflect their risk and equity characteristics, they also reflect the liquidity of the underlying asset. Liquidity is arguably best represented in third generation monetary search models, such as in Lagos and Wright (2005). Geromichalos, Licari, and Suárez-Lledó (2007) introduced risk-free assets in fixed supply into the Lagos-Wright model of monetary exchange. They find that an asset price reflects the asset's fundamental value, as well as a liquidity premium.[1] The case of a risky asset is introduced in Lagos (2010). In his model, risk-free and risky assets coexist to explain the risk-free rate and equity premium puzzles following the methodology of Mehra and Prescott (1985).

Liquidity differences among assets can be explained via asymmetrically-informed agents, such as in Rocheteau (2011). In his model, buyers have private information regarding the future value of their risky asset, whereas the seller is uncertain as to the quality of the buyer's asset. As a result, buyers in the high-dividend state retain a fraction of their asset holdings in order to signal their quality. Additionally, a pecking-order theory of payments arises. Other explanations of liquidity differences across assets can be found in Kiyotaki and Moore (2005), who assume that the transfer of ownership of capital exhibits a delay so that an agent can steal a fraction of his capital before the transfer takes effect. Additionally, Lester, Postlewaite, and Wright (2012) explain the illiquidity of capital goods by assuming that claims on capital can be costlessly counterfeited and can only be authenticated in a fraction of meetings.

My paper also touches on models of competing media of exchange, including the coexistence of interest-bearing assets

[1] Assets can carry liquidity even if they are not used directly as media of exchange. Frictions in an over-the-counter market are considered in Geromichalos and Herrenbrueck (2012), where agents can allocate their wealth between a liquid asset and an illiquid asset. When a consumption shock is realized, agents can visit an over-the-counter (OTC) market to readjust their portfolios. The illiquid asset still carries a liquidity premium due to its liquidity properties in the OTC, even though it is illiquid in the DM.

and fiat money as means of payment. Kiyotaki and Wright (1989) consider storable commodities that can serve as methods of payment, but include heterogeneous storage costs. In such an environment, the liquidity of each commodity is dependent on the storage costs, as well as preferences and technologies through the pattern of specialization. Models that study the coexistence of money and bonds as media of exchange can be found in Bryant and Wallace (1979), Wallace et al. (1980), Aiyagari et al. (1996), Kocherlakota (2003), Shi et al. (2004), Shi (2005), and Zhu and Wallace (2007). In an international macroeconomics context, Geromichalos and Simonovska (2014) considers a two-country monetary model of competing media of exchange, which explains the positive relationship between consumption and asset home bias coupled with higher turnover rates of foreign over domestic assets.

3 Differences of Opinion on Risky Assets

3.1 A Risky Lucas Tree

Consider an infinitely-living Lucas tree with a risky dividend. At time t, the price of a claim on a Lucas tree, or asset, is ψ_t and pays dividend d_t if the asset was held last period. The dividend can take any value from the set $S = \{d_1, ..., d_n\}$ and evolves according to a Markov process described by the transition function $Q : S \times \mathcal{S} \to [0, 1]$, where $\mathcal{S}$ is a σ-algebra of S and $Q(d_i, J)$ is the probability that the next dividend is in $J \subseteq S$ when the current dividend is d_i.

A probability mass function λ on S can be defined recursively using the operator $T\lambda$ such that

$$T\lambda(d_i) = \sum_{d_j \in S} Q(d_j, d_i)\lambda(d_j)$$

By Theorems 11.1 and 11.2 in Stokey and Lucas (1989), there exists a unique invariant probability mass function γ such that $\gamma(d_i) = T\gamma(d_i) > 0$ for every $d_i \in S$. Denote the conditional expectation of the period $t + 1$ dividend when the period t dividend is d_i as

$$d_i^e \equiv \mathbb{E}_t[d_{t+1}|d_t = d_i] = \sum_{d_j \in S} Q(d_i, d_j)d_j$$

3.2 Theories on Future Asset Prices

There is a collection of agents with heterogeneity in how they determine next period's asset price. Each type of agent is said to have a different theory as to how assets are prices, by which the collection of theories is denoted $\Omega = \{1, ..., m\}$. These theories are formulated such that they prevail over an infinite horizon. Consider a stochastic steady state in which the asset price is state-dependent. When the current state is $d \in S$, an agent with theory $i \in \Omega$ formulates her expectation of next period's asset price, $\psi_i'(d)$, according to

$$\psi_i'(d) \equiv \mathbb{E}_i[\psi'|\psi(d)] = f_i\big(\psi(d_1), ..., \psi(d_n)\big)$$

where ψ' is the asset price next period, $\psi(d)$ is the asset price in state $d \in S$, and f_i is continuously differentiable.

As a candidate for the set of theories and each f_i for $i \in \Omega$, I consider the belief structure in Eyster and Piccione (2013). In their framework, agents have a limited understanding as to how the next-period asset price depends on the current dividend realization. Specifically, each agent forms a partition $\mathcal{F}$ of S in which for every element $F \in \mathcal{F}$ and any $d, d' \in F$, an agent with theory $\mathcal{F}$ forms the same beliefs about next period's asset price when the current dividend is d as when it is d'. In this case, the set of theories can be defined as $\Omega = \{\mathcal{F}_1, ..., \mathcal{F}_m\}$, where I refer to an agent with theory $\mathcal{F}_i$ as an i-trader. Given a current-period dividend realization d, an i-trader forms a conditional expectation $\psi_i'(d)$, where $d \in F_i$, such that

$$\psi_i'(d) = \frac{\sum_{d' \in F_i} \sum_{d'' \in S} Q(d', d'')\psi(d'')\gamma(d')}{\sum_{d' \in F_i} \gamma(d')}$$

This expectation corresponds to the long-run empirical average of $\psi'(d)$ given $\mathcal{F}_i$. Hence, each agent's expectation can be interpreted as the limit point of a statistical learning process[2]. If an agent's theory is the set of singletons of S, then their expectation coincides with the conditional expectation of $\psi'(d)$. A theory with a partition of only singletons is the *complete theory*, whereas all other theories are considered *incomplete theories*[3].

3.3 Benchmark Walrasian Asset Prices

Consider the collection of theories described by Ω as in the previous section and a set of traders of measure one. Let $\sigma_i \in (0, 1)$ be the measure of i-traders for $i \in \Omega$, where $\sum_{i \in \Omega} \sigma_i = 1$. Time is infinite, discrete, and each period consists of one stage: a centralized (Walrasian) market (CM). Each agent can produce one unit of a non-storable generalized good per unit of labor. The agent can then choose to consume or sell her production. For $i \in \Omega$, an i-trader's per-period utility function is $x_i - h_i$, where x_i are her units of consumption and h_i are her units of labor. Additionally, agents may purchase claims on a risky Lucas tree, as defined in Section 3.1, where the price ψ is measured in units of the generalized good. Agents' theories reflect their subjective expectations of next period's asset prices conditional on the current period's dividend, as explained in the previous section. The dividend is realized and paid at the beginning of each period. For a given dividend realization $d \in S$, an i-trader who carries a_i asset holdings at the start of the period has a value function

$$W_i(a_i, d) = \max_{(a_i', x_i, h_i) \in \mathbb{R}_+^3} \left\{ x_i - h_i + \beta \mathbb{E}_i[W_i'(a_i', d')|d] \right\} \tag{1}$$

s.t.

$$x_i + \psi a_i' = h_i + [d + \psi]a_i \tag{2}$$

[2]See Eyster and Piccione (2013) for a theoretical explanation of this interpretation. It is essentially an application of Theorem 14.7 in Stokey and Lucas (1989).

[3]It should be noted that Eyster and Piccione (2013) consider trader sophistication, in which a trader is more sophisticated than another trader if her partition has higher cardinality. The complete theory is of the highest cardinality, so it is the most sophisticated theory. I do not consider trader sophistication in my model, as I am more interested in optimism versus pessimism for the sake of liquidity considerations.

According to (1) and (2), an i-buyer chooses asset holdings a_i' to carry into the next period along with consumption x_i and labor h_i in order to maximize her current-period utility and a subjective expectation of next period's value, subject to a resource constraint. Given the Markov process described in Section 3.1 and the conditional expectation of next period's asset price formed by an i-trader as described in Section 3.2, the expectation in (1) can be rewritten as $\mathbb{E}_i[W_i'(a_i', d')|d] = [d^e + \psi_i'(d)]a_i' + \Lambda_i$, where Λ_i is the discounted expectation of the value two periods from today, which is not a function of a_i' or d. Plugging in the constraint (2) into (1), an i-trader's asset holding decision is represented by

$$\max_{a_i' \in \mathbb{R}_+} \left\{ -\psi a_i' + \beta[d^e + \psi_i'(d)]a_i' \right\} \tag{3}$$

If $\psi > \beta[d^e + \psi_{t+1}^i(d)]$, then an i-trader does not purchase any assets. If $\psi = \beta[d^e + \psi_i'(d)]$, then she is indifferent in her asset holding decision. If $\psi < \beta[d^e + \psi_i'(d)]$, then her asset demand is unbounded and so there is no solution to (3). Therefore, it must be that $\psi \geq \beta[d^e + \psi_i'(d)]$ for each $i \in \Omega$.

Consider a steady state equilibrium in which the asset price only depends on the current-period dividend realization. The supply of the asset is fixed and equal to A. In equilibrium, only the most optimistic trader in a given period purchases assets. To see this, define trader $o(d)$ as the most optimistic trader given a current-period dividend of $d \in S$, i.e., $o(d) = \{i \in \Omega : \psi_i'(d) \geq \psi_j'(d) \text{ for every } j \ \in \Omega\}$. If any other trader besides $o(d)$ were to demand assets, then it must be that $\psi = \beta[d^e + \psi_i'(d)]$ for some $i \neq o(d)$. However, this is impossible since this implies $\psi < \beta[d^e + \psi_{o(d)}'(d)]$, which implies there is no solution to (3) for trader $o(d)$. A steady state equilibrium can then be characterized by the following proposition.

Proposition 1. *A steady state equilibrium exists and is characterized by a list of prices* $\{\psi(d)\}_{d \in S}$ *and asset demand correspondences* $\{a_i(d)\}_{i \in \Omega}$ *which satisfy*

$$\psi(d) = \beta[d^e + \psi_{o(d)}'(d)] \tag{4}$$

$$A = \sigma_{o(d)} a_{o(d)}(d) \tag{5}$$

The asset price takes on a total of n possible values; one per each dividend realization. Specifically, the asset price is equal to the subjective fundamental value of the most optimistic trader of the current period. Hence, the trader who values the asset more is the sole participant in the financial sector of the CM. This result serves as a benchmark in which the asset is only valued for its return value. In the next section, each period will contain an additional subperiod in which the asset can be used as a medium of exchange. Hence, the asset will be valued for its return value as well as its liquidity value.

4 A Monetary Approach to Asset Prices with Differences of Opinion

Consider a world in which time is discrete and agents live forever. Unlike the benchmark model of Section 3.3, each period can be divided into two stages: a centralized market (CM) and a decentralized market (DM). There are two types of agents, buyers and sellers, each of measure one and share the same distribution over the set of theories Ω. Specifically, a fraction $\sigma_i \in (0,1)$ of buyers and sellers are i-traders, where $\sum_i \sigma_i = 1$. For notational simplicity, I will refer to a buyer or seller with theory $i \in \Omega$ as an i-buyer or i-seller, respectively.

Each period begins with the CM stage, by which agents (buyers and sellers) interact as price-takers in a Walrasian market. The CM stage is described identically as in Section 3.3, except that the traders of measure one from Section 3.3 are replaced with buyers and sellers, each of measure one.

The DM follows the CM, where buyers observe spending shocks in which they meet sellers bilaterally to trade assets for a non-storable specialized good that only sellers can produce. For any given level of production q of the specialized good, a buyer receives utility $u(q)$ at the expense $c(q)$ of the seller, where I assume that $u'(q) > 0, u''(q) < 0, u(0) = c(0) = c'(0) = 0, u'(0) = +\infty, c'(q) > 0, c''(q) > 0$, and $u'(q^*) = c'(q^*)$ for some $q^* > 0$. The terms of trade are determined by a take-it-or-leave-it (TIOLI) offer by the buyer. A buyer participates in the DM with probability λ. If a buyer participates, the probability distribution over the set of sellers (of measure one) she could match with is uniform. Hence, a buyer matches with a j-seller with probability σ_j for $j \in \Omega$.

If an i-buyer enters the the current-period CM with a_i claims of the asset at a current dividend realization d, her CM value function is

$$W_{b,i}(a_i, d) = \max_{(x_i, h_i, a_i') \in \mathbb{R}_+^3} \big\{x_i - h_i + V_{b,i}(a_i', d)\big\}$$

s.t.

$$x_i + \psi a_i' = h_i + [d + \psi] a_i$$

Given the current-period asset claims a_i' and dividend realization d, $V_{b,i}(a_i', d)$ is an i-buyer's DM value function. Substituting $x_i - h_i$ from the resource constraint into the objective function yields

$$W_{b,i}(a_i, d) = [d + \psi] a_i + \max_{a_i' \in \mathbb{R}_+} \big\{ - \psi a_i' + V_{b,i}(a_i', d)\big\} \tag{6}$$

The CM value function is linear in last period's asset choice a_i. Hence, $W_{b,i}(a_i, d) = [d + \psi] a_i + W_{b,i}(0, d)$. Similarly, an i-seller's CM value function can be written as

$$W_{s,i}(\tilde{a}_i, d) = [d + \psi_t] \tilde{a}_i + \max_{\tilde{a}_i' \in \mathbb{R}_+} \big\{ - \psi \tilde{a}_i{}' + V_{s,i}(\tilde{a}_i{}', d)\big\} \tag{7}$$

where tildes denote seller choice variables. Net consumption, $x_i - h_i$ absorbs all of the wealth effects in this model. Hence, the distribution of asset holdings per theory is degenerate since a_i' does not depend on a_i.

4.1 Agree-to-Disagree Bargaining

If a spending shock is observed by an i-buyer, she meets a j-seller in the DM with probability σ_j, for $j = 1, ..., m$. A buyer trades claims of the asset α_{ij} for the seller's specialized DM good q_{ij}, where the terms of trade are determined by a TIOLI offer proposed by the buyer. Specifically, an i-buyer who meets with a j-seller solves the following problem:

$$\max_{q_{ij},\alpha_{ij}} \left\{u(q_{ij}) + \beta\mathbb{E}_i[W'_{b,i}(a_i - \alpha_{ij}, d')|d]\right\} \tag{8}$$

s.t.

$$-c(q_{ij}) + \beta\mathbb{E}_j[W'_{s,j}(\tilde{a}_j + \alpha_{ij}, d')|d] \geq \beta\mathbb{E}_j[W'_{s,j}(\tilde{a}_j, d')|d] \tag{9}$$

$$\alpha_{ij} \in [0, a_i] \tag{10}$$

The i-buyer maximizes her expected surplus, which includes her utility from consuming q_{ij} and her expected valuation of next period's CM given her new portfolio of asset holdings $a_i - \alpha_{ij}$, subject to the j-seller's participation constraint. Notice that the participation constraint is in terms of the j-seller's beliefs, rather than the i-buyer's beliefs. The i-buyer takes her own beliefs into account when considering her expected surplus from the trade. However, she need not guarantee a trade if she uses her own beliefs in the j-seller's participation constraint, as the i-buyer may be more optimistic than the j-seller about next period's asset price. Hence, the i-buyer must agree to disagree with the j-seller about next period's asset price in order to guarantee that the TIOLI offer is accepted. (10) is merely a resource constraint on the i-buyer's choice of traded asset holdings.

By plugging in (6) and (7) into (8) and (9), the i-buyer's TIOLI problem can be rewritten as follows:

$$\max_{q_{ij},\alpha_{ij}} \left\{u(q_{ij}) - \beta[d^e + \psi'_i(d)]\alpha_{ij}\right\} \tag{11}$$

s.t.

$$-c(q_{ij}) + \beta[d^e + \psi'_j(d)]\alpha_{ij} \geq 0 \tag{12}$$

and (10). Let $R_i(d) = [d^e + \psi'_i(d)]$ be an i-trader's gross expected return on the asset next period. The solution to the buyer's TIOLI problem is presented in Proposition 2.

Proposition 2. *Consider a match between an i-buyer and j-seller, for $i, j \in \Omega$, where the terms of trade are determined by a TIOLI offer represented by (8) subject to (9) and (10). Then, the terms of trade are expressed as follows:*

$$q_{ij}(a_i, d) = \min\left\{\bar{q}_{ij}, \tilde{q}_{ij}\right\} \tag{13}$$

$$\alpha_{ij}(a_i, d) = \min\left\{a_i, \tilde{\alpha}_{ij}\right\} \tag{14}$$

where

$$\bar{q}_{ij} = \left\{ q \in \mathbb{R}_+ : c(q) = \beta R_i(d) a_i \right\}$$

$$\tilde{q}_{ij} = \left\{ q \in \mathbb{R}_+ : \frac{u'(q)}{c'(q)} = \frac{R_i(d)}{R_j(d)} \right\}$$

$$\tilde{\alpha}_{ij} = \left\{ \alpha \in \mathbb{R}_+ : \alpha = \frac{c(\tilde{q}_{ij})}{\beta R_j(d)} \right\}$$

Disagreement regarding next period's asset price is key in determining the terms of trade of a bilateral match in the DM. In the case where the j-seller is more optimistic than the i-buyer regarding next period's asset price, the level of DM output produced in the match may exceed q^*. In order to understand the terms of trade in this situation from an intuitive level, the total surplus resulting from a DM bilateral match is

$$S_{ij}(a_i, d) = u\big(q_{ij}(a_i, d)\big) - c\big(q_{ij}(a_i, d)\big) - \beta[\psi_i'(d) - \psi_j'(d)]\alpha_{ij}(a_i, d) \tag{15}$$

Total surplus in this framework is the sum of two components. The first component is the gain from trade due to the production of output, i.e., the difference between the buyer's utility of consumption and the seller's disutility of production. In the majority of the monetary search literature, total surplus is exclusively this term. Disagreement over next period's asset price introduces a second term to the total surplus of a match, which either rewards or penalizes surplus depending on the level of disagreement, the identity of the pessimist (or optimist), and the amount of assets that change hands. If the i-buyer is more pessimistic than the j-seller, then this term is positive, whereas if the i-buyer is more optimistic than the j-seller, then this term is negative. If both agents share the same opinion, then this term drops out and total surplus is reflected by the gains from trade due to output. Consider the case where the i-buyer is more pessimistic than the j-seller. While the total surplus directly increases as a function of more assets changing hands, it decreases due to the increase in output at levels greater than q^* (the level that maximizes $u(q) - c(q)$), which reduces the difference between $u(q)$ and $c(q)$. The maximum total surplus is represented by the equality of the marginal rate of substitution $u(q)/c(q)$ and the ratio of returns between the i-buyer and j-seller, $R_i(d)/R_j(d)$. The marginal rate of substitution represents the marginal change in the first component of total surplus, i.e., the gap between the utility of the buyer and disutility of the seller, whereas the ratio of returns represents the marginal change in the second component of total surplus, i.e., the gains from trade of the asset changing hands to the agents that has a higher valuation. The level of output implicit in the equality $u(q)/c(q) = R_i(d)/R_j(d)$ is $\tilde{q}_{ij} > q^*$. If the i-buyer can afford $\tilde{q}_{ij}$, then she purchases exactly this amount and holds onto any leftover assets. If the i-buyer cannot afford $\tilde{q}_{ij}$, then she spends all of her asset holdings to get $\bar{q}_{ij} < \tilde{q}_{ij}$.

In the case where the i-buyer is more optimistic than the j-seller, then the intuition of the previous paragraph carries over, but $\tilde{q}_{ij} < q^*$. Hence, when sellers are more pessimistic, the surplus-maximizing level of output is less than q^*. This results from the second component of total surplus, which has a negative sign. Intuitively, the asset is valued more by the

i-buyer, so total surplus decreases when the asset changes hands from the more optimistic i-buyer to the more pessimistic j-seller.

When both parties share the same opinion regarding next period's asset price, the terms of trade are identical to a benchmark Lagos and Wright (2005) model. Specifically, $\tilde{q}_{ij} = q^*$ maximizes total surplus if the i-buyer brings enough assets to afford this level of output. The second component of total surplus drops out, as both parties agree about next period's asset price.

4.2 Equilibrium Asset Prices

The DM value function of an i-buyer takes the following form:

$$V_{b,i}(a_i, d) = \lambda \sum_{j \in \Omega} \sigma_j \big\{ u\big(q_{ij}(a_i, d)\big) + \beta \mathbb{E}_i \big[W'_{b,i}(a_i - \alpha_{ij}(a_i, d), d') | d \big] \big\} \tag{16}$$

The DM value for an i-buyer is the expected value of surplus extracted from the TIOLI offer. Taking advantage of the linearity in asset holdings of the CM value function, (16) can be rewritten as

$$V_{b,i}(a_i, d) = \beta R_i(d) a_i + \lambda \sum_{j \in \Omega} \sigma_j \big\{ u\big(q_{ij}(a_i, d)\big) - \beta R_j(d) \alpha_{i,j}(a_i, d) \big\} + \beta \mathbb{E}_i [W'_{b,i}(0, d') | d] \tag{17}$$

Recall that an i-buyer solves the following problem to determine her current-period asset holdings:

$$\max_{a_i \in \mathbb{R}_+} \big\{ -\psi a_i + V_{b,i}(a_i, d) \big\}$$

Using (17), this problem can be rewritten as

$$\max_{a_i \in \mathbb{R}_+} \left\{ -[\psi - \beta R_i(d)] a_i + \lambda \sum_{j \in \Omega} \sigma_j S_{ij}(a_i, d) \right\} \tag{18}$$

The term $\psi - \beta R_i(d)$ is an i-buyer's subjective cost of investing in the asset. The second term is the expected surplus in the DM. According to (18), an i-buyer chooses her asset holdings in order to maximize her expected surplus in the DM, net of the cost of investing in the asset.

In a bilateral match in the DM, the buyer enjoys all of the surplus, whereas the seller receives none of the surplus from trade due to the structure of the bargaining mechanism. Hence, an i-seller's asset holding problem is

$$\max_{\tilde{a}_i \in \mathbb{R}_+} \left\{ -[\psi - \beta R_i(d)] \tilde{a}_i \right\}$$

As long as $\psi > \beta R_i(d)$ for every $i \in \Omega$, then none of the sellers will demand asset holdings. If $\psi = \beta R_i(d)$ for some $i \in \Omega$, then the i-seller is indifferent between any non-negative quantity of asset holdings. Without loss of generality, I assume all sellers choose zero asset holdings for this case.

Before proceeding, a steady state equilibrium cannot exist if $\psi = \beta R_i(d)$ for some $i \neq o(d)$. In words, the subjective cost of investing in the asset for any trader who is not the most optimistic trader in the market cannot be equal to zero. Suppose this were true for a k-trader, where $k \in \Omega$ and $k \neq o(d)$. Then, for any trader more optimistic than a k-trader, the subjective cost of investing in the asset is negative. Hence, there is no solution to (18) for such traders and a steady state equilibrium cannot exist. Therefore, a necessary condition for a steady state equilibrium is $\psi \geq \beta R_{o(d)}(d)$.

Clearing of the asset market requires

$$\sum_{k \in \Omega} \sigma_k a_k(d) = A \tag{19}$$

where $a_k(d)$ is a solution to a k-buyer's asset holding problem represented by (13) and A is the exogenous supply of assets.

Definition 1. *A steady state equilibrium is a list of asset holdings, terms of trade in the DM, and the price of assets,* $\langle [a_k(d)]_{k\in\Omega}, [q_{ij}(a_i, d), \alpha_{ij}(a_i, d)]_{i,j\in\Omega}, \psi(d)\rangle_{d\in S}$, *such that* $a_k(d)$ *is a solution to (18) for any* $k \in \Omega$, $[q_{ij}(a_i, d), \alpha_{ij}(a_i, d)]$ *is a solution to (8) subject to (9) and (10) for every* $i, j \in \Omega$, *and* $\psi(d)$ *solves (19).*

In order to characterize the set of equilibria, necessary and sufficient conditions must be established for optimal asset holdings for each i-buyer, $i \in \Omega$. Then, the current-period asset prices can be recovered from (19). Notice that $q_{ij}(a_i, d)$ is not differentiable if $\bar{q}_{ij} = \tilde{q}_{ij}$. Therefore, define

$$\tilde{\psi}^{ij}(d) = \left\{\psi(d) \in \mathbb{R}_+ : \tilde{\alpha}_{ij} \text{ is a solution to (18) } \right\} \tag{20}$$

In words, $\tilde{\psi}^{ij}(d)$ is the price at which an i-buyer purchases exactly $\tilde{\alpha}_{ij}$, which will yield the surplus-maximizing level of output in the DM when matched with a j-seller. At such a price, (18) is not differentiable and so such an asset price must be considered as a separate case from the analysis of the corresponding first-order conditions.

Proposition 3. *If* $\psi(d) \geq \beta R_i(d)$ *and* $\psi(d) \neq \tilde{\psi}^{ij}(d)$ *for every* j, *then* a_i *is a solution to the* i*-buyer's asset holding problem (18) if and only if*

$$-\left[\psi(d) - \beta R_i(d)\right] + \lambda \sum_{j\in\Omega} \sigma_j S^a_{ij} \leq 0, \text{ ”=” if } a_i > 0 \tag{21}$$

where

$$S^a_{ij} \equiv \frac{\partial S_{i,j}(a_i, d)}{\partial a_i} = \begin{cases} \beta\lambda\left[\frac{u'(\bar{q}_{ij})}{c'(\bar{q}_{ij})} R_j(d) - R_i(d)\right] & \text{if } \bar{q}_{ij} < \tilde{q}_{ij} \\ 0 & \text{if } \bar{q}_{ij} > \tilde{q}_{ij} \end{cases} \tag{22}$$

If $\psi(d) > \beta R_i(d)$, *then* a_i *is unique. If* $\psi(d) = \beta R_i(d)$, *then* $a_i \geq \bar{a}_i$ *where*

$$\bar{a}_i = \max_{j\in\Omega} \left\{ \frac{c(\tilde{q}_{ij})}{\beta R_j(d)} \right\} \tag{23}$$

If $\psi(d) = \tilde{\psi}^{ij}(d)$, *then* $a_i = \tilde{\alpha}_{ij}$ *for any* j.

According to (21), in order for an i-buyer to hold an asset, its cost must be equal to the expected marginal benefit it yields in the DM. According to (22), the marginal benefit an asset yields in the DM is zero in matches where the i-buyer can attain the surplus-maximizing quantity of output $\tilde{q}_{ij}$, whereas it is positive in matches where the surplus-maximizing quantity of output is not attainable. (23) describes the choice of asset holdings that ensures an i-buyer can obtain the surplus-maximizing level of output in every possible match.

Proposition 4. *A steady state equilibrium exists and it is such that the asset price and asset demand for every i-buyer, $i \in \Omega$, are state-dependent and uniquely determined if $\psi(d) > \beta R_{o(d)}(d)$. For each $i \in \Omega$, it must satisfy*

$$\psi(d) = \beta R_i(d) + \beta\lambda \sum_{j\in\Omega} \sigma_j \min\left\{0, \frac{u'(\bar{q}_{ij})}{c'(\bar{q}_{ij})} R_j(d) - R_i(d)\right\} \tag{24}$$

Implicit in the asset pricing equation (24) is the asset demand for each i-buyer, $a_i(d)$. Aggregating asset demand across the set of theories in (19) yields the equilibrium asset price, which depends on the current-period dividend. The asset price is the sum of two components, each of which varies depending on the buyer's theory. The first term on the right-hand side of (24) is an i-buyer's subjective fundamental value of the asset. This fundamental value is subjective because it depends completely on the i-buyer's opinion of next period's asset price. The second term on the right-hand side of (24) is an i-buyer's speculative liquidity premium. Specifically, it represents the liquidity value of assets in the DM, which is precisely the marginal benefit from bringing more assets into the DM. I refer to this liquidity premium as speculative, as it depends entirely on the differences of opinion expressed in the set of traders who meet in the DM. Consider two buyers, an i-buyer and a k-buyer, by which the former is more optimistic than the latter. While the i-buyer has a greater subjective fundamental value of the asset than the k-buyer, the i-buyer has a smaller speculative liquidity premium than the k-buyer. Hence, a more optimistic buyer values the asset more for its returns, whereas a more pessimistic buyer values the asset more for its liquidity.

The asset price (24) is strictly greater than the benchmark asset price (4). This follows from the fact that the DM provides the possibility of using the asset as a medium of exchange. Unlike the benchmark economy, traders other than the most optimistic actively participate in the investment of assets, even though it is priced above their subjective fundamental values. It should be noted that as $\lambda \to 0$, the asset price (24) approaches the benchmark asset price (4), as its liquidity value diminishes.

Welfare in a steady state equilibrium can be measured by aggregate expected DM surplus:

$$S(d) = \lambda \sum_{i\in\Omega} \sum_{j\in\Omega} \sigma_i \sigma_j S_{ij}(a_i(d), d) \tag{25}$$

Aggregate expected DM surplus is maximized when the surplus-maximizing level of output is affordable in every match. For this to occur, an i-buyer must demand at least $\bar{a}_i$, for every $i \in \Omega$. Specifically, let $\tilde{S}(d)$ denote the maximum aggregate expected DM surplus when the current-period dividend is $d \in S$, where

$$\tilde{S}(d) = \lambda \sum_{i \in \Omega} \sum_{j \in \Omega} \sigma_i \sigma_j S_{ij}(\tilde{\alpha}_{ij}, d) \tag{26}$$

The following proposition states that $\tilde{S}(d)$ is unobtainable in a steady state equilibrium.

Proposition 5. *In a steady state equilibrium, $S(d) < \tilde{S}(d)$ for every $d \in S$.*

The proof of Proposition 5 follows immediately from the lower bound requirement of an asset price in a steady state equilibrium, namely that $\psi(d) \geq \beta R_{o(d)}(d)$. In equilibrium, the asset price must be at least as large as the most optimistic buyer's fundamental value. If it is lower, then the most optimistic buyer's demand for the asset will be unbounded and a steady state equilibrium will fail to exist. In order for any other buyer to demand enough assets to reach her optimal expected DM surplus, the asset must be priced at her fundamental value. Since the most optimistic buyer's fundamental value is strictly greater than every other agents' fundamental values, then it is impossible for any other agent to achieve optimal expected DM surplus. Differences of opinion are at the heart of this result, as the optimal level of expected DM output would be obtainable if all traders in the market shared the same opinion regarding next period's asset price. In such a scenario, an asset price equal to the universal fundamental value would result in the maximal level of aggregate DM surplus[4].

4.3 Asset Demand Among Optimists and Pessimists

In many models of disagreement in financial markets, the asset price reflects the fundamental value of the most optimistic trader, whom which is the only participant in the market. As seen in Section 3.3, omitting the DM stage eliminates the liquidity value of the asset and so its price only reflects the subjective fundamental value of the most optimistic trader. Miller (1977) finds that when short-selling constraints are present, pessimists sit out of the market entirely. My model includes short-selling constraints, but pessimists do not sit out of the market since they value the asset for its liquidity in the DM. Additionally, the asset price reflects the sum of the subjective fundamental value and the speculative liquidity premium for any trader's theory. While my model serves as the contrary to much of the differences of opinion literature with regards to the determination of asset prices and asset demand of pessimists, there is still more to be said regarding the demand of a pessimist relative to that of an optimist. To accommodate a comparison of asset demand between optimists and pessimists, I simplify the set of theories to two types in which there is an optimist and a pessimist for every dividend realization in a stationary equilibrium. The demand of a pessimist relative to that of an optimist is described in the Proposition 6.

Proposition 6. *Consider a steady state equilibrium such that $\Omega = \{1, 2\}$, $\lambda \in (0, 1]$, and $\psi_i'(d) > \psi_j'(d)$ for some $d \in S$ and $i, j \in \Omega$. If $\lambda \in (0, 1)$, then $a_i(d) > a_j(d)$. If $\lambda = 1$, then*

[4]This asset price would be achieved if the aggregate supply of the asset, A, was sufficiently large. Hence, this is essentially the Friedman rule, but in the context of asset supply.

(i) For prices $\psi(d) \leq \tilde{\psi}_i(d)$, $a_i(d) > a_j(d)$.

(ii) For prices $\psi(d) > \tilde{\psi}_i(d)$, $a_i(d) = a_j(d)$.

where

$$\tilde{\psi}_i(d) = \max\{\tilde{\psi}^{i,i}(d), \tilde{\psi}^{i,j}(d)\}$$

The probability of a spending shock, λ, is pivotal in determining the relationship between the asset demand of an optimist and a pessimist. If there is uncertainty regarding the opportunity to consume in the DM, i.e., $\lambda \in (0,1)$, then an optimist demands more assets than a pessimist for any steady state prices. However, if there is certainty that all buyers will participate in the DM, then an optimist demands more assets than a pessimist only for prices sufficiently low. For prices sufficiently high, their asset demands coincide.

To understand the intuition behind Proposition 6, consider (24) in the current context for an optimist:

$$\psi(d) = \beta\big[1 - \lambda[1(\bar{q}_{ii} < \tilde{q}_{ii})\sigma_i + 1(\bar{q}_{ij} < \tilde{q}_{ij})\sigma_j]\big]R_i(d) + \beta\lambda\left[1(\bar{q}_{ii} < \tilde{q}_{ii})\sigma_i\frac{u'(\bar{q}_{ii})}{c'(\bar{q}_{ii})}R_i(d) + 1(\bar{q}_{ij} < \tilde{q}_{ij})\sigma_j\frac{u'(\bar{q}_{ij})}{c'(\bar{q}_{ij})}R_j(d)\right] \quad (27)$$

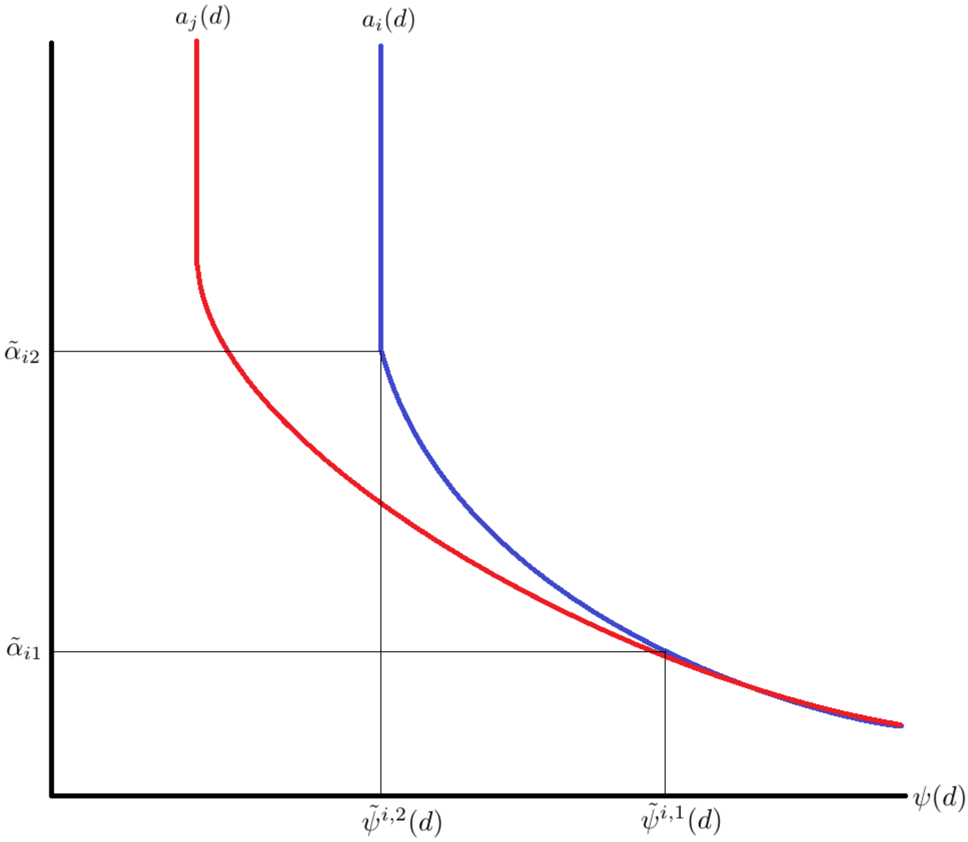
where $1(\bar{q}_{ij} < \tilde{q}_{ij})$ is an indicator function that takes on a value of one if $\bar{q}_{ij} < \tilde{q}_{ij}$ and zero otherwise. A corresponding asset pricing equation involving the pessimist's beliefs is identical to (27), but with the i and j reversed. First, consider prices $\psi(d) > \max\{\tilde{\psi}^{i,1}(d), \tilde{\psi}^{i,2}(d)\}$. At such prices, both indicator functions in (27) are nonzero. Hence, (27) can be rewritten as

$$\psi(d) = \beta[1-\lambda]R_i(d) + \beta\lambda\left[\sigma_i\frac{u'(\bar{q}_{ii})}{c'(\bar{q}_{ii})}R_i(d) + \sigma_j\frac{u'(\bar{q}_{ij})}{c'(\bar{q}_{ij})}R_j(d)\right] \quad (28)$$

The corresponding equation with respect to a pessimist's beliefs is identical, but again with the i and j reversed. According to (28), an optimist participates in the DM, in which she spends all of her assets, with probability λ. In contrast, she does not participate in the DM and instead enjoys the return value of her assets with probability $1-\lambda$. If $\lambda = 1$, then she will never leave the current period with assets and so the price in (28) only reflects a liquidity premium. The same holds for a pessimist. Hence, the liquidity premia of optimists and pessimists coincide. Since both types of buyers observe the same matching probabilities in the DM, it follows that their asset demands coincide. If instead $\lambda \in (0,1)$, the asset is still valued for its return value by both buyers. In such a case, since an optimist's return value is strictly greater than a pessimist's return value, it follows that a pessimist's liquidity value is greater than that of an optimist. Since they face the same matching probabilities in the DM, it follows that an optimist demands more assets than a pessimist since $u'(q)/c'(q)$ is decreasing in q.

Now, consider prices such that $\psi(d) \leq \max\{\tilde{\psi}^{i,1}(d), \tilde{\psi}^{i,2}(d)\}$. In this case, one of the indicator functions in (27) is equal to zero. In such a case, even if $\lambda = 1$, there is still some positive probability that the optimist will enter the next period with a positive quantity of assets. This follows because the optimist has more than enough assets to achieve the optimal level of output in one of her possible DM matches, so she will have leftover assets after such a match. Therefore, for any

Figure 1: Asset demand of optimist versus pessimist.



$\lambda \in (0, 1]$, the return value of the asset for an optimist dominates that of a pessimist, whereas the liquidity value of the asset for a pessimist dominates that of an optimist. It follows that an optimist demands more assets than a pessimist. The case by which $\lambda = 1$ is illustrated in Figure 1. The red asset demand correspondence is for a pessimist, whereas the blue is for an optimist. For Figure 1, I assume WLOG that $\max\{\tilde{\psi}^{i,1}(d), \tilde{\psi}^{i,2}(d)\} = \tilde{\psi}^{i,1}(d)$.

In contrast to Miller (1977) and many other models in the differences of opinion in financial markets literature, not only do pessimists actively participate in the financial market, but their asset demand coincides with that of optimists for prices sufficiently high and when DM participation is certain. Assets are valued not only for their returns, but they are also valued for facilitating trade in the DM. As the price of an asset increases, it is valued less for its returns and more for its liquidity. Intuitively, for prices sufficiently low, an optimist will always demand more assets than a pessimist because at such prices, the return value of the asset dominates its liquidity value, where the former is in the optimist's favor. If instead there is certainty of participating in the DM, then only for prices sufficiently high enough will a pessimist demand as many assets as an optimist. In such a case, the asset is only valued for its liquidity value, by which this value must coincide between optimists and pessimists since the return value will never be enjoyed in the next period due to full liquidation in the DM.

5 Monetary Policy and Asset Prices

In a steady state equilibrium of the economy in the previous section, it is impossible to achieve the optimal level of expected DM output. For this reason, I introduce money into the model in order to study its impact on social welfare. Money will serve alongside the asset as a medium of exchange in the DM. In this section, I will show what kind of impact money has in bilateral DM matches. Additionally, I will derive an optimal monetary policy on interest rates that Pareto dominates the welfare results of Proposition 6.

Money is perfectly divisible and storable in any quantity. The stock of money is assumed to grow at a rate $\pi_t = M_{t+1}/M_t$, where M_t is the supply of money in period t. The gross inflation rate π_t is chosen such that real balances are constant over time, i.e., $\phi_t M_t = \phi_{t+1} M_{t+1}$, where ϕ_t is the price of one unit of money at time t[5]. Unlike the next-period price of the asset, ϕ_{t+1} is common knowledge among all traders, so there are no differences of opinion regarding money. To facilitate the forthcoming analysis, let $z_i = \phi_t m_i$ be the current-period real balances of an i-buyer, where m_i is the i-buyer's current-period money holdings.

The value function for an i-buyer holding z_i units of real balances and a_i claims of the asset at the beginning of the CM when the current dividend realization is $d \in S$ satisfies

$$W_{b,i}(z_i, a_i, d) = \max_{(x_i, h_i,, z_i', a_i') \in \mathbb{R}_+^4} \left\{ x_i - h_i + V_{b,i}(z_i', a_i', d) \right\}$$

s.t.

$$x_i + \pi z_i' + \psi a_i' = h_i + z_i + [d + \psi]a_i + T_{b,i}$$

$T_{b,i}$ are lump sum transfers of real money balances for an i-buyer. Substituting $x_i - h_i$ from the resource constraint into the objective function yields

$$W_{b,i}(z_i, a_i, d) = z_i + [d + \psi]a_i + T_{b,i} + \max_{(z_i', a_i') \in \mathbb{R}_+^2} \left\{ -\pi z_i' - \psi a_i' + V_{b,i}(z_i', a_i', d) \right\} \tag{29}$$

Analogously, an i-seller's CM value function is

$$W_{s,i}(\tilde{z}_i, \tilde{a}_i, d) = \tilde{z}_i + [d + \psi]\tilde{a}_i + T_{s,i} + \max_{(\tilde{z}_i', \tilde{a}_i') \in \mathbb{R}_+^2} \left\{ -\pi \tilde{z}_i' - \psi \tilde{a}_i' + V_{s,i}(\tilde{z}_i', \tilde{a}_i', d) \right\} \tag{30}$$

These CM value functions have the same properties as in Section 4.

5.1 Agree-to-Disagree Bargaining with Money

A meeting between an i-buyer and a j-seller in the DM is once against described by a TIOLI offer by the buyer, but now the asset and money are viable media of exchange. Specifically, the i-buyer makes a TIOLI offer to the j-seller where the terms

[5]This choice of the gross inflation rate ensures that a steady state can be reached. See Nosal and Rocheteau (2011) Chapter 6 for details.

of trade are represented by the triple $(q_{ij}, \mu_{ij}, \alpha_{ij})$. The i-buyer offers a portfolio of real balances, μ_{ij}, and asset holdings, α_{ij}, for q_{ij} units of production of the j-seller's DM good. The TIOLI offer is represented by the following problem:

$$\max_{q_{ij},\mu_{ij},\alpha_{ij}} \left\{u(q_{ij}) + \beta\mathbb{E}_i[W'_{b,i}(z_i - \mu_{ij}, a_i - \alpha_{ij}, d')|d]\right\} \tag{31}$$

s.t.

$$-c(q_{ij}) + \beta\mathbb{E}_j[W'_{s,j}(\tilde{z}_j + \mu_{ij}, \tilde{a}_j + \alpha_{ij}, d')|d] \geq \beta\mathbb{E}_j[W'_{s,j}(\tilde{z}_j, \tilde{a}_j, d')|d] \tag{32}$$

$$\mu_{ij} \in [0, z_i], \alpha_{ij} \in [0, a_i] \tag{33}$$

By plugging in (29) and (30) into (31) and (32), the i-buyer's TIOLI offer to the j-seller can be rewritten as

$$\max_{q_{ij},\mu_{ij},\alpha_{ij}} \left\{u(q_{ij}) - \beta[\mu_{ij} + R_i(d)\alpha_{ij}]\right\} \tag{34}$$

s.t.

$$-c(q_{ij}) + \beta[\mu_{ij} + R_j(d)\alpha_{ij}] \geq 0 \tag{35}$$

and (33). This problem has an analogous interpretation as the one represented by (11) subject to (12) and (10), but with real balances as an additional medium of exchange. However, the solution to this problem is a bit more complicated, as this additional medium of exchange has an agreed-upon value between the i-buyer and the j-seller, even though they disagree about the asset's value next period. Due to the fact that assets are opinion-sensitive as a medium of exchange and money is not opinion-sensitive, a pecking order theory of payments is established in Proposition 7.

Proposition 7. *Consider a match between an i-buyer and j-seller, for $i, j \in \Omega$, where the terms of trade are determined by a TIOLI offer represented by (31) subject to (32) and (33). Then, the terms of trade are expressed as follows:*

(i) If $\psi'_i(d) > \psi'_j(d)$, then

$$q_{ij}(z_i, a_i, d) = \begin{cases} \min\{\bar{q}^z q^*\} & \text{if } \bar{q}^z \geq \tilde{q}_{ij} \\ \min\{\bar{q}_{ij}, \tilde{q}_{ij}\} & \text{if } \bar{q}^z < \tilde{q}_{ij} \end{cases} \tag{36}$$

$$\mu_{ij}(z_i, a_i, d) = \min\{z_i, \mu^*\} \tag{37}$$

$$\alpha_{ij}(z_i, a_i, d) = \begin{cases} 0 & \text{if } \bar{q}^z \geq \tilde{q}_{ij} \\ \min\{a_i, \hat{\alpha}_{ij}\} & \text{if } \bar{q}^z < \tilde{q}_{ij} \end{cases} \tag{38}$$

(ii) If $\psi'_i(d) < \psi'_j(d)$, then

$$q_{ij}(z_i, a_i, d) = \begin{cases} \min\{\bar{q}^a_j, \tilde{q}_{ij}\} & \bar{q}^a_{ij} \geq q^* \\ \min\{\bar{q}_{ij}, q^*\} & \text{if } \bar{q}^a_{ij} < q^* \end{cases} \tag{39}$$

$$\mu_{ij}(z_i, a_i, d) = \begin{cases} 0 & \text{if } \bar{q}^a_{ij} \geq q^* \\ \min\{z_i, \hat{\mu}_{ij}\} & \text{if } \bar{q}^a_{ij} < q^* \end{cases} \tag{40}$$

$$\alpha_{ij}(z_i, a_i, d) = \min\{a_i, \tilde{\alpha}_{ij}\} \tag{41}$$

,

(iii) If $\psi_i(d) = \psi'_j(d)$, then

$$q_{ij}(z_i, a_i, d) = \min\{\bar{q}_{ij}, q^*\} \tag{42}$$

$$\big(\mu_{ij}(z_i, a_i, d), \alpha_{ij}(z_i, a_i, d)\big) = \begin{cases} (z_i, a_i) & \text{if } \bar{q}_{ij} < q^* \\ (\bar{\mu}, \bar{\alpha}) & \text{if } \bar{q}_{ij} \geq q^* * \end{cases} \tag{43}$$

where

$$\bar{q}_{ij} = \Big\{ q \in \mathbb{R}_+ : c(q) = \beta\big[z_i + R_j(d)a_i\big] \Big\}$$

$$\bar{q}^z_i = \Big\{ q \in \mathbb{R}_+ : c(q) = \beta z_i \Big\}$$

$$\bar{q}^a_{ij} = \Big\{ q \in \mathbb{R}_+ : c(q) = \beta R_j(d) a_i \Big\}$$

$$\tilde{q}_{ij} = \Big\{ q \in \mathbb{R}_+ : \frac{u'(q)}{c'(q)} = \frac{R_i(d)}{R_j(d)} \Big\}$$

$$\mu^* = \Big\{ \mu \in \mathbb{R}_+ : \beta\mu = c(q^*) \Big\}$$

$$\tilde{\mu}_{ij} = \Big\{ \mu \in \mathbb{R}_+ : \beta\mu = c(\tilde{q}_{ij}) \Big\}$$

$$\hat{\mu}_{ij} = \Big\{ \mu \in \mathbb{R}_+ : \beta\big[\mu + R_j(d)a_i\big] = c(q^*) \Big\}$$

$$\alpha^*_{ij} = \Big\{ \alpha \in \mathbb{R}_+ : \beta R_j(d) = c(q^*) \Big\}$$

$$\hat{\alpha}_{ij} = \Big\{ \alpha \in \mathbb{R}_+ : \beta\big[z_i + R_j(d)\alpha\big] = c(\tilde{q}_{ij}) \Big\}$$

$$(\bar{\mu}, \bar{\alpha}) = \Big\{ (\mu, \alpha) \in \mathbb{R}^2_+ : \beta\big[\mu + R_j(d)\alpha\big] = c(q^*) \Big\}$$

A pecking order theory of payments is established in Proposition 7. Specifically, when a buyer meets a more pessimistic seller, she prefers to use her money in order to finance the production of DM output. When a buyer meets a more optimistic seller, she prefers to use her assets. This result is most similar to Rocheteau (2011)[6], Jacquet and Tan (2012)[7] and

[6]Rocheteau (2011) assumes asymmetric information regarding a risky asset, in which the buyer knows its true value, whereas the seller does not. Buyers can use risk-free bonds (fiat money) and the risky asset in the DM. The terms of trade in this environment consist of a pecking order in which buyers use the risk-free bond first, then only use the risky asset if risk-free bonds are depleted and the optimal level of output is not met. This result is most similar to case (i) of Proposition 7. However, the clear difference is that assets become useless beyond a certain threshold.

[7]Jacquet and Tan (2012) use an overlapping generations model in order to induce disagreement regarding the return value of two assets. They obtain a similar pecking order of payments.

Geromichalos and Simonovska (2014)[8].

Due to the TIOLI bargaining mechanism, the buyer enjoys all of the surplus in the DM. Surplus in a match between an i-buyer and a j-seller, for $i, j \in \Omega$, when the current-period dividend is $d \in S$ is

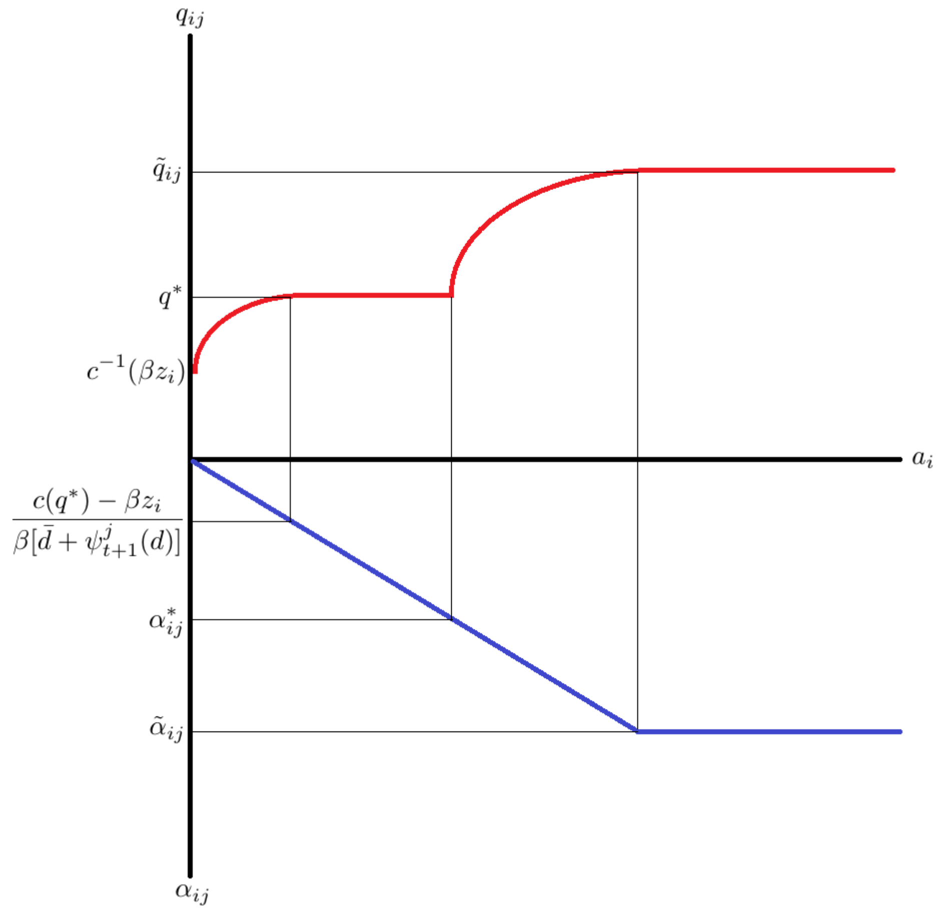
$$S_{ij}(z_i, a_i, d) = u\big(q_{ij}(z_i, a_i, d)\big) - c\big(q_{ij}(z_i, a_i, d)\big) - \beta[\psi_i'(d) - \psi_j'(d)]\alpha_{ij}(z_i, a_i, d) \tag{44}$$

Surplus is identical to that in Section 3, except that it is now a function of real money balances of the buyer, alongside the buyer's asset holdings and the current-period dividend realization. Just as in Section 3, the surplus can be divided into the sum of two parts: (i) the difference between the utility of consumption by the buyer and the disutility of production by the seller, and (ii) the penalty (or reward) from exchanging assets due to differences of opinion. I will call the first term the "output effect" and the second term the "asset effect" of surplus. While the exchange of assets affects both the output effect and asset effect, the exchange of money only affects the output effect. This follows because assets are opinion-sensitive, whereas money is opinion-insensitive. Hence, the asset carries an additional effect in surplus, whereas money does not. There are three cases to consider regarding the terms of trade in a bilateral match, namely when the buyer is more optimistic than the seller, when the buyer is more pessimistic than the seller, and when the buyer and seller share the same opinion regarding next period's asset price.

First, consider the case in which the buyer is more optimistic than the seller. In this case, the asset effect is negative and so it is costly to trade assets. However, since trading real money balances yields no penalty, it is the preferred method of payment by the buyer. If she is unable to afford $\tilde{q}_{ij}$ with both real balances and asset holdings, then she spends the entirely of both methods of payment in exchange for $\bar{q}_{ij} < \tilde{q}_{ij}$. When this is the case, the marginal cost of spending real balances and the marginal cost of spending assets are each less than the marginal benefit from consuming more output, i.e., $R_i(d)/R_j(d) < u'(\bar{q}_{ij})/c'(\bar{q}_{ij})$ and $1 < u'(\bar{q}_{ij})/c'(\bar{q}_{ij})$, respectively. If the buyer is able to afford strictly more than $\tilde{q}_{ij}$ with real balances and assets, but not with real balances alone, then she purchases exactly $\tilde{q}_{ij}$ with all of her real balances and the remainder with her asset holdings. This pecking order exists because the marginal cost of exchanging real balances is less than the marginal cost of exchanging assets, i.e., $1 < R_i(d)/R_j(d)$. The buyer is still willing to use assets though because the marginal cost associated with exchanging them is equal to the marginal benefit from consuming $\tilde{q}_{ij}$. If the buyer is able to purchase $\tilde{q}_{ij}$ with real balances alone, then she no longer uses assets as a method of payment. In this case, she spends all of her real balances up to the point where she can afford $q^* > \tilde{q}_{ij}$ with real balances alone. For quantities greater than $\tilde{q}_{ij}$, the marginal cost of exchanging assets is greater than the marginal benefit from additional output. At a level q^*, then marginal

[8] Geromichalos and Simonovska (2014) consider a two-country monetary model in which home and foreign assets can each be used in a home or foreign DM. They assume that foreign assets exhibit a policy friction in which its return value is reduced. The case in which the buyer is from the home country is most similar to case (iii) in Proposition 7. The case in which the buyer is from the foreign country is most similar to case (ii) in Proposition 7.

Figure 2: Bilateral Meeting with Pessimistic Buyer and Optimistic Seller.



cost of exchanging real balances and the marginal benefit of consuming output are equalized.

Consider now the case in which the buyer is more pessimistic than the seller. The asset effect is positive, as the seller values the asset more than the buyer. Figure 2 expresses the terms of trade for a pessimistic buyer and an optimistic seller. The upper half of the vertical axis represents the level of output that is exchanged with a fixed level of real balances, z_i, as a function of asset holdings, a_i. The lower half of the vertical axis shows the amount of assets that are exchanged for each associated level of output. Note that in this case, $\tilde{q}_{ij} > q^*$. If the buyer cannot afford q^* with both real balances and asset holdings, then she spends the entirely of both on $\bar{q}_{ij} < q^*$. This occurs because the marginal cost of exchanging assets and real balances are less than the marginal benefit of an additional unit of output. This region is the first concave portion of the output curve (in red). If the buyer is able to afford q^* with both real balances and asset holdings, but not with asset holdings alone, then she purchases q^* with all of her assets and funds the remainder with real balances. This region is the first flat portion of the output curve. The marginal cost of exchanging real balances is greater than the marginal cost of exchanging asset holdings, so asset holdings are the preferred method of payment. If the buyer can afford at least q^* with assets alone, then she spends all of her assets until she can afford $\tilde{q}_{ij}$, which is the level of output in which the marginal cost of exchanging assets is equal to the marginal benefit from an additional unit of output. This region is the second concave portion of the output curve. In this region, the marginal cost of exchanging real balances is greater than the marginal benefit from an additional unit of output, so the buyer does not spend her real balances. The second flat portion of the output curve represents the maximum level of output a buyer is willing to purchase, namely $\tilde{q}_{ij}$. Notice that the buyer spends all of her assets up to this point, as indicated by the blue curve in Figure 2.

If the buyer and the seller have the same opinion regarding next period's asset price, then the marginal cost of exchanging assets and real balances are equal. In this case, the buyer is indifferent between using real balances and assets as a methods of payment. The asset effect is zero in this case, as both methods of payment are opinion-insensitive when both the buyer and seller share the same opinions. Without loss of generality, I assume that the buyer prefers to use her money first in this case.

5.2 Money and Assets in Equilibrium

An i-buyer's DM value function for a portfolio (z_i, a_i) and realized dividend d is

$$V_{b,i}(z_i, a_i, d) = \sum_{j\in\Omega} \sigma_j \big\{ u\big(q_{ij}(z_i, a_i, d)\big) + \beta \mathbb{E}_i \big[W'_{b,i}(z_i - \mu_{ij}(z_i, a_i, d), a_i - \alpha_{ij}(z_i, a_i, d), d') | d \big] \big\} \tag{45}$$

Using the linearity of (29) in the i-buyer's portfolio and the fact that (35) binds, (45) can be rewritten as

$$V_{b,i}(z_i, a_i, d) = \beta[z_i + R_i(d)a_i] + \sum_{j\in\Omega} \sigma_j S_{ij}(z_i, a_i, d) + \beta \mathbb{E}_i[W'_{b,i}(0, 0, d')|d] \tag{46}$$

Plugging in (46) into (29), the i-buyer's portfolio decision during the CM can be stated as

$$\max_{(z_i,a_i)\in\mathbb{R}^2_+} \Big\{ - n z_i - \Big[\frac{\psi}{\beta} - R_i(d) \Big] a_i + \frac{\lambda}{\beta} \sum_{j\in\Omega} \sigma_j S_{ij}(z_i, a_i, d) \Big\} \tag{47}$$

where $n \equiv (\pi - \beta)/\beta$ is the cost of holding real balances, or the nominal interest rate. According to (47), an i-buyer chooses a portfolio of real money balances and asset holdings in order to maximize her expected surplus in the DM, net of the cost of holding money and the cost of investing in assets.

Clearly, if $n < 0$, then an i-buyer's solution for real balances will be unbounded. Hence, a solution for the i-buyer's real balances exists as long as $n \geq 0$. Following the reasoning from Section 4, it must be that $\psi \geq \beta R_{o(d)}(d)$ for a solution to exist, where the $o(d)$-buyer is the most optimistic buyer from the set of theories Ω when the current-period dividend is $d \in S$. It follows that if $n > 0$ and $\psi > \beta R_{o(d)}(d)$, then every seller demands zero real balances and zero asset holdings. Without loss of generality, I assume that sellers exhibit the same demand if any of these pricing conditions hold with equality.

Clearing of the money and asset markets require

$$\sum_{k\in\Omega} \sigma_k z_k(d) = Z \tag{48}$$

$$\sum_{k\in\Omega}\sigma_k a_k(d) = A \tag{49}$$

where $\langle z_k(d), a_k(d)\rangle$ is a solution to a k-buyer's portfolio problem (47). Z and A represent the aggregate supply of real money balances and asset holdings, respectively.

Definition 2. *A steady state equilibrium is a list of real money balances and asset holdings, terms of trade in the DM, and the price of assets,* $\langle[z_k(d), a_k(d)]_{k\in\Omega}, [q_{ij}(z_i, a_i, d), \mu_{ij}(z_i, a_i, d), \alpha_{ij}(z_i, a_i, d)]_{i,j\in\Omega}, n(d), \psi(d)\rangle_{d\in S}$*, such that* $[z_k(d), a_k(d)]$ *is a solution to (47) for any* $k \in \Omega$*,* $[q_{ij}(z_i, a_i, d), \mu_{ij}(z_i, a_i, d), \alpha_{ij}(z_i, a_i, d)]$ *is a solution to (31) subject to (32) and (33) for every* $i, j \in \Omega$*, and* $[n(d), \psi(d)]$ *solves the system (48) and (49).*

Necessary and sufficient conditions must be established for optimal real money balances and assets holdings for each i-buyer, $i \in \Omega$. Similarly to Section 4, the i-buyer's objective function is not differentiable for various prices $[n(d), \psi(d)]$. The solution functions of (31) subject to (32) and (33) have various kinks, so the first-order necessary and sufficient conditions do not apply to these points. Specifically, the set of prices in which (47) is not differentiable is

$$\Lambda^{i,d} = \left\{(n(d), \psi(d)) \in \mathbb{R}^2_+ : \bar{q}^z_i = q^* \text{ or } \bar{q}^z_i = \tilde{q}_{ij} \text{ or } \bar{q}_{ij} = \tilde{q}_{ij} \text{ or } \bar{q}^a_{ij} = q^* \text{ or } \bar{q}^a_{ij} = \tilde{q}_{ij} \text{ or } \bar{q}_{ij} = q^*\right\} \tag{50}$$

The next result establishes the necessary and sufficient conditions of a solution to (47).

Proposition 8. *If* $\psi(d) \geq \beta R_i(d)$*,* $n(d) \geq 0$*, and* $(n(d), \psi(d)) \notin \Lambda^{i,d}$*, then* (z_i, a_i) *is a solution to the* i*-buyer's portfolio problem (47) if and only if*

$$-n(d) + \frac{1}{\beta}\sum_{j\in\Omega}\sigma_j S^z_{ij} \leq 0 \text{ ”=” if } z_i > 0 \tag{51}$$

$$-\left[\frac{\psi(d)}{\beta} - R_i(d)\right] + \frac{1}{\beta}\sum_{j\in\Omega}\sigma_j S^a_{ij} \leq 0 \text{ ”=” if } a_i > 0 \tag{52}$$

where S^z_{ij} *and* S^a_{ij} *are the partial derivatives of* $S_{ij}(z_i, a_i, d)$*, respectively. If* $\psi(d) > \beta R_i(d)$ *and* $n(d) > 0$*, then* (z_i, a_i) *is unique if either* $z_i > 0$ *and* $a_i = 0$ *or* $z_i = 0$ *and* $a_i > 0$*. If* $\psi(d) = \beta R_i(d)$*, then* $a_i \geq \bar{a}_i$*. If* $n(d) = 0$*, then* $z_i \geq \mu^*$*.*

According to (51), an i-buyer demands real money balances up to the point where the expected marginal benefit of DM surplus from bringing an additional unit of real balances is equal to the nominal interest rate. If the nominal interest rate exceeds the expected marginal benefit of DM surplus from bringing a single unit of real balances, then the i-buyer will not purchase any real money balances. The interpretation of (52) is similar to (51), except the cost of investing in the asset is the difference between the asset's price and the i-buyer's subjective fundamental value. Existence of a steady state equilibrium as well as the corresponding nominal interest rates and asset prices, given the current-period dividend, are described in Proposition 9.

Proposition 9. *A steady state equilibrium exists if $n(d) \geq 0$ and $\psi(d) \geq \beta R_{o(d)}(d)$ and it is such that the prices $(n(d), \psi(d))$ and portfolio demand $(z_i, a_i)_{i\in\Omega}$ are state-dependent. For each $i \in \Omega$ and prices such that $n(d) > 0$ and $\psi(d) > \beta R_{o(d)}(d)$, one of the following must be satisfied:*

(i) If

$$\psi(d) - \beta\Big[1 - \lambda\sum_{j\in\Omega}\sigma_j 1(\bar{q}_i^z < \tilde{q}_{ij})\Big]R_i(d) > \beta[n(d) + \lambda]\sum_{j\in\Omega}\sigma_j 1(\bar{q}_i^z < \tilde{q}_{ij})R_j(d) \tag{53}$$

then $z_i(d) > 0$ and $a_i(d) = 0$, where $z_i(d)$ uniquely solves

$$n(d) = \lambda\left[\frac{u'(\bar{q}_i^z)}{c'(\bar{q}_i^z)} - 1\right] \tag{54}$$

(ii) If

$$\psi(d) - \beta\Big[1 - \lambda\sum_{j\in\Omega}\sigma_j 1(\bar{q}_i^z < \tilde{q}_{ij})\Big]R_i(d) < \beta[n(d) + \lambda]\sum_{j\in\Omega}\sigma_j 1(\bar{q}_i^z < \tilde{q}_{ij})R_j(d) \tag{55}$$

then $z_i(d) = 0$ and $a_i(d) > 0$, where $a_i(d)$ uniquely solves

$$\psi(d) = \beta R_i(d) + \beta\lambda\sum_{j\in\Omega}\sigma_j 1(\bar{q}_{ij}^a < \tilde{q}_{ij})\left[\frac{u'(\bar{q}_{ij}^a)}{c'(\bar{q}_{ij}^a)}R_j(d) - R_i(d)\right] \tag{56}$$

(iii) If

$$\psi(d) - \beta\Big[1 - \lambda\sum_{j\in\Omega}\sigma_j 1(\bar{q}_i^z < \tilde{q}_{ij})\Big]R_i(d) = \beta[n(d) + \lambda]\sum_{j\in\Omega}\sigma_j 1(\bar{q}_i^z < \tilde{q}_{ij})R_j(d) \tag{57}$$

then $z_i(d) > 0$ and $a_i(d) > 0$, where $[z_i(d), a_i(d)]$ satisfy

$$n(d) = \lambda\sum_{j\in\Omega}\sigma_j\left\{1(j \in \Omega_1^{i,d} \cup \Omega_3^{i,d} \cup \Omega_4^{i,d})\left[\frac{u'(q_{ij})}{c'(q_{ij})} - 1\right] + 1(j \in \Omega_2^{i,d})\left[\frac{R_i(d)}{R_j(d)} - 1\right]\right\} \tag{58}$$

$$\psi(d) = \beta R_i(d) + \beta\lambda\sum_{j\in\Omega}\sigma_j\left\{1(j \in \Omega_1^{i,d} \cup \Omega_4^{i,d} \cup \Omega_6^{i,d})\left[\frac{u'(q_{ij})}{c'(q_{ij})}R_j(d) - R_i(d)\right] + 1(j \in \Omega_5^{i,d})\big[\psi_j'(d) - \psi_i'(d)\big]\right\} \tag{59}$$

where

$$\Omega_1^{i,d} = \Big\{j \in \Omega : \psi_i'(d) \geq \psi_j'(d) \text{ and } \bar{q}_{ij} < \tilde{q}_{ij}\Big\} \tag{60}$$

$$\Omega_2^{i,d} = \Big\{j \in \Omega : \psi_i'(d) \geq \psi_j'(d) \text{ and } \bar{q}_i^z < \tilde{q}_{ij} < \bar{q}_{ij}\Big\} \tag{61}$$

$$\Omega_3^{i,d} = \Big\{j \in \Omega : \psi_i'(d) \geq \psi_j'(d) \text{ and } \tilde{q}_{ij} < \bar{q}_i^z < q^*\Big\} \tag{62}$$

$$\Omega_4^{i,d} = \Big\{j \in \Omega : \psi_i'(d) < \psi_j'(d) \text{ and } \bar{q}_{ij} < q^*\Big\} \tag{63}$$

$$\Omega_5^{i,d} = \Big\{j \in \Omega : \psi_i'(d) < \psi_j'(d) \text{ and } \bar{q}_{ij}^a < q^* < \bar{q}_{ij}\Big\} \tag{64}$$

$$\Omega_6^{i,d} = \Big\{j \in \Omega : \psi_i'(d) < \psi_j'(d) \text{ and } q^* < \bar{q}_{ij}^a < \tilde{q}_{ij}\Big\} \tag{65}$$

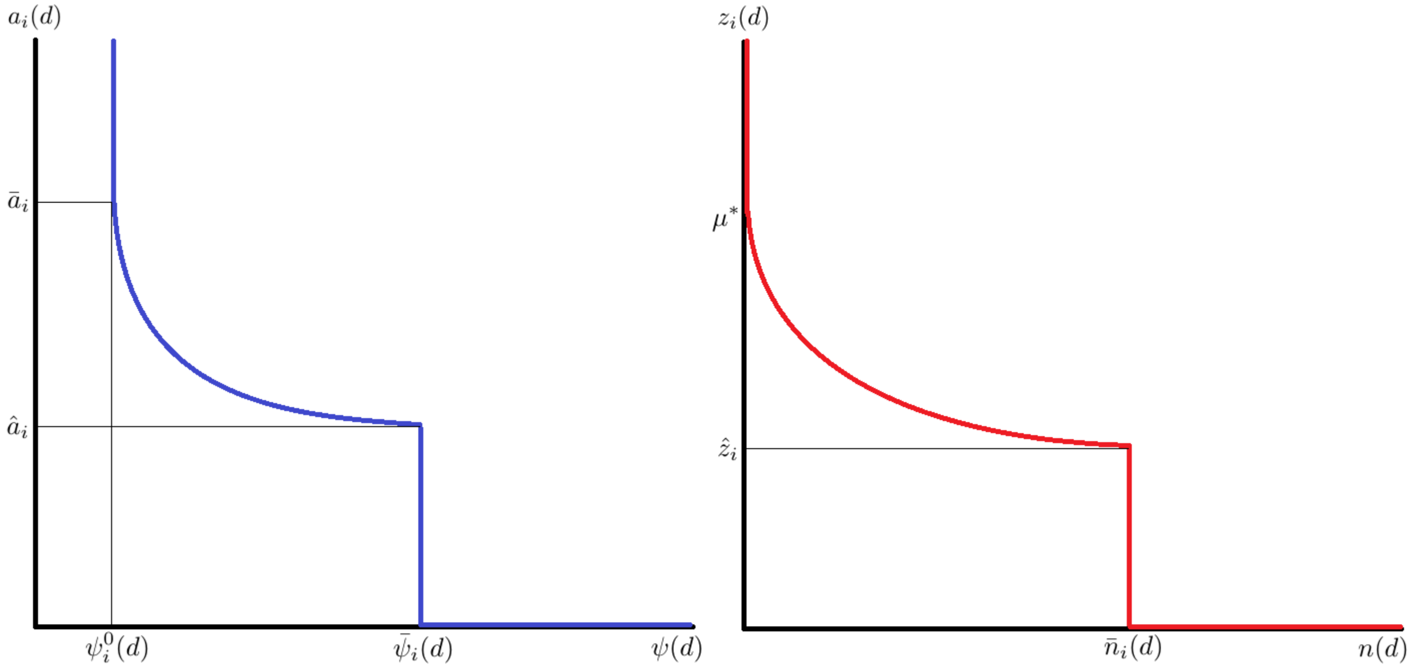
The left-hand side of (53) can be interpreted as the net cost of holding the asset between on CM and the next. I use the term "net" because a buyer might not carry assets into the next CM, as she may liquidate them in the DM. For example, if a buyer liquidates her assets with certainty in the DM, then her expected return value is not discounted from the cost of holding the asset between CMs, and so this cost is just the price of the asset. The term on the right-hand side of (53) is thus the net cost of holding the equivalent amount of real balances between one CM and the next. If the net cost of holding an asset is higher than carrying an equivalent amount of real balances, then a buyer will only purchase real balances in the CM, as indicated by case (i) in Proposition 9. In such a scenario, the demand for real balances is determined by (54), which is identical to the demand for real balances in a benchmark Lagos-Wright monetary model.

In contrast, if the net cost of holding the asset is less than the net cost of holding an equivalent amount of real balances, then a buyer will only demand assets in the CM, as indicated in case (ii) of Proposition 9. A buyer chooses her asset demand according to (56), which is identical to (24). Hence, a non-monetary equilibrium in this model is equivalent to an equilibrium in the asset-only economy of Section 4.

If the net costs of holding assets and real balances are equal, then a buyer is indifferent between purchasing assets and real balances in the CM. There exists a continuum of possible portfolios in this case. Specifically, these portfolios satisfy (58) and (59). The set of portfolios for this case can be described by the set $[0, \hat{a}_i] \times [0, \hat{z}_i]$, where $\hat{a}_i$ satisfies (58) and (59) when $z_i = 0$ and $\hat{z}_i$ satisfies (58) and (59) when $a_i = 0$. The nominal interest rate and asset price for any state can be intuitively explained by (58) and (59), respectively. When viewing (59) as an asset pricing equation, the second term on the right-hand side is a speculative liquidity premium just as in (24) and (56), except there are additional terms. The first term in this speculative liquidity premium represents the marginal increase to DM surplus when bringing an additional asset to the DM increases the level of output produced in a match. This can occur with sellers who belong to the sets represented by (60), (63), and (65). This coincides with the segments of the red contract curve in Figure 2 with positive slope. The second term in this speculative premium represents the marginal increase to surplus when real balances are replaced with assets when a buyer meets a more optimistic seller. This is represented by the first flat portion of the red contract curve in Figure 2. In this region of the contract curve, output doesn't increase, but assets are changing hands to someone who values them more. The nominal interest rate described by (58) has a similar interpretation, except that real balances are only valued for liquidity, so the right-hand side of (58) is purely a liquidity premium.

The asset demand and real balance demand correspondences for an i-buyer for any $d \in S$ are depicted in Figure 3. The asset demand correspondence as a function of the current asset price is on the left, whereas the real balance demand correspondence as a function of the current nominal interest rate is on the right. A non-monetary equilibrium for an i-buyer, or case (ii) of Proposition 9, is represented by the downward-sloping portion of the asset demand correspondence and the flat

Figure 3: Indivudual Asset and Real Balance Demand Correspondences



portion of the real balance demand correspondence. A monetary equilibrium for an i-buyer in which assets are also purchased is represented by the vertical portions of the asset demand and real balance demand correspondences in the intervals $[0, \hat{a}_i]$ and $[0, \hat{z}_i]$, respectively. A monetary equilibrium for an i-buyer in which no assets are purchased is represented by the flat portion of the asset demand correspondence and the downward-sloping portion of the real balance demand correspondence. The prices $\psi_i^0(d)$, $\hat{\psi}(d)$, and $\bar{n}_i(d)$ are defined as

$$\psi_i^0(d) = \{\psi(d) \in \mathbb{R}_+ : \psi(d) = \beta R_i(d)\} \tag{66}$$

$$\bar{\psi}_i(d) = \{\psi(d) \in \mathbb{R}_+ : \psi(d) \text{ satisfies } (57)\} \tag{67}$$

$$\bar{n}_i(d) = \{n(d) \in \mathbb{R}_+ : n(d) \text{ satisfies } (57)\} \tag{68}$$

Notice that the nominal interest rate defined in (68) depends on i. This implies that the vertical portion of the real balance demand correspondence at this price may vary by the theory of the buyer.

The comparative statics regarding the demand correspondences and prices can be recovered using the Implicit Function theorem on the system of equations defining a steady state equilibrium, but intuition is lost due to the computational rigor. This occurs because asset demand in a particular state, as well as prices, are functions of prices in *every* state due to the subjective expected prices entering the demand correspondences, and hence the market-clearing conditions which determine prices. However, a non-explicit description of some comparative statics are available using Figure 3. First, note that asset prices are strictly increasing in the subjective expected prices of each agent. Also, in a monetary equilibrium, asset prices and nominal interest rates move in the same direction, as implied by (57). Specifically, as $n(d)$ decreases, the vertical portion of the asset demand correspondence at a price $\bar{\psi}_i(d)$ shifts to the left. If the asset price in a different state decreases, then both vertical portions of the asset demand correspondence shift to the left. Since opinions are a function of prices in each state, it follows that an i-buyer may need to spend more assets in order to get the same level of output in the DM. Hence, the vertical portions not only shift to the left due to a decrease in another state's price, but $\bar{a}_i$ and $\hat{a}_i$ may increase. This also alters the slope of the downward-sloping portion of the asset demand correspondence, which depends on what happens to the vertical portions. Note that there is additional feedback between nominal interest rates and asset prices in various states, so there are many channels involved in such changes. The only portion of the real balance demand correspondence that changes is the location of the vertical portion at $\bar{n}_i(d)$, which can shift as a result of price changes in any state or every nominal interest rate changes in other states.

5.3 Monetary Policy in Equilibrium

Now that a steady state equilibrium has been established, the notion of optimal monetary policy can be explored. In general, the definition of optimality with regards to monetary policy is highly subjective and ultimately left to the appropriate monetary authority. With respect to this model though, I define monetary policy to be optimal if it maximizes social welfare, which is the expected DM surplus. A Lagos and Wright (2005) benchmark for optimality is the Friedman rule, in which the nominal interest rate is set to zero. At this nominal interest rate, each agent can afford the optimal level of output in every match. In my model, the optimal level of output may be above q^*, which occurs when a pessimistic buyer meets an optimistic seller and the buyer has sufficient assets. While this level of output can't be achieved with money, the Friedman rule is still the optimal monetary policy in this differences-of-opinion framework, as stated in the next proposition. Without loss of generality, I simplify the set of theories and states to have a cardinality of two[9].

Proposition 10. *Let* $|\Omega| = |S| = 2$, $\psi_i'(d) > \psi_j'(d)$ *for* $i, j \in \Omega$, *and* $\max\{\tilde{\alpha}_{ii}, \tilde{\alpha}_{ij}\} = \tilde{\alpha}_{ij}$[10]. *Consider a steady state equilibrium and a monetary authority who can choose* Z. *If a monetary authority chooses* Z *to maximize expected DM surplus in each state* $d \in S$, *then the optimal* Z *is chosen such that* $n(d) = 0$ *for each* $d \in S$.

Under the Friedman rule, every possible DM meeting results in a level q^* of output in exchange for μ^* units of real

[9] A cardinality greater than two for either of these two sets requires a numerical proof, which takes away from the intuition of a closed-form proof.

[10] If instead $\max\{\tilde{\alpha}_{ii}, \tilde{\alpha}_{ij}\} = \tilde{\alpha}_{ii}$, then a numerical proof is required. If one considers a utility function of the constant relative risk aversion type, then the restriction in Proposition 10 is equivalent to the assumption that the coefficient of relative risk aversion is greater than one. Most empirical measures of the coefficient are greater than one, so this is not a restrictive assumption. I omit a numerical proof for the alternative case, but note that it does hold for an array of parameter choices.

balances. The liquidity premium of the asset for the most optimistic buyers becomes zero when a zero interest rate policy is enacted. The most optimistic buyers do not meet any sellers who are more optimistic than themselves, so q^* is the absolute best they can achieve in terms of output. Consider the asset holding decision of an i-buyer in which $i \neq o(d)$. Such an i-buyer demands zero asset holdings if

$$\psi(d) \geq \beta R_i(d) + \beta \sum_{j \in \Omega} \sigma_j 1(\psi_i'(d) < \psi_j'(d))[\psi_j'(d) - \psi_i'(d)] \tag{69}$$

In a steady state equilibrium, it must be that $\psi(d) \geq \beta R_{o(d)}(d)$, which satisfies (69) with strict inequality for every $i \neq o(d)$. Therefore, in a steady state equilibrium under the Friedman rule, aggregate asset demand is composed strictly of the most optimistic traders. A zero interest rate thus eliminates the asset's value as liquidity, while it maintains its value as an investment, but only for the most optimistic traders. The asset prices under the Friedman rule thus coincide with (4), which are the prices in which the DM doesn't exist. It should be noted that the nominal interest rate is zero in every state under the Friedman rule, as the vertical portion of the real balance demand correspondence in Figure 3 at zero is always on the interval $[\mu^*, \infty)$ in ever state. A monetary authority only needs to adjust Z to reach this point.

5.4 The Effects of Monetary Policy on Optimists and Pessimists

Monetary policy can have an impact on the composition of traders in the asset market. Consider an economy with the same belief structure as in Section 4.3, where there is an optimist and a pessimist in each period. As implied by the analysis of Section 5.2, optimists value the asset more than pessimists for its expected return value, but less for its liquidity value. Therefore, money serves as a better medium of exchange than the asset from the perspective of an optimist, whereas the reverse is true for a pessimist. Hence, altering the nominal exchange rate should alter the liquidity premium between assets and money. Proposition 11 describes the extension to Proposition 6 in which money and assets are used as media of exchange in the DM.

Proposition 11. *Consider a steady state equilibrium such that $\Omega = \{1, 2\}$ and $\psi_i'(d) > \psi_j'(d)$. Asset demand in a non-monetary equilibrium (in which $z_i(d) = z_j(d) = 0$) is described identically as in Proposition 6. A monetary equilibrium (in which $z_k(d) > 0$ for at least one $k \in \Omega$) can be described as follows:*

(i) If $\lambda = 1$, then

$$\bar{\psi}_i(d) \begin{cases} = \bar{\psi}_j(d) & \text{if } 1 + n(d) \geq \frac{R_1(d)}{R_2(d)} \\ > \bar{\psi}_j(d) & \text{if } 1 + n(d) < \frac{R_1(d)}{R_2(d)} \end{cases} \tag{70}$$

$$\bar{n}_i(d) \begin{cases} = \bar{n}_j(d) & \text{if } 1 + n(d) \geq \frac{R_1(d)}{R_2(d)} \\ < \bar{n}_j(d) & \text{if } 1 + n(d) < \frac{R_1(d)}{R_2(d)} \end{cases} \tag{71}$$

(ii) If $\lambda \in (0,1)$, then

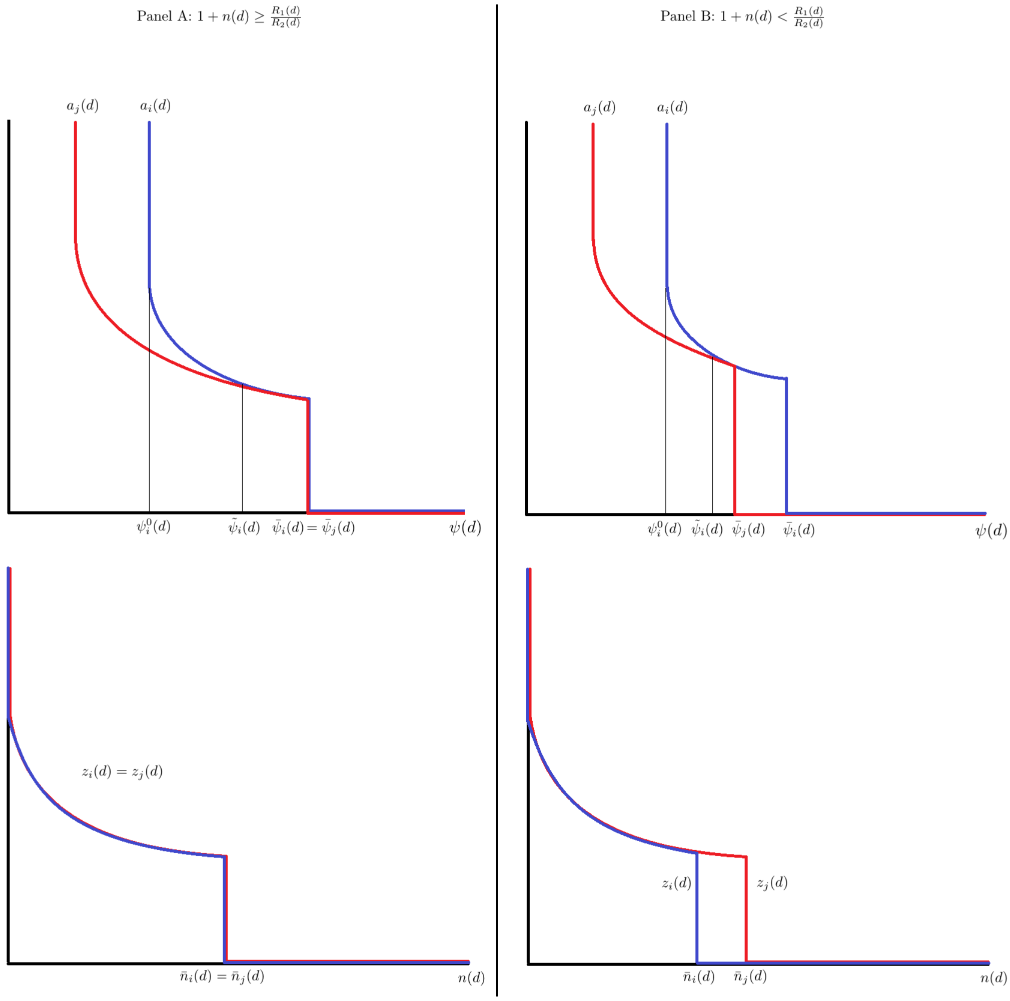
$$\bar{\psi}_i(d) > \bar{\psi}_j(d) \tag{72}$$

$$\bar{n}_i(d) < \bar{n}_j(d \tag{73}$$

Recall that $(\bar{\psi}_i(d), \bar{n}_i(d))$ is the asset price and nominal interest rate pair in which an i-buyer is indifferent between purchasing assets and real balances in the DM. These prices correspond to the vertical portions of the asset demand and real balance demand correspondences that touch the horizontal axes, as seen in Figure 3. According to Proposition 11, the downward-sloping portions of the asset demand correspondences of optimists and pessimists share the same relationship as in Proposition 6. Any asset price along the downward-sloping portions of both asset demand correspondences of optimists and pessimists is associated with a non-monetary equilibrium. Any asset price associated with the vertical or flat portions of a buyer's asset demand correspondence is associated with a monetary equilibrium. The asset demand correspondences in the case where $\lambda = 1$ are depicted in Figure 4. If $\lambda = 1$ and $1 + n(d) \geq \frac{R_1(d)}{R_2(d)}$, then the asset demand correspondences in a monetary equilibrium coincide for an optimist and a pessimist, as shown at the top of Panel A in Figure 4. Their real money demand correspondences coincide completely, as shown at the bottom of Panel A in Figure 4. If instead $\lambda = 1$ and $1+n(d) < \frac{R_1(d)}{R_2(d)}$, then an optimist will tolerate higher asset prices than a pessimist before shifting her demand strictly to real balances, as shown in the top of Panel B in Figure 4. Correspondingly, a pessimist will tolerate a higher nominal interest rate than an optimist before shifting her demand strictly to assets, as shown in the bottom of Panel B in Figure 4. If $\lambda\ in(0,1)$, then an optimist will tolerate a higher asset price than a pessimist before switching completely to real balances, whereas a pessimist will tolerate a higher nominal interest rate than an optimist before switching completely to assets. This occurs regardless of the relationship between the nominal interest rate and subjective expected returns.

Consider the case where $\lambda = 1$. When $1 + n(d) \geq \frac{R_1(d)}{R_2(d)}$, an optimist at most affords the optimal level of output in a match with a pessimist, $\tilde{q}_{ij}$, with real balances alone at the nominal interest rate $n(d)$. To verify this, recall that $1 + n(d) = u'(\bar{q}_i^z)/c'(\bar{q}_i^z)$ when an optimist only purchases real balances, and $\frac{R_1(d)}{R_2(d)} = u'(\tilde{q}_{ij})/c'(\tilde{q}_{ij})$. Hence, $1 + n(d) \geq \frac{R_1(d)}{R_2(d)}$ is equivalent to $u'(\bar{q}_i^z)/c'(\bar{q}_i^z) \geq u'(\tilde{q}_{ij})/c'(\tilde{q}_{ij})$, or $\bar{q}_i^z \leq \tilde{q}_{ij}$. Note that since $\tilde{q}_{ij} < q^* < \tilde{q}_{ji}$, it follows that the optimal level of output cannot be achieved in any other match with real balances alone. Therefore, if $1 + n(d) \geq \frac{R_1(d)}{R_2(d)}$, then assets have liquidity value in ever match under a monetary equilibrium. Since $\lambda = 1$, this guarantees that each buyer matches with a seller, so assets are guaranteed to be spent. Therefore, the asset demand correspondences of an optimist and pessimist coincide in a monetary equilibrium. If instead $1 + n(d) < \frac{R_1(d)}{R_2(d)}$, then an optimist's real balances alone exceed the amount needed to purchase $\tilde{q}_{ij}$. An optimist thus enjoys the return value of her assets in a monetary equilibrium in the case where she matches with a pessimist, as she won't spend any of her assets on production, in favor of spending all of her real balances. On the other hand, a pessimist does not enjoy the return value of her assets since she spends them in the DM in every match.

Figure 4: Asset and Real Money Demand of Optimists vs. Pessimists ($\lambda = 1$)



Therefore, an optimist is willing to tolerate higher asset prices than a pessimist before switching to a wallet filled with only real balances because the asset carries a positive net return value to the optimist and a zero net return value to pessimist. Notice that if $\bar{\psi}_i(d) > \bar{\psi}_j(d)$, then $\bar{\psi}_i(d) < \bar{\psi}_j(d)$. In words, if an optimist is willing to pay a higher price for assets than a pessimist before switching to a portfolio of real balances only, then a pessimist is willing to pay a higher nominal interest rate than an optimist before switching to a portfolio of assets only. If instead $\bar{\psi}_i(d) > \bar{\psi}_j(d)$ and $\bar{\psi}_i(d) > \bar{\psi}_j(d)$, then a pessimist would demand zero assets and real money, and so an equilibrium would not exist. If $\bar{\psi}_i(d) > \bar{\psi}_j(d)$ and $\bar{\psi}_i(d) = \bar{\psi}_j(d)$, then the latter implies $\bar{\psi}_i(d) = \bar{\psi}_j(d)$, which is a contradiction. Intuitively, optimists value assets more for returns, whereas pessimists value them more for liquidity. At asset prices sufficiently high, real money becomes a more attractive source of liquidity, so pessimists will abandon assets at a lower price than optimists. At sufficiently high nominal interest rates, optimists are willing to abandon real balances in favor of assets sooner than pessimists, as the competing media of exchange holds a net return value and a liquidity value to the optimist, whereas it only holds a liquidity value to the pessimist.

5.5 Asset Price Volatility and Monetary Policy

Asset price volatility is an important indicator of market risk for investors in asset markets. In a world of sufficiently risk averse investors, a higher level of volatility may be associated with a decrease in market participation. However, is an increase in volatility associated with a decrease in social welfare? If this relationship holds, then can a monetary authority improve social welfare by employing a policy that reduces asset price volatility? My model shows the Friedman rule maximizes social welfare, so unless a zero nominal interest rate is also associated with the lowest level of asset price volatility, then the answer to the previous question is "no." In this section, I seek to answer this question.

I first consider a measure of asset price volatility. The variance of asset prices is defined as

$$Var(\psi) = \sum_{d \in S} \gamma(d)[\psi(d) - \bar{\psi}]^2 \tag{74}$$

where

$$\bar{\psi} = \sum_{d \in S} \gamma(d)\psi(d) \tag{75}$$

The variance of asset prices is clearly minimized when $\psi(d) = \psi(d')$ for every $d, d' \in S$. The next proposition states a necessary condition to guarantee that the Friedman rule is not only welfare-maximizing, but also variance-minimizing.

Proposition 12. *If $d_i^e = d_j^e$ for every $d_i, d_j \in S$, then there exists a steady state equilibrium under the Friedman rule in which $Var(\psi) = 0$.*

Proposition 12 implies that if the conditional expectations of future dividends are not equal for every state, then asset price volatility under the Friedman rule cannot be zero. This severely limits the possibility of the Friedman rule coinciding with a monetary policy aimed at minimizing asset price volatility. To formalize the forthcoming argument, a monetary authority concerned with minimizing asset price volatility solves the following problem:

$$\min_{Z}\{Var(\psi)\} \tag{76}$$

A monetary authority concerned with maximizing social welfare solves the following problem:

$$\max_{Z}\left\{\sum_{i\in\Omega}\sum_{j\in\Omega}\sigma_i\sigma_j S_{ij}(z_i(d), a_i(d), d)\right\} \tag{77}$$

for each $d \in S$. Note that the solution to (77) is to choose Z large enough to induce $n(d) = 0$ for every $d \in S$, so (77) is equivalent to maximizing social welfare in aggregate across states. Proposition (12) states that the solutions to (76) and (77) coincide if the conditional expectations of dividends in every state coincide. If not, then the solution to (76) need not coincide with (77). Therefore, a welfare-maximizing monetary policy is not necessarily the same as a volatility-minimizing monetary policy. According to my model, a policymaker should not be concerned with asset price volatility if maximizing social welfare is the objective of monetary policy. An explanation for this result is that the beliefs of traders in my model only depend on the first moment of asset prices, not the second moment. If instead beliefs depended on higher moments, then it may be possible for (76) and (77) to be equivalent problems without the assumption in Proposition 12.

6 Concluding Remarks

Differences of opinion regarding an asset's future value matter when it comes to asset prices, liquidity, demand composition, and monetary policy. An opinion-sensitive asset's price reflects not only a subjective fundamental value, but also a speculative liquidity premium for each trader who demands a positive quantity of the asset. An optimist has a higher subjective fundamental value than a pessimist, whereas a pessimist has a higher speculative liquidity premium for the asset. This asset pricing formulation yields an intuitive description of how increased pessimism, such as during the financial crisis of 2007-2008, can lead to the evaporation of liquidity and hence, lower asset prices. When agents become more pessimistic regarding an asset's future value, they are willing to accept less of the asset as a method of payment (or collateral). In fact, not only does an asset's speculative liquidity premium shrink, but its subjective return value decreases as well, since returns are a function of asset prices in all states of the world, which are also sensitive to opinions. My model describes a channel for which increased pessimism can lead to liquidity abatement and decreased asset prices, which are hallmarks of many financial market recessions. Since speculation is prominent in financial markets more so than a majority of other markets, my framework could shed some light on the findings by Jordà, Schularick, and Taylor (2013) that financial market recessions are more severe than typical recessions.

A central bank who is concerned with maximizing social welfare need not concern themselves with asset price volatility, as explained in my model. A lowering of the nominal interest rate is welfare-improving, which may or may not decrease asset price volatility. Bernanke and Gertler (2000) recommend a monetary policy of inflation targeting, which is essentially built into my model in a steady state. Inflation targeting eliminates speculation regarding the cost of holding money. If the value of money in my model were subject to speculation, then this would add an additional speculative component to the liquidity premium of assets, since assets and money are competing media of exchange. Since the belief structure in this model only takes first moments into consideration, a natural next step would be to allow subjective expectations of future asset prices to depend on second moments. In such an environment, there may be more of a link between welfare-maximizing monetary policy and asset price volatility. However, one should not consider such an extension if the impact of second moments on price forecasts is statistically insignificant, as such a consideration would not reflect an empirically-relevant phenomenon.

7 Appendix

7.1 Proof of Proposition 1

Proof. Consider problem (3). The first-order conditions yield

$$\psi(d) \geq \beta[d^e + \psi_i'(d)], \text{ "=" if } a_i > 0 \tag{78}$$

Since $\psi_{o(d)}'(d) \geq \psi_i'(d)$ for every $i \neq o(d)$, it follows by (78) that $a_i = 0$ for every $i \neq o(d)$ and $a_{o(d)} \geq 0$. Since the supply of the asset is strictly positive, it must be that $a_{o(d)}(d) > 0$ in equilibrium, so (5) holds. This immediately implies (4). □

7.2 Proof of Proposition 2

Proof. Consider (11) subject to (12) and (10). The first-order conditions are

$$u'(q_{ij}) - \lambda_{ij} c'(q_{ij}) \leq 0, \text{ "=" if } q_{ij} > 0 \tag{79}$$

$$-\beta R_i(d) + \lambda_{ij} \beta R_j(d) - \gamma_{ij} \leq 0, \text{ "=" if } \alpha_{ij} > 0 \tag{80}$$

$$\lambda_{ij}\big[-c(q_{ij}) + \beta R_j(d)\alpha_{ij}\big] = 0 \tag{81}$$

$$\gamma_{ij}[a_i - \alpha_{ij}] = 0 \tag{82}$$

$$\lambda_{ij} \geq 0, \gamma_{ij} \geq 0 \tag{83}$$

where the participation constraint is binding, since the objective function is decreasing in α_{ij}. Since $u(q)/c(q) \to +\infty$ as $q \to 0$, it follows that (79) holds with equality. There are three cases to consider.

Case 1: $\alpha_{ij} = 0$

By (82) and (83), it follows that $\gamma_{ij} = 0$. Since the participation constraint is binding, it follows that $c(q_{ij}) = 0$, which is impossible since $c(0) = 0$ and so (79) does not hold.

Case 2: $\alpha_{ij} \in (0, a_i)$

By (82), it follows that $\gamma_{ij} = 0$. Also, (80) holds with equality. Combining (79) and (80) yields

$$\frac{u'(q_{ij})}{c'(q_{ij})} = \frac{R_i(d)}{R_j(d)} \tag{84}$$

The optimal level of q_{ij} is given by (84). This coincides with $\tilde{q}_{ij}$ from Proposition 1. The optimal level of α_{ij} coincides with $\tilde{\alpha}_{ij}$.

Case 3: $\alpha_{ij} = a_i$

It follows that (80) is binding. Given that $\gamma_{ij} \geq 0$, combining (79) and (80) yields

$$\frac{u'(q_{ij})}{c'(q_{ij})} \geq \frac{R_i(d)}{R_j(d)} \tag{85}$$

The solution for q_{ij} in this case is $\bar{q}_{ij} \leq \tilde{q}_{ij}$ by (85).

Combining these cases yields solutions (13) and (14). □

7.3 Proof of Proposition 3

Proof. The first-order conditions to (18) are represented by (21) and (22). Since $u'(q)/c'(q)$ approaches infinity as q approaches zero, it follows that $a_i > 0$. Hence, (21) holds with equality. There are three cases to consider for prices. First, if $\psi(d) > \beta R_i(d)$, then (21) becomes (24) and so a_i is unique since $u'(q)/c'(q)$ is monotonic. If instead $\psi(d) = \beta R_i(d)$, then (18) is strictly increasing in a_i on the interval $[0, \bar{a}_i]$, and remains constant for $a_i \in [\bar{a}_i, \infty)$. Hence, the solution set is $a_i \geq \bar{a}_i$. Finally, at $\psi(d) = \tilde{\psi}^{ij}(d)$ for some $j \in \Omega$, (18) is not differentiable. However, by definition, this is the price that gives $a_i = \tilde{\alpha}_{ij}$. □

7.4 Proof of Proposition 4

Proof. (24) is precisely a rewriting of (21) with equality, which holds by the previous proof. It must now be shown that $\psi(d) > \beta R_{o(d)}(d)$ in a unique steady state equilibrium. Suppose instead that $\psi(d) \leq \beta R_{o(d)}(d)$. Then, an $o(d)$-buyer's asset demand would be unbounded if this held with inequality. Similarly if this holds with equality, then by Proposition 3, an $o(d)$-buyer's asset demand is not unique. Hence, it must be that $\psi(d) > \beta R_{o(d)}(d)$. Since $R_{o(d)}(d) \geq R_i(d)$ for every $i \neq o(d)$, then $\psi(d) > \beta R_i(d)$ for every $i \neq o(d)$, and so a_i is unique for every $i \neq o(d)$. Therefore, this steady state is unique for each $d \in S$. □

7.5 Proof of Proposition 5

Proof. A steady state equilibrium, uniqueness aside, requires that $\psi(d) \geq \beta R_{o(d)}(d)$. Since $R_{o(d)}(d) \geq R_i(d)$ for every $i \neq o(d)$, it follows that $\psi(d) > \beta R_i(d)$ for every $i \neq o(d)$. Therefore, $a_i < \max_{j\in\Omega}\{\tilde{\alpha}_{ij}\}$. Hence, $S_{ij}(a_i(d), d) < S_{ij}(\tilde{\alpha}_{ij}, d)$ for $i \neq o(d)$ and some $j \in \Omega$. Thus, $S(d) < \tilde{S}(d)$. □

7.6 Proof of Proposition 6

Proof. There are two regions to consider: (i) $\psi(d) \leq \max\{\tilde{\psi}^{i,1}(d), \tilde{\psi}^{i,2}(d)\}$ and (ii) $\psi(d) > \max\{\tilde{\psi}^{i,1}(d), \tilde{\psi}^{i,2}(d)\}$. WLOG, assume $i = 1$ and consider $\max\{\tilde{\psi}^{i,1}(d), \tilde{\psi}^{i,2}(d)\} = \tilde{\psi}^{i,1}(d)$ where $\tilde{\psi}^{i,1}(d) \neq \tilde{\psi}^{i,2}(d)$. This implies $\tilde{\alpha}_{11} < \tilde{\alpha}_{12}$. Also, since $j = 2$, it follows that $\tilde{\alpha}_{11} < \tilde{\alpha}_{21}$, $\tilde{\alpha}_{12} < \tilde{\alpha}_{22}$, and $\tilde{\alpha}_{21} < \tilde{\alpha}_{22}$.

(i) $\psi(d) \leq \tilde{\psi}^{1,1}(d)$

In this region, the asset price equation involving the optimist's beliefs is

$$\psi(d) = \beta[1-\lambda\sigma_2]R_1(d) + \lambda\beta\sigma_2\frac{u'(\bar{q}_{12})}{c'(\bar{q}_{12})}R_2(d) \tag{86}$$

For the pessimist, there are two possibilities for the asset price equation involving their beliefs:

$$\psi(d) = \beta[1-\lambda\sigma_2]R_2(d) + \lambda\beta\sigma_2\frac{u'(\bar{q}_{22})}{c'(\bar{q}_{22})}R_2(d) \tag{87}$$

or

$$\psi(d) = \beta[1-\lambda]R_2(d) + \lambda\beta\left[\sigma_1\frac{u'(\bar{q}_{21})}{c'(\bar{q}_{21})}R_1(d) + \sigma_2\frac{u'(\bar{q}_{22})}{c'(\bar{q}_{22})}R_2(d)\right] \tag{88}$$

First, suppose (87) is the correct asset price equation involving the pessimist's beliefs. Subtracting (87) from (86) yields

$$[1-\lambda\sigma_2][R_1(d)-R_2(d)] + \lambda\sigma_2\left[\frac{u'(\bar{q}_{12})}{c'(\bar{q}_{12})} - \frac{u'(\bar{q}_{22})}{c'(\bar{q}_{22})}\right]R_2(d) = 0 \tag{89}$$

Since $R_1(d) > R_2(d)$, the second term on the right-hand-side of (89) must be negative. Since $u'(q)/c'(q)$ is a decreasing function, it follows that $a_1(d) > a_2(d)$.

Next, suppose instead (88) is the correct asset price equation involving the pessimist's beliefs. Subtracting (88) from (86) yields

$$\frac{R_1(d)-R_2(d)}{\lambda} = \left[\sigma_1\frac{u'(\bar{q}_{21})}{c'(\bar{q}_{21})} + \sigma_2\right]R_1(d) - \left[1+\sigma_2\left[\frac{u'(\bar{q}_{12})}{c'(\bar{q}_{12})} - \frac{u'(\bar{q}_{22})}{c'(\bar{q}_{22})}\right]\right]R_2(d) \tag{90}$$

Since $\lambda \in (0,1]$, it follows that

$$R_1(d)-R_2(d) \leq \left[\sigma_1\frac{u'(\bar{q}_{21})}{c'(\bar{q}_{21})} + \sigma_2\right]R_1(d) - \left[1+\sigma_2\left[\frac{u'(\bar{q}_{12})}{c'(\bar{q}_{12})} - \frac{u'(\bar{q}_{22})}{c'(\bar{q}_{22})}\right]\right]R_2(d) \tag{91}$$

Suppose $a_1(d) \leq a_2(d)$, then the right-hand side of (91) is less than or equal to

$$\left[\sigma_1\frac{u'(\bar{q}_{11})}{c'(\bar{q}_{11})} + \sigma_2\right]R_1(d) - R_2(d)$$

Since $\bar{q}_{11} < q^*$, this is strictly less than $R_1(d) - R_2(d)$, a contradiction. Therefore, it must be that $a_1(d) > a_2(d)$.

(ii) $\psi(d) > \tilde{\psi}^{1,1}(d)$

In this region, the asset price equation involving the optimist's beliefs is

$$\psi(d) = \beta[1-\lambda]R_1(d) + \lambda\beta\left[\sigma_1\frac{u'(\bar{q}_{11})}{c'(\bar{q}_{11})}R_1(d) + \sigma_2\frac{u'(\bar{q}_{12})}{c'(\bar{q}_{12})}R_2(d)\right] \tag{92}$$

Additionally, the asset price satisfies (88) regarding the pessimist's beliefs. Subtracting (88) from (92) yields

$$[1-\lambda][R_1(d)-R_2(d)] + \lambda\sigma_1\left[\frac{u'(\bar{q}_{11})}{c'(\bar{q}_{11})} - \frac{u'(\bar{q}_{21})}{c'(\bar{q}_{21})}\right]R_1(d) + \lambda\sigma_2\left[\frac{u'(\bar{q}_{12})}{c'(\bar{q}_{12})} - \frac{u'(\bar{q}_{22})}{c'(\bar{q}_{22})}\right]R_2(d) = 0 \tag{93}$$

If $\lambda \in (0,1)$, then the first term on the right-hand side of (93) is strictly positive. The second and third terms of the right-hand side of (91) have the same sign. Therefore, it must be that $a_1(d) > a_2(d)$. If $\lambda = 1$, then the first term on the right-hand side of (93) is zero and so it must be that $a_1(d) = a_2(d)$.

Note that if instead $\tilde{\psi}^{i,1}(d) = \tilde{\psi}^{i,2}(d)$, then the non-vertical portion of an optimist's asset demand correspondence coincides entirely with prices $\psi(d) > \tilde{\psi}^{1,1}(d)$ and so the analysis follows from that region. □

7.7 Proof of Proposition 7

Proof. Consider (34) subject to (35) and (33). Since the objective function is decreasing in both μ_{ij} and α_{ij}, it follows that (35) holds with equality. The first-order conditions are

$$u'(q_{ij}) - \lambda_{ij} c'(q_{ij}) \leq 0, \text{ "=" if } q_{ij} > 0 \tag{94}$$

$$\beta[\lambda_{ij} - 1] - \eta_{ij} \leq 0, \text{ "=" if } \mu_{ij} > 0 \tag{95}$$

$$\beta[\lambda_{ij} R_j(d) - R_i(d)] - \zeta_{ij} \leq 0, \text{ "=" if } \alpha_{ij} > 0 \tag{96}$$

$$\lambda\big[\beta[\mu_{ij} + R_j(d)\alpha_{ij}] - c(q_{ij})\big] = 0 \tag{97}$$

$$\eta_{ij}[z_i - \mu_{ij}] = 0, \zeta_{ij}[a_i - \alpha_{ij}] = 0 \tag{98}$$

$$\lambda_{ij} \geq 0, \eta_{ij} \geq 0, \zeta_{ij} \geq 0 \tag{99}$$

Since $u'(0) = +\infty$, it follows immediately that $q_{ij} > 0$ and so (94) holds with equality. Additionally, it must be that $\mu_{ij} > 0$ or $\alpha_{ij} > 0$ or both. There are three main cases to consider regarding subjective returns: $R_i(d) > R_j(d), R_i(d) < R_j(d)$, and $R_i(d) = R_j(d)$.

Case 1: $R_i(d) > R_j(d)$

Subcase 1.1: $\mu_{ij} = 0$, $\alpha_{ij} > 0$

In this case, $\eta_{ij} = 0$ by (98). Combining (94) and (95) yields

$$\frac{u'(q_{ij})}{c'(q_{ij})} \leq 1 \tag{100}$$

Combining (94) and (96) yields

$$\frac{u'(q_{ij})}{c'(q_{ij})} = \frac{R_i(d) + \zeta_{ij}/\beta}{R_j(d)} \tag{101}$$

Since (100) and (101) contradict each other by the fact that the right-hand side of (101) is greater than 1, this case is not possible.

Subcase 1.2: $\mu_{ij} \in (0, z_i)$, $\alpha_{ij} = 0$

In this case, $\eta_{ij} = \zeta_{ij} = 0$ by (98). Combining (94) and (95) yields

$$\frac{u'(q_{ij})}{c'(q_{ij})} = 1 \tag{102}$$

Combining (94) and (96) yields

$$\frac{u'(q_{ij})}{c'(q_{ij})} \leq \frac{R_i(d)}{R_j(d)} \tag{103}$$

It follows that (102) and (103) do not contradict each other since the right-hand side of (103) is strictly greater than one. Therefore, the solution in this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (q^*, \mu^*, 0)$.

Subcase 1.3: $\mu_{ij} = z_i$, $\alpha_{ij} = 0$

In this case, $\zeta_{ij} = 0$ by (98). Combining (94) and (95) yields

$$\frac{u'(q_{ij})}{c'(q_{ij})} = 1 + \eta_{ij}/\beta \geq 1 \tag{104}$$

Combining (94) and (96) yields (103). The solution in this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (\bar{q}_i^z, z_i, 0)$.

Subcase 1.4: $\mu_{ij} \in (0, z_i)$, $\alpha_{ij} > 0$

In this case, $\eta_{ij} = 0$ by (98). Combining (94) and (95) yields (102), while combining (94) and (96) yields (101). This is a contradiction, so this case is not possible.

Subcase 1.5: $\mu_{ij} = z_i$, $\alpha_{ij} \in (0, a_i)$

In this case, $\zeta_{ij} = 0$ by (98). Combining (94) and (95) yields (104), whereas combining (94) and (96) yields (103) with equality. Therefore, the solution in this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (\tilde{q}_{ij}, \tilde{\mu}_{ij}, \hat{\alpha}_{ij})$.

Subcase 1.6: $\mu_{ij} = z_i$, $\alpha_{ij} = a_i$

Combining (94) and (95) yields (104), whereas combining (94) and (96) yields (101). Therefore, the solution in this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (\bar{q}_{ij}, z_i, a_i)$.

Case 2: $R_i(d) < R_j(d)$

Subcase 2.1: $\mu_{ij} > 0$, $\alpha_{ij} = 0$

In this case, $\zeta_{ij} = 0$ by (98). Combining (94) and (95) yields (104), whereas combining (94) and (96) yields (104). However, since $R_i(d)/R_j(d) < 1$, both (103) and (104) cannot hold simultaneously. Hence, this case is not possible.

Subcase 2.2: $\mu_{ij} = 0$, $\alpha_{ij} \in (0, a_i)$

In this case, $\eta_{ij} = \zeta_{ij} = 0$ by (98). Combining (94) and (95) yields (100), whereas combining (94) and (96) yields (103) with equality. The solution for this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (\tilde{q}_{ij}, 0, \tilde{\alpha}_{ij})$.

Subcase 2.3: $\mu_{ij} = 0$, $\alpha_{ij} = a_i$

In this case, $\eta_{ij} = 0$ by (98). Combining (94) and (95) yields (100), whereas combining (94) and (96) yields (101). Therefore, the solution for this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (\bar{q}_{ij}^a, 0, a_i)$.

Subcase 2.4: $\mu_{ij} > 0$, $\alpha_{ij} \in (0, a_i)$

In this case, $\zeta_{ij} = 0$ by (98). Combining (94) and (95) yields (104), whereas combining (94) and (96) yields (103) with equality. However, since $R_i(d)/R_j(d) < 1$, (103) and (104) cannot hold simultaneously. Hence, this case is impossible.

Subcase 2.5: $\mu_{ij} \in (0, z_i)$, $\alpha_{ij} = a_i$

In this case, $\eta_{ij} = 0$ by (98). Combining (94) and (95) yields (102), whereas combining (94) and (96) yields (101). Since $\zeta_{ij} \geq 0$, then there are no contradictions in this case. Therefore, the solution for this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (q^*, \hat{\mu}_{ij}, a_i)$.

Subcase 2.6: $\mu_{ij} = z_i$, $\alpha_{ij} = a_i$

Combining (94) and (95) yields (104), whereas combining (94) and (96) yields (101). Therefore, the solution in this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (\bar{q}_{ij}, z_i, a_i)$.

Case 3: $R_i(d) = R_j(d)$

For this case, $R_i(d) = R_j(d) \equiv R(d)$.

Subcase 3.1: $\mu_{ij} > 0$, $\alpha_{ij} = 0$

In this case, $\zeta_{ij} = 0$ by (98). Combining (94) and (95) yields (104), whereas combining (94) and (96) yields (100). Hence, it must be that $(u'(q_{ij})/c'(q_{ij}) = 1$. Therefore, the solution in this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (q^*, z_i, 0)$.

Subcase 3.2: $\mu_{ij} = 0$, $\alpha_{ij} > 0$

In this case, $\eta_{ij} = 0$ by (98). Combining (94) and (95) yields (100), whereas combining (94) and (96) yields

$$\frac{u'(q_{ij})}{c'(q_{ij})} = 1 + \zeta_{ij}/\beta R(d) \geq 1 \tag{105}$$

Therefore, it must be that $u'(q_{ij})/c'(q_{ij}) = 1$, so the solution for this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (q^*, 0, a_i)$.

Subcase 3.3: $\mu_{ij} \in (0, z_i)$, $\alpha_{ij} \in (0, a_i)$

In this case, $\eta_{ij} = \zeta_{ij} = 0$ by (98). Therefore, combining (94) with (95) or (96) yields (102). There are infinitely many solutions for this case. Specifically, $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (q^*, \bar{\mu}, \bar{\alpha})$

Subcase 3.4: $\mu_{ij} \in (0, z_i)$, $\alpha_{ij} = a_i$

In this case, $\eta_{ij} = 0$ by (98). It follows that both (102) and (105) hold. Therefore, it must be that $u'(q_{ij})/c'(q_{ij}) = 1$. This solution is a subset of the solution set in Subcase 3.3, namely where $\bar{\alpha} = a_i$.

Subcase 3.5: $\mu_{ij} = z_i$, $\alpha_{ij} \in (0, a_i)$

This subcase is analogous to Subcase 3.4.

Subcase 3.6: $\mu_{ij} = z_i$, $\alpha_{ij} = a_i$

In this case, both (104) and (105) hold. Therefore, the solution in this case is $(q_{ij}, \mu_{ij}, \alpha_{ij}) = (\bar{q}_{ij}, z_i, a_i)$

□

7.8 Proof of Proposition 8

Proof. The first-order necessary and sufficient conditions are (51) and (52). As long as $(n(d), \psi(d)) \notin \Lambda^{i,d}$, necessity holds since the objective function is differentiable. Additionally, each S_ij is strictly quasiconcave, so sufficiency holds. Let $n(d) > 0$ and $\psi(d) > \beta R_i(d)$. Clearly, if $n(d) = 0$, then $S^z_{ij} = 0$ and if $\psi(d) = \beta R_i(d)$, then $S^a_{ij} = 0$. Since $u'(0) = +\infty$, it follows that $z_i = a_i = 0$ is not a solution. There are three cases to consider.

Case 1: $z_i > 0, a_i = 0$

$$S^z_{ij} = \beta \left[\frac{u'(\bar{q}^z_i)}{c'(\bar{q}^z_i)} - 1 \right] \tag{106}$$

$$S^a_{ij} = \beta 1(\bar{q}^z_i < \tilde{q}_{ij}) \left[\frac{u'(\bar{q}^z_i)}{c'(\bar{q}^z_i)} R_j(d) - R_i(d) \right] \tag{107}$$

Case 2: $z_i = 0, a_i > 0$

$$S^z_{ij} = 1(\bar{q}^a_{ij} < q^*) \left[\frac{u'(\bar{q}^a_{ij})}{c'(\bar{q}^a_{ij})} - 1 \right] \tag{108}$$

$$S^a_{ij} = 1(\bar{q}^a_{ij} < \tilde{q}_{ij}) \left[\frac{u'(\bar{q}^a_{ij})}{c'(\bar{q}^a_{ij})} R_j(d) - R_i(d) \right] \tag{109}$$

Case 3: $z_i > 0, a_i > 0$

$$S^z_{ij} = 1(j \in \Omega^{i,d}_1 \cup \Omega^{i,d}_3 \cup \Omega^{i,d}_4) \left[\frac{u'(q_{ij})}{c'(q_{ij})} - 1 \right] + 1(j \in \Omega^{i,d}_2) \left[\frac{R_i(d)}{R_j(d)} - 1 \right] \tag{110}$$

$$S^a_{ij} = 1(j \in \Omega^{i,d}_1 \cup \Omega^{i,d}_4 \cup \Omega^{i,d}_6) \left[\frac{u'(q_{ij})}{c'(q_{ij})} R_j(d) - R_i(d) \right] + 1(j \in \Omega^{i,d}_5) \left[\psi'_j(d) - \psi'_i(d) \right] \tag{111}$$

where the sets $\{\Omega^{i,d}_k\}_{k=1,2,3,4,5,6}$ are defined in Proposition 9. A solution under Case 1 or Case 2 is unique, whereas a solution under Case 3 is part of a continuum of solutions. This holds because (51) and (52) both hold with equality and can be combined into one equation. This leads to one equation in two unknowns. Cases 1 and 2 have unique solutions, as they each involve one equation in one unknown. □

7.9 Proof of Proposition 9

Proof. Using the explicit partial derivatives of S_{ij} from the proof of Proposition 8, (51) and (52) can be rewritten as follows:

Case 1: $z_i > 0, a_i = 0$

$$n(d) = \lambda \left[\frac{u'(\bar{q}^z_i)}{c'(\bar{q}^z_i)} - 1 \right] \tag{112}$$

$$\psi(d) \geq \beta R_i(d) + \beta\lambda \sum_{j \in \Omega} \sigma_j 1(\bar{q}^z_i < \tilde{q}_{ij}) \left[\frac{u'(\bar{q}^z_i)}{c'(\bar{q}^z_i)} R_j(d) - R_i(d) \right] \tag{113}$$

Case 2: $z_i = 0, a_i > 0$

$$n(d) \geq \lambda \sum_{j \in \Omega} \sigma_j 1\big(\bar{q}^a_{ij} < \min\{q^*, \tilde{q}_{ij}\}\big) \left[\frac{u'(\bar{q}^a_{ij})}{c'(\bar{q}^a_{ij})} - 1 \right] \tag{114}$$

$$\psi(d) = \beta R_i(d) + \beta\lambda \sum_{j \in \Omega} \sigma_j 1(\bar{q}^a_{ij} < \tilde{q}_{ij}) \left[\frac{u'(\bar{q}^a_{ij})}{c'(\bar{q}^a_{ij})} R_j(d) - R_i(d) \right] \tag{115}$$

Case 3: $z_i > 0, a_i > 0$

$$n(d) = \lambda \sum_{j \in \Omega} \sigma_j \left\{ 1(j \in \Omega_1^{i,d} \cup \Omega_3^{i,d} \cup \Omega_4^{i,d}) \left[\frac{u'(q_{ij})}{c'(q_{ij})} - 1 \right] + 1(j \in \Omega_2^{i,d}) \left[\frac{R_i(d)}{R_j(d)} - 1 \right] \right\} \tag{116}$$

$$\psi(d) = \beta R_i(d) + \beta\lambda \sum_{j \in \Omega} \sigma_j \left\{ 1(j \in \Omega_1^{i,d} \cup \Omega_4^{i,d} \cup \Omega_6^{i,d}) \left[\frac{u'(q_{ij})}{c'(q_{ij})} R_j(d) - R_i(d) \right] + 1(j \in \Omega_5^{i,d}) \left[\psi_j'(d) - \psi_i'(d) \right] \right\} \tag{117}$$

When $a_i = 0$ in Case 3, (116) and (112) are equivalent, while (117) is equivalent to (113) with equality. Similarly, when $z_i = 0$ in Case 3, (117) and (115) are equivalent, while (116) is equivalent to (114) with equality. Hence, a solution in Case 1 or Case 2 is also a solution in Case 3 when the inequality first-order conditions hold with equality. Therefore, the asset price in Case 3 coincides with the asset price in (115) with equality and (115). Each case can be divided by analyzing the relationship between the nominal interest rate and this asset price. Combining (112) and (113) with equality yields (57), which implies Case 3 solutions. It follows immediately that (53) implies Case 1 unique solution and (55) implies a Case 2 unique solution. □

7.10 Proof of Proposition 10

Proof. First, consider the class of monetary equilibria in which only money is demanded. If $n(d) > 0$, then every buyer is able to afford some $q < q^*$ in every match. Since ever buyer is able to afford q^* when $n(d) = 0$, it follows that surplus is strictly greater under the Friedman rule than in any other monetary equilibrium in which only money is demanded.

Now, consider a non-monetary equilibrium in which A is sufficiently large such that the asset price is as low as possible, i.e., $\psi(d) = \beta R_{o(d)}(d)$ for some $d \in S$. In this case, expected aggregate DM surplus is

$$\begin{aligned} S(n(d), \psi_{o(d)}(d)) =& \sigma_1^2[u(q^*) - c(q^*)] + \sigma_2^2[u(\bar{q}_{22}) - c(\bar{q}_{22})] + \sigma_1\sigma_2\left[u(\tilde{q}_{12}) - c(\tilde{q}_{12}) + u(\bar{q}_{21}) - c(\tilde{q}_{21}) + \beta[\psi_1'(d) - \psi_2'(d)][a_2 - \tilde{\alpha}_{12}]\right] \\ &< u(q^*) - c(q^*) + \beta[\psi_1'(d) - \psi_2'(d)][a_2 - \tilde{\alpha}_{12}] \\ &< u(q^*) - c(q^*) \\ &= S(0, \psi_{o(d)}(d)) \end{aligned} \tag{118}$$

where the last inequality follows from the fact that $a_2 < \tilde{\alpha}_{12}$. (118) states that surplus under the Friedman rule is strictly greater than surplus under a monetary equilibrium by which the price of the asset as at a minimum. Since this asset price is associated with the highest surplus in a non-monetary equilibrium, it follows that the Friedman rule yields a higher surplus than any non-monetary equilibrium.

Finally, consider a monetary equilibrium in which asset and money demand is non-zero in aggregate. The Friedman rule yields higher surplus for all buyers who either demand only money or only assets, as implied by the previous two paragraphs. Now, consider traders who are indifferent between assets and money. Suppose an i-buyer chooses a portfolio from the set

$[0, \hat{z}_i] \times [0, \hat{a}_i]$. Since every portfolio from this set yields the same level of surplus, it suffices to consider the boundary points $(\hat{z}_i, 0)$ and $(0, \hat{a}_i)$. The point $(\hat{z}_i, 0)$ coincides with an i-buyer's monetary equilibrium in which she only chooses money, when corresponding prices are $(\bar{n}_i(d), \bar{\psi}_i(d))$. As already explained, the surplus under the Friedman rule for any match dominates any other monetary equilibrium in which only real balances are demanded. Similarly, since $(0, \hat{a}_i)$ coincides with an i-buyer in a non-monetary equilibrium under prices $(\bar{n}_i(d), \bar{\psi}_i(d))$ and since the Friedman rule dominates all non-monetary equilibria in terms of surplus for any match, it follows that the Friedman rule dominates this case in terms of surplus as well.

Since all possible cases have been exhausted, it follows that the Friedman rule yields the maximum expected aggregate DM surplus among the set of steady state equilibria.

□

7.11 Proof of Proposition 11

Proof. Without loss of generality, let $i = 1$ and $j = 2$. The relationship between asset demand in a non-monetary equilibrium is described by Proposition 6. Therefore, it suffices to compare a monetary equilibrium among a 1-buyer and a 2-buyer. Let $n(d) > 0$. Note that with $n(d) > 0$, it follows that $\bar{q}_1^z < \tilde{q}_{11}$ and $\bar{q}_2^z < \tilde{q}_{22} < \tilde{q}_{21}$ because $\tilde{q}_{11} = \tilde{q}_{22} = q^*$. There are two cases to consider.

Case 1: $\bar{q}_1^z < \tilde{q}_{12}$

The asset price in a non-monetary equilibrium must satisfy

$$\psi(d) \geq \beta[1-\lambda]R_1(d) + \beta\lambda\left[\sigma_1 \frac{u'(\bar{q}_1^z)}{c'(\bar{q}_1^z)}R_1(d) + \sigma_2 \frac{u'(\bar{q}_1^z)}{c'(\bar{q}_1^z)}R_2(d)\right] \tag{119}$$

$$\psi(d) \geq \beta[1-\lambda]R_2(d) + \beta\lambda\left[\sigma_1 \frac{u'(\bar{q}_2^z)}{c'(\bar{q}_2^z)}R_1(d) + \sigma_2 \frac{u'(\bar{q}_2^z)}{c'(\bar{q}_2^z)}R_2(d)\right] \tag{120}$$

The vertical portion of the asset demand correspondence is associated with an asset price that satisfies (119) or (120). The asset prices that satisfy (119) and (120) with equality are $\bar{\psi}_1(d)$ and $\bar{\psi}_2(d)$, respectively. It thus suffices to compare these asset prices in order to compare asset demand in a monetary equilibrium. Subtracting $\bar{\psi}_2(d)$ from $\bar{\psi}_1(d)$ yields

$$\bar{\psi}_1(d) - \bar{\psi}_2(d) = \beta[1-\lambda][R_1(d) - R_2(d)] + \beta\lambda\left\{\sigma_1\left[\frac{u'(\bar{q}_1^z)}{c'(\bar{q}_1^z)} - \frac{u'(\bar{q}_2^z)}{c'(\bar{q}_2^z)}\right]R_1(d) + \sigma_2\left[\frac{u'(\bar{q}_1^z)}{c'(\bar{q}_1^z)} - \frac{u'(\bar{q}_2^z)}{c'(\bar{q}_2^z)}\right]R_2(d)\right\} \tag{121}$$

Suppose first that $\bar{\psi}_1(d) - \bar{\psi}_2(d) = 0$. If $\lambda = 1$, then it must be that $u'(\bar{q}_1^z)/c'(\bar{q}_1^z) = u'(\bar{q}_2^z)/c'(\bar{q}_2^z)$, so $\hat{z}_1(d) = \hat{z}_2(d)$ and $\bar{n}_1(d) = \bar{n}_2(d)$. If instead $\lambda < 1$, then it must be that $u'(\bar{q}_1^z)/c'(\bar{q}_1^z) < u'(\bar{q}_2^z)/c'(\bar{q}_2^z)$, so $\hat{z}_1(d) > \hat{z}_2(d)$. However, this implies $\bar{n}_1(d) < \bar{n}_2(d)$, which is impossible since $n(d)$ cannot take on two values. To see this explicitly, note that if $\psi(d) = \bar{\psi}_1(d) = \bar{\psi}_2(d) = 0$, then (57) must hold for both the 1-buyer and the 2-buyer:

$$\psi(d) = \beta[1-\lambda]R_1(d) + \beta[n(d) + \lambda][\sigma_1 R_1(d) + \sigma_2 R_2(d)] \tag{122}$$

$$\psi(d) = \beta[1-\lambda]R_2(d) + \beta[n(d)+\lambda][\sigma_1 R_1(d) + \sigma_2 R_2(d)] \tag{123}$$

With $\lambda < 1$, then (122) and (123) cannot hold simultaneously.

Now, suppose that $\bar{\psi}_1(d) - \bar{\psi}_2(d) > 0$. If $\lambda = 1$, then $u'(\bar{q}_1^z)/c'(\bar{q}_1^z) > u'(\bar{q}_2^z)/c'(\bar{q}_2^z)$, so $\hat{z}_1(d) < \hat{z}_2(d)$ and $\bar{n}_1(d) > \bar{n}_2(d)$. However, note that if the equilibrium asset price is $\bar{\psi}_1(d)$, then the equilibrium nominal interest rate is $\bar{n}_1(d)$. At these prices, the 2-buyer's demand for assets and real balances is zero. This is can never be a solution to (47), as $u'(q)/c'(q)$ approaches infinity as q approaches zero. Therefore, this case is impossible. It must be that $\bar{n}_1(d) < \bar{n}_2(d)$. If $\lambda < 1$, then this is possible as long as $\hat{z}_1(d) > \hat{z}_2(d)$.

Finally, suppose $\bar{\psi}_1(d) - \bar{\psi}_2(d) < 0$. For any $\lambda \in (0, 1]$, it follows that $u'(\bar{q}_1^z)/c'(\bar{q}_1^z) < u'(\bar{q}_2^z)/c'(\bar{q}_2^z)$, so $\hat{z}_1(d) > \hat{z}_2(d)$ and $\bar{n}_1(d) < \bar{n}_2(d)$. However, this is impossible by an analogous argument to the previous paragraph.

The condition $\bar{q}_1^z < \tilde{q}_{12}$ implies $u'(\bar{q}_1^z)/c'(\bar{q}_1^z) > u'(\tilde{q}_{12})/c'(\tilde{q}_{12})$, which implies $1 + n(d)/\lambda > R_1(d)/R_2(d)$. To summarize, if $1 + n(d)/\lambda > R_1(d)/R_2(d)$, then

(i) if $\lambda = 1$, then $\bar{\psi}_1(d) = \bar{\psi}_2(d)$ and $\bar{n}_1(d) = \bar{n}_2(d)$

(ii) if $\lambda \in (0, 1)$, then $\bar{\psi}_1(d) > \bar{\psi}_2(d)$ and $\bar{n}_1(d) < \bar{n}_2(d)$

Case 2: $\bar{q}_1^z \geq \tilde{q}_{12}$

This case is equivalent to $1 + n(d)/\lambda \leq R_1(d)/R_2(d)$ by the same argument as the previous case. For a 1-buyer, prices associated with a non-monetary equilibrium must satisfy

$$\psi(d) \geq \beta[1-\lambda\sigma_1]R_1(d) + \beta\lambda\sigma_1 \frac{u'(\bar{q}_1^z)}{c'(\bar{q}_1^z)} R_1(d) \tag{124}$$

whereas (123) still holds for a 2-buyer. Subtracting (123) at equality from (124) at equality yields

$$\bar{\psi}_1(d) - \bar{\psi}_2(d) = \beta\big[[1-\sigma_1\lambda]R_1(d) - [1-\lambda]R_2(d)\big] + \beta\lambda\sigma_1 \left[\frac{u'(\bar{q}_1^z)}{c'(\bar{q}_1^z)} - \frac{u'(\bar{q}_2^z)}{c'(\bar{q}_2^z)}\right] R_1(d) - \beta\lambda\sigma_2 \frac{u'(\bar{q}_2^z)}{c'(\bar{q}_2^z)} R_2(d) \tag{125}$$

Note that $[1-\sigma_1\lambda]R_1(d) - [1-\lambda]R_2(d) > 0$ for every $\lambda \in (0, 1]$ since $[1-\sigma_1\lambda]R_1(d) - [1-\lambda]R_2(d) > [1-\lambda][R_1(d) - R_2(d)]$ for every $\lambda \in (0, 1]$.

First, suppose $\hat{z}_1(d) = \hat{z}_2(d)$, so $\bar{n}_1(d) = \bar{n}_2(d)$. Then, (125) becomes

$$\bar{\psi}_1(d) - \bar{\psi}_2(d) = \beta[1-\sigma_1\lambda]R_1(d) - \beta\left[1 - \lambda + \lambda\sigma_2 \frac{u'(\bar{q}_1^z)}{c'(\bar{q}_1^z)}\right] R_2(d) \tag{126}$$

If $\lambda = 1$, then this becomes

$$\bar{\psi}_1(d) - \bar{\psi}_2(d) = \beta\sigma_2 \left[R_1(d) - \frac{u'(\bar{q}_1^z)}{c'(\bar{q}_1^z)} R_2(d)\right] \geq \beta\sigma_2 \left[R_1(d) - \frac{u'(\tilde{q}_{12})}{c'(\tilde{q}_{12})} R_2(d)\right] \tag{127}$$

It follows that $\bar{\psi}_1(d) - \bar{\psi}_2(d) > 0$ if $1 + n(d) < R_1(d)/R_2(d)$, and $\bar{\psi}_1(d) - \bar{\psi}_2(d) = 0$ if $1 + n(d) = R_1(d)/R_2(d)$. Only the latter is possible since both buyers must be indifferent between assets and real balances.

Now, suppose $\hat{z}_1(d) > \hat{z}_2(d)$, so $\bar{n}_1(d) < \bar{n}_2(d)$. For $\lambda = 1$, (125) becomes

$$\bar{\psi}_1(d) - \bar{\psi}_2(d) = \beta\sigma_2\left[R_1(d) - \frac{u'(\bar{q}_1^z)}{c'(\bar{q}_1^z)}R_2(d)\right] + \beta\sigma_1\left[\frac{u'(\bar{q}_1^z)}{c'(\bar{q}_1^z)} - \frac{u'(\bar{q}_2^z)}{c'(\bar{q}_2^z)}\right]R_1(d) \tag{128}$$

Since $\bar{n}_1(d) < \bar{n}_2(d)$, it must be that $\bar{\psi}_1(d) > \bar{\psi}_2(d)$. This is only possible if $1 + n(d) < R_1(d)/R_2(d)$ since the second term on the right-hand side of (128) is strictly negative. A similar argument shows that this is only possible if $1+n(d)/\lambda \leq R_1(d)/R_2(d)$ for $\lambda < 1$.

Finally, suppose $\hat{z}_1(d) < \hat{z}_2(d)$, so $\bar{n}_1(d) > \bar{n}_2(d)$. Let $\lambda = 1$. This implies that the right-hand side (128) must be negative. However, this is impossible since the first term is non-negative and the second term is strictly positive. A similar argument shows that this is also impossible for $\lambda < 1$.

Therefore, this case can be summarized as follows: If $1 + n(d)/\lambda = R_1(d)/R_2(d)$, then

(i) if $\lambda = 1$, then $\bar{\psi}_1(d) = \bar{\psi}_2(d)$ and $\bar{n}_1(d) = \bar{n}_2(d)$

(ii) if $\lambda \in (0, 1)$, then $\bar{\psi}_1(d) > \bar{\psi}_2(d)$ and $\bar{n}_1(d) < \bar{n}_2(d)$

Furthermore, if $1 + n(d)/\lambda < R_1(d)/R_2(d)$, then $\bar{\psi}_1(d) > \bar{\psi}_2(d)$ and $\bar{n}_1(d) < \bar{n}_2(d)$ for any $\lambda \in (0, 1]$. □

7.12 Proof of Proposition 12

Proof. Suppose $d_i^e = d_j^e = \bar{d}$ for every $d_i, d_j \in S$. Then, asset prices under the Friedman rule satisfy

$$\psi(d) = \beta[\bar{d} + \psi_{o(d)}(d)] \tag{129}$$

for every $d \in S$. Note that $Var(\psi) = 0$ if and only if $\psi(d) = \bar{\psi}$ for every $d \in S$ and some $\bar{\psi}$. In this case, it follows that $\psi_{o(d)}(d) = \bar{\psi}$ for every $d \in S$. Hence, (129) is satisfied for every $d \in S$. It also follows that $\bar{\psi} = \bar{d}/(1 - \beta)$. □

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