Analysis of tax effects on household debts of a nation in a monetary union under classical dichotomy

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Abstract

Unlike many theoretical analysis of tax effects on household debts in a monetary union, this paper builds up analysis from a household budget constraint, instead of starting from a model. By a monetary union, it is assumed that all nations in the union share same currency. Also, if taxes are assumed to be in real values, or if one assumes that government targets real value of taxes T/P, then it is also possible to produce the size of fiscal multiplier on real value of household debts, if relaxed version of classical dichotomy - that agents' decisions are only affected by real variables - is assumed. This paper argues that size is $db/dt_r = -1$ where t_r is real value of taxes or "real taxes" and $b = B/(PR)$ where $B = -D$ with D nominal debt, P price and $R-1$ nominal interest rate , or in terms of real debts, $dd/dt_r = 1$.

1 Budget Constraint Analysis

A nation being analyzed is in a monetary union with some other nations. Thus, inside these nations, there is no exchange rate mechanism. For simplification, there are only consumption goods in an economy, without any capital goods. The nation faces the following households budget constraint, assuming such an emergent budget constraint exists:

$$P_t C_t + T_t + \frac{B_t}{R_t} \leq W_t N_t + \Pi_t + B_{t-1} \tag{1}$$

where P is price level, C is consumption, T is net taxes, W is nominal wage, $R_t - 1$ is nominal interest rate, Π_t is firms' profits all distributed as dividends, X_t is net export. B_t is net one-time bond holding, with $B_t < 0$ implying net indebtedness. It will be assumed that the agents in the economy do not hold any bond for simplification purposes. It will be assumed for rest of analysis that $T \geq 0$ with assumption of zero government spending. Also, for simplification, import M will be assumed to be zero, and all nations are assumed to be in a monetary union. $P > 0$ for an obvious reason. B_{t-1} is assumed to be given. Assuming that the markets clear, $W_t N_t + \Pi_t = P_t (C_t + X_t)$. Thus, Equation 1 becomes with equality:

$$P_t C_t + T_t + \frac{B_t}{R_t} = P_t (C_t + X_t) + B_{t-1} \tag{2}$$

Thus,

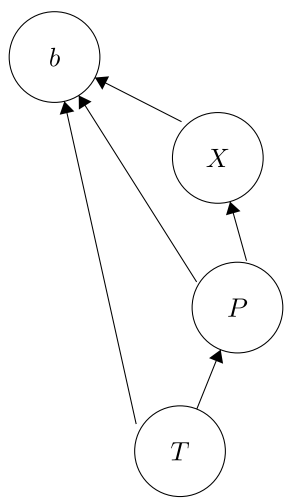
$$T_t + \frac{B_t}{R_t} = P_t X_t + B_{t-1} \tag{3}$$

$$\frac{T_t}{P_t} + \frac{B_t}{P_t R_t} = X_t + \frac{B_{t-1}}{P_t} \tag{4}$$

Let us define $b_t = \frac{B_t}{P_t R_t}$.

$$b_t = X_t + \frac{B_{t-1}}{P_t} - \frac{T_t}{P_t} \tag{5}$$

Define relationships as in the Figure:



The above diagram shows that $b = b(X, P, T)$, $X = X(P)$, $P = P(T)$. The underlying idea is that increase or decrease in taxes affect P, exports are assumed to only depend on price of goods - which is a reasonable assumption given that all export demands are honored, that all nations are in a monetary union, and that quality of goods or technology does not suddenly improve solely by increasing taxes, and inverse net indebtedness obviously depends on X, P, T. Thus,

$$\frac{db_t}{dT_t} = \frac{\partial b_t}{\partial X_t}\frac{\partial X_t}{\partial P_t}\frac{\partial P_t}{\partial T_t} + \frac{\partial b_t}{\partial P_t}\frac{\partial P_t}{\partial T_t} + \frac{\partial b_t}{\partial T_t} \tag{6}$$

Recall Equation 5:

$$b_t = X_t + \frac{B_{t-1}}{P_t} - \frac{T_t}{P_t}$$

$$\frac{\partial b_t}{\partial T_t} = -\frac{1}{P_t} \tag{7}$$

$$\frac{\partial b_t}{\partial X_t} = 1 \tag{8}$$

$$\frac{\partial b_t}{\partial P_t} = -\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} \tag{9}$$

Thus,

$$\frac{db_t}{dT_t} = \left[-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} + \frac{\partial X_t}{\partial P_t} \right] \frac{\partial P_t}{\partial T_t} - \frac{1}{P_t} \tag{10}$$

It is assumed that $\frac{\partial X_t}{\partial P_t} < 0$ and for our interests, $T_t \geq 0$.

- $\frac{\partial P_t}{\partial T_t} < 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} > -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$. Then, $\frac{db_t}{dT_t} < 0$. Real value of debts increase when taxes are raised.
- $\frac{\partial P_t}{\partial T_t} < 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} < -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$.
  Also, $\left[-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} + \frac{\partial X_t}{\partial P_t} \right] \frac{\partial P_t}{\partial T_t} < \frac{1}{P_t}$. Then still $\frac{db_t}{dT_t} < 0$.
- $\frac{\partial P_t}{\partial T_t} < 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} < -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$.
  Also, $\left[-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} + \frac{\partial X_t}{\partial P_t} \right] \frac{\partial P_t}{\partial T_t} > \frac{1}{P_t}$. Then, $\frac{db_t}{dT_t} > 0$.
- If $\frac{\partial P_t}{\partial T_t} = 0$, then $\frac{db_t}{dT_t} < 0$.
- $\frac{\partial P_t}{\partial T_t} > 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} < -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$. Then, $\frac{db_t}{dT_t} < 0$.
- $\frac{\partial P_t}{\partial T_t} > 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} > -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$.
  Also, $\left[-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} + \frac{\partial X_t}{\partial P_t} \right] \frac{\partial P_t}{\partial T_t} < \frac{1}{P_t}$. Then still $\frac{db_t}{dT_t} < 0$.
- $\frac{\partial P_t}{\partial T_t} > 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} > -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$.
  Also, $\left[-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} + \frac{\partial X_t}{\partial P_t} \right] \frac{\partial P_t}{\partial T_t} > \frac{1}{P_t}$. Then, $\frac{db_t}{dT_t} > 0$.

Now, let us change Equation 5 into:

$$b_t = X_t + \frac{B_{t-1}}{P_t} - t_{r,t} \tag{11}$$

where $t_{r,t} = T_t/P_t$, real taxes. $b = b(X, P, t_r)$, $X = X(P)$, $P = P(t_r)$.

$$\frac{db_t}{dt_{r,t}} = \frac{\partial b_t}{\partial X_t}\frac{\partial X_t}{\partial P_t}\frac{\partial P_t}{\partial t_{r,t}} + \frac{\partial b_t}{\partial P_t}\frac{\partial P_t}{\partial t_{r,t}} + \frac{\partial b_t}{\partial t_{r,t}} \tag{12}$$

$$\frac{\partial b_t}{\partial t_{r,t}} = -1 \tag{13}$$

$$\frac{\partial b_t}{\partial X_t} = 1 \tag{14}$$

$$\frac{\partial b_t}{\partial P_t} = -\frac{B_{t-1}}{{P_t}^2} \tag{15}$$

$$\frac{db_t}{dt_{r,t}} = \left[-\frac{B_{t-1}}{{P_t}^2} + \frac{\partial X_t}{\partial P_t}\right]\frac{\partial P_t}{\partial t_{r,t}} - 1 \tag{16}$$

Now I will introduce a relaxed version of classical dichotomy. Even under many sticky-price models, price level itself is irrelevant, as long as some real variables remain adjusted. Let new $P_{n,t} = kP_t$. Then, one needs to adjust B_{t-1}, B_t and T_t to reflect the change. $B_{n,t-1} = kB_{t-1}$, $B_{n,t} = kB_t$, $T_{n,t} = kT_t$, with initial equilibria adjusted appropriately. But $t_{r,t}$ does not change under price level transformation, and similarly b_t does not change. Let $k \to \infty$. Then, $-\frac{B_{n,t-1}}{{P_{n,t}}^2} \to 0$. Thus,

$$\frac{db_t}{dt_{r,t}} = \frac{\partial X_t}{\partial P_t}\frac{\partial P_t}{\partial t_{r,t}} - 1 \tag{17}$$

But $\frac{\partial X_t}{\partial P_t}\frac{\partial P_t}{\partial t_{r,t}}$ is invariant relative to price level transformation. Then the problem appears: $-\frac{B_{t-1}}{{P_t}^2}$ is non-zero and finite whenever $k \neq 0, \infty$. If $\frac{\partial P_t}{\partial t_{r,t}}$ is finite too, then $\frac{db_t}{dt_{r,t}}$ no longer remains invariant relative to price level transformation. Thus, three choices:

- $|\frac{\partial P_t}{\partial t_{r,t}}| = \infty$ all the time
- $\frac{\partial P_t}{\partial t_{r,t}} = 0$ all the time
- $\frac{\partial X_t}{\partial P_t} = -\infty$ all the time

If the second choice is made,

$$\frac{db_t}{dt_{r,t}} = -1 \tag{18}$$

The first choice brings an interesting conclusion: that raising real taxes explodes nominal value, and if the government cares also about monetary/nominal stability, then real taxes cannot be set exogenously and thus are endogenous.

The third choice implies that exports are extremely sensitive to price change. This can be expected in an economy where every good is homogeneous and market is perfectly competitive. However, if any form of monopolistic competition exists, then the third choice is likely not true.