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Abstract. The welfare effects of entry are studied in a model of consumer search. Potential entrants differ in quality, with high quality sellers being more likely to meet consumer needs. Contrary to the standard view in economics that more entry benefits consumers, we find that free entry is excessive for *both* consumer welfare and total welfare when entry cost is relatively low, and consumer welfare has an inverted-U relationship with entry cost. We explain why these results may arise naturally in search markets due to the search variety and search quality effects of entry, and discuss their business and policy implications.

Keywords: Entry, entry cost, search, search variety, search quality

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1. INTRODUCTION

Entry is of central importance to competition and market performance. While it has long been known that free entry is efficient under perfect competition, economists have more recently recognized that the impact of unencumbered entry on total welfare is ambiguous when firms possess market power, due to consumers' gain and competitors' loss that the entrant does not internalize (e.g., Von Weizsacker, 1980; Mankiw and Whinston, 1986; and Cabral, 2004). The standard view in economics, however, is still that more entry will boost *consumer welfare*. In homogeneous-product markets, industry output under Cournot competition generally expands with entry (e.g., Seade, 1980).[1] Even in markets with differentiated products, where it has been argued that price-increasing entry is theoretically unexceptional, the consumer gain from greater product variety will usually dominate any potential adverse price effect (e.g., Chen and Riordan, 2008).

This paper conducts a new analysis of entry and welfare in an important class of markets—those with consumer search, focusing especially on how entry affects consumer welfare, measured by aggregate consumer surplus. Our interest in search markets is partly motivated by the reflection that, despite the substantial progress in the economics of search,[2] little attention has been paid to the effects of changes in entry conditions, and yet technological progress such as the Internet has drastically reduced entry costs in many search markets. We focus on consumer welfare because, as we shall demonstrate, the common belief that unfettered entry benefits consumers is actually misguided. This will have ramifications for business practices as well as for antitrust and regulation policies.

We consider a model where potential entrants differ in quality—the probability that a seller's product will match a consumer's need. This probability is larger for a high quality

[1] Amir and Lambson (2000) demonstrate that price can increase in the number of firms under Cournot competition. Nevertheless, as the authors point out, the assumptions needed for such an outcome, which involves an unstable equilibrium in a certain sense, are restrictive.

[2] Starting from the seminal work of Stigler (1961), the literature has advanced in the directions of search for the best price among competing homogeneous sellers (e.g., Stahl, 1989) and of search for the best value among competing differentiated sellers (e.g., Wolinsky, 1986).

firm, whose product thus has a high expected value to consumers. Each firm privately learns its quality, and can choose to enter the market by incurring an entry cost. After firms simultaneously make their entry decisions, those who have entered the market simultaneously choose prices, whereas each consumer, observing the number of firms in the market, can conduct sequential search to find out the price and product value of one or more sellers. A consumer receives zero utility from a non-matched product, while her utility from a matched product is a random draw from a known distribution and is identical for all her matches.[3] The model is thus a dynamic game of incomplete information, and the type-contingent nature of the entry decision makes the model different from an otherwise standard two-stage entry game (e.g., Mankiw and Whinston, 1986).

Under certain conditions and for a given entry cost, the model has a unique symmetric (perfect Bayesian) equilibrium, where every potential entrant will choose to enter the market if and only if its quality exceeds some threshold, and, similarly as in Diamond (1971), the equilibrium price is invariant with respect to the number of sellers.[4] We are interested in two related welfare questions at this equilibrium. First, given an entry cost, how will the expected number of entrants under free entry compare to those that maximize consumer or total welfare? Second, how will an exogenous change in entry conditions, such as the entry cost, affect welfare in the free entry equilibrium?

We find that, holding everything else constant, free entry leads to an excessive number of firms for consumer welfare, and hence also for total welfare,[5] when entry cost is below some critical value; whereas entry is deficient for consumer welfare when entry cost is above this critical value. More strikingly, we find that consumer welfare is an inverted-U function

[3] This formulation follows several recent papers on consumer search (e.g., Athey and Ellison, 2011; Chen and He, 2011; and Eliaz and Spiegler, 2011).

[4] In search markets, more sellers can cause price to rise (e.g., Satterthwaite, 1979; Stahl, 1989), to fall (e.g., Wolinsky, 1986), or to either increase, decrease, or unchange (Janssen and Moraga-González, 2004). Our model provides a useful baseline case, making it transparent that the mechanism through which entry affects consumer welfare in our setting differs from the usual price effect.

[5] Since entry also has the business-stealing effect (Mankiw and Whinston, 1986), if it is excessive for consumer welfare, it must also be for total welfare.

of entry cost, first increasing and then decreasing, maximized at the critical entry cost. We obtain these results in our model by first identifying two externalities of entry on consumer search, which we term as the search variety and search quality effects: The entry of a firm expands the search options available to each consumer, which is the positive search variety effect that the entrant does not internalize. But a marginal entrant also lowers search quality, because it reduces the expected quality of sellers in the market and makes a search less likely to produce a match. This negative search quality effect is also not internalized by an entrant.[6]

The existence of the search variety and quality effects of entry suggests that free entry may not maximize consumer welfare. But this by itself does not tell us whether on balance entry will be too much or too little for consumers. While the analysis to establish our consumer welfare results is rather involved, the intuition behind it is simple. When entry cost is low, the expected number of entrants is large while the marginal entrant's quality is much below the average quality. Hence, the positive search variety effect of entry is small but the negative search quality effect is large. Consequently, free entry, under which an entrant does not internalize these two externalities, is excessive for consumer welfare. Moreover, for a small increase in the entry cost, the increase in search quality is significant but the decrease in search variety is not, and thus consumer welfare rises. Conversely, when entry cost is high, the positive variety effect is substantial but the negative quality effect is negligible, so that free entry is deficient for consumers, and a marginal reduction in entry cost will increase consumer welfare. Remarkably, in our model the trade off between these two effects of entry varies smoothly so that consumer welfare is a single peaked function of entry cost.[7]

Our result on how entry affects consumer welfare, while unconventional, is quite natural

[6] More generally, entry can also affect consumer welfare through a price effect. We will study a variant of our model, where the price effect is present, to check the robustness of our welfare results.

[7] In the literature on information congestion in two-sided markets (e.g., Anderson and de Palma, 2009), externalities created by both information senders and receivers can also lead to deficient or excessive entries. By analyzing a different trade-off in a novel model, we offer new insights on how entry affects consumer welfare and total welfare in search markets.

for search markets. In fact, we can interpret search markets very broadly, to include any market where firms have private information about product quality and consumers can obtain costly quality information before purchase. Consider, for example, Akerlof (1970)'s classic model of used-car market, where, under adverse selection, low quality sellers drive out high quality sellers, and the market may shut down completely. One may view our paper as taking Akerlof's model a step further by adding consumer search to it, so that a buyer can incur a search (inspection) cost to find out, possibly with the help of an auto mechanic, whether a car has a defect.[8] A high quality seller, whose car is less likely to be defective, then has a higher probability to succeed in trading, and hence more incentive to incur the (entry) cost to list its car for sale. The buyers' ability to detect a car's flaw through costly search may thus mitigate the adverse selection problem.[9] But if entry cost is very low, it will not prevent low quality sellers from entering the market; buyers' search efficiency will then be too low and the market is likely to perform poorly. On the other hand, if entry cost is too high, very few sellers will enter the market, and even if their expected quality is high, it will be hard for buyers with heterogeneous preferences to find a match under the very limited search opportunities. This, in essence, is the trade off between the search variety and search quality effects of entry, as our analysis uncovers.[10]

In search markets, therefore, it will not be unusual for entry restrictions to benefit consumers. This can shed light on many business practices. Consider, for instance, the market of apps for iPhones and iPads. Apple clearly has the incentive to increase consumer surplus in this market, which would boost its profits from the sale of iPhones and iPads. Whereas

[8] This, together with the consumer's idiosyncratic taste, may then determine whether the car will meet her need.

[9] There are related studies of search and product quality in the literature. For example, in Wolinsky (1983), prices are observable before consumer search and may serve as signals of product quality that is privately known by firms. Dranove and Satterthwaite (1992) consider a search model where consumers can imperfectly observe prices and qualities after incurring search costs. They find that an improvement of price or quality information may either increase or decrease welfare.

[10] Importantly, the search quality effect arises only because asymmetric information on product quality: if consumers had perfect quality information, the entry of more firms, even of those with low qualities, would not reduce search efficiency, because consumers could always choose to search high quality sellers first.

more entrants of app developers will offer users more choices, the entry of low quality sellers can reduce search quality and make it harder for consumers to find a desired app. Apple appears to balance this trade off by both increasing entry cost and maintaining a minimum quality standard: it charges a fixed fee to each entrant ($99/year), and the entrant's product has to go through a stringent review process. Only products that are approved by Apple can be offered for sale to consumers. In addition to entry barriers created by private entities (as we shall discuss further in the concluding section), government policies can also limit entry, as for example, with a minimum quality requirement. A license fee that acts as a transfer payment can also positively impact both consumer and total welfare by raising the quality of the marginal entrant. On the other hand, an entry barrier that adds to physical cost of entry (such as transaction cost) might benefit consumers but reduce total welfare.

We describe our model in Section 2, and characterize its equilibrium in Section 3. In Section 4, we establish our main results on how the free entry outcomes compare with those that maximize consumer or total welfare, and how welfare may vary with entry conditions. In Section 5, we analyze a variant of the main model, in which a consumer's value for each match is an independent random draw, and hence the matched sellers of any consumer are horizontally differentiated. Entry then also has a price effect, which complicates the analysis; nevertheless, in the numerical examples that we consider the welfare results of our main model continue to hold.[11] In addition to serving as a robustness check, this section also contains a result of independent interest: we find that equilibrium market price *decreases* in the expected quality of sellers in the market. Section 6 offers concluding remarks. Proofs that are more technical in nature are relegated to the appendix.

[11] This variant of our model is closely related to Anderson and Renault (1999), who study a standard consumer search model with horizontally differentiated products and find that market entry is always excessive for total welfare. We shall explain in section 5 why the presence of vertical differentiation leads to very different results and new insights.

2. THE MODEL

The market contains a unit mass of consumers, each demanding one unit of a product. There are $N \geq 2$ potential entrants who can choose to become active sellers, and the entry cost for each seller is $k > 0$. Each consumer is ex ante uncertain about whether a particular firm offers a product that she desires and how much she is willing to pay for such a product. Specifically, with probability β_i, potential entrant i's product, $i = 1, 2, ...N$, meets a consumer's need. The consumer derives utility u from consuming the product of all her matched sellers, and u is an independent draw from distribution F with density f on support $[\underline{u}, \bar{u}]$, where $\bar{u} > \underline{u} \geq 0$.[12] With probability $1 - \beta_i$, i's product does not meet the consumer's need, in which case the consumer utility from the product is normalized to zero.[13] Thus, we consider β_i as a measure of $i's$ quality.[14] Potential sellers differ in their quality. In particular, we assume that β_i draws from cumulative distribution function G with density function $g > 0$ on support $[0, 1]$. Finally, the production cost of each seller is normalized to zero.[15]

The timing of the model is as follows. First, β_i is realized and is known privately by i. Second, potential entrants simultaneously choose either to enter the market or to stay out. Third, the market structure is determined, with n entrants as sellers. Consumers are informed of the number of sellers. Although $n = 0$ is always a possibility, our analysis will

[12] Note that β_i is firm-specific, but its realization is consumer-specific (in the sense that a match for one consumer does not necessarily imply a match for another). Also, u is consumer-specific, but for each consumer, it is equal across her matched sellers.

[13] For example, a consumer may have a specific requirement for a product, such as a certain quality feature for a car, and a high-quality seller is more likely to meet the requirement. Or, a consumer may need to improve a product's performance (such as the energy efficiency of a house), and a high-quality firm is more likely to find the right solution to the problem. It could also be that the consumers are input purchasers on an intermediate-good market, and a high-quality supplier is more likely to meet each buyer's quality standard.

[14] In fact, the expected value of seller $i's$ product to a consumer is simply $\beta_i \int_{\underline{u}}^{\bar{u}} u dF(u)$, which increases in β_i. Thus, a high-quality seller is more likely to offer consumers a high-value product.

[15] More precisely, the marginal cost of production is equal to the consumer utility from a non-matched product, which has been assumed to be zero.

focus on situations where $n \geq 1$, and we assume that k is relatively small so that a potential entrant with a sufficiently high β_i will enter the market. Fourth, sellers simultaneously and independently set their prices, after which each consumer, without knowing whether any particular seller is a match, her value u for the match, and the seller's price, chooses whether and how to conduct sequential search. Each search will enable the consumer to discover the aforementioned information from a seller, with search cost s. We study symmetric perfect Bayesian equilibrium of this game.

Throughout the paper, we maintain the assumption that $1 - F(u)$ and $1 - G(\beta)$ are log-concave. Let

$$p^m = \arg \max_p \{p [1 - F(p)]\}; \quad \pi^m = p^m [1 - F(p^m)].$$

Then, p^m exists uniquely and is interior, due to the log-concavity of $1-F$. The logconcavity of $1 - G$ will be used in Lemma 1 to show that the equilibrium profit of an entrant is increasing in its quality type.

3. MARKET EQUILIBRIUM

Suppose for a moment that, given k, a potential entrant will enter the market if and only if its quality exceeds some threshold t. We first study equilibrium for any given threshold t. We then show that in equilibrium the expected profit of potential entrant i indeed increases in β_i, thereby confirming the optimality of the threshold-based entry strategy for each potential entrant. The equilibrium threshold t_f is then determined, which is shown to increase in k.

For any given t, the expected quality of an entrant is

$$\gamma \equiv \gamma(t) = \frac{\int_t^1 xg(x)\,dx}{1 - G(t)}, \tag{1}$$

where $\gamma > t$ for all $t \in [0, 1)$ since $\int_t^1 xg(x)\,dx > t[1 - G(t)]$.

First, consider the sellers' price strategy and consumers' search strategy. If there is only one seller ($n = 1$), its equilibrium price will be p^m, and consumers will search if

$$\gamma \int_{p^m}^{\bar{u}} (u - p^m) f(u)\,du - s \geq 0. \tag{2}$$

Condition (2) is satisfied if s is not too large, which we assume throughout the paper.

With $n \geq 2$ sellers, from standard arguments (e.g., Diamond, 1971; Chen and He, 2011), there is a unique equilibrium where each seller sets $p = p^m$, each consumer will search sequentially and will purchase from the first match, provided that $u \geq p^m$. The consumer will exit the market without purchase if $u < p^m$ or if she has searched all n sellers without finding a match.

Thus, in equilibrium, seller $i's$ expected profit for any given t when there are n entrants in the market (including i) is

$$\pi_n\left(\beta_i\right) = \beta_i \pi^m \phi_n, \tag{3}$$

where

$$\phi_n = \frac{1}{n} \sum_{j=0}^{n-1} (1-\gamma)^j = \frac{1-(1-\gamma)^n}{n\gamma} \tag{4}$$

is the expected number of consumers who visit seller i when n firms ($n-1$ rivals) enter the market.

We next determine the endogenous number of sellers. Consider a potential seller's entry decision. From (3), a seller's expected profit, when there are n entrants, is increasing in β_i. To determine the equilibrium t, we consider the decision of i with β_i. The post-entry expected profit for i is

$$E\left(\pi | \beta_i\right) = \sum_{n=1}^{N} \delta_n\left(t\right) \pi_n\left(\beta_i\right), \tag{5}$$

where

$$\delta_n\left(t\right) = \tbinom{N-1}{n-1} \left[1 - G\left(t\right)\right]^{n-1} G\left(t\right)^{N-n} \tag{6}$$

is the probability that $n-1$ other potential entrants enter and $\pi_n\left(\beta_i\right)$ is the expected profit for i if it chooses entry simultaneously as the $n-1$ others. Our analysis will utilize Lemma 1 below, which states that (i) an increase in the marginal entrant's quality will raise the average quality of all entrants in the market, but (ii) the marginal increases relatively more than the average. Part (i) is straightforward, and while (ii) is also intuitive, it relies on the log-concavity of $1-G$.

Lemma 1 *For all $t \in [0,1)$:*

$$(i)\ \frac{d\gamma}{dt} = \frac{g(t)}{1-G(t)}(\gamma - t) > 0; \quad (ii)\ \frac{d(t/\gamma(t))}{dt} = \frac{\gamma - \frac{g(t)t(\gamma-t)}{1-G(t)}}{\gamma^2} > 0. \tag{7}$$

By Lemma 1, the proof of which is contained in the Appendix,

$$\pi_n(t) = \pi^m \frac{t}{\gamma}\frac{1-(1-\gamma)^n}{n} \tag{8}$$

increases in t. That is, given n, the expected profit for the marginal entrant is higher if it has a higher quality. It can also be verified that $\pi_n(t)$ decreases in n. Lemma 2, which is also proved in the Appendix, establishes that the expected post-entry profit for the marginal entrant is increasing in its quality:

Lemma 2 *$E(\pi|t)$ increases in t.*

Notice that the marginal entrant will earn zero if it has $\beta_i = 0$, and will earn π^m if it has $\beta_i = 1$. Therefore, for any given $k \in [0, \pi^m)$, there exists a unique threshold $t_f \equiv t_f(k) \in [0,1)$ that satisfies

$$E(\pi|t_f) = k, \tag{9}$$

and $t_f = t_f(k)$ increases in k, with $t_f = 0$ for $k = 0$ and $t_f \to 1$ as $k \to \pi^m$. We have thus shown that there exists a symmetric equilibrium where each potential entrant will enter if and only if its quality reaches the threshold t_f, and t_f monotonically increases in k. Moreover, it is straightforward to check that there can be no other symmetric equilibrium.

Summarizing the above discussion, we have:

Proposition 1 *For any given $k \in (0,\ \pi^m)$, there exists a unique symmetric equilibrium where: (i) potential entrant i, $i = 1, 2, ..., N$, will enter the market if and only if $\beta_i \geq t_f$, with $t_f \in (0,1)$, defined in (9), being an increasing function of k, and each seller will charge price p^m; (ii) each consumer will search sequentially in random order, purchase from the first match if $u \geq p^m$, and make no purchase if either she finds no match or $u < p^m$.*

4. WELFARE ANALYSIS

In our model, the number of entrants (n) is uncertain, depending on the number of potential entrants (N), the realizations of β_i, and entry cost (k). Hence a proper measure of entry is the expected number of entrants, which is determined by t, the minimum possible quality of actual entrants. A lower t corresponds to a higher expected number of sellers in the market. In equilibrium, through the dependence of t_f on k, the expected number of sellers in turn will be determined by k. We can then compare it with the number that maximizes consumer or total welfare, and explore how welfare may vary with entry conditions.

4.1 Consumer Welfare

For a given t, consumer welfare, measured by expected aggregate consumer surplus (net of search cost), is

$$V = \sum_{n=1}^{N} \lambda_n(t) V_n(\gamma, p^m), \tag{10}$$

where

$$\lambda_n(t) = \binom{N}{n} [1 - G(t)]^n G(t)^{N-n} \tag{11}$$

is the probability that exactly n sellers have entered, and

$$V_n(\gamma, p^m) = \sum_{i=1}^{n} (1-\gamma)^{i-1} \gamma \int_{p^m}^{\bar{u}} (u - p^m) f(u)\, du - \sum_{i=1}^{n} (1-\gamma)^{i-1} \gamma i s - (1-\gamma)^n ns \tag{12}$$

is the consumer welfare with $n \geq 1$ sellers when their expected quality is γ. In V_n above, the first term is the (weighted) sum of benefit when a consumer has searched and purchased from the i^{th} seller, while the second and the third terms are the expected search cost when the consumer ends up with and without purchase, respectively. We define:

$$\Phi = \int_{p^m}^{\bar{u}} (u - p^m) f(u)\, du; \qquad M(t) = 1 - \gamma [1 - G(t)], \tag{13}$$

where Φ is a consumer's expected surplus from a match, and $M(t)$ indicates the probability that a potential entrant will not be a match when the entry threshold is t.

Lemma 3 *Consumer welfare $V \equiv V(t)$ can be expressed as:*

$$V = \left[1 - M(t)^N\right]\left(\Phi - \frac{s}{\gamma}\right). \tag{14}$$

Equation (14) has an intuitive interpretation. The probability that a consumer will (eventually) find a match is $1 - M(t)^N$. Since Φ is the expected surplus to a consumer from a match and s/γ is the search cost adjusted by the expected match probability per seller, $\Phi - \frac{s}{\gamma}$ reflects the expected net benefit from a search that yields a match. With a unit mass of consumers, consumer welfare is the consumer's expected net benefit from the entry of firms under threshold t.

Notice that given the distribution of u, search cost s, and the number of potential entrants N, V is entirely determined by t through $\gamma = \gamma(t)$ and $M(t)$. Totally differentiating (14) with respect to t, we have

$$\frac{dV}{dt} = \underbrace{-NM(t)^{N-1}\frac{dM}{dt}\left(\Phi - \frac{s}{\gamma}\right)}_{\text{search variety effect}} + \underbrace{\left[1 - M(t)^N\right]\frac{s}{\gamma^2}\frac{d\gamma}{dt}}_{\text{search quality effect}}. \tag{15}$$

Thus, the impact of increased entry (i.e., a decrease in t) on consumer welfare can be decomposed into two parts: a search variety effect and a search quality effect. The first term in (15), the variety effect, is the change in V due to dM/dt: a decrease in t raises the expected number of entrants, providing consumers with more search opportunities to obtain the expected net benefit $\left(\Phi - \frac{s}{\gamma}\right)$. From (13) and by Lemma 1,

$$\frac{dM(t)}{dt} = -\frac{d\gamma}{dt}\left[1 - G(t)\right] + \gamma g(t) = g(t)\,t > 0. \tag{16}$$

The second term, the quality effect, is the change in V due to $d\gamma/dt$. Noticing from (7)

$$\frac{d\gamma}{dt} = \frac{g(t)(\gamma - t)}{1 - G(t)} > 0, \tag{17}$$

and hence more entry has a negative quality effect: a decrease in t reduces the average match probability of sellers in the market, lowering consumer search efficiency. The change in consumer welfare from a marginal entrant depends on the balance of these two opposing effects. Since $t_f(k)$ is monotonically increasing, a reduction in k has the same two effects as a reduction in t_f in equilibrium.

Define $V_f \equiv V(t_f)$ as the consumer welfare in the free-entry equilibrium. We can now state our main result concerning free entry and consumer welfare. Its proof first establishes that $V(t)$ is single-peaked, and then uses the fact that t_f monotonically increases in k.

Theorem 1 *There exists some $k^* \in (0, \pi^m)$ such that, relative to what maximizes consumer welfare, the expected number of entrants under free entry is too high when $k < k^*$ and too low when $k > k^*$. Furthermore, consumer welfare is an inverted-U function of k, first increasing and then decreasing, maximized at k^*.*

Proof. From (15), utilizing (16) and (17), and noticing $\frac{1}{1-G(t)} = \frac{\gamma}{1-M(t)}$, we have

$$\begin{aligned}\frac{dV}{dt} &= \left[1 - M(t)^N\right]\left(\frac{s}{\gamma^2}\right)\frac{g(t)}{1-G(t)}(\gamma - t) - NM(t)^{N-1}g(t)\,t\left(\Phi - \frac{s}{\gamma}\right) \\ &= g(t)\left[\frac{1-M(t)^N}{1-M(t)}\frac{s}{\gamma}(\gamma - t) - NM(t)^{N-1}t\left(\Phi - \frac{s}{\gamma}\right)\right]. \end{aligned} \tag{18}$$

Therefore, for $t \in (0,1)$, $\frac{dV}{dt} = 0$ if

$$\frac{t}{\gamma} = \frac{1}{1 + NM(t)^{N-1}\frac{1-M(t)}{1-M(t)^N}\left(\frac{\gamma\Phi - s}{s}\right)}. \tag{19}$$

If $t = 0$, the LHS of (19) $<$ the RHS of (19); if $t \to 1$, the LHS of (19) $>$ the RHS of (19). Furthermore, from Lemma 1, the LHS of (19) monotonically increases in t. Since $\frac{dM(t)}{dt} \geq 0$, $\frac{d\gamma}{dt} \geq 0$, and

$$\frac{d\left(M^{N-1}\frac{1-M}{1-M^N}\right)}{dM} = \frac{M^{N-2}}{\left(1-M^N\right)^2}\left(N - NM + M^N - 1\right) = \frac{M^{N-2}(1-M)}{\left(1-M^N\right)^2}\left(N - \sum_{j=0}^{N-1} M^j\right) \geq 0,$$

the RHS of (19) decreases in t. Therefore, there exists a unique $t^* \in (0,1)$ that solves (19), with $\frac{dV}{dt} > 0$ if $t < t^*$ and $\frac{dV}{dt} < 0$ if $t > t^*$. Furthermore, since $t_f = t(k)$ is monotonically increasing and $\frac{dV_f}{dk} = \frac{dV_f}{dt_f}t'_f(k)$, V_f first increases and then decreases in k, maximized at some $k^* \in (0, \pi^m)$. Finally, since the entry threshold associated with the maximum consumer welfare is $t^* = t_f(k^*)$, relative to what maximizes consumer welfare, free entry is excessive when $k < k^*$ but deficient when $k > k^*$. ■

As t, or entry cost k, decreases, more potential entrants choose to enter the market, but the marginal entrant has a lower quality. When k is high, entry is deficient and a decrease in k benefits consumers, both because the opportunity to search an additional entrant is highly valuable when the expected number of entrants is small and because the margin entrant has a relatively high quality, so that the positive variety effect dominates.[16] When k is relatively low, entry is excessive and an increase in k benefits consumers, because in this case the negative quality effect of entry dominates.[17]

We note that search cost is crucial for our consumer welfare results in Theorem 1. As search cost approaches zero, from (15) the search quality effect vanishes so that $dV/dt < 0$; consequently both t^* and k^* approach 0, and more entrants will (almost) always benefit consumers because of the positive variety effect. Then, free entry will (almost) always be deficient from consumers' perspective, and our novel result, that there is too much equilibrium entry for consumers in search markets when entry cost is small, becomes irrelevant.[18]

The result below shows more generally how search cost and the number of potential entrants affect k^*, the entry cost that maximizes consumer welfare.

Corollary 1 k^*, *or* t^*, *increases in search cost* (s) *and in the number of potential entrants* (N).

Proof. Since $t^* = t(k^*)$, it suffices to show that t^* increases in s and in N. Since LHS of (19) increases in t and is independent of s while RHS decreases in t and increases in s, t^* increases in s. Moreover, since $M < 1$, $d\left[N \ln M - M^N + 1\right]/dM = \frac{N}{M} - NM^{N-1} > 0$,

[16] When $t \to 1$, t and γ are close. So the first term in the square bracket of (18) approaches zero, while the second term is positive and increasing in t. Hence $dV/dt < 0$ when t is high.

[17] When $t \to 0$, the first term in the square bracket of (18) approaches some positive constant, while the second term approaches zero. Hence $dV/dt > 0$ when t is low.

[18] Our analysis for Theorem 1 remains valid for arbitrarily small s, as long as it is strictly positive. If $s = 0$, there would be no pure-strategy price equilibrium, because firms would want to undercut each other to compete for consumers with multiple matches, but also to raise price to consumers with only one match. The analysis would then be very different from ours. Thus, again, costly search plays a crucial role in our model.

and $N \ln 1 - 1^N + 1 = 0$, we have

$$d\left(NM^{N-1}\frac{1-M}{1-M^N}\right)/dN = \frac{M^{N-1}}{(1-M^N)^2}(1-M)\left(N\ln M - M^N + 1\right) < 0.$$

Therefore, t^* increases in N. ■

The entry cost (or the quality threshold) that maximizes consumer welfare increases in search cost and in the number of potential entrants. Intuitively, with a high search cost, it is more costly for consumers to search more varieties. It follows that fewer sellers with higher quality tend to be better for consumers, and hence k^* (or t^*) is higher. Also, when the number of potential sellers is high, the variety effect is less significant because for a given k the expected number of entrants is large, and hence an increase in t tends to be more beneficial to consumers. Therefore, t^* also increases in the number of potential entrants.

4.2 Total Welfare

We next consider total welfare. For given k and t, the (expected) industry profit is

$$\Pi = \sum_{n=0}^{N} \lambda_n(t)\, n\left[\pi_n(\gamma) - k\right], \tag{20}$$

where $\pi_n(\gamma) - k$ is the expected profit for a seller of quality γ in a market with n sellers.[19] The result below is proved in the appendix:

Lemma 4 *For any given t, industry profit is*

$$\Pi(t) = \pi^m\left[1 - M(t)^N\right] - kN\left[1 - G(t)\right], \tag{21}$$

and the free-entry equilibrium industry profit is

$$\Pi_f = \left(1 - \frac{t_f}{\gamma}\right)\left[1 - M(t_f)^N\right]\pi^m. \tag{22}$$

Notice that $\left[1 - M(t_f)^N\right]\pi^m$ is the expected industry revenue when at least one seller's product matches a consumer's need. Since the marginal entrant with t_f earns zero profit, $1 - \frac{t_f}{\gamma}$ reflects the expected profit margin of each entrant.

[19] Note that in our model, in the free entry equilibrium only the marginal entrant earns zero profit, while the other entrants earn positive profits.

From (14) and (22), total welfare at the free entry equilibrium is

$$W_f = \left[1 - M\left(t_f\right)^N\right]\left[\left(\Phi - \frac{s}{\gamma}\right) + \left(1 - \frac{t_f}{\gamma}\right)\pi^m\right]. \tag{23}$$

Proposition 2 *In equilibrium: (i) industry profit decreases in k; (ii) total welfare decreases in k when s is sufficiently small or k is sufficiently high.*

Proof. From (22) and (23), since k affects Π_f and W_f only through t_f, and since t_f increases in k, it suffices to show that the stated relationships for k hold for t_f. (i) Recall from (7) and (16) that $\frac{d(t/\gamma)}{dt} > 0$ and $\frac{dM}{dt} \geq 0$. Thus $\frac{d\Pi_f}{dt_f} < 0$. (ii) From Proposition 1, consumer welfare decreases in t when t is high. Thus, since $\frac{d\Pi_f}{dt_f} < 0$, $W_f = V_f + \Pi_f$ must decrease in t_f when t_f is sufficiently high. Furthermore,

$$\begin{aligned}\frac{dW_f}{dt_f} &= -NM\left(t_f\right)^{N-1} g\left(t_f\right) t_f \left[\left(\Phi - \frac{s}{\gamma}\right) + \left(1 - \frac{t_f}{\gamma}\right)\pi^m\right] \\ &\quad + \left[1 - M\left(t_f\right)^N\right]\left[\frac{s}{\gamma^2}\frac{d\gamma}{dt_f} - \frac{d\left(t_f/\gamma\right)}{dt_f}\pi^m\right].\end{aligned}$$

Recall that $\frac{d\left(t_f/\gamma\right)}{dt_f} > 0$. Hence, $\frac{dW_f}{dt_f} < 0$ if $s \to 0$. ■

A marginal increase in entry cost raises t_f, which reduces the expected number of sellers, and, hence, the probability of sales. Additionally, a higher entry cost reduces an inframarginal seller's profit margin. Consequently, industry profit is reduced with a higher entry cost. On total welfare, a higher k will increase consumer welfare by raising t_f when $k < k^*$, which can potentially outweigh the profit effect. But when k is large, profit and consumer welfare move in the same direction, and hence W is lower with an even higher k. Also, when s is small, the low search cost can largely offset the reduction in sellers' quality to provide search incentives, so that the profit change will dominate and hence an increase in entry cost will lower total welfare.

Example 1 below illustrates how the equilibrium consumer welfare, industry profit and total welfare vary with entry cost k.

Example 1 *Suppose that $N = 3$, $s = 0.05$, with β_i and u being uniformly distributed on $[0, 1]$. Then, from (1) and (13), $\gamma = \frac{1+t}{2}$, $M = \frac{1+t^2}{2}$, $\Phi = \frac{1}{8}$ and $\pi^m = \frac{1}{4}$. From (5), t_f solves*

$E(\pi|t) = \frac{1}{48}t\left(4t^2 + t^4 + 7\right) = k$. From (14), $V_f = \frac{1}{320}(5t_f + 1)(1 - t_f)\left(4t_f^2 + t_f^4 + 7\right)$, $t^* = 0.497$, and $k^* = 0.083$. Moreover, from (22), $\Pi_f = \frac{1}{32}(1 - t_f)^2\left(4t_f^2 + t_f^4 + 7\right)$, and thus $W_f = \frac{1}{320}(11 - 5t_f)(1 - t_f)\left(4t_f^2 + t_f^4 + 7\right)$. In Figure 1, consumer welfare is the inverted-U curve, while both industry profit and total welfare decrease with k.

Consumer welfare - solid curve, Industry profit - dashed curve, Total welfare - dotted curve

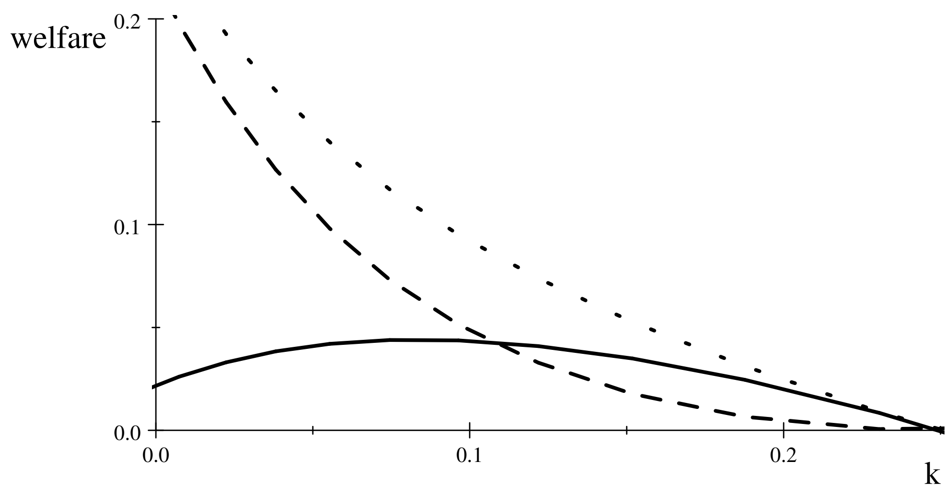

Figure 1

Now consider the socially optimal t, denoted as $t^o \equiv t^o(k)$, for which we do not impose the free-entry condition $E(\pi|t) = k$. From (21) and (14), for any given t, total welfare is given by

$$W(t) = \left(\Phi - \frac{s}{\gamma} + \pi^m\right)\left[1 - M(t)^N\right] - kN\left[1 - G(t)\right]. \tag{24}$$

Thus,

$$\frac{dW}{dt} = \frac{s}{\gamma^2}\frac{d\gamma}{dt}\left[1 - M(t)^N\right] - \left(\Phi - \frac{s}{\gamma} + \pi^m\right)NM(t)^{N-1}tg(t) + kNg(t). \tag{25}$$

At the free entry equilibrium, since the marginal entrant has zero net profit due to $E(\pi|t_f) = k$, the marginal entrant must reduce industry profit due to the business-stealing effect. From Proposition 1, for $t_f < t^* \equiv t_f(k^*)$, free entry is excessive for consumer welfare.

Therefore, when $k \leq k^*$ (or $t_f \leq t^*$), free entry must be socially excessive, with $t^o > t_f$. When $k > k^*$, entry is deficient for consumer welfare, but it can still be socially excessive when the negative profit effect is considered. However, when k is large, the profit effect is small relative to the effect on consumers, and entry is socially deficient, as we establish in the result below.

Proposition 3 *Free entry is socially excessive (i.e., $t^o > t_f$) when $k \leq k^*$, and it is socially deficient (i.e., $t^o < t_f$) when k is sufficiently large (but still smaller than π^m).*

Proof. We have already argued $t^o > t_f$ when $k \leq k^*$. It remains to show $t^o < t_f$ when k ($< \pi^m$) is sufficiently large. From the proof of Lemma 4, any $t \geq t_f$ satisfies $k \leq \frac{t}{\gamma}\pi^m \left[\frac{1-M(t)^N}{N[1-G(t)]}\right]$. Substituting this into (25), we have

$$\left.\frac{dW}{dt}\right|_{t\geq t_f} \leq \left[\frac{1-M(t)^N}{1-G(t)}\left(\frac{s(\gamma-t)}{\gamma^2}+\frac{t}{\gamma}\pi^m\right)-\left(\Phi-\frac{s}{\gamma}+\pi^m\right)NM(t)^{N-1}t\right]g(t).$$

Thus, when $k \to \pi^m$, $t \to 1$, $\gamma \to 1, M(t) \to 1$ and, from (16), $\lim_{t\to1}\frac{1-M(t)^N}{1-G(t)} = \lim_{t\to1}\frac{-N\cdot M(t)^{N-1}g(t)t}{-g(t)} = N$. Hence, the right-hand side of the above inequality approaches

$$-(\Phi-s)Ng(1) < 0.$$

Therefore, when k is sufficiently large (but still smaller than π^m), $\left.\frac{dW}{dt}\right|_{t\geq t_f} < 0$, so that free entry is socially deficient (i.e., $t^o < t_f$). ■

As in the case of consumer welfare, search cost also plays an important role for our total welfare results. As $s \to 0$ and hence $k^* \to 0$, the result in Proposition 3 that free entry is excessive for total welfare when $k \leq k^*$, while still valid, is virtually irrelevant. More entry will then increase consumer welfare due to the search variety effect but reduce industry profit due to business stealing, with the net impact on total welfare potentially ambiguous. When k ($< \pi^m$) is sufficiently high, however, Proposition 3 applies and the equilibrium number of entrants will be too small relative to the social optimum.

5. DIFFERENTIATION AMONG MATCHED SELLERS

So far, we have assumed that a consumer has the same value (u) from all of her matched sellers, even though u is ex ante uncertain to the consumer. As we mentioned earlier, one advantage of this formulation is that equilibrium price will then be invariant to the number of sellers, which substantially simplifies the analysis. We now consider an alternative setting where a consumer has heterogeneous values for sellers who match her need. Specifically, as in Wolinsky (1986), we assume that a consumer's value for each matched seller i, u_i, is independently drawn from distribution F on support $[0, \bar{u}]$, with density f.[20] Thus, there is horizontal differentiation among matched sellers.[21] Everything else is the same as in the main model.

A key aspect in which this variant differs from the main model is that entry will now also affect market price. Our analysis in this section proceeds as follows: First, we characterize the equilibrium pricing strategy given the number of active sellers (n) and their average quality (γ). Next, we show that the equilibrium market price (p_n) *decreases* in γ. This additional price effect introduces a complication to the expected profit for a seller. In particular, unlike in the main model, it is no longer clear that a potential entrant's expected profit will increase in t, because a higher t, which results in a higher average quality γ, now also leads to a lower equilibrium price. After presenting the equilibrium analysis for a given n, we will turn to numerical analysis to show that the welfare results of the main model still hold under additional functional and parameter conditions.

Suppose first that there are $n \leq N$ sellers in the market who all set price p_n.[22] Following

[20] That is, in contrast to our main model in which the values of a consumer's matched sellers are perfectly dependent, this formulation considers the other polar case where these values are independent. More realistically, the values of a consumer's matched sellers may be neither perfectly dependent nor independent; but, like others in the literature, we focus on these two polar cases for analytical tractability.

[21] Search models with horizontally differentiated sellers following Wolinsky (1986) include, for example, Anderson and Renault (1999), Armstrong, Vickers and Zhou (2009), Haan and Moraga-González (2011), and Bar-Isaac, Caruana, and Cuñat (2012).

[22] Even though sellers differ in match probabilities, a seller is either a match or no match after the search by a consumer, and the seller competes in prices only with other matched sellers for the consumer. Since

Kohn and Shavell (1974) and Wolinsky (1986), consumers' optimal search strategy is to sample sellers sequentially, with reservation value $a(\gamma)$ from matched seller i that satisfies

$$\gamma \int_a^{\bar{u}} (u_i - a) f(u_i)\, du_i = s. \tag{26}$$

Note that the market is active only when sellers are expected to charge $p_n \leq a$. A consumer stops searching when she finds a match with $u_i \geq a$; if no such product is found after she searches all sellers, she buys the product from the matched seller with the highest $u_i \geq p_n$, and she buys nothing if no match is found or if $u_i < p_n$ for all matches. Since u_i is independently and identically distributed for each of a consumer's matched sellers, for convenience we shall drop the subscript i for the rest of the section.

Total differentiation of (26) with respect to γ and rearranging terms, we have

$$\frac{\partial a}{\partial \gamma} = \frac{\int_a^{\bar{u}} (u-a) f(u)\, du}{\gamma \int_a^{\bar{u}} f(u)\, du} = \frac{s}{\gamma^2 [1 - F(a)]} > 0. \tag{27}$$

Hence, a increases with γ. That is, the benefit of search is larger if the expected quality of sellers is higher. We assume that s is sufficiently small such that consumers will indeed search in equilibrium.

Next, we characterize the condition for the equilibrium where sellers charge the same price despite differences in match probabilities.[23] If there is only one seller ($n = 1$), then it optimally charges $p_1 = p^m$. So suppose that $n \geq 2$. If other sellers charge p_n in equilibrium, given the search strategy by consumers, a seller with β_i charges p to maximize

$$\pi_{ni}(p, p_n) = p \left\{ \beta_i [1 - F(p + a - p_n)] \varphi_n + \beta_i R_n(p, p_n) \right\}, \tag{28}$$

where

$$\varphi_n = \frac{1}{n} \sum_{j=0}^{n-1} \sum_{h=0}^{j} \binom{j}{h} (1-\gamma)^h [\gamma F(a)]^{j-h} = \frac{1 - [1 - \gamma + \gamma F(a)]^n}{n\gamma [1 - F(a)]} \tag{29}$$

all matched sellers of a consumer are horizontally differentiated and are ex ante identical to the consumer, it is appropriate to consider a symmetric price equilibrium. Alternatively, one may consider a possible asymmetric price equilibrium where firms with higher match probabilities change higher prices, but it does not appear to be analytically tractable in the model here.

[23] Intuitively, all matched sellers of a consumer are horizontally differentiated as in Wolinsky (1986), and thus the equilibrium has a similar structure.

is the number of consumers who come to seller i for the first time after sampling $j \in \{0, 1, ..., n-1\}$ other sellers and finding no match or the valuation is below a, and

$$\begin{aligned} R_n(p, p_n) &= \int_p^{p+a-p_n} \left[\sum_{j=0}^{n-1} \binom{n-1}{j} (1-\gamma)^{n-1-j} [\gamma F(u-p+p_n)]^j \right] f(u)\,du \quad (30) \\ &= \int_p^{p+a-p_n} [1-\gamma+\gamma F(u-p+p_n)]^{n-1} f(u)\,du \end{aligned}$$

is the number of returning consumers who have sampled all sellers and have not found any value above a, while seller i is a match that gives the highest valuation. It follows that

$$R_n(p_n, p_n) = \int_{p_n}^{a} [1-\gamma+\gamma F(u)]^{n-1} f(u)\,du. \quad (31)$$

From the first-order condition of (28), at an equilibrium with $p_i = p_n$ for all $i = 1, ..., n$, the equilibrium p_n satisfies $\pi'_{ni} \equiv \frac{\partial \pi_{ni}}{\partial p}|_{p=p_n} = 0$:

$$[1-F(a)]\varphi_n + \int_{p_n}^{a} [1-\gamma+\gamma F(u)]^{n-1} dF(u) - p_n \{ f(a)\varphi_n - \int_{p_n}^{a} [1-\gamma+\gamma F(u)]^{n-1} df(u) \} = 0. \quad (32)$$

If $p_n = 0$, the LHS of (32) is positive. If $p_n = a$, the LHS of (32) becomes $\{[1-F(a)] - af(a)\}\varphi_n$, which is negative because $a - \frac{1-F(a)}{f(a)} > p_1 - \frac{1-F(p_1)}{f(p_1)} = 0$, where the inequality holds due to $p_1 < a$ and $\frac{1-F(a)}{f(a)} < \frac{1-F(p_1)}{f(p_1)}$. Thus there exists some $p_n \in (0, a)$ that solves (32), and p_n is given by

$$p_n = \frac{[1-F(a)]\varphi_n + \int_{p_n}^{a} [1-\gamma+\gamma F(u)]^{n-1} f(u)\,du}{f(a)\varphi_n - \int_{p_n}^{a} [1-\gamma+\gamma F(u)]^{n-1} f'(u)\,du}. \quad (33)$$

Lemma 5 below, which is proved in the appendix, states that sellers will charge p_n at any symmetric price equilibrium and provides a sufficient condition for the unique existence of such an equilibrium.[24]

Lemma 5 *At any symmetric price equilibrium of the alternative model, each seller sets p_n according to (33) and consumers search with reservation value $a(\gamma)$ that satisfies (26). If F follows a uniform distribution, then the symmetric price equilibrium exists and is unique.*

[24] Search models generally also have a trivial equilibrium where firms are expected to and indeed charge very high prices, and no consumer engages in search. As in the literature, we do not consider such trivial cases.

We next state a result on how equilibrium price may vary with the average quality of sellers in the market. The proof is also contained in the appendix.

Proposition 4 *In the alternative model where each consumer's value is independent for every match, given the number of sellers* (n)*, an increase in* γ *leads to a decrease in* p_n*.*

It may seem surprising that a higher average quality would lead to a lower market price, but in a search market this result is quite natural, for the following reason. An increase in the average quality of sellers in the market induces a higher consumer reservation value in their search decision, because the expected benefit from another search is higher. This forces sellers to lower prices in order to induce consumers to purchase without further search.[25]

For a given entry cost, when the equilibrium price is given by p_n in (33), there exists a free-entry equilibrium that is similar to the one in the main model, with the marginal entrant's quality, t_f, now defined by (34) below.

Proposition 5 *In the alternative model, suppose that the equilibrium price is given by* p_n *in (33). Then, for any* $k \in (0, \pi^m)$ *, there exists an equilibrium for the entire model where: (i) potential entrant i will enter the market if and only if* $\beta_i \geq t_f$ *, each entrant will charge* p_n*, and* t_f *satisfies*

$$\sum_{n=1}^{N} \delta_n (t_f)\, t_f p_n \frac{1 - [1 - \gamma + \gamma F(p_n)]^n}{n\gamma} = k; \tag{34}$$

(ii) consumers will search sequentially with reservation value a that satisfies (26).

Proof. For a given t and thus γ, from (28) and (36), in the symmetric equilibrium with n sellers the profit for seller i is

$$\begin{aligned} \pi_{ni} &= p_n \left\{ \beta_i [1 - F(a)] \frac{1 - [1 - \gamma + \gamma F(a)]^n}{n\gamma [1 - F(a)]} + \beta_i \int_{p_n}^{a} [1 - \gamma + \gamma F(u)]^{n-1} f(u)\, du \right\} \\ &= \beta_i p_n \frac{1 - [1 - \gamma + \gamma F(p_n)]^n}{n\gamma}. \end{aligned}$$

[25] Anderson and Renault (1999) study the effect of consumer taste for diversity on equilibrium price in a search model. They show that, when the preference for diversity is low, equilibrium price may decrease as the preference for diversity increases due to the increased consumer search.

Thus, the expected post-entry profit for entrant i is $E(\pi|\beta_i) = \sum_{n=1}^{N} \delta_n(t)\,\pi_{ni}$, which increases in β_i. For the seller with match probability t, its expected profit from entry is

$$E(\pi|t) = \sum_{n=1}^{N} \delta_n(t)\, t p_n \frac{1 - [1 - \gamma + \gamma F(p_n)]^n}{n\gamma},$$

which is a continuous function of t. Since the marginal entrant with $t = 0$ has zero profit, and the marginal entrant with $t = 1$ has profit π^m, for any $k \in (0, \pi^m)$, $E(\pi|0) < k$ and $E(\pi|1) > k$. Therefore, there exists some $t_f \in [0, 1)$ such that $E(\pi|t_f) = k$. That is, given k, there exists some t_f such that potential entrants with $\beta_i \geq t_f$ will enter. Finally, from Proposition 5, the pricing strategy and consumer search behavior are optimal when there are n sellers. ■

Different from the main model, here we have not proven that t_f is an increasing function of k. The complication is that, as t increases, equilibrium price decreases and thus the impact on the expected profit of the marginal seller with quality t is unclear.[26] For the rest of this section, we assume that (i) $N = 3$ and (ii) F and G are both uniform distributions on $[0, 1]$.

Then, it can be verified numerically that t_f increases in k for various values of s. Furthermore, consumer welfare initially increases but eventually decreases in t. The intuition is similar as in the main model: a lower t_f leads to a higher expected number of sellers in the market but to a lower sellers' average quality. The increase in variety benefits consumers by expanding their search opportunities, whereas the decrease in quality harms consumers by reducing their search efficiency. However, here price is also affected, in two opposing directions: greater variety acts to reduce equilibrium prices, whereas lower quality works in the opposite direction as consumers search less due to the lower search benefit. Nevertheless, as in the main model, when t_f is high, and thus the number of active sellers is low, the variety effect tends to dominate, so that a further increase in t_f results in lower consumer welfare. On the other hand, when t_f is low, the quality effect tends to dominate, so that an increase in t_f results in higher consumer welfare. Since t_f increases in k, it follows that

[26] Recall that in the main model, equilibrium price is independent of γ.

consumer welfare also first increases and then decreases in k. Figure 2 below shows how consumer welfare varies with k for three different values of search cost.

$s = 0.05$ (solid curve), $s = 0.04$ (dashed curve), $s = 0.03$ (dotted curve)

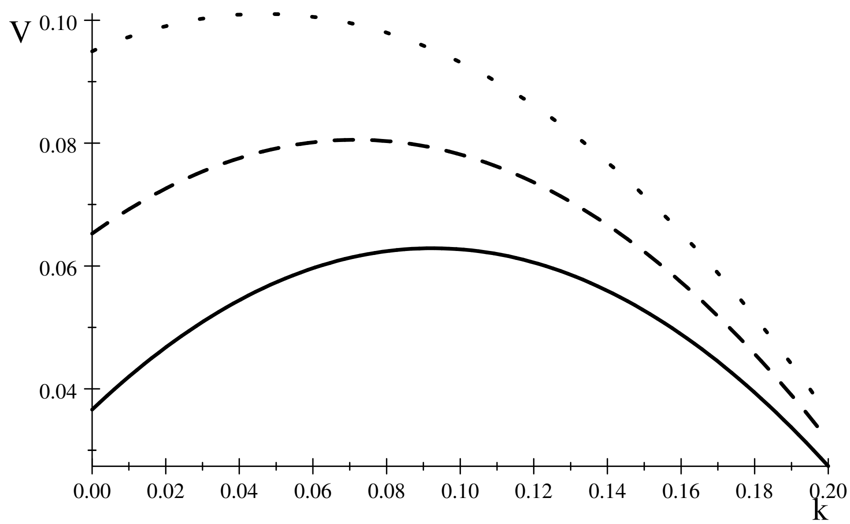

Figure 2

For a given s, let k^* be the entry cost that maximizes consumer welfare. Then, same as in the main model (Theorem 1), from the standpoint of consumer welfare entry is excessive when $k \leq k^*$ but deficient when $k > k^*$. Notice from Figure 2 that for a higher s, k^* (or t^*) is higher, also same as in the main model (Corollay 1). This is because with a higher search cost, the option to search more varieties is less valuable while the lower quality of sellers (or lower search efficiency) is more detrimental to consumers.

Consider next total welfare, W. Same as in the main model (Proposition 3), when $k \leq k^*$, entry is socially excessive, again because of the overentry for consumers and the additional negative effect on industry profit of the marginal entrant. Furthermore, when k is sufficiently large (but still less than π^m), entry can be socially deficient (as in the main model, too). For instance, if $s = 0.03$ and $k = 0.2$, we can compute that the quality cutoff is $t_f = 0.9$ under free entry but $t^o = 0.823$ for the social optimum, and the intuition is also similar to that in the main model: since $k = 0.2 > k^* \simeq 0.045$, entry is too low from consumers' perspective, and although the marginal entrant has the negative externality on industry profit, the consumer effect dominates the profit effect when k is large, despite the price effect of entry in this case.

Therefore, the results of our main model continue to hold in this alternative setting, under additional functional and parameter restrictions.[27]

While our most novel result is about the effects of entry on consumer welfare, our finding about the total welfare effects of entry also differs from those in several closely-related papers. In particular, Wolinsky (1984) studies the optimality of entry in a circle model with consumer search. Similar to our main model, entry has no price effect in his model, where for simplicity he assumes that the price is exogenously given. He reports an overentry result: the market will offer excessive variety from the standpoint of total welfare when entry cost is sufficiently low. This is because in his model the socially optimal variety is bounded: when the number of varieties is sufficiently high (or the entry cost is sufficiently low), consumers will find a brand satisfying the sequential search stopping rule and not search further. In this case, an extra entrant will not benefit consumers and, consequently, will reduce total welfare due to the negative externality on other sellers' profits.

Our models also predict that entry is socially excessive when the entry cost is sufficiently small, but in our case entry can also be insufficient when the entry cost is relatively large. In both of our models, overentry in terms of total welfare occurs under low entry costs because the marginal entrant reduces the average quality of sellers in the market, leading to lower search efficiency (reenforcing the negative externality on profits); whereas deficient entry can arise under high entry costs because consumers benefit from more search opportunities to find a match, which can overcome the negative externality on profits.

Our model with horizontal differentiation in this section is more closely related to Anderson and Renault (1999), which studies a standard consumer search model with differentiated products. They find that market entry is always excessive, because in their model entry is

[27] We have also studied the case where $N = 2$, with F and G being standard uniform. The results are qualitatively the same.

excessive in the limiting case of zero search cost, and the presence of positive search costs exacebate the distortion: as search cost rises, equilibrium prices and the number of entrants increase, while the socially optimal number of firms falls. One may then wonder why in our model entry can be insufficient or excessive. The key difference is that in our model firms have different match probabilities that are all less than 1, whereas in theirs these match probabilities are the same and equal to 1. The difference in match probabilities (i.e., the vertical differentiation) in our model creates a search quality effect of entry: the marginal entrant lowers the expected product quality in the market so that entry can be excessive for consumers, and excessive entry for total welfare is still more likely due to the marginal entrant's negative effect on industry profit. On the other hand, in our model since the sellers' match probabilities are all less than 1, entry also has a positive output expansion effect (the marginal entrant may be a match for some consumers who would otherwise find no match), which explains why there can be underentry in the market equilibrium.

If $s = 0$, it is now possible that a pure-strategy price equilibrium exists, because the matched sellers of a consumer are now horizontally differentiated. The equilibrium price would be higher than in a standard model of product differentiation, because here each seller also has a captured mass of consumers who have no other match. Entry would still have a positive variety effect, but would no longer have the negative quality effect, so it would always benefit consumers. For total welfare, however, the consumer benefits would need to be balanced against the negative externality on industry profit, and thus entry could be socially excessive. If all sellers had the same match probability that also approaches 1, then the positive variety effect of entry also vanishes, and we would expect entry to be socially excessive, same as in the limiting case of $s = 0$ in Anderson and Renault (1999).

We have assumed $g(\cdot) > 0$ on $[0, 1]$, so that firms always differ in match probabilities. If all firms had the same match probability, then the negative quality externality from a marginal entrant would disappear, but the positive variety effect remains if the match probability is less than 1. Since the marginal entrant still has the negative externality on industry profits, entry may still be either deficient or excessive for total welfare. However, if all firms had the same match probability that also approaches 1, then the positive variety effect also vanishes, and we would expect entry to become socially excessive, same as in Anderson and Renault (1999).

6. CONCLUDING REMARKS

In parallel to how free entry may lead to social inefficiency when firms possess market power, this paper has shown that unfettered entry can be detrimental to consumers when they have imperfect information about sellers' quality. If we extend Akerlof (1970)'s classic model of adverse selection by adding consumer search to it, low quality sellers may no longer drive out high quality ones—in fact, a high quality seller will have more incentive to enter the market. But there is a different form of market failure: there can be either excessive or deficient entry, for both consumers and the society, because an entrant internalizes neither the search variety nor the search quality effect that it generates. In contrast to the standard view in economics, in our model free entry is excessive for consumers when entry cost is relatively small, and consumer welfare has an inverted-U relationship with entry cost.

To illustrate our idea most transparently, we have abstracted from various market institutions that respond to the information problem and potentially improve the search variety vs. quality trade-off. For example, firms may engage in costly advertising to convey quality information to consumers. While advertising cost has often been viewed as a barrier to competition, it may actually improve search quality and benefit consumers by deterring low quality entrants.[28] Also, market intermediaries can simultaneously lower the number of entrants and raise their average quality. Various accreditation agencies can serve this purpose, as, for instance, the accreditation of business schools could potentially help applicants search for the right MBA programs. An Internet platform may prominently display sellers who are more likely to meet consumers' needs, based on either organic search results or paid placement, as is done by the three largest search engines (Google, Yahoo! and Microsoft Bing). This can enhance consumer search efficiency, but also raise entry hur-

[28] For simplicity, we have assumed that consumers have no prior information about the match probability of a seller. More realistically, consumers may have certain information about the qualities of different firms before conducting search, possibly through observing their ads or prices, or through repeat purchases.

dles for less relevant sellers. Moreover, the organization of firms may also be motivated by such considerations. For instance, a hotel chain under a brand name may impose certain quality standards on its member hotels, and a merger between two firms might enable the merged firm to offer products that better meet consumer needs,[29] both of which could help consumer search. To the extent that antirust and regulation can influence these business practices, it would be important for policy makers to recognize their beneficial roles.

Policies may also impact consumer welfare directly by either facilitating or impeding entry. However, since it is unlikely that a policy maker will know the precise entry cost or entry scale that would be optimal for consumers, it is not obvious that government intervention would improve market outcomes, especially given the institutional arrangements that the market itself can make, as discussed above. Nevertheless, policies such as a minimum safety standard or truth-in-advertising regulation will likely improve search efficiency and benefit consumers.

APPENDIX

The Appendix contains proofs for Lemmas 1, 2, 4, 5 and Proposition 4.

Proof of Lemma 1. (i) From (1), for all $t \in [0, 1)$,

$$\frac{d\gamma}{dt} = \frac{-tg(t)[1 - G(t)] + g(t)\int_t^1 xg(x)\,dx}{[1 - G(t)]^2} = \frac{g(t)}{1 - G(t)}(\gamma - t) > 0.$$

(ii) Since

$$\frac{d(t/\gamma)}{dt} = \frac{1}{\gamma^2}\left(\gamma - t\frac{d\gamma}{dt}\right) = \frac{1}{\gamma^2}\left[\gamma - \frac{g(t)\,t(\gamma - t)}{1 - G(t)}\right] = \frac{\mu(t)}{\gamma^2}, \tag{35}$$

where $\mu(t) \equiv \gamma - \frac{g(t)t(\gamma - t)}{1 - G(t)}$, to prove $\frac{d(t/\gamma(t))}{dt} > 0$, it suffices to show $\mu(t) > 0$ for all $t \in [0, 1)$.

Notice that $\mu(0) = \gamma > 0$. Also, since

$$\lim_{t\to 1}\frac{d\gamma}{dt} = \lim_{t\to 1}\frac{g(t)(\gamma - t)}{1 - G(t)} = g(1)\lim_{t\to 1}\frac{(\gamma - t)}{1 - G(t)} = g(1)\frac{\lim_{t\to 1}\frac{d\gamma}{dt} - 1}{-\lim_{t\to 1}g(t)} = 1 - \lim_{t\to 1}\frac{d\gamma}{dt},$$

[29] Moraga-González and Petrikaite (2013) point out that mergers can benefit consumers by reducing consumer search costs, when products from the merging firms are sold in one store after the merger. This "demand-side" economies of mergers, arising from the reduction of consumer search costs, differ from the "supply-side" benefit of mergers through the increase of product quality.

we have $\lim_{t\to 1}\frac{d\gamma}{dt}=\frac{1}{2}$. It follows that

$$\lim_{t\to 1}\mu(t)=1-g(1)\frac{\lim_{t\to 1}\frac{d\gamma}{dt}-1}{-g(1)}=\frac{1}{2}.$$

Now, suppose to the contrary that $\mu(t)\leq 0$ for some $t\in(0,1)$. Then there must exist at least one $\hat{t}\in(0,1)$ such that $\mu\left(\hat{t}\right)=0$ and $\mu'\left(\hat{t}\right)>0$. Our proof will be complete if we can show that this leads to a contradiction.

Rewrite $\mu(t)=(\gamma-t)\left[\frac{\gamma}{\gamma-t}-t\frac{g(t)}{1-G(t)}\right]$, then

$$\mu'(t)=\left(\frac{d\gamma}{dt}-1\right)\left[\frac{\gamma}{\gamma-t}-t\frac{g(t)}{1-G(t)}\right]+(\gamma-t)\left[\frac{\frac{1}{\gamma^2}\mu(t)}{\left(1-\frac{t}{\gamma}\right)^2}-\frac{d\left(\frac{tg(t)}{1-G(t)}\right)}{dt}\right].$$

But for any $\hat{t}\in(0,1)$ such that $\mu\left(\hat{t}\right)=0$, $\left[\frac{\gamma}{\gamma-\hat{t}}-\hat{t}\frac{g(\hat{t})}{1-G(\hat{t})}\right]=0$, and thus

$$\mu'\left(\hat{t}\right)=-\left(\gamma\left(\hat{t}\right)-\hat{t}\right)\left.\frac{d\left(\frac{tg(t)}{1-G(t)}\right)}{dt}\right|_{t=\hat{t}}\leq 0$$

because $d\left(\frac{g(t)}{1-G(t)}\right)/dt\geq 0$ by the log-concavity of $1-G$. This is a contradiction. ■

Proof of Lemma 2. First, we show that $\sum_{n=1}^{l}\delta_n(t)$ increases in t for $l=1,2,...,N$. Integrating by parts, we have

$$\begin{aligned}
&\frac{(N-1)!}{(N-1-l)!\,(l-1)!}\int_0^{G(t)}\left(\xi^{N-1-l}\right)(1-\xi)^{l-1}\,d\xi\\
=\;&\frac{(N-1)!}{(N-l)!\,(l-1)!}\int_0^{G}(1-\xi)^{l-1}\,d\left(\xi^{N-l}\right)\\
=\;&\frac{(N-1)!}{(N-l)!\,(l-1)!}(1-G)^{l-1}G^{N-l}+\frac{(N-1)!}{(N-l)!\,(l-2)!}\int_0^{G}\xi^{N-l}(1-\xi)^{l-2}\,d\xi.
\end{aligned}$$

Repeatedly performing integration by parts for $\int_0^G\xi^{N-l}(1-\xi)^{l-2}\,d\xi$, $\int_0^G\xi^{N-l+1}(1-\xi)^{l-3}\,d\xi$, and so on, we obtain:

$$\begin{aligned}
&\frac{(N-1)!}{(N-1-l)!\,(l-1)!}\int_0^{G(t)}\left(\xi^{N-1-l}\right)(1-\xi)^{l-1}\,d\xi\\
=\;&\sum_{n=1}^{l}\frac{(N-1)!}{(n-1)!\,(N-n)!}(1-G(t))^{n-1}G(t)^{N-n}=\sum_{n=1}^{l}\delta_n(t).
\end{aligned}$$

Since $\int_0^{G(t)} \left(\xi^{N-1-l}\right)(1-\xi)^{l-1} d\xi$ increases in $G(t)$, which in turn increases in t, $\sum_{n=1}^{l} \delta_n(t)$ increases in t.

Then, for any $t' > t$, recalling $\pi_n(t') > \pi_n(t)$ and $\pi_n(t)$ decreases in n, we have

$$\begin{aligned} E(\pi|t') - E(\pi|t) &= \sum_{n=1}^{N} \delta_n(t')\,\pi_n(t') - \sum_{n=1}^{N} \delta_n(t)\,\pi_n(t) > \sum_{n=1}^{N} \left[\delta_n(t') - \delta_n(t)\right]\pi_n(t) \\ &\geq \sum_{n=1}^{N} \left[\delta_n(t') - \delta_n(t)\right]\pi_N(t) = \left[\sum_{n=1}^{N} \delta_n(t') - \sum_{n=1}^{N} \delta_n(t)\right]\pi_N(t) > 0. \end{aligned}$$

Hence, $E(\pi|t)$ increases in t. ■

Proof of Lemma 3. From (12), consumer surplus when n sellers are active is

$$\begin{aligned} V_n &= \left[\frac{1-(1-\gamma)^n}{\gamma}\right]\gamma\int_{p^m}^{1}(u-p^m)\,f(u)\,du - \left[\frac{1-(1-\gamma)^n}{\gamma^2} - \frac{n(1-\gamma)^n}{\gamma}\right]\gamma s - (1-\gamma)^n ns \\ &= \left[\frac{1-(1-\gamma)^n}{\gamma}\right]\left[\gamma\int_{p^m}^{1}(u-p^m)\,f(u)\,du - s\right] = \left[1-(1-\gamma)^n\right]\left(\Phi - \frac{s}{\gamma}\right), \end{aligned}$$

where we have used the fact that $\sum_{i=1}^{n} x^{i-1} i = \frac{1-x^n}{(1-x)^2} - \frac{nx^n}{1-x}$. Hence, from (10), consumer welfare is

$$\begin{aligned} V &= \left(\Phi - \frac{s}{\gamma}\right)\left(\sum_{n=1}^{N}\binom{N}{n}\left[1-G(t)\right]^n G(t)^{N-n} - \sum_{n=1}^{N}\binom{N}{n}\left[1-G(t)\right]^n G(t)^{N-n}(1-\gamma)^n\right) \\ &= \left\{1-\left[1-\gamma(1-G(t))\right]^N\right\}\left(\Phi - \frac{s}{\gamma}\right) = \left[1-M(t)^N\right]\left(\Phi - \frac{s}{\gamma}\right). \end{aligned}$$

■

Proof of Lemma 4. Given that there are n sellers and each seller's expected match probability is γ, the expected industry profit is

$$n\pi_n(\gamma) = \pi^m\left[1-(1-\gamma)^n\right].$$

Then, from (20) and (11),

$$\begin{aligned} \Pi(t) &= \pi^m \sum_{n=0}^{N} \lambda_n(t)\left[1-(1-\gamma)^n\right] - k\sum_{n=0}^{N}\lambda_n(t)\,n \\ &= \pi^m\left\{1-\sum_{n=0}^{N}\frac{N!}{n!\,(N-n)!}\left[1-G(t)\right]^n G(t)^{N-n}(1-\gamma)^n\right\} - kN\left[1-G(t)\right] \\ &= \pi^m\left[1-M(t)^N\right] - kN\left[1-G(t)\right]. \end{aligned}$$

Moreover, from (9), under free-entry t_f satisfies

$$
\begin{aligned}
k &= \frac{t\pi^m}{\gamma}\sum_{n=1}^{N}\binom{N-1}{n-1}[1-G(t)]^{n-1}G(t)^{N-n}\frac{1-(1-\gamma)^n}{n} \\
&= \frac{t\pi^m}{\gamma}\left\{\sum_{n=1}^{N}\frac{(N-1)!}{n!(N-n)!}[1-G(t)]^{n-1}G(t)^{N-n}-\sum_{n=1}^{N}\frac{(N-1)!}{n!(N-n)!}[1-G(t)]^{n-1}G(t)^{N-n}(1-\gamma)^n\right\} \\
&= \frac{t\pi^m}{\gamma}\left[\frac{1-G(t)^N}{N[1-G(t)]}-\frac{M(t)^N-G(t)^N}{N[1-G(t)]}\right]=\frac{t}{\gamma}\pi^m\left[\frac{1-M(t)^N}{N[1-G(t)]}\right].
\end{aligned}
$$

Therefore, the free-entry equilibrium industry profit is

$$
\begin{aligned}
\Pi_f &= \sum_{n=0}^{N}\lambda_n(t_f)\,n\,[\pi_n(\gamma)-k]=\pi^m\sum_{n=1}^{N}\lambda_n(t_f)[1-(1-\gamma)^n]-N[1-G(t_f)]k \\
&= \pi^m\left[1-M(t_f)^N\right]-\frac{t_f}{\gamma}\left[1-M(t_f)^N\right]\pi^m \\
&= \left(1-\frac{t_f}{\gamma}\right)\left[1-M(t_f)^N\right]\pi^m.
\end{aligned}
$$

■

Proof of Lemma 5. First, from our argument leading to (33), at any symmetric price equilibrium of the alternative model, each seller sets p_n according to (33) and consumers search with reservation value $a(\gamma)$ that satisfies (26).

Next, we show that p_n, defined by (33), is unique, which would be the case if $\pi_{ni}(p_n,p_n)$ is a strictly concave function of p_n, or

$$
\pi''_{ni}=-[1-\gamma+\gamma F(p_n)]^{n-1}\left[f(p_n)+p_nf'(p_n)\right]-\{f(a)\varphi_n-\int_{p_n}^{a}[1-\gamma+\gamma F(u)]^{n-1}f'(u)\,du\}
$$

is negative. From (29), $\varphi_n=\frac{1}{n}\sum_{j=0}^{n-1}[1-\gamma+\gamma F(a)]^j\geq[1-\gamma+\gamma F(a)]^{n-1}$. Hence

$$
\begin{aligned}
&\int_{p_n}^{a}[1-\gamma+\gamma F(u)]^{n-1}f'(u)\,du \\
\leq\ & [1-\gamma+\gamma F(a)]^{n-1}\int_{p_n}^{a}f'(u)\,du=[1-\gamma+\gamma F(a)]^{n-1}[f(a)-f(p_n)]\leq f(a)\varphi_n.
\end{aligned}
$$

Therefore, $\pi''_{ni}<0$ if $f(p_n)+p_nf'(p_n)\geq 0$. When $f'\geq 0$, clearly $\pi''_{ni}<0$.

Finally, we show that $\pi_{ni}(p, p_n)$ is strictly concave in p if F follows a uniform distribution, and hence p_n indeed defines a unique symmetric price equilibrium. From (28),

$$\begin{aligned}
\frac{\partial \pi_{ni}^2(p,p_n)}{\partial p^2} &= \beta_i \left\{ 2\left[-f(p+a-p_n)\varphi_n + \frac{\partial R_n(p,p_n)}{\partial p} \right] + p\left[-f'(p+a-p_n)\varphi_n + \frac{\partial^2 R_n(p,p_n)}{\partial p^2} \right] \right\} \\
&= \beta_i \left\{ -\left[2f(p+a-p_n) + f'(p+a-p_n)p \right]\varphi_n + 2\frac{\partial R_n(p,p_n)}{\partial p} + p\frac{\partial^2 R_n(p,p_n)}{\partial p^2} \right\} \\
&= \beta_i \left\{ \begin{array}{c} -\left[2f(p+a-p_n) + f'(p+a-p_n)p\right]\varphi_n \\ + \int_p^{p+a-p_n} \left[1-\gamma+\gamma F(u-p+p_n)\right]^{n-1} \left[2f'(u) + pf''(u)\right] du \end{array} \right\},
\end{aligned}$$

where the last equality follows because

$$\begin{aligned}
\frac{\partial R_n(p,p_n)}{\partial p} &= \frac{\partial \left(\int_p^{p+a-p_n} \left[1-\gamma+\gamma F(u-p+p_n)\right]^{n-1} f(u)\, du \right)}{\partial p} \\
&= \frac{\partial \left(\int_{p_n}^{a} \left[1-\gamma+\gamma F(v)\right]^{n-1} f(v+p-p_n)\, dv \right)}{\partial p} \\
&= \int_{p_n}^{a} \left[1-\gamma+\gamma F(v)\right]^{n-1} f'(v+p-p_n)\, dv \\
&= \int_p^{p+a-p_n} \left[1-\gamma+\gamma F(u-p+p_n)\right]^{n-1} f'(u)\, du
\end{aligned}$$

and, similarly,

$$\frac{\partial R_n^2(p,p_n)}{\partial p^2} = \int_p^{p+a-p_n} \left[1-\gamma+\gamma F(u-p+p_n)\right]^{n-1} f''(u)\, du.$$

Clearly, if F follows a uniform distribution, then $f' = 0$ and $\partial \pi_{ni}^2(p,p_n)/\partial p^2 < 0$. ■

Proof of Proposition 4. From (32), since $\frac{\partial \alpha}{\partial \gamma} > 0$ and $\pi_{ni}'' < 0$,

$$\frac{dp_n}{d\gamma} = -\frac{\frac{\partial \pi_{ni}'}{\partial \gamma} + \frac{\partial \pi_{ni}'}{\partial a}\frac{\partial \alpha}{\partial \gamma}}{\pi_{ni}''} < 0 \text{ if } \frac{\partial \pi_{ni}'}{\partial \gamma} < 0 \text{ and } \frac{\partial \pi_{ni}'}{\partial a} < 0.$$

First, π_{ni}' in (32) can be rewritten as

$$\begin{aligned}
\pi_{ni}' &= \left[1-F(a)\right]\varphi_n + \int_{p_n}^{a} \left[1-\gamma+\gamma F(u)\right]^{n-1} dF(u) - p_n \left\{ f(a)\varphi_n - \int_{p_n}^{a} \left[1-\gamma+\gamma F(u)\right]^{n-1} df(u) \right\} \\
&= \left[1-F(a)-p_n f(a)\right]\varphi_n + \int_{p_n}^{a} \left[1-\gamma+\gamma F(u)\right]^{n-1} \left[f(u)+p_n f'(u)\right] du.
\end{aligned}$$

From (29), $\varphi_n = \frac{1}{n}\sum_{j=0}^{n-1} \left[1-\gamma+\gamma F(a)\right]^j$. Hence, $\frac{\partial \varphi_n}{\partial \gamma} < 0$ and thus $\frac{\partial \pi_{ni}'}{\partial \gamma} < 0$.

Second, substituting φ_n from (29), we have

$$\begin{aligned} & [1-F(a)]\,\varphi_n + \int_{p_n}^{a} [1-\gamma+\gamma F(u)]^{n-1} f(u)\,du \\ = \;& [1-F(a)]\,\frac{1-[1-\gamma+\gamma F(a)]^n}{n\gamma\,[1-F(a)]} + \frac{1}{n\gamma}\left\{[1-\gamma+\gamma F(a)]^n - [1-\gamma+\gamma F(p_n)]^n\right\} \\ = \;& \frac{1-[1-\gamma+\gamma F(p_n)]^n}{n\gamma}. \end{aligned} \tag{36}$$

Thus, Letting $x \equiv 1-\gamma+\gamma F(a)$, we have

$$\gamma\pi'_{ni} = \frac{1-[1-\gamma(1-F(p_n))]^n}{n} - p_n\left\{\frac{f(a)(1-x^n)}{n[1-F(a)]} - \gamma\int_{p_n}^{a}[1-\gamma(1-F(u))]^{n-1}\,df(u)\right\},$$

$$\begin{aligned} \frac{\gamma}{p_n}\frac{\partial\pi'_{ni}}{\partial a} = \;& [1-\gamma+\gamma F(a)]^{n-1}\gamma f(a)\frac{f(a)}{1-F(a)} - \frac{1}{n}\left[1-[1-\gamma+\gamma F(a)]^n\right]\frac{d\left(\frac{f(a)}{1-F(a)}\right)}{da} \\ & +\gamma\,[1-\gamma+\gamma F(a)]^{n-1} f'(a) \end{aligned}$$

$$\begin{aligned} = \;& \frac{1}{n}\left\{\gamma n x^{n-1}\frac{f(a)^2 + f'(a)[1-F(a)]}{1-F(a)} - \frac{1-x^n}{1-F(a)}\frac{f'(a)[1-F(a)]+f^2(a)}{[1-F(a)]}\right\} \\ = \;& \frac{1}{n}\frac{f'(a)[1-F(a)]+f^2(a)}{1-F(a)}\left[\gamma n x^{n-1} - \frac{1-x^n}{1-F(a)}\right] \\ = \;& \frac{\gamma}{n}\frac{f'(a)[1-F(a)]+f^2(a)}{1-F(a)}\left[n x^{n-1} - \frac{1-x^n}{1-x}\right] < 0 \text{ for } x \in (0,1) \end{aligned}$$

since $\frac{1-x^n}{1-x} = \sum_{j=0}^{n-1} x^j > n x^{n-1}$ for $x \in (0,1)$. Hence, $\frac{\partial \pi'_{ni}}{\partial a} < 0$. ■

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