Moral Hazard, Bertrand Competition, and Natural Monopoly

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Abstract

In the traditional model of Bertrand price competition among symmetric firms, there is no restriction on the number of firms that are active in equilibrium. A symmetric equilibrium exists with the different firms sharing the market. I show that this does not hold if we preserve the symmetry between firms but introduce moral hazard with a customer-sensitive probability of exposure; competition necessarily results in a natural monopoly with only one active firm. Sequential price announcements and early adoption are some equilibrium selection mechanisms that help to pin down the identity of the natural monopolist. If we modify the standard Bertrand assumptions to introduce decreasing returns to scale, a natural oligopoly will emerge instead of a natural monopoly. The insights of the basic model are robust to many extensions.

Keywords: Bertrand competition, active firms, moral hazard, natural monopoly.

JEL Codes: D82, D43, C73.

1. Introduction

The traditional Bertrand model of price competition between identical firms producing a homogenous product yields a straightforward symmetric equilibrium in which consumers divide their demand among these firms, which price at cost. The model imposes no restrictions on the number of active firms in equilibrium. The present paper shows that if we allow identical Bertrand competitors to experience moral hazard (given an opportunity to cheat their customers) then given a customer-sensitive probability of detection of wrong-doing, there is *only one active firm* in any non-collusive equilibrium. Moreover, this is so despite the fact that all firms have the same costs *and* that no firm is assumed to have an incumbency advantage. The single active firm does not charge the monopoly price, but rather the lowest price compatible with maintaining its credibility; we refer to it as a natural monopolist rather than a standard monopolist.When we

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relax some of our assumptions, for instance, when we allow for imperfect information transmission between customers, we still find that at least some potential entrants remain inactive in equilibrium, obtaining either a natural monopoly or a natural oligopoly with an endogenously determined number of active firms. This is also the case when the traditional assumptions of the Bertrand model are modified by, for instance, allowing for price competition within the framework of increasing costs and not just constant costs. Thus, this paper contributes both to the literature on Bertrand competition and to that on the impact of asymmetric information on market structure. It also touches more tangentially on two other literatures – the network effects and the multimarket contact literature.

While much of the literature on Bertrand competition has concentrated on the Bertrand paradox – the implication that price drops sharply from the monopoly price to the competitive price when the number of firms in the industry increases from one to two, and that the equilibrium price remains insensitive to further increases in the number of firms – this is not the focus of the present paper. This paper is more closely connected to the rather more limited literature that touches on the number of active firms within Bertrand competition. This includes Rasmusenand Janssen (2002), Novshek and Roy Chowdhury (2003), and Ledvina and Sircar (2011). My paper differs from theirs both in terms of its approach and its results.

In Rasmusenand Janssen (2002), each firm may be inactive with some probability; the authors motivate this in terms of firms endogenously choosing whether to incur a fixed cost of activity. In Novshek and Roy Chowdhury (2003), under Bertrand competition with free entry, some firms may choose to set a price that generates no sales, thus remaining inactive. In both of these papers, the inactivity of some firms in equilibrium stems from actions taken by these firms (whether or not to incur a fixed cost of activity, or what price to charge). In my paper, in contrast, all potential entrants take identical actions, but customers choose to patronize only one firm; the others perforce remain inactive.[2] (In the case of imperfect information transmission or diseconomies of scale, they may choose to patronize a determinate number of firms greater than one).In Ledvina and Sircar (2011), asymmetry of costs between firms producing a homogenous good under Bertrand competition is critical for some firms to be inactive; with symmetric costs,

[2]The reason why customers behave this way is endogenous, and the intuition is explained in the following paragraph.

either all firms are active, or all firms are inactive. In my paper, in contrast, all firms have *symmetric* costs, but in the main model only one firm is active. There are no equilibria in which all firms are active or all firms inactive.

I now briefly explain the intuition underlying the customers' behavior in my model. This stems from an interaction between firm-customer moral hazard problems and inter-firm Bertrand competition. Each potential entrant has the opportunity to cheat his customers by promising a high-quality product but supplying a low-quality one. The product's quality is unverifiable on inspection. However, a low-quality product sold to any one customer has a certain probability of failure. The more customers the firm sells its inferior product to, the greater the chance that a piece sold to at least one customer fails. Customers publicize product failures, ensuring that a cheating firm is punished. For the threat of punishment to constitute an effective deterrent, firms who supply high quality must earn a premium over cost (whose existence would be threatened by a punishment), and must charge a minimum "credibility price" in the spirit of Klein and Leffler (1981).[3] Unlike Klein and Leffler, in my model this threshold price is sensitive to the number of customers a firm has; a firm with more customers faces a higher probability of being caught if it cheats, weakening its cheating incentives. Therefore, it can guarantee high quality provision even if it charges a relatively low price; the minimum credibility price is decreasing in the firm's customer base. Now, under Bertrand competition, the credible threat of undercutting by rivals ensures that the price is driven down to this minimum. However, when all firms charge this minimum price, any outcome in which customers go to more than one firm is not credible and hence cannot be sustained as an equilibrium. (Charging a low price already weakens incentives for high quality provision, and a low number of customers per firm would further weaken these incentives).

Papers that deal with the effect of asymmetric information (moral hazard or adverse selection) on market structure include Farrell (1986) on moral hazard as an entry barrier, and Dell'Ariccia et al (1999) on adverse selection in a Bertrand competition model. In Farrell (1986), an incumbent firm is known to its customers as a supplier of high quality; a potential entrant's quality is unknown. The paper derives conditions under which the possibility of moral hazard ensures that the incumbent retains its monopoly. In Dell'Ariccia et al, a pair of incumbent banks

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[3]Further developed by Shapiro (1983).

engage in Bertrand competition, and have an informational advantage over later potential entrants; they have better information about risky borrowers whom they have previously encountered. This advantage then blocks further entry. My paper, which involves moral hazard, differs from these papers in not assuming a pre-existing incumbent. This eliminates the issue of an incumbency advantage in information or reputation. All potential entrants enter the market at the same time in my model; customers, however, will choose to buy from only one in any equilibrium (or from a determinate small number in the extensions).

The discussion so far shows that my paper emphasizes how the addition of moral hazard, with a customer-sensitive probability of detection and exposure, changes the standard Bertrand competition equilibrium, in particular leading to the emergence of a natural monopoly in equilibrium even when all firms have symmetric costs and no firm is an incumbent.A symmetric equilibrium where demand is divided among all the entrants no longer obtains. I now briefly attempt to compare my paper with two other literatures – that on network effects and that on multi-market contact – to which it is more tangentially related.

Farrell and Klemperer (2007) survey the literature on network effects. Most network effects are technological, and arise when the nature of the product ensures that a user's benefit to adopting the product increases when the number of other users rises. For example, this may happen due to compatibility issues. Network effects also occur due to complementarities with other products. Indirect network effects arise when a larger number of buyers also attracts a large number of sellers, so that each individual buyer then gains from being able to interact with more parties on the other side of the market.

While these technological network effects – both direct and indirect – create herding behavior, which also occurs in our model, they are quite different from the forces that we model. In our model, no assumptions are needed on the technical nature of the product. Different potential entrants in our model are in fact all selling exactly the same product, so there is no issue of compatibility. However, it is still optimal for consumers to flock to the same firm, because not doing so generates moral hazard on the part of the firm. Firms cannot counter this increased moral hazard by charging higher prices, due to the presence of Bertrand competition and the risk of being undercut.

Somewhat closer to our model are pecuniary network effects (Liebowitz and Margolis 1994, Farrell and Klemperer 2007). Here, increased adoption by other users benefits all users by lowering the price of the product. However, the underlying micro-foundations of pecuniary network effects are unclear (Farrell and Klemperer 2007). In our model, the minimum price that would render a producer a credible high-quality seller is decreasing in the producer's number of customers; due to the increased probability of getting caught, a seller with many customers will fear to cheat even if the profits he foregoes from exposure are relatively modest. Bertrand competition ensures that the price is driven down to this minimum, while only one firm survives the process, as all the customers in the industry flock to it.The equilibrium price therefore depends negatively on the total number of customers in the *industry.* Thus, our model provides some micro-foundations for pecuniary external economies of scale (we should not call them "network effects" as different sellers sell the same product in our model).

My paper differs from the multi-market contact literature[4] in several important respects. First, while this literature shows how multiplicity (across markets or products) enables a seller to maintain a *high* (monopoly) price, my paper (which deals with multiplicity across clients or transactions rather than markets) shows that a seller with multiple clients can credibly sustain a *low* price, thereby becoming a *natural monopolist* (rather than a standard monopolist) in an environment with price competition and moral hazard.[5] Secondly, these papers, unlike mine, do not deal with Bertrand competition. Third, I do not deal with umbrella branding.

The rest of the paper is organized as follows. Section 2 contains the basic version of the model and the main result. After presenting the main result, we perform several robustness exercises in Section 3, relaxing the assumptions of the basic model. Specifically, we investigate (i) what happens if the rate of information transmission decays among customers, (ii) what happens if making a product failure public knowledge becomes costly, (iii) what happens if there is a small probability that a truly high quality product may fail, as well, (iv) how the results are affected by dropping the traditional Bertrand assumption of constant costs, and allowing for decreasing returns within the price competition framework, and (v) what happens if firms do not have identical costs. Section 4 concludes with a discussion of possible policy implications. The appendices contain further robustness exercises. While the paper concentrates on non-

[4]Including Rasmusen (2012), Andersson (2002), Cabral (2009), and Hakenes and Peitz (2008).
[5]The more clients a seller acquires, the lower the price he charges.

cooperative equilibria, in which firms compete rather than collude, the appendix also contains a brief discussion of the conditions under which an equilibrium with collusive price-fixing among different potential entrants will not be sustainable.[6]

2. The Game : the basic model and main results

2.1 Players and Technology

The players in the game are N firms – all potential entrants into a market – and a total of X customers, where X is larger than N. Firms and customers are infinitely lived and have a common discount factor of δ. In our basic model, all firms have identical costs (we will later briefly consider the effects of introducing asymmetry) and produce the same product. They incur a cost $c_H$in order to produce a single unit of a high-quality product; their corresponding cost for low-quality provision is $c_L<c_H$. Product quality is however not verifiable on inspection.

Customers, who are also all identical, each buy a single unit of the product from a firm of their choice. They value a unit of the high-quality product at $v_H>c_H$, so that high quality provision is efficient. Low quality is valued at $v_L<c_L$, so that no consumer would willingly buy low quality. Therefore, all agreements between firms and customers involve high-quality production.

Suppose however that a firm cheats by supplying a low-quality product instead of a high-quality one. Low quality products probabilistically fail after they have been purchased, revealing their low quality. Let $q(x)$be the probability that a firm with x customers that cheats by supplying low quality instead of high is detected and exposed. It is easy to see that q is an increasing function of x; the probability that the product sold to at least one customer fails, revealing its low quality, is higher when it has been sold to a relatively large number of customers.[7] [As an example, if we assume that the probability of a low-quality product sold to *any one* customer failing is q, then if product failures are independent across customers, we would have $q(x) = 1-$

[6]This happens when the number of potential entrants is large relative to the discount factor.

[7]An example is a product with a defective safety valve. Even though the valve is defective, a specific piece sold to any one customer may not fail. If sufficiently many pieces are sold, however, the probability that at least one of these pieces will fail becomes high.

$(1-q)^x$.[8]] We assume that any such information instantly becomes public knowledge; any dissatisfied customer can instantly post a web review complaining about his or her experience, destroying the firm's reputation (we will relax the assumption of perfect and costless information transmission in Section 3). All customers avoid a firm which has lost its reputation.

A1: $q(X) < 1$,

A2: $v_H > c_H + (1-\delta)(c_H - c_L)/\delta q(X)$,

recalling that X is the total number of customers.

2.2 Timing and equilibrium concept

The timing of moves is as follows.

1. Each firm publicly announces a price P_i, $i \in [1, \ldots .N]$, at which it will sell its high-quality product.[9]
2. Customers observe these announcements and sign purchase orders, choosing which firms to order (high-quality) products from. Each customer orders a single unit.
3. Each firm observes the volume of its purchase orders, decides on product quality, and sells at its announced price.
4. Customers observe whether the products they have purchased fail. If they do, they announce the failure on a public forum.
5. Steps 2 to 4 are repeated indefinitely.

Our equilibrium concept is pure strategy Nash equilibrium. We will focus on the class of symmetric pure strategy NE.

We now make a clarification about notation. As clear from the timing of the game, firms announce prices before observing the volume of purchase orders, x. However, in what follows, I use the notation $\underline{P}(x)$ to denote a threshold "credibility" price. This denotes the mathematical fact

[8]In this example, q(x) is increasing and concave; q'(x) = $-(1-?)^? \ln(1-?) > 0$; q''(x) = $-(1-?)^?$ [ln(1 − ?)]2<0.

[9]As no one will buy the low quality product willingly, firms do not bother to announce prices for these.

that the threshold is affected by the number of customers but *does not* imply that a firm observes the number of customers before announcing a price. While one can think of a firm rationally anticipating the number of its clients in equilibrium and announcing a price that reflects this, the unique equilibrium of our model does not even rely on this, but only depends on firms knowing the total client size, *X*.As we will see later, each firm's dominant strategy leads it to announce a price that is only a function of the total number of clients X; this is regardless of the way in which customers are expected to divide their custom among firms.

2.3 A Preliminary Result

Lemma 1.*Define* $\underline{P}(x) = c_H + (1-\delta)(c_H - c_L)/\delta q(x)$*. If a firm with x customers charges less than* $\underline{P}(x)$*, it cannot convince its customers that it will supply high quality. Moreover, this "minimum credibility price" is decreasing in the firm's customer base.*

Proof: Consider the cheating incentives of a firm with *x* customers. By supplying low quality instead of high quality, the firm saves on its cost of production, obtaining one-time cheating gains of $c_H - c_L$. However, by cheating, the firm runs a risk $q(x)$ of being exposed and being punished by consumers. It is subgame perfect for consumers to respond with a punishment which represents their strategy in a one-shot Nash equilibrium; boycotting the cheating firm.[10] Therefore, in the event of exposure the firm would lose the present discounted value of its future profits from honest high quality supply. The no-cheating constraint is therefore

$$c_H - c_L < q(x)\ \delta(P_i - c_H)/(1-\delta) \tag{1}$$

or

$$P_i \geq c_H + (1-\delta)(c_H - c_L)/\delta q(x) = \underline{P}(x) \tag{1a}$$

A firm with *x* customers must charge a price of at least $\underline{P}(x)$ to convince customers that it is not going to cheat. If not, no one buys from it, as consumers never willingly buy low quality from our assumptions. The RHS of(1a)is decreasing in $q(x)$, and therefore in *x*. A firm with a higher number of customers, therefore, is able to signal credibility at a lower price than one with fewer customers.***QED***

We now use Lemma 1 to obtain our main result.

[10]In a one-shot game, firms would always cheat as they cannot be punished; knowing this, customers would never buy – the classic hold-up problem.

2.4 A Natural Monopoly with Bertrand Competition

So far, we have not discussed competition between firms. Firms compete through Bertrand price competition. While consumers are interested in ensuring that the firm(s) they buy from are credible suppliers of high quality, subject to this, they prefer buying from a firm that charges a lower price than its competitors. As is traditional in the Bertrand model, our main focus is on equilibria that result from active competition, rather than on possibilities of collusive price-fixing between firms; however, for completeness, we discuss the latter theme in an appendix.

It is well-known that with Bertrand competition among firms with symmetric costs, an equilibrium exists in which these firms share the market. However, I find that adding moral hazard with a customer-sensitive probability of exposure, as I have done, drastically changes this result, so that *any* non-collusive equilibrium involves *only one* active firm.[11] This is a surprising result in view of the fact that all firms have symmetric costs, and moreover, that since all firms enter at the same time, there is no pre-existing incumbent. The single active firm would most accurately be described as a natural monopolist, and *not* as a standard monopolist (it does not charge the monopoly price, but the lowest price compatible with high-quality supply), though the presence of moral hazard means that it earns above-normal profits. (If not, it would have an incentive to cheat by supplying low quality; the presence of above-normal rents which it would forego by cheating is necessary to ensure credible high-quality supply). We now prove our chief result, using Lemma 1.

Proposition 1.*Any non-collusive equilibrium of the game described in section 2.2 involves only one active firm servicing the entire market. This firm charges $\underline{P}(X)$.*

Proof: The proof proceeds in a couple of steps.

*Step 1.*In this step, we show that it is optimal for each of the N potential entrants to announce $P_i = \underline{P}(X) = c_H+(1-\delta)(c_H-c_L)/\delta q(X)$(obtained by substituting X for x from equation 1(a)), regardless of how they expect consumers to divide their demand across firms. Suppose, to the contrary, that firm i charges a lower price, P_i'. Now, from Lemma 1, for any feasible x_i , we have $\underline{P}(x_i) \geq$

[11]A later sub-section shows that similar results can be extended to some contexts where price competition occurs but traditional Bertrand assumptions, e.g constant cost, do not hold. For example, if.decreasing returns to scaleare incorporated into the firms' cost functions, a natural oligopoly will emerge, with the number of active firms being endogenously determined.

$\underline{P}(X)$(with strict inequality for any $x_i < X$), therefore, we have $P_i' < \underline{P}(x_i)$. But then, individual customers know that regardless of firm i's total number of customers, it has the incentive to supply low quality. They therefore do not buy from firm i. Hence, this is an unprofitable strategy for firm i. Next, suppose that firm i charges some $P_i' > \underline{P}(X)$. Then, any competitor can undercut firm i by charging a price slightly below P_i', thus luring away all of firm i's potential customers. Knowing this, the firm will not charge a higher price, either.

*Step 2.*In this step, we show that when all firms announce $\underline{P}(X)$, every possible equilibrium involves all customers signing up with the same firm, so that there is only one active firm in equilibrium. Suppose, to the contrary, that there is an equilibrium with L active firms, where L is any integer in the range [2, N]. Assume, without loss of generality, that market demand is evenly divided among these L firms, so that each active firm services X/L customers. But then, the no-cheating constraint (1) is violated; with only X/L customers, each firm must charge a price of at least $\underline{P}(X/L)$ to be credible. However, they are charging only $\underline{P}(X) < \underline{P}(X/L)$, and therefore have the incentive to cheat. Knowing this, customers do not buy from them, a contradiction. However, it is an equilibrium for all customers to sign up with the same firm; as the firm has X customers, it will supply high quality at price $\underline{P}(X)$. From A2, this price is always strictly less than the highest price that customers are willing to pay, so that such an equilibrium always exists. Moreover, this firm earns supernormal profits in equilibrium, since $\underline{P}(X) > c_H$ (by definition). ***QED***

The intuition underlying Proposition 1 stems from the interaction of price competition and the sensitivity of cheating incentives to the customer base. If customers split their demand among rival firms, each firm has a relatively small customer base, which implies that an individual firm's chances of being detected and exposed in low-quality provision are smaller. To offset this increased moral hazard, firms would then need to charge a higher price to convince customers of their intentions to supply high quality. However, if they do so, they are vulnerable to being undercut by a rival. The highest price which rules out undercutting is sufficiently low that credibility requires all customers to sign up with just one firm, which thus becomes a natural monopolist.

2.5 Equilibrium selection

Proposition 1 shows that any non-collusive equilibrium involves a single active firm. Given the symmetry between firms, there are *N* such equilibria, which only differ from each other in the identity of the firm that becomes the natural monopolist. This multiplicity can be eliminated by using either of two plausible equilibrium selection devices.

The first possibility is allowing firms to make *sequential* price announcements in step 1 of the game.[12] One possible mechanism would be to allow all potential entrants to draw an integer from 1 to *N*, and make their announcements in the order specified by the integer they drew. In this case, all customers will flock to the first firm that announces $\underline{P}(X)$. The order of announcement serves as a co-ordinating device allowing all customers to decide which firm to collectively patronize.

The second possibility, often referred to in the literature on network effects, is *early adoption.* This device assumes that some customers are "leaders" who decide, before other customers do, which firm they wish to buy from. Other customers can observe the actions of the leaders, and follow them. In our model, they would always do so; if there is a designated "early adopter" among the customers, and he signs up with a particular firm, all other customers will find it in their interest to sign up with the same firm.

3. Robustness

We now modify some of the assumptions in the previous analysis to see how our results are affected.

3.1 Imperfect transmission of information about product failure

So far we assumed that when low quality is detected by an individual customer, he is able to make this knowledge public with probability 1. However, in communities in which the customer base does not have access to sophisticated technology (i.e. the internet), *and* extends beyond a small network (where personal ties would suffice to spread information), this would not necessarily be the case. Now, let the probability that a discovery of cheating is publicized be

[12]For an example of another game in which firms under Bertrand competition announce prices sequentially, see Roy Chowdhury and Sengupta (2004).

*τ(x)*where *τ' < 0* (the larger the customer base, the less easy it is to transmit information) and *τ" < 0* (the rate of information transmission decays at an increasing rate). The probability of *detection* continues to be denoted by $q(x)$, $q' > 0$. Then, a firm with x customers runs an overall risk of $q(x)\tau(x)$ of being punished by its entire customer base for supplying low quality. Now, the no-cheating constraint (1) changes to

$$c_H - c_L < q(x)\tau(x)\,\delta(P_i - c_H)/(1-\delta) \tag{2}$$

yielding

$$P_i \geq c_H + (1-\delta)(c_H - c_L)/\delta q(x)\tau(x) \;=\; \underline{P}(x) \tag{2a}$$

Observation 1.*Suppose that (i) q"< 0, and (ii)* |? ? ? ?/? ? | ? / ? ? ?/? ? < ? ' (?/?) ? / ? (?/?) . *Then the equilibrium outcome is either a natural monopoly or a natural oligopoly with an endogenously determined scale.*

Proof: Differentiating (2a) with respect to *x,* we obtain

$$\underline{P}'(x) = \frac{???}{?}(?_? - ?_?)\,?\frac{???(?)}{?(?)(?(?))^?} - \frac{??(?)}{?(?)(?(?))^?}? \tag{3}$$

The derivative is zero at x^* such that

$$q'(x^*)/q(x^*) = -\,\tau'(x^*)/\tau(x^*) \tag{4}$$

Differentiating (3) a second time, we obtain

$$\underline{P}''(x) = \frac{???}{?}(?_? - ?_?)\,?\frac{??"}{???} + \frac{?(??)^?}{???} + \frac{?????}{????} - \frac{?"}{???} + \frac{?(??)^?}{???}? > 0 \tag{5}$$

given $q' > 0$, $q'' < 0$, $\tau' < 0$, $\tau'' < 0$. Thus $\underline{P}(x)$ is minimized at x^*. Moreover, from condition (ii)[13] of the Observation, $x^* > X/N$ (which would be the number of clients per firm if *all N* firms were active in a symmetric outcome). Now, there are two possibilities. In the first, $x^* > X$, so that $\underline{P}'(X) < 0$, in which case, just as in Lemma 1, the minimum credibility price is unambiguously decreasing in the customer base, and a natural monopoly obtains by mimicking the proof of Proposition 1. In the second possibility, there is some $x^* < X$ which solves (4). In this case, it is easy to prove that a natural *oligopoly* emerges with X/x^* active firms[14], each with x^* customers, and charging $\underline{P}(x^*)$. To see this, note that Step 1 of Proposition 1 now applies to $\underline{P}(x^*)$;

[13]Condition (ii) is identical with $\underline{P}'(X/N) < 0$. Note that X/N is the smallest feasible client size per firm in a symmetric outcome. This along with the convexity of $\underline{P}$ implies that $x^* > X/N$.
[14]With integer constraints, the number of active firms will be the greatest integer less than or equal to X/x^*.

announcing this price is a dominant strategy for all firms. As $\underline{P}(x^*) \leq \underline{P}(x)$ for all feasible x, charging a price below $\underline{P}(x^*)$ convinces customers that the firm will cheat, while charging a higher price leaves the possibility of undercutting open. Confining our attention to symmetric equilibria, next note that if customers divide themselves among *more* than X/x^* firms, each firm will have a customer base smaller than x^*, which implies that the no-cheating constraint (2) would be violated at price $\underline{P}(x^*)$. However, if customers divide themselves among *less* than X/x^* firms, each firm has a customer base *larger* than x^*, which again violates the no-cheating constraint, as the price required for credibility with such a large customer base is strictly more than $\underline{P}(x^*)$. Thus the equilibrium cannot involve either too many or too few active firms. ***QED***

The parameter restrictions imposed in Observation 1 have the following interpretation. Condition (ii) ensures that the rate at which information diffusion decays with the customer base must not overpower the rate at which detection probability of low quality increases with the customer base, for very low client sizes. Condition (i) requires that the detection probability be concave in client size, which is true of many detection technologies; for instance, it is the case when probabilities of failure are independent across customers, so that $q(x) = 1- (1-q)^x$. Note however that condition (i) is only a sufficient, and not a necessary, condition for Observation 1 to hold. Thus, subject to these conditions, we find again that Bertrand competition, even with symmetric firms, always results in an equilibrium with either one active firm, or with a few active firms where the number of firms is determined by (4). There is no equilibrium in which customers symmetrically divide their demand among all N potential entrants.

3.2 Costly information transmission

In this subsection, we look at the implications of assuming that dissatisfied customers incur a cost to publicize a product failure, relaxing the assumption of costless information transmission in the basic model.

Assume that spreading information about a product failure involves a cost of θ>0. However, disgruntled customers obtain some satisfaction from ensuring that a low quality provider is punished by others. Suppose this "vindication" yields a customer-specific utility of $?_?$, where the subscript denotes an individual customer, and $\kappa$is distributed with a density function of f and a cdf of F. Then, it is straightforward to see that customers who experience product failures will publicize them if and only if $?_? > ?$ for these customers. Therefore, the probability that a cheating firm with a clientele of *x* is exposed changes from *q(x)* in the main model to *(1-F(θ))q(x)* in this modified model. It is easy to see that with this modification, the threshold minimum credibility price continues to be a decreasing function of firm clientele, and that Proposition 1 goes through.

3.3 A small chance of failure of a high quality product

In the main model, truly high quality products do not fail. However, suppose there is a small margin of error or aberration causing a truly high quality product to fail with a small probability *μ*. The probability that a high quality product sold to *x* customers fails at least for one customer is denoted by *μ(x)*. The corresponding probability for a low quality product continues to be denoted by *q(x)*. Since a high quality product failure is an aberration, while a low quality product failure is a common occurrence, we have *μ(x)<q(x)* for all *x*.

In what follows, we see that given an additional, reasonable condition, our natural monopoly result remains unchanged. The condition implies that the ratio of the probabilities of at least one low quality product failure to at least one high quality product failure increases in the number of customers buying products. Essentially, the relative probability of an aberration decreases with a rising number of trials.

Observation 2.*Let* $\frac{?(?)}{?(?)}$ *be an increasing function of x. Then the equilibrium outcome is a natural monopoly.*

Proof: The condition above implies that

$$?^{?}(?)?(?) < ?(?)?^{?}(?) \qquad (6)$$

Since *q(x)>μ(x)*, (6) also implies that

$$?^{?}(?) < ?^{?}(?) \qquad (7)$$

The no-cheating constraint, (1), is modified in two ways when high quality products may fail. First, the lifetime payoff to a firm from honest high quality supply decreases to reflect the probability that it might go out of business for no fault of its own; a high quality product failure results in the same collective boycott, as customers cannot tell from a product failure whether the product was high or low quality. Secondly, the expected penalty of a cheat also changes; while it realizes that it will lose its future payoffs with probability *q(x),* it also knows that with a lower probability *μ(x),* it would lose these payoffs anyway, even if it did not cheat. Thus (1) changes to

$$?_? - ?_? < (?(?) - ?(?))\frac{?(???_?)}{(??????(?))} \quad (8)$$

yielding

$$? \geq ?_? + \frac{(?_? ??_?)(??????(?))}{?(?(?)??(?))} = \underline{P}(x) \quad (8a)$$

Differentiating (8a) with respect to x, and simplifying, we find that $\underline{P}'(x)$ has the same sign as

$$???^?(?)?(?) - ?(?)?^?(?)? - (1-?)??^?(?) - ?^?(?)? < 0$$

where the negative sign follows from (6) and (7). Thus, we have $\underline{P}'(x)<0$. As this holds for all x, we also have $\underline{P}'(X)<0$, and the natural monopoly result follows from mimicking the proof of Proposition 1. ***QED***

3.4 Decreasing returns to scale in the production function

We now modify the standard Bertrand assumptions of constant cost. Assume that the unit cost of production is increasing in the scale of production (decreasing returns to scale), so that we have $c_H'(x) > 0$. Following standard assumptions on marginal cost, we also have $c_H'' \geq 0$. Also, for simplicity, assume that the scale of production affects unit costs similarly for both high and low quality production, so that the cheating gains $c_H - c_L$remain independent of x. For instance, this would be the case if producing high rather than low quality required a fixed investment, such as on a machine, so that the quality cost differential were independent of scale. This is however not an essential assumption; appendix B works out the case where we allow the marginal cost of high quality production to exceed that of low quality production. As in the previous subsection, assume that the detection technology is concave in x.

It turns out that this modification also leads to the emergence of a natural oligopoly, again with an endogenously determined number of firms. (1a) is now modified to

$$P_i \geq c_H(x)+(1-\delta)(c_H-c_L)/\delta q(x) = \underline{P}(x) \quad (9)$$

Differentiating (9) with respect to x, we obtain

$$\underline{P}'(x) = ?_?^?(?) - \frac{(???)(?_? ??_?)}{?}\frac{??(?)}{(?(?))^?} \quad (10)$$

Differentiating again, we obtain

$$\underline{P}''(x) = ?_?'' - \frac{(???)(?_? ??_?)}{?}[\frac{?''}{??} - \frac{?(?^?)^?}{??}] > 0 \quad (11)$$

The convexity of $\underline{P}(x)$ ensures that the solution obtained by setting (10) equal to zero represents a minimum. Again, the interesting parameter space is where this solution, x^{**}, is an interior minimum.[15] Then, any symmetric equilibrium involves X/x^{**} active firms, each charging $\underline{P}(x^{**})$. The proof is very similar to that of Observation 1.

This shows that our results are qualitatively robust to allowing for increasing costs.

3.5 Asymmetric firms

Instead of assuming that all firms have identical costs, if we allow for one firm *k* to have lower costs than the other *N-1* firms, it is easy to see that Bertrand competition again leads to a natural monopoly, with firm *k* emerging as the natural monopolist. Specifically, suppose that $c_H(k) = c_H - \varepsilon$, $c_L(k) = c_L - \varepsilon$, while c_H and c_L continue to represent the other *N-1* firms' costs of high and low quality provision respectively. (The results also hold if firm *k* has an advantage only in high quality production and not in low quality production). Then, firm *k* can charge any price in the semi-open interval $[\underline{P}(X)-\varepsilon, \underline{P}(X))$ where $\underline{P}(X) = c_H+(1-\delta)(c_H-c_L)/\delta q(X)$ (obtained by substituting *X* for *x* in 1(a)). By doing so, it maintains its credibility at a price which cannot be credibly matched by its competitors, and therefore emerges as the natural monopolist.

That Bertrand competition leads to a natural monopoly when one firm has a cost advantage is not as surprising as our main result that this happens even when *all* firms are symmetric, and that, moreover, any non-collusive equilibrium has only one active firm (Proposition 1). Nonetheless, we include this result for completeness.

4. Conclusion

This paper integrates a model of moral hazard in the product market with one of Bertrand price competition between potential entrants in a market where no one firm has an incumbency advantage. It shows that provided a firm's probability of getting exposed cheating is increasing in the number of its customers, all possible equilibria involve only one active firm. This firm is a natural monopolist and charges the lowest price compatible with credibility. In particular, the standard symmetric Bertrand equilibrium in which customers divide their demand among the different firms (which price at cost) will never obtain. Though, in our model, the equilibrium

[15] As long as diseconomies of scale at very small client sizes (X/N) are not too strong, either a natural oligopoly or a natural monopoly obtains – the latter if $x^{**} > X$.

price is higher than the competitive price (due to moral hazard considerations), our emphasis is not on this, but on the fact that just one firm operates in equilibrium. Moreover, this result holds regardless of the fact that all firms are symmetric in their costs, and no firm has an incumbency advantage.

When we modify our assumptions, allowing for imperfect information dissemination among customers (so that the probability of getting exposed is not monotonic in the customer base), we obtain a natural oligopoly instead of a natural monopoly. The number of firms is endogenously determined and it remains true that it is not an equilibrium for customers to divide their demand among all potential entrants. Moreover, our insights are robust to a number of other modifications in assumptions, as shown in Section 3.

In terms of policy implications, our results suggest that concerns about promoting greater competition in an industry with one or very few active players may be misguided or unproductive, especially if the good being sold is an experience good subject to quality concerns. Even if the competitive environment is already very favorable, as with Bertrand price competition, moral hazard may ensure that customers direct all their demand to one (or a very few) firms, so that few active players operate in equilibrium. On the other hand, improving the rate of information transmission, by increasing the probability that a cheat is exposed, lowers the minimum credibility price, which, along with price competition, lowers the equilibrium price $\underline{P}(X)$; therefore, achieving this would result in the natural monopolist charging a lower equilibrium price. It would, however, not raise the number of active firms in equilibrium.

This paper contributes both to the literature on Bertrand competition and to that on the effect of asymmetric information on market structure. It also provides some possible micro-foundations for pecuniary external economies of scale, showing how moral hazard concerns combined with competitive mechanisms lead to an inverse relationship between *industry size* and the equilibrium price charged by a competitive firm.

Appendix A: collusion

The body of the paper concentrates on non-collusive equilibria, in which firms compete actively and do not co-operate. Here, we examine their incentives to collude, agreeing to collectively share the market while charging the monopoly price. This price is v_H in our model.

Appendix result 1: Suppose that δ≤ (N-1)/N. Then, an equilibrium where firms collude cannot be sustained.

Proof: Suppose the N potential entrants agree to collude, charging the monopoly price $v_H$for a high quality product. Note that for collusion to work, all N entrants must agree to collude; if not, they can always be undercut by a potential entrant who is not party to the agreement. Each firm's discounted profits from adhering to the monopoly agreement are $[v_H\text{-}c_H]X/N(1\text{-}\delta)$. A single participant firm's one-time profits from undercutting the others and grabbing the whole market is $[v_H\text{-}c_H\text{-}\alpha]X$, where α is any very small number. As $\alpha \rightarrow 0$, the one-time profit from deviation approaches $[v_H\text{-}c_H]X$. Deviation would imply the breakdown of the agreement, so that all N firms compete in subsequent periods. From our previous results, in this outcome firms charge $\underline{P}(X)$ and expect, with equal probability, to be the natural monopolist. Expected discounted profits in this outcome are then $\delta[\underline{P}(X)\text{-}c_H]X/N(1\text{-}\delta) = (c_H\text{-}c_L)X/q(X)N$ (using 1(a)). Thus, the collusive agreement breaks down if

$$\frac{?_? - ?_?}{1 - ?}\frac{?}{?} < [?_? - ?_?]? + \frac{?_? - ?_?}{?(?)}\frac{?}{?}$$

which simplifies to

$$?(?)?\frac{?}{???} - ?? < \frac{?_? ? ?_?}{?_? ? ?_?} \quad (12)$$

If the condition mentioned in Appendix Result 1 holds, that is, if $\delta \leq (N\text{-}1)/N$, the LHS of (12) becomes nonpositive; since its RHS is positive, (12) therefore necessarily holds. Thus the condition is sufficient (though not necessary) to rule out a collusive equilibrium. *QED*

Appendix B: Decreasing returns to scale with $c_H' > c_L'$

Consider the model of section 3.4, but now suppose the marginal cost of high quality production exceeds that of low quality production, so that the cheating gains $c_H - c_L$ are an increasing function of x. Moreover, suppose that $c_H'' \geq c_L''$; the rate at which marginal cost rises with production is weakly higher for high quality products. Now, instead of equations (10) and (11), we have

$$\underline{P}'(x) = ?_?^?(?) + \frac{(???)}{?}\left[\frac{?_?^? ? ?_?^?}{?(?)} - \frac{(?_? ? ?_?)?^?(?)}{??(?)?^?}\right] \quad (13)$$

and

$$\underline{P}''(x) = ?_{?}\,'' - \frac{(???)}{?}\left[\{?_{?} - ?_{?}\}\left\{\frac{?''}{??} - \frac{?(?^{?})^{?}}{??}\right\} - \frac{?_{?}\,''??_{?}''}{?} + \frac{???(?_{?}^{?}\,?\,?_{?}^{?})}{??}\right] \tag{14}$$

Appendix result 2: Suppose diseconomies of scale are small at low levels of x, so that $\underline{P}'(X/N)<0$. Let x^ be the value of x obtained by setting (13) equal to zero. Then, a sufficient condition for either a natural monopoly or a natural oligopoly with X/x^ firms to emerge in equilibrium is that the probability of detection q be more elastic with respect to customer base x than the cheating gains are.

Proof: The condition on elasticities implies that

$$\frac{?_{?}^{?}\,?\,?_{?}^{?}}{?_{?}\,?\,?_{?}}? < \frac{??}{?}? \tag{15}$$

Manipulating and cross-multiplying, we obtain

$$\frac{???(?_{?}^{?}\,?\,?_{?}^{?})}{??} < \frac{?(?_{?}\,?\,?_{?})(?^{?})^{?}}{??} \tag{15'}$$

From (15'), we see that the expression in square brackets in (14) is clearly negative, so that we have $\underline{P}''(x)>0$. From this, we see that x^ minimizes $\underline{P}(x)$. Moreover, as $\underline{P}'(X/N)<0$and $\underline{P}''(x)>0$, we must have x^>X/N. If x^>X, we have $\underline{P}'(X)<0$ and a natural monopoly obtains; if x^<X, we have a natural oligopoly with X/x^ firms. The proof for this part of the result mimics that of the latter part of Observation 1. *QED*

References

Andersson, F. (2002) "Pooling Reputations", International Journal of Industrial Organization, 20: 715-730.

Cabral, L. (2009) "Umbrella branding with Imperfect Observability and Moral Hazard", International Journal of Industrial Organization, 27: 206-213.

Dell'Ariccia, G., E. Friedman and R. Marquez (1999) "Adverse Selection as a Barrier to Entry in the Banking Industry", RAND Journal of Economics, 30: 515-534.

Farrell, J. (1986) "Moral Hazard as an Entry Barrier", RAND Journal of Economics 17: 440-449.

Farrell, J. and P. Klemperer (2007) "Coordination and Lock-In: Competition with Switching Costs and Network Effects" in the *Handbook of Industrial Organization Volume 3,* eds Armstrong, M. and Porter, R., pp 1967-2072. North-Holland.

Hakenes, H. and M. Peitz (2008) "Umbrella Branding and the Provision of Quality", International Journal of Industrial Organization, 26: 546-556.

Klein, B. and K. Leffler (1981) “The Role of Market Forces in Assuring Contractual Performance”, *Journal of Political Economy*, 89: 615-641.

Ledvina, A.F. and R. Sircar (2011) “Bertrand and Cournot Competition under Asymmetric Costs: Number of Active Firms in Equilibrium”. SSRN Working Paper. Available at SSRN: http://ssrn.com/abstract=1692957 or http://dx.doi.org/10.2139/ssrn.1692957.

Liebowitz, S.J. and S.E. Margolitz (1994) “Network externality: An uncommon tragedy”, *Journal of Economic Perspectives*, 8: 133-150.

Novshek, W. and P. Roy Chowdhury (2003) “Bertrand Equilibria with Entry: Limit Results”, *International Journal of Industrial Organization* 21: 795-808.

Rasmusen, E. (2012) “Leveraging of Reputation through Umbrella Branding: the Implications for Market Structure”, working paper, Indiana University.

Rasmusen, E. and M. Janssen (2002) “Bertrand Competition under Uncertainty”, *Journal of Industrial Economics* 50: 11-21.

Roy Chowdhury, P. and K. Sengupta (2004) “Coalition-proof Bertrand Equilibria”, *Economic Theory* 24: 307-324.

Shapiro, C. (1983) “Premiums for High Quality Products as Returns to Reputations”, *Quarterly Journal of Economics,* 98: 659-679.