Aspects of Stickiness in Understanding Inflation

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30 April 2016

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2016/04/30

Abstract

This paper examines the question of how conventional understanding of inflation relates to stickiness. Several results that often go unnoticed are re-examined.

1 Examining equations

Suppose that economy has constant natural real interest rate $r_{t,n} = r$ all the time. For exposition, I will assume that even under a sticky price model, $r_t = r$. This assumption is indeed restrictive, but this will not damage the analysis. Let Taylor rule be,

$$i_t = r + \phi_\pi \pi_t + \phi_y \tilde{y_t} \tag{1}$$

where $\tilde{y_t}$ is output gap, π_t is inflation rate.
By the Fisher equation,

$$i_t = r + E_t \left[\pi_{t+1}\right] \tag{2}$$

Combining two and assuming $\phi_y = 0$, as in [1],

$$E_t \left[\pi_{t+1}\right] = \phi_\pi \pi_t \tag{3}$$

If equilibrium real variables are unaffected by π_t alone, then one may say that while real explosions are ruled out by some conditions, nominal explosions cannot be ruled out and thus Equation 3 allows explosive solutions. Unless $\pi_t = 0$, any expected path explodes if $|\phi_\pi| > 1$, as [1] says.
Now introduce New Keynesian Phillips Curve (NKPC), which assumes some degree of price stickiness:

$$\pi_t = \beta E_t \left[\pi_{t+1}\right] + \kappa \tilde{y_t} \tag{4}$$

Substituting the equation into Equation 3,

$$\tilde{y_t} = \frac{(1 - \beta\phi_\pi)\pi_t}{\kappa} \tag{5}$$

- If $\beta\phi_\pi = 1$, then $\tilde{y_t} = 0$ at every t. Ruling out real variable explosions does not rule out nominal explosions.

- If $|\phi_\pi| > 1$ but $\beta\phi_\pi \neq 1$, then either $\pi_t = 0$ at every t which ensures $\tilde{y}_t = 0$ at every t, or $\tilde{y}_t$ explodes. If real variable explosions are to be always ruled out, then $|\phi_\pi| > 1$ ensures $\pi_t = 0$, $\tilde{y}_t = 0$.
- If $|\phi_\pi| < 1$, then real variable explosions are avoided always. But nominal variable explosions cannot be ruled out by ruling out real variable explosions. Under the standard consumption Euler equation, an additional constraint is created, thus rules out nominal and real explosions.

Thus, this gives us another type of divine coincidence - if $0 < \beta < 1$ and assuming that real explosions are eliminated automatically, then just setting $|\phi_\pi| > 1$ is consistent with completely eliminating sticky price distortions. While NKPC is only an approximation, its validity stands, as it is approximated around zero inflation steady state. Monetary policy is basically invisible. Thus, there is no need in this theoretical economy to adjust interest rate based on output gap and doing so only complicates the matter by creating an unnecessary business cycle.
The problem now is this: as demonstrated above, now the theory of inflation relies on existence of some type of stickiness that allows one to affect real variables by nominal variable changes. Here, $\tilde{y}_t$ is affected by π_t. Thus, ruling out explosion of $\tilde{y}_t$ allows one to rule out explosion of π_t, even without the standard consumption Euler equation.
But in an economy that does not allow real variables to be affected by nominal variable changes, such an understanding cannot be used to rule out explosion of π_t. And under classical economy, $r_t = r$ should be true under a certain cases (possibly constant technology growth) - or one can assume to be so.
This now brings the question of faith: one may believe that there must be one rule/unified interpretation that allows how inflation rate or price level is determined for both flexible/frictionless and price-sticky or any sticky economies. Some others may believe that different rules can apply for frictionless and flexible economies. I will examine the former only.
For the former belief, one possible stance may be that "Standard New Keynesian theory of inflation is invalid." This stance would argue that there is no way to reconcile the equations into providing unique inflation path for a flexible economy. The following classical logic argument follows independent of the stance.

banning real explosions is valid $\rightarrow$ NK understanding of inflation is valid

By "banning real explosions is valid," it is meant that selection of a unique equilibrium by eliminating real-explosive equilibria is always valid, provided the model is consistent. Then assuming the model is true, "NK understanding of inflation is valid."
The aforementioned stance states that this understanding is invalid even if the model is true. Thus, a believer in this stance must be ready to say that obtaining a unique equilibrium by banning real explosions may not always be a valid way.

1.1 Can real explosion elimination be justified?

In any case, New Keynesian models need real explosion elimination to obtain a unique equilibrium. But why would an agent wish to have non-explosive equilibrium? The reason may be found on utility based on consumption-leisure tradeoff. Since actual labor quantity equilibrium depends on the specification of supply side, one cannot generalize completely, but it is in general safe to say that keeping magnitude of output gap under a certain bound is a welfare-wise good idea. The question then is whether the agents have power to enforce the hope as part of equilibrium selection. The households in general are constrained by the supply side too, so it is natural to say that there must be expectation that central bank can control output gap and inflation. But there is nothing in the model that justifies such an expectation.

1.2 Is $|\phi_\pi| > 1$ a threat to explode the economy if a target equilibrium is not met?

Cochrane in [1] argues that setting $|\phi_\pi| > 1$ for $E_t[\pi_{t+1}] = \phi_\pi \pi_t$ amounts to central bank ensuring nominal uniqueness by promising to explode the nominal economy if π_t does not turn out to be the unique inflation rate obtained after elimination procedures. But as Cochrane says,

$$\pi_{t+1} = \phi_\pi \pi_t + \delta_{t+1} \tag{6}$$

where δ_{t+1} can be any arbitrary random variable with $E_t \delta_{t+1} = 0$. In many rational expectation models, an agent is forward-looking - thus, it is $E_t \pi_{t+1}$ that affects π_t, not π_t affecting $E_t \pi_{t+1}$. After π_t is decided, it is mainly sunspot δ_{t+1} that decides π_{t+1}. This sunspot reflects the fact that expectation of future inflation is not anchored. Thus, even if $|\phi_\pi| > 1$, there is no reason to see why the economy would necessarily go explosive if the target equilibrium is not met. The fact π_t is not the target inflation rate reflects the fact that the household expectation for π_{t+1} was different from the central bank's forecast. If this gap can be eliminated, then even if the economy started out with the gap, the nominal economy will not explode.
Thus, the actual problem again is whether central bank can make the agents believe that it has control over π_t.

2 Conclusion

It is also known that if price level has floor and ceiling, then explosive solutions can be eliminated. Thus, all the aforementioned discussions do rely on the space of price level not being constrained. One can, as Cochrane in [1] argues, justify the space of price level not being constrained by hyperinflation episodes in history, but whether this is justifiable remains outside the scope of this paper. To summarize, what matters in the end is whether central bank and agents can agree on expected future inflation rate and output gap. As learning mechanism

is missing from the model, one is still left with possibility of exploding real and nominal equilibria. And if justification of ruling out explosive real equilibria is made, then at least in simple New Keynesian models, one loses the reason to set ϕ_y other than zero, since $|\phi_\pi| > 1$ can fix inflation rate to zero and output gap to zero.

References

[1] Cochrane, J. (2011). "Determinacy and Identification with Taylor Rules", Journal of Political Economy 119 (3): 565-615.