Analysis of tax effects on household debts of a nation in a monetary union

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Abstract

Unlike many theoretical analysis of tax effects on household debts in a monetary union, this paper builds up analysis from a household budget constraint, instead of starting from a model. By a monetary union, it is assumed that all nations in the union share same currency. The size of tax multiplier is analyzed.

1 Budget Constraint Analysis

A nation being analyzed is in a monetary union with some other nations. Thus, inside these nations, there is no exchange rate mechanism. For simplification, there are only consumption goods in an economy, without any capital goods. The nation faces the following households budget constraint, assuming such an emergent budget constraint exists:

$$P_tC_t + T_t + \frac{B_t}{R_t} \leq W_tN_t + \Pi_t + B_{t-1} \tag{1}$$

where P is price level, C is consumption, T is net taxes, W is nominal wage, $R_t - 1$ is nominal interest rate, Π_t is firms' profits all distributed as dividends, X_t is net export. B_t is net one-time bond holding, with $B_t < 0$ implying net indebtedness. It will be assumed that the agents in the economy do not hold any bond for simplification purposes. It will be assumed for rest of analysis that $T \geq 0$ with assumption of zero government spending. Also, for simplification, import M will be assumed to be zero, and all nations are assumed to be in a monetary union. $P > 0$ for an obvious reason. B_{t-1} is assumed to be given.
Assuming that the markets clear, $W_tN_t + \Pi_t = P_t\left(C_t + X_t\right)$. Thus, Equation 1 becomes with equality:

$$P_tC_t + T_t + \frac{B_t}{R_t} = P_t\left(C_t + X_t\right) + B_{t-1} \tag{2}$$

Thus,

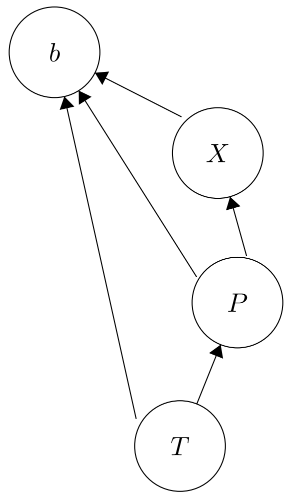
$$T_t + \frac{B_t}{R_t} = P_tX_t + B_{t-1} \tag{3}$$

$$\frac{T_t}{P_t} + \frac{B_t}{P_t R_t} = X_t + \frac{B_{t-1}}{P_t} \tag{4}$$

Let us define $b_t = \frac{B_t}{P_t R_t}$.

$$b_t = X_t + \frac{B_{t-1}}{P_t} - \frac{T_t}{P_t} \tag{5}$$

Define relationships as in the Figure:



The above diagram shows that $b = b(X, P, T)$, $X = X(P)$, $P = P(T)$. The underlying idea is that increase or decrease in taxes affect P, exports are assumed to only depend on price of goods - which is a reasonable assumption given that all export demands are honored, that all nations are in a monetary union, and that quality of goods or technology does not suddenly improve solely by increasing taxes, and inverse net indebtedness obviously depends on X, P, T. Thus,

$$\frac{db_t}{dT_t} = \frac{\partial b_t}{\partial X_t}\frac{\partial X_t}{\partial P_t}\frac{\partial P_t}{\partial T_t} + \frac{\partial b_t}{\partial P_t}\frac{\partial P_t}{\partial T_t} + \frac{\partial b_t}{\partial T_t} \tag{6}$$

Recall Equation 5:

$$b_t = X_t + \frac{B_{t-1}}{P_t} - \frac{T_t}{P_t}$$

$$\frac{\partial b_t}{\partial T_t} = -\frac{1}{P_t} \tag{7}$$

$$\frac{\partial b_t}{\partial X_t} = 1 \tag{8}$$

$$\frac{\partial b_t}{\partial P_t} = -\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} \tag{9}$$

Thus,

$$\frac{db_t}{dT_t} = \left[-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} + \frac{\partial X_t}{\partial P_t}\right]\frac{\partial P_t}{\partial T_t} - \frac{1}{P_t} \tag{10}$$

It is assumed that $\frac{\partial X_t}{\partial P_t} < 0$ and for our interests, $T_t \geq 0$.

- $\frac{\partial P_t}{\partial T_t} < 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} > -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$. Then, $\frac{db_t}{dT_t} < 0$. Real value of debts increase when taxes are raised.
- $\frac{\partial P_t}{\partial T_t} < 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} < -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$.
  Also, $\left[-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} + \frac{\partial X_t}{\partial P_t}\right] \frac{\partial P_t}{\partial T_t} < \frac{1}{P_t}$. Then still $\frac{db_t}{dT_t} < 0$.
- $\frac{\partial P_t}{\partial T_t} < 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} < -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$.
  Also, $\left[-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} + \frac{\partial X_t}{\partial P_t}\right] \frac{\partial P_t}{\partial T_t} > \frac{1}{P_t}$. Then, $\frac{db_t}{dT_t} > 0$.
- If $\frac{\partial P_t}{\partial T_t} = 0$, then $\frac{db_t}{dT_t} < 0$.
- $\frac{\partial P_t}{\partial T_t} > 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} < -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$. Then, $\frac{db_t}{dT_t} < 0$.
- $\frac{\partial P_t}{\partial T_t} > 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} > -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$.
  Also, $\left[-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} + \frac{\partial X_t}{\partial P_t}\right] \frac{\partial P_t}{\partial T_t} < \frac{1}{P_t}$. Then still $\frac{db_t}{dT_t} < 0$.
- $\frac{\partial P_t}{\partial T_t} > 0$, $-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} > -\frac{\partial X_t}{\partial P_t}$ at initial $X_t, P_t, T_t, B_t, B_{t-1}$.
  Also, $\left[-\frac{B_{t-1}}{{P_t}^2} + \frac{T_t}{{P_t}^2} + \frac{\partial X_t}{\partial P_t}\right] \frac{\partial P_t}{\partial T_t} > \frac{1}{P_t}$. Then, $\frac{db_t}{dT_t} > 0$.

Now, let us change Equation 5 into:

$$b_t = X_t + \frac{B_{t-1}}{P_t} - t_{r,t} \tag{11}$$

where $t_{r,t} = T_t/P_t$, real taxes. $b = b(X, P, t_r)$, $X = X(P)$, $P = P(t_r)$.

$$\frac{db_t}{dt_{r,t}} = \frac{\partial b_t}{\partial X_t}\frac{\partial X_t}{\partial P_t}\frac{\partial P_t}{\partial t_{r,t}} + \frac{\partial b_t}{\partial P_t}\frac{\partial P_t}{\partial t_{r,t}} + \frac{\partial b_t}{\partial t_{r,t}} \tag{12}$$

$$\frac{\partial b_t}{\partial t_{r,t}} = -1 \tag{13}$$

$$\frac{\partial b_t}{\partial X_t} = 1 \tag{14}$$

$$\frac{\partial b_t}{\partial P_t} = -\frac{B_{t-1}}{{P_t}^2} \tag{15}$$

$$\frac{db_t}{dt_{r,t}} = \left[-\frac{B_{t-1}}{{P_t}^2} + \frac{\partial X_t}{\partial P_t}\right]\frac{\partial P_t}{\partial t_{r,t}} - 1 \tag{16}$$

Simplify as:

$$\frac{db_t}{dt_{r,t}}|_{P_i,t_i,X_i} = \left[-\frac{b_{t-1}}{P_i} + \gamma\right]\lambda - 1 \tag{17}$$

where $\gamma = \frac{\partial X_t}{\partial P_t}|_{P_i,X_i}$, $\lambda = \frac{\partial P_t}{\partial t_{r,t}}{}_{P_i,t_i}$ and P_i, t_i, X_i represent initial equilibrium points. $b_{t-1} = B_{t-1}/P_i$.
Thus, assuming $\gamma < 0$:

- If $-b_{t-1} > -P_i\gamma$ and $\lambda < 0$, then $db_t/dt_{r,t} < -1$.
- If $-b_{t-1} > -P_i\gamma$ and $\lambda > 0$, then $db_t/dt_{r,t} > -1$.
- If $\lambda = 0$, $db_t/dt_{r,t} = 0$.

Let us now rewrite the budget equation into

$$b_t = X_t + b_{t-1} - t_{r,t} \tag{18}$$

Now b_{t-1} is not B_{t-1}/P_t, but rather past debt is denominated in real term. Then,

$$\frac{db_t}{dt_{r,t}} = \frac{\partial X_t}{\partial P_t}\frac{\partial P_t}{\partial t_{r,t}} - 1 = \frac{dX_t}{dt_{r,t}} - 1 \tag{19}$$

Let $\gamma = \frac{\partial X_t}{\partial P_t}_{X_i,P_i}$, $\lambda = \frac{\partial P_t}{\partial t_{r,t}}_{P_i,t_i}$. If $\gamma < 0$ and $\lambda < 0$, then unlike in the previous cases, $\frac{db_t}{dt_{r,t}} > -1$.